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COMMON RANDOM FIXED POINT THEOREMS FOR A PAIR OF GENERALIZED WEAKLY CONTRACTIVE MAPPINGS IN POLISH SPACES

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Abstract. In this paper we obtain a new common random fixed point theorem for a pair of random mappings satisfying weakly contractive condition under generalized altering distance function in polish spaces.

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1. Introduction

Random fixed point theory has receive much attention in recent years and it is needed for the study of various classes of random equations. The study of random fixed point theorems was initiate by the Prague school of probabilistic in the 1950s. The interest in this subject enhanced after publication of the survey paper of Bharucha Reid [6].

Obtaining the existence and uniqueness of fixed points for the self-maps of a metric space by altering distances between the points with the use of a control function is an interesting aspect in the classical fixed point theory. In this direction, Khan et al. [10] introduced a new category of fixed point problems for a single self-map with the help of a control function that alters the

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distance between two points in a metric space which they called an altering distance function. However, similar type of function was already in use in the fixed point theory under the title $\mathscr{D}$-function and the details may be found in Dhage [7].

Definition 1.1. [Dhage [7]] A function $\phi : [0,\infty) \to [0,\infty)$ is called a $\mathscr{D}$-function if it is a continuous and monotone nondecreasing function satisfying $\phi(0) = 0$.

There do exist $\mathscr{D}$-function useful in the fixed point theory and applications and commonly used $\mathscr{D}$-functions are $\phi(r) = kr$ and $\psi(r) = \dfrac{Lr}{K+r}$. The $\mathscr{D}$-functions ϕ and ψ are respectively used in the fixed point theory for linear and nonlinear contraction mappings in metric spaces (cf. Dhage [7] and the references cited therein).

Definition 1.2. (Weakly contractive mapping): Let X be a metric space. A mapping $T : X \to X$ is called weakly contractive if for each $x, y \in X$,

$$d(Tx,Ty) \leq d(x,y) - \phi(d(x,y)) \tag{1.1}$$

where $\phi : [0,\infty) \to [0,\infty)$ is positive on $(0,\infty)$ and $\phi(0) = 0$.

In fact, Alber and Guerre-Delabriere [2] assumed an additional condition on ϕ that is $\lim_{t\to\infty} \phi(t) = \infty$. But Rhoades [11] obtained the result noted in following theorem without using this particular assumption.

Theorem 1.1. (Rhoades [11]) *If $T : X \to X$ is a weakly contractive mapping, where (X,d) is a complete metric space, then T has a unique fixed point.*

It may be observed that though the function φ has been defined in the same way as the $\mathscr{D}$-function, the way it has been used in Theorem 2.1 is completely different from the use of $\mathscr{D}$-function.

Definition 1.3. A self mapping T of a metric space (X,d) is said to be weakly contractive with respect to a self mapping $S : X \to X$, if for each $x, y \in X$,

$$d(Tx,Ty) \leq d(Sx,Sy) - \psi(d(Sx,Sy)),$$

where $\psi : [0,\infty) \to [0,\infty)$ is a continuous and nondecreasing function such that ψ is positive on $(0,\infty)$, $\psi(0) = 0$ and $\lim_{t\to\infty} \psi(t) = \infty$.

Recently, Beg and Abbas [4] proved a generalization of the corresponding theorems of Rhoades [11] for a pair of mapping in which one is weakly contractive with respect to the other which is further generalized by Azam and Shakeel [3] in convex metric spaces. Combining the generalization of Banach contraction principle given by Khan et al. [9] and the generalization given by Rhoades [11], Dutta and Choudhury [8] obtained a result which is further extended by Abbas and Khan [1]. Choudhury [6] also proved similar type of works for generalized $\mathscr{D}$-functions. Recently, Beg et al. [5] obtained random version of these results in convex separable complete metric spaces.

2. Random Common Fixed Point Theorem For Generalized Weakly Contractions

Throughout this paper, let (X,d) be a polish space, i.e., a separable complete metric space and $(\Omega,\mathscr{A})$ be a measurable space (i.e., $\mathscr{A}$ is σ-algebra of subsets of Ω). A function $\xi:\Omega\to X$ is said to be a $\mathscr{A}$-measurable if for any open subset B of $X, \xi^{-1}(B)\in\mathscr{A}$.

A mapping $S:\Omega\times X\to X$ is said to be a random map if and only if for each fixed $x\in X$, the mapping $S(\cdot,x):\Omega\to X$ is measurable. A random map $S:\Omega\times X\to X$ is continuous if for each $\omega\in\Omega$, the mapping $S(\Omega,\cdot):X\to X$ is continuous. A measurable mapping $\xi:\Omega\to X$ is a random fixed point of the random map $S:\Omega\times X\to X$ if and only if $S(\omega,\xi(\omega))=\xi(\omega)$ for each $\omega\in\Omega$.

Definition 2.1. A measurable mapping $\xi:\Omega\to K$, is said to be a random common fixed point of random operators $S:\Omega\times K\to K$ and $T:\Omega\times K\to K$ if for each $\omega\in\Omega$, $\xi(\omega)=S(\omega,\xi(\omega))=T(\omega,\xi(\omega))$.

In [6], Choudhury introduced the concept of a generalized altering distance function for three variables. In the following we generalized this notion for five variables.

Definition 2.2. A function $\phi:[0,\infty)^5\to[0,\infty)$ is said to be a generalized $\mathscr{D}$-function if the following conditions are satisfied:

(i) ϕ is continuous,

(ii) ϕ is monotone increasing for every variables, and

(iii) $\phi(t_1,t_2,t_3,t_4,t_5)=0$ if and only if $t_1=t_2=t_3=t_4=t_5=0$.

Define $\psi(x)=\phi(x,x,x,x,x)$ for $x\in[0,\infty)$. Clearly, $\psi(x)=0$ if and only if $x=0$. Some nice examples of the generalized $\mathscr{D}$-functions ϕ which may be used in metric fixed point theory are

$$\phi(t_1,t_2,t_3,t_4,t_5)=k\max\{t_1,t_2,t_3,t_4,t_5\},\ \text{for } k>0,$$

and

$$\phi(t_1,t_2,t_3,t_4,t_5)=t_1^{a_1}+t_2^{a_2}+t_3^{a_3}+t_4^{a_4}+t_5^{a_5};\, a_1,a_2,a_3,a_4,a_5\geq 1.$$

Now we prove a random common fixed point theorem for a pair of mappings.

Theorem 2.1 *Let X be a metric space and K be a nonempty Polish subspace of X. Let $S,T:\Omega\times K\to K$ be continuous map satisfying for each $\omega\in\Omega$,*

$$\begin{aligned}\psi(d(S(\omega,x),T(\omega,y)))\leq\ &\phi_1\Big(d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x)),d(y(\omega),S(\omega,y)),\\ &\quad d(x(\omega),S(\omega,y)),d(y(\omega),T(\omega,x))\Big)\\ &-\phi_2\Big(d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x)),d(y(\omega),S(\omega,y)),\\ &\quad d(x(\omega),S(\omega,y)),d(y(\omega),T(\omega,x))\Big)\end{aligned}\tag{2.1}$$

for each $x,y\in K$, where $\phi_i(i=1,2)$ are generalized $\mathscr{D}$-functions and the function ψ is defined by $\psi(x)=\phi(x,x,x,x,x)$. Then there exists a measurable mapping $\xi:\Omega\to K$ such that $\xi(\omega)=S(\omega,\xi(\omega))=T(\omega,\xi(\omega))$.

Proof. Let $\xi_0:\Omega\to K$ be a measurable but fixed mapping in K, we get

$$\xi_1(\omega)=T(\omega,\xi_0(\omega))\quad\text{and}\quad\xi_2(\omega)=S(\omega,\xi_1(\omega)).$$

Similarly, we get

$$\xi_3(\omega)=T(\omega,\xi_2(\omega))\quad\text{and}\quad\xi_4(\omega)=S(\omega,\xi_3(\omega)).$$

Inductively, we construct a sequence of measurable maps $\{\xi_n\}$ from Ω to K such that

$$\xi_{2n+1}(\omega)=S(\omega,\xi_{2n}(\omega))\quad\text{and}\quad\xi_{2n+2}(\omega)=T(\omega,\xi_{2n+1}(\omega)).\tag{2.2}$$

Since S and T are continuous, by a result of Himmelberg [9], $\{\xi_n\}$ is a measurable sequence. First we will prove that

$$d(\xi_n(\omega),\xi_{n+1}(\omega)) \leq d(\xi_{n-1}(\omega),\xi_n(\omega)).$$

Consider, the following estimate:

$$\begin{aligned}
&\psi\Big(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big)\\
&\quad \leq \psi\Big(d(T(\omega,\xi_{2n}(\omega)),S(\omega,\xi_{2n+1}(\omega)))\Big)\\
&\quad \leq \phi_1\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),T(\omega,\xi_{2n}(\omega))),d(\xi_{2n+1}(\omega),S(\omega,\xi_{2n+1}(\omega))),\\
&\qquad\quad d(\xi_{2n}(\omega),S(\omega,\xi_{2n+1}(\omega))),d(\xi_{2n+1}(\omega),T(\omega,\xi_{2n}(\omega)))\Big)\\
&\quad - \phi_2\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),T(\omega,\xi_{2n}(\omega))),d(\xi_{2n+1}(\omega),S(\omega,\xi_{2n+1}(\omega))),\\
&\qquad\quad d(\xi_{2n}(\omega),S(\omega,\xi_{2n+1}(\omega))),d(\xi_{2n+1}(\omega),T(\omega,\xi_{2n}(\omega)))\Big)\\
&\quad = \phi_1\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&\qquad\quad d(\xi_{2n}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+1}(\omega))\Big)\\
&\quad - \phi_2\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&\qquad\quad d(\xi_{2n}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+1}(\omega))\Big)\\
&\quad \leq \phi_1\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&\qquad\quad d(\xi_{2n}(\omega),\xi_{2n+1}(\omega))+d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big)\\
&\quad - \phi_2\Big(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\
&\qquad\quad d(\xi_{2n}(\omega),\xi_{2n+1}(\omega))+d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big).
\end{aligned} \tag{2.3}$$

If

$$d\big(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)\big) > d\big(\xi_{2n}(\omega),\xi_{2n+1}(\omega)\big),$$

then,

$$\begin{aligned}\psi\Big(d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big) < \phi_1\Big(&d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),\\ &d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)),d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big) \quad (2.4)\\ = \psi\Big(&d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))\Big)\end{aligned}$$

which is a contradiction. Since ϕ_1 is monotone increasing for all variables and

$$\phi_2[d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega))] \neq 0$$

whenever

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)).$$

So, we have

$$d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)) \leq d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)) \tag{2.5}$$

for all $n = 0,1,\ldots$. Putting $x = \xi_{2n}(\omega)$, $y = \xi_{2n-1}(\omega)$ in (2.1), we have

$$\begin{aligned}&\psi(d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)))\\ &= \psi(d(T(\omega,\xi_{2n-1}(\omega)),d(S(\omega,\xi_{2n}(\omega)))))\\ &\leq \phi_1\Big(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),T(\omega,\xi_{2n-1}(\omega))),d(\xi_{2n}(\omega),S(\omega,\xi_{2n}(\omega))),\\ &\qquad d(\xi_{2n-1}(\omega),S(\omega,\xi_{2n}(\omega))),d(\xi_{2n}(\omega),T(\omega,\xi_{2n-1}(\omega)))\Big)\\ &\quad -\phi_2\Big(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),T(\omega,\xi_{2n-1}(\omega))),d(\xi_{2n}(\omega),S(\omega,\xi_{2n}(\omega))),\\ &\qquad d(\xi_{2n-1}(\omega),S(\omega,\xi_{2n}(\omega))),d(\xi_{2n}(\omega),T(\omega,\xi_{2n-1}(\omega)))\Big)\\ &= \phi_1\Big(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),\\ &\qquad d(\xi_{2n-1}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n}(\omega))\Big)\\ &\quad -\phi_2\Big(d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n-1}(\omega),\xi_{2n}(\omega)),d(\xi_{2n}(\omega),\xi_{2n+1}(\omega)),\\ &\qquad d(\xi_{2n-1}(\omega),\xi_{2n+1}(\omega)),d(\xi_{2n}(\omega),\xi_{2n}(\omega))\Big).\end{aligned} \tag{2.6}$$

By similar arguments, we have

$$d(\xi_{2n+2}(\omega),\xi_{2n+3}(\omega)) \leq d(\xi_{2n+1}(\omega),\xi_{2n+2}(\omega)) \tag{2.7}$$

for all $n \in \mathbb{N}$. From (2.5) and (2.7) we obtain

$$d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \leq d(\xi_n(\omega),\xi_{n+1}(\omega)) \tag{2.8}$$

for all $n \in \mathbb{N}$. From (2.3) and (2.8), we have for all integers $n \geq 0$

$$\psi(d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))) \leq \phi_1(d(\xi_n,\xi_{n+1}(\omega))) - \phi_2(d(\xi_n(\omega),\xi_{n+1}(\omega)))$$

or, equivalently,

$$\phi_2(d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))) \leq \phi_1(d(\xi_n(\omega),\xi_{n+1}(\omega))) - \phi_1(d(\xi_n(\omega),\xi_{n+1}(\omega))).$$

Summing up from (2.8), we obtain

$$\sum_{n=0}^{\infty} \phi(d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))) \leq \phi_1(d(\xi_0(\omega),\xi_1(\omega))) < \infty.$$

This implies,

$$\phi_2(d(\xi_n(\omega),\xi_{n+1}(\omega))) \to 0 \ as \ n \to \infty. \tag{2.9}$$

Again, from (2.8), the sequence $\{d(\xi_n(\omega),\xi_{n+1}(\omega))\}$ is monotone non-increasing and bounded. Hence there exists a real number $r(\omega) \geq 0$ such that,

$$\lim_{n\to\infty} d(\xi_n(\omega),\xi_{n+1}(\omega)) = r(\omega).$$

Then, by continuity of ϕ, from (2.9), we obtain $\phi_2(r(\omega)) = 0$ which implies that by the property of function ϕ, we have $r(\omega) = 0$. Thus,

$$\lim_{n\to\infty} d(\xi_n(\omega),\xi_{n+1}(\omega)) = 0. \tag{2.10}$$

Now we claim that $\{\xi_n(\omega)\}$ is a Cauchy sequence in K. If possible, let $\{\xi_n(\omega)\}$ is not a Cauchy sequence then there exists $\varepsilon > 0$ for which we can find subsequences $\{\xi_{n_i}(\omega)\}$ and $\{\xi_{m_i}(\omega)\}$ with $n_i > m_i > i$ such that

$$d(\xi_{m_i}(\omega),\xi_{n_i}(\omega)) < \varepsilon. \tag{2.11}$$

Further we can choose n_i corresponding m_i, in such a way that it is smallest integer with $n_i > m_i$ satisfying

$$d(\xi_{m_i}, \xi_{n_i-1}(\omega)) < \varepsilon. \tag{2.12}$$

Using (2.11), (2.12) and the triangle inequality, we have

$$\begin{aligned} \varepsilon &\leq d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)) \\ &\leq d(\xi_{m_i}(\omega), \xi_{n_i-1}(\omega)) + d(\xi_{n_i-1}(\omega), \xi_{n_i}(\omega)) \\ &< \varepsilon + d(\xi_{n_i-1}(\omega), \xi_{n_i}(\omega)). \end{aligned} \tag{2.13}$$

Letting $i \to \infty$ and using (2.10),

$$\lim_{i\to\infty} d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)) = \varepsilon \tag{2.14}$$

Again, from the triangle inequality we get

$$\begin{aligned} d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)) &\leq d(\xi_{m_i}(\omega), \xi_{m_i-1}(\omega)) + d(\xi_{m_i-1}(\omega), \xi_{n_i-1}(\omega)) \\ &\quad + d(\xi_{n_i-1}(\omega), \xi_{n_i}(\omega)) \\ d(\xi_{m_i-1}(\omega), \xi_{n_i-1}(\omega)) &\leq d(\xi_{m_i-1}(\omega), \xi_{m_i}(\omega)) + d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)) \\ &\quad + d(\xi_{n_i}(\omega), \xi_{n_i-1}(\omega)). \end{aligned} \tag{2.15}$$

Letting $i \to \infty$ and using the inequalities (2.10) and (2.14), we obtain

$$\lim_{i\to\infty} d(\xi_{m_i-1}(\omega), \xi_{n_i-1}(\omega)) = \varepsilon. \tag{2.16}$$

Setting $x = \xi_{m_i}(\omega)$ and $y = \xi_{n_i}(\omega)$ in (2.1), we obtain

$$\begin{aligned}
&\psi\Big(d(\xi_{m_i-1}(\omega), \xi_{n_i-1}(\omega))\Big) \\
&\quad = \psi\Big(d(T(\omega, \xi_{m_i}(\omega)), S(\omega, \xi_{n_i}(\omega)))\Big) \\
&\quad \le \phi_1\Big((d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), T(\omega, \xi_{m_i}(\omega))), d(\xi_{n_i}(\omega), S(\omega, \xi_{n_i}(\omega))), \\
&\qquad\qquad d(\xi_{m_i}(\omega), S(\omega, \xi_{n_i}(\omega))), d(\xi_{n_i}(\omega), T(\omega, \xi_{m_i}(\omega))))\Big) \\
&\qquad - \phi_2\Big((d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), T(\omega, \xi_{m_i}(\omega))), d(\xi_{n_i}(\omega), S(\omega, \xi_{n_i}(\omega))), \\
&\qquad\qquad d(\xi_{m_i}(\omega), S(\omega, \xi_{n_i}(\omega))), d(\xi_{n_i}(\omega), T(\omega, \xi_{m_i}(\omega))))\Big).
\end{aligned} \tag{2.17}$$

Letting $i \to \infty$ in (2.17) and using the inequalities (2.2), (2.11) and (2.12), we obtain

$$\begin{aligned}
\psi(\varepsilon) &\le \lim_{i\to\infty} \psi(d(T(\omega, \xi_{m_i}(\omega)), S(\omega, \xi_{n_i}(\omega)))) \\
&\le \lim_{i\to\infty} \phi_1\Big(d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), T(\omega, \xi_{m_i}(\omega))), d(\xi_{n_i}(\omega), S(\omega, \xi_{n_i}(\omega))), \\
&\qquad\qquad d(\xi_{m_i}(\omega), S(\omega, \xi_{n_i}(\omega))), d(\xi_{n_i}(\omega), T(\omega, \xi_{n_i}(\omega)))\Big) \\
&\quad - \lim_{i\to\infty} \phi_2\Big(d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), T(\omega, \xi_{m_i}(\omega))), d(\xi_{n_i}(\omega), S(\omega, \xi_{n_i}(\omega))), \\
&\qquad\qquad d(\xi_{m_i}(\omega), S(\omega, \xi_{n_i}(\omega))), d(\xi_{n_i}(\omega), T(\omega, \xi_{n_i}(\omega)))\Big) \\
&= \lim_{i\to\infty} \phi_1\Big(d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), \xi_{m_i+1}(\omega)), d(\xi_{n_i}(\omega), \xi_{n_i+1}(\omega)), \\
&\qquad\qquad d(\xi_{m_i}(\omega), \xi_{n_i+1}(\omega)), d(\xi_{n_i}(\omega), \xi_{m_i+1}(\omega))\Big) \\
&\quad - \lim_{i\to\infty} \phi_2\Big(d(\xi_{m_i}(\omega), \xi_{n_i}(\omega)), d(\xi_{m_i}(\omega), \xi_{m_i+1}(\omega)), d(\xi_{n_i}(\omega), \xi_{n_i+1}(\omega)), \\
&\qquad\qquad d(\xi_{m_i}(\omega), \xi_{n_i+1}(\omega)), d(\xi_{n_i}(\omega), \xi_{m_i+1}(\omega))\Big)
\end{aligned} \tag{2.18}$$

Using inequalities (2.10), (2.12) and (2.14), we have

$$\psi(\varepsilon) \le \phi_1(\varepsilon, 0, 0, 0, 0) - \phi_2(\varepsilon, 0, 0, 0, 0) < \phi_1(\varepsilon)$$

Since ϕ_1 is monotone increasing in its variables and by the property ϕ_2 that

$$\phi(t_1, t_2, t_3, t_4, t_5) = 0 \quad \text{if and only if} \quad t_1 = t_2 = t_3 = t_4 = t_5.$$

Thus we arrive at a contradiction as $\varepsilon > 0$.

Hence $\{\xi_{n_i}(\omega)\}$ is Cauchy sequence in K, there exists $\xi : \Omega \to K$ such that $\xi_n(\omega) \to \xi(\omega)$ for all $\omega \in \Omega$. We show that $\xi(\omega)$ is random common fixed point of S and T.

$$T(\omega,\xi(\omega)) = \lim_{n\to\infty} T(\omega,\xi_{2n}(\omega)) = \lim_{n\to\infty} \xi_{2n+1}(\omega) = \xi(\omega)$$

Similarly, we can prove $\xi(\omega) = S(\omega,\xi(\omega))$. Hence, $T(\omega,\xi(\omega)) = \xi(\omega) = S(\omega,\xi(\omega))$ and consequently $\xi(\omega)$ is common fixed point of $S(\omega)$ and $T(\omega)$.

Finally, we prove the uniqueness of the common random fixed point ξ of S and T. Let $\zeta(\omega)$ and $\xi(\omega)$ be two random fixed points of S and T i.e.

$$S(\omega,\xi(\omega)) = \xi(\omega) = T(\omega,\xi(\omega))$$

and

$$T(\omega,\zeta(\omega)) = \zeta(\omega) = S(\omega,\zeta(\omega))$$

for each $\omega \in \Omega$. Using inequality (2.1), we have

$$\begin{aligned}
\psi(d(\xi(\omega),\zeta(\omega))) &= \psi(d(T(\omega,\xi(\omega)),T(\omega,\xi(\omega)))) \\
&\leq \phi_1\Big(d(\xi(\omega),\zeta(\omega)),d(\xi(\omega),T(\omega,\xi(\omega))),d(\zeta(\omega),T(\omega,\zeta(\omega))), \\
&\qquad d(\xi(\omega),T(\omega,\zeta(\omega))),d(\zeta(\omega),T(\omega,\xi(\omega)))\Big) \\
&\quad - \phi_2\Big(d(\xi(\omega),\zeta(\omega)),d(\xi(\omega),T(\omega,\xi(\omega))),d(\zeta(\omega),T(\omega,\zeta(\omega))), \\
&\qquad d(\xi(\omega),T(\omega,\zeta(\omega))),d(\zeta(\omega),T(\omega,\xi(\omega)))\Big) \\
&= \phi_1\Big(d(\xi(\omega),\zeta(\omega)),0,0,d(\xi(\omega),\zeta(\omega)),0,d(\zeta(\omega)\xi(\omega))\Big) \\
&\quad - \phi_2\Big(d(\xi(\omega),\zeta(\omega)),0,0,d(\xi(\omega),\zeta(\omega)),0,d(\zeta(\omega)\xi(\omega))\Big) \\
&< \phi_1\Big(d(\xi(\omega),\zeta(\omega))\Big)
\end{aligned} \tag{2.19}$$

which is possible only when $\xi(\omega) = \zeta(\omega)$, since ϕ_1 is monotone increasing in all its variables and $\phi(t_1,t_2,t_3,t_4,t_5) \leq 0$ if at least one of t_1,t_2,t_3,t_4,t_5 is nonzero. Hence, $\xi(\omega)$ is the unique random common fixed point of S and T, i.e., $S(\omega,\xi(\omega)) = \xi(\omega) = T(\omega,\xi(\omega))$ for all $\omega \in \Omega$.

Remark 2.1. (i) Theorem 2.1 is a generalization of Theorem 2.2 [10] with correction in the proof. The part of proof showing that the limit of the sequence of iterations is a random fixed point of the random mapping $T(\omega)$ is superfluous, because the required conclusion follows by virtue of continuity of $T(\omega)$ on X.

(ii) Theorem 2.1 presents random version improvement, extension and generalization of Abbas and Khan [1], Dutta and Choudhury [8, Theorem 2.1] and Rhoades [11] by considering generalized $\mathscr{D}$-function.

(iii) Theorem 2.1 is generalization of Theorem 2.1 [5] for two mappings considering the generalized altering distance function.

If we take $S = T$ in Theorem 2.1, then we have the following result as a particular case.

Corollary 2.1. *Let X be a metric space and K be a nonempty Polish subspace of X. Let $T : \Omega \times K \to K$ be continuous map satisfying for each $\omega \in \Omega$,*

$$\begin{aligned}\psi(d(T(\omega,x),T(\omega,y))) \le{}& \phi_1\Big(d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x)),d(y(\omega),T(\omega,y)),\\ &\qquad d(x(\omega),T(\omega,y)),d(y(\omega),T(\omega,x))\Big)\\ &- \phi_2\Big(d(x(\omega),y(\omega)),d(x(\omega),T(\omega,x)),d(y(\omega),T(\omega,y)),\\ &\qquad d(x(\omega),T(\omega,y)),d(y(\omega),T(\omega,x))\Big)\end{aligned} \tag{2.20}$$

for each $x, y \in K$, where $\phi_i (i = 1, 2)$ are generalized $\mathscr{D}$-functions and the $\mathscr{D}$-function ψ is defined by $\psi(x) = \phi(x,x,x,x,x)$. Then there exists a measurable mapping $\xi : \Omega \to K$ such that $\xi(\omega) = T(\omega,\xi(\omega))$.

An immediate consequence of Theorem 2.1 is the following:

Corollary 2.2. *Let X be a metric space and K be a nonempty Polish subspace of X. Let $S,T : \Omega \times K \to K$ be continuous map satisfying for each $\omega \in \Omega$,*

$$\begin{aligned}\Big[d(S(\omega,x),T(\omega,y))\Big]^s \le{}& k_1\Big[d(x,y)\Big]^s + k_2\Big[d(x,T(\omega,x))\Big]^s + k_3\Big[d(y,S(\omega,y))\Big]^s\\ &+ k_4\Big[d(x,S(\omega,y))\Big]^s + k_5\Big[d(y,T(\omega,x))\Big]^s\end{aligned} \tag{2.21}$$

for all $x, y \in X$ where, $0 < k_1 + k_2 + k_3 + k_4 + k_5 < 1$ and $s > 0$. Then S and T have a random common fixed point.

Proof. We make particular choices of ϕ_1 and ϕ_2 given by

$$\phi_1(t_1,t_2,t_3,t_4,t_5) = k_1t_1^s + k_2t_2^s + k_3t_3^s + k_4t_4^s + k_5t_5^s \tag{2.22}$$

and

$$\phi_2(t_1,t_2,t_3,t_4,t_5) = (k-1)\left[k_1t_1^s + k_2t_2^s + k_3t_3^s + k_4t_4^s + k_5t_5^s\right] \tag{2.23}$$

with $k = k_1 + k_2 + k_3 + k_4 + k_5$, then (2.1) is implied by (2.21). The corollary then follows by an application of Theorem 2.1.

Conflict of Interests

The authors declare that there is no conflict of interests.

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