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# ON COMPLETELY GENERALIZED RANDOM VARIATIONAL INCLUSIONS WITH RANDOM FUZZY MAPPING

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**Abstract.** In this paper, we consider the completely generalized random variational inclusions for random fuzzy mappings and define an Ishikawa type algorithm. We prove existence of solutions of our inclusions involving random relaxed Lipschitz and random relaxed monotone mappings and study the convergence of the iterative sequences generated by the proposed algorithm. The result presented in this paper improve and generalize some known corresponding results in the literature.

**Keywords:** Random variational inclusions; Algorithm; Random fuzzy mappings; Existence and convergence.

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## 1. Introduction-preliminaries

The theory of variational inequalities was introduced in early sixties. This theory arise in models for a wide class of optimization and control, mechanics, elasticity, physics, transportation and engineering sciences. For the physical formulation, numerical methods, applications

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and other aspects of variational inequalities, see for example [1]-[22] and references therein. An useful and important generalization of the variational inequalities is a variational inclusions. In 1994, Hassouni and Moudafi [8] introduced and studied a class of variational inclusions and developed a perturbed algorithm for finding approximate solutions of the variational inclusions.

In 1965, Professor Lotfi Zadeh [22] at the University of California introduced the concept of fuzzy sets. This theory has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of physical, mathematical and engineering sciences, see for example [2, 10, 11, 12, 14, 15, 16, 17, 19] and references therein.

In 1999, Huang [12] introduced the concept of random fuzzy mapping. The random variational inclusion problem involving random fuzzy mapping is studied by Petrot and Balooee [19]. They consider a new class of general nonlinear random set-valued variational inclusion problem. By using the resolvent operator technique for $A$ maximal $m$relaxed $\eta$-accretive mappings they constuct a new iterative algorithm for finding the approximate solutions of this class of nonlinear random equations. Very recently Ahmad and Farajzadeh [3] introduced and study random variational inclusions with random fuzzy and random relaxed cocoercive mappings. In support of their results they also provide some examples.

Motivated and inspired by the resent research work in this fascinating area, in this paper we consider the completely generalized random variational inclusions for random fuzzy mappings and define an Ishikawa type algorithm. We prove existence of random solution for completely generalized random variational inclusions problem and the convergence of iterative sequence generated by the algorithm.

Throughout the paper, let $(\Omega,\Sigma)$ be a measurable space, where $\Omega$ is a set and $\Sigma$ is a $\sigma$-algebra of subsets of $\Omega$. Let $H$ be a real Hilbert space whose norm and inner product are denoted by $\|.\|$ and $\langle .,.\rangle$, respectively. We denote $\mathscr{B}(H), 2^H, CB(H)$ and $\mathscr{H}(.,.)$ the class of Borel $\sigma$-fields in $H$, the family of all nonempty subsets of $H$, the family of all nonempty closed bounded subsets of $H$ and the Housdorff metric on $CB(H)$ respectively.

**Definition 1.1.** A mapping $g : \Omega \times H \to H$ is called a *random operator* if for any $x \in H,\ g(t,x) = x(t)$ is measurable. A random operator $g$ is said to be *continuous* if for any $t \in \Omega$, the mapping $g(t,.) : H \to H$ is continuous.

**Definition 1.2.** A multivalued mapping $T : \Omega \to 2^H$ is said to be *measurable* if for any $B \in \mathscr{B}(H),\ T^{-1}(B) = \{t \in \Omega :\ T(t) \cap B \neq \phi\} \in \Sigma$.

**Definition 1.3.** A mapping $u : \Omega \to H$ is called a *measurable selection* of a multivalued measurable mapping $T : \Omega \to 2^H$ if $u$ is a measurable and for any $t \in \Omega,\ u(t) \in T(t)$.

**Definition 1.4.** A mapping $T : \Omega \times H \to 2^H$ is called a *random multivalued mapping* if for any $x \in H,\ T(.,x)$ is measurable. A random multivalued mapping $T : \Omega \times H \to CB(H)$ is said to be $\mathscr{H}$*-continuous* if for any $t \in \Omega,\ T(t,.)$ is continuous in the Houdorff metric.

Let $Q$ be any set and $\mathscr{F}(H)$ be a collection of fuzzy sets over $H$. A mapping $F$ from $Q$ into $\mathscr{F}(H)$ is called a fuzzy mapping. If $F$ is a fuzzy mapping on $H$, then for any $x \in Q,\ F(x)$ (denote it by $F_x$, in the sequel) is a fuzzy set on $H$ and $F_x(y)$ is the membership function of $y$ in $F_x$.

Let $N \in \mathscr{F}(H),\ q \in [0,1]$. Then the set $(N)_q = \{x \in H : N(x) \geq q\}$ is called a $q$-cut set of $N$.

**Definition 1.5.** A fuzzy mapping $F : \Omega \to \mathscr{F}(H)$ is called *measurable,* if for any $\alpha \in (0,1],\ (F(.))_\alpha : \Omega \to 2^H$ is a measurable multivalued mapping.

**Definition 1.6.** A fuzzy mapping $F : \Omega \times H \to \mathscr{F}(H)$ is called a *random fuzzy mapping* if for any $x \in H, F(.,x) : \Omega \to \mathscr{F}(H)$ is a measurable fuzzy mapping.

Clearly, the random fuzzy mapping include multivalued mappings, random multivalued mappings and fuzzy mappings as the special cases.

Let $M,S,T : \Omega \times H \to \mathscr{F}(H)$ be random fuzzy mappings satisfying the following condition:
$(*)$ : There exist three mappings $a,b,c : H \to (0,1]$ such that

$$(M_{t,x})_{a(x)} \in CB(H),\ (S_{t,x})_{b(x)} \in CB(H),\ (T_{t,x})_{c(x)} \in CB(H),\ \forall\ (t,x) \in \Omega \times H.$$

By using the random fuzzy mappings $M,S$ and $T$, we can define three random multivalued mappings $\tilde{M},\tilde{S}$ and $\tilde{T}$ respectively as follows:

$$\tilde{M}:\Omega\times H\to CB(H),\ x\to(M_{t,x})_{a(x)},\ \forall\,(t,x)\in\Omega\times H;$$

$$\tilde{S}:\Omega\times H\to CB(H),\ x\to(S_{t,x})_{b(x)},\ \forall\,(t,x)\in\Omega\times H;$$

and

$$\tilde{T}:\Omega\times H\to CB(H),\ x\to(T_{t,x})_{c(x)},\ \forall\,(t,x)\in\Omega\times H.$$

In the sequel, $\tilde{M},\tilde{S}$ and $\tilde{T}$ are called the random multivalued mappings induced by the random fuzzy mapping $M,S$ and $T$ respectively.

Given mappings $a,b,c:H\to(0,1]$, random fuzzy mappings $M,S,T:\Omega\times H\to\mathscr{F}(H)$ and random operators $g,h,F,G,P:\Omega\times H\to H$ with Img$\cap$dom$(\partial\phi(t,.,.))\neq\phi$ and the random map $\eta:\Omega\times H\times H\to H$. We consider the following problem:

Find measurable mappings $x,u,w,q:\Omega\to H$ such that for all $t\in\Omega$, $x(t),y(t)\in H$, $M_{t,x(t)}(u(t))\geq a(x(t))$, $S_{t,x(t)}(w(t))\geq b(x(t))$, $T_{t,x(t)}(q(t))\geq c(x(t))$, $g(t,x(t))\cap$ dom$(\partial\phi(t,.,.))\neq\phi$ and

$$\begin{aligned}&\langle P(t,h(t,u(t)))-(F(t,w(t))-G(t,q(t))),\eta(t,y(t),g(t,x(t)))\rangle\\ &\geq\ \phi(t,g(t,x(t)),x(t))-\phi(t,y(t),x(t)),\end{aligned}\tag{1.1}$$

where $\partial\phi$ denotes the subdifferential of a proper, convex and lower semicontinuous function $\phi:\Omega\times H\times H\to R\cup\{+\infty\}$. Problem (1.1) is called *completely generalized random variational inclusion problem for random fuzzy mappings*. The set of measurable mappings $(x,u,w,q)$ is called a random solution of (1.1).

**Special Cases.**

(i) If $h=I$ identity mapping $\eta(t,y(t),g(t,x(t))=y(t)-g(t,x(t))$ and $\phi(t,g(t,x(t)),x(t))-\phi(t,y(t),x(t)=\phi(t,g(t,x(t))-\phi(y(t))$ then (1.1) reduces to the problem of finding measurable mappings $x,u,w,q:\Omega\to H$ such that for all $t\in\Omega,x(t),y(t)\in H$, $M_{t,x(t))}(u(t))$ $\geq a(x(t))$, $S_{t,x(t))}(w(t))\geq b(x(t))$, $T_{t,x(t))}(q(t))\geq c(x(t))$, $g(t,x(t))\bigcap$ dom$(\partial\phi)\neq\phi$, and

$$\langle P(t,u(t))-((F(t,w(t))-G(t,q(t)),y(t)-g(t,x(t))\rangle\geq\phi(g(t,x(t))-\phi(y(t)).\tag{1.2}$$

Problem (1.2) is introduced and studied by Ahmad and Bazán [2].

(ii) If $F, G, h = I$ are identity mappings and

$$\phi(t, g(t, x(t)), x(t))) - \phi(t, y(t), x(t)) = \phi(t, g(t, x(t)) - \phi(t, y(t)),$$

then (1.1) reduces to the problem of finding measurable mapping $x : \Omega \to H$ such that for all $t \in \Omega, x(t), y(t) \in H$, and

$$\langle P(t, u(t)) + q(t) - w(t), \eta(t, y(t), g(t, x(t))) \rangle \geq \phi(t, g(t, x(t)) - \phi(t, y(t)). \tag{1.3}$$

Which is called the generalized nonlinear random variational inclusions for random multivalued operators in Hilbert spaces. The determinate form of the problem (1.3) was studied by Agarwal *et al.* [1].

(iii) If $\eta(t, y(t), g(t, x(t))) = y(t) - g(t, x(t))$ for all $t \in \Omega$ $x(t), y(t) \in H$, then problem (1.3) reduces to the problem of finding measurable mappings $x, u : \Omega \to H$ such that $u(t) \in M(t, x(t))$, and

$$\langle P(t, u(t)) + q(t) - w(t), y(t) - g(t, x(t)) \rangle \geq \phi(t, g(t, x(t)) - \phi(t, y(t)). \tag{1.4}$$

The determinate form is a generalization of the problem (1.4) considered in [7].

**Definition 1.7.** A random mapping $\eta : \Omega \times H \times H \to H$ is said to be:

(i) *monotone* if

$$\langle x(t) - y(t), \eta(t, x(t), y(t)) \rangle \geq 0, \ \forall\, x(t), y(t) \ \in H, \ t \in \Omega; \tag{1.5}$$

(ii) *strictly monotone* if the equality holds in (1.5) only when $x(t) = y(t)$;

(iii) *strongly monotone* if there exists a measurable function $q : \Omega \to (0, \infty)$ such that

$$\langle x(t) - y(t), \eta(t, x(t), y(t)) \rangle \geq q(t)\|x(t) - y(t)\|^2, \ \forall\, x(t), y(t) \ \in H, \ t \in \Omega;$$

(iv) *Lipschitz continuous* if there exists a measurable function $z : \Omega \to (0, \infty)$ such that

$$\|\eta(t, x(t), y(t))\| \leq z(t)\|x(t) - y(t)\|, \ \forall\, x(t), y(t) \ \in H, \ t \in \Omega.$$

**Definition 1.8.** If $G : \Omega \times H \to 2^H$ is a maximal monotone mapping. Then the *resolvent operator* $J_{\rho(t)}^{G_t}$ for $G$ is defined as follows:

$$J_{\rho(t)}^{G_t}(x) = (I + \rho(t)G_t)^{-1}(x)$$

where $G_t(x) = G(t, x(t)),\ \forall\, t \in \Omega,\ x(t) \in H$, and $\rho : \Omega \to (0, \infty)$ is a measurable function and $I$ is the identity mapping on $H$.

Furthermore, the resolvent operator $G_t$ is single-valued and nonexpansive that is

$$\|J^{G_t}_{\rho(t)}(x(t)) - J^{G_t}_{\rho(t)}(y(t))\| \leq \|x(t) - y(t)\|,\ \forall\, t \in \Omega \text{ and } x(t), y(t) \in H.$$

Since the subdifferential $\partial\phi$ of proper, convex and lower semicontinuous function, $\eta : \Omega \times H \times H \to H$ is strictly monotone, $\phi : \Omega \times H \times H \to R \cup \{+\infty\}$ is a maximal monotone multivalued mapping, it follows that resolvent operator $J^{\partial_\eta \phi_t}_{\rho(t)}$ of $\partial_\eta \phi_t$ is given by

$$J^{\partial_\eta \phi_t}_{\rho(t)} = (I + \rho(t)\partial_\eta \phi_t)^{-1}(x(t))\ \forall\, t \in \Omega \text{ and } x(t) \in H.$$

**Lemma 1.1.** Let $\{a_n(t)\}_{n\geq 0}$, $\{b_n(t)\}_{n\geq 0}$ and $\{c_n(t)\}_{n\geq 0}$ be non-negative sequences satisfying

$$a_{n+1}(t)\ \leq\ (1-\chi_n)a_n(t) + b_n(t)\chi_n + c_n(t)\ \forall\, n \geq 0$$

where $0 \leq \chi_n \leq 1$, $\sum\limits_{n=0}^{\infty} \chi_n = \infty$, $\lim\limits_{n\to\infty} b_n(t) = 0$ and $\sum\limits_{n=0}^{\infty} c_n(t) < \infty$. Then $\lim\limits_{n\to\infty} a_n(t) = 0$.

**Assumption 1.1.** the random operator $\eta : \Omega \times H \times H \to H$ satisfies the condition

$$\eta(t, y(t), x(t)) + \eta(t, x(t), y(t)) = 0,\ \forall\, x(t), y(t) \in H,\ t\ \in \Omega.$$

## 2. Ishikawa type iterative algorithm

To suggest the random three step iterative algorithm for computing the approximate solutions of problem (1.1), we mention the following equivalence between (1.1) and a fixed point problem which can be easily proved by using Definition 1.8.

**Lemma 2.1** [4]. Let $T : \Omega \times H \to CB(H)$ be a $\mathscr{H}$-continuous random multivalued mapping. Then for any measurable mapping $q : \Omega \to H$, the multivalued mapping $T(., q(.)) : \Omega \to CB(H)$ is measurable.

**Lemma 2.2** [4]. Let $S, T : \Omega \times H \to CB(H)$ be two measurable multivalued mappings, $\varepsilon > 0$ be a constant and $w : \Omega \to H$ be a measurable selection of $S$. Then there exist a measurable selection $q : \Omega \to H$ of $T$ such that for all $t \in \Omega$

$$\|w(t) - q(t)\| \leq (1+\varepsilon)\mathscr{H}(S(t), T(t)).$$

**Lemma 2.3.** The set of measurable mappings $x,u,w,q:\Omega\to H$ is a random solution of problem (1.1) if and only if $\forall\, t\in\Omega, x(t)\in H,\ u(t)\in\tilde{M}(t,x(t)),\ w(t)\in\tilde{S}(t,x(t)),\ q(t)\in\tilde{T}(t,x(t))$ and

$$g(t,x(t)) \;=\; J^{\partial_\eta\phi_t}_{\rho(t)}[g(t,x(t))-\rho(t)(P(t,h(t,u(t)))-(F(t,w(t))-G(t,q(t))))], \tag{2.1}$$

where $\rho:\Omega\to(0,\infty)$ is a measurable function.

**Proof.** From the difinition of $J^{\partial_\eta\phi_t}_{\rho(t)}$, it follows that

$$g(t,x(t))-\rho(t)[P(t,h(t,u(t)))-(F(t,w(t))-G(t,q(t)))]\in g(t,x(t))+\rho(t)\partial_\eta\phi_t$$

and hence

$$(F(t,w(t))-G(t,q(t))-P(t,h(t,u(t))))\in\rho(t)\partial_\eta\phi_t.$$

By using the definition of $\eta$-subdifferentiable, we have
$\langle(F(t,w(t))-G(t,q(t))-P(t,h(t,u(t)))),\eta(t,y(t),g(t,x(t))\rangle$
$\le\phi(t,y(t),x(t))-\phi(t,g(t,x(t)),x(t)),\ \forall\, y(t)\ \in H,\ t\in\Omega.$
Thus $(x,u,w,q)$ is a random solution of problem (1.1).

Based on Lemma 2.3, we define the following random three step iterative algorithm for solving problem (1.1).

**Algorithm 2.1.** Let $h,g,F,G,P:\Omega\times H\to H$ be random mappings and $M,S,T:\Omega\times H\to\mathscr{F}(H)$ be three random fuzzy mappings satisfying condition $(*)$. Let $\tilde{M},\tilde{S},\tilde{T}:\Omega\times H\to CB(H)$ be random multivalued mappings induced by $M,S$ and $T$ respectively. For any given measurable mapping $x_0:\Omega\to H$, the multivalued mappings $\tilde{M}(.,x_0(.)),\tilde{S}(.,x_0(.)),\tilde{T}(.,x_0(.)):\Omega\to CB(H)$ are measurable by Lemma 3.1. Hence there exist measurable selection $u_0:\Omega\to H$ of $\tilde{M}(.,x_0(.)),\ w_0:\Omega\to H$ of $\tilde{S}(.,x_0(.))$ and $q_0:\Omega\to H$ of $\tilde{T}(.,x_0(.))$ by Himmelberg [9]. Let

$$\begin{aligned}
z_n(t) &= \alpha''_n(t)x_n(t)+\beta''_n(t)[x_n(t)-g(t,x_n(t))+J^{\partial_\eta\phi_n(t,.,x_n(t))}_{\rho(t)}\{g(t,x_n(t))\\
&\quad -\rho(t)(P(t,h(t,u_n(t))-(F(t,w_n(t))-G(t,q_n(t)))))\}]+\gamma''_n(t)\chi_n(t)\\
y_n(t) &= \alpha'_n(t)x_n(t)+\beta'_n(t)[z_n(t)-g(t,z_n(t))+J^{\partial_\eta\phi_n(t,.,z_n(t))}_{\rho(t)}\{g(t,z_n(t))\\
&\quad -\rho(t)(P(t,h(t,\bar{u}'_n(t))-(F(t,\bar{w}'_n(t))-G(t,\bar{q}'_n(t)))))\}]+\gamma'_n(t)f_n(t)\\
x_{n+1}(t) &= \alpha_n(t)x_n(t)+\beta_n(t)[y_n(t)-g(t,y_n(t))+J^{\partial_\eta\phi_n(t,.,y_n(t))}_{\rho(t)}\{g(t,y_n(t))\\
&\quad -\rho(t)(P(t,h(t,\bar{u}_n(t)))-(F(t,\bar{w}_n(t))-G(t,\bar{q}_n(t)))))\}]+\gamma_n(t)s_n(t)
\end{aligned} \tag{2.2}$$

for all $n \geq 0$, where $u_n(t) \in \tilde{M}(.,x_n(t)), w_n(t) \in \tilde{S}(.,x_n(t)), q_n(t) \in \tilde{T}(.,x_n(t)), \bar{u}'_n(t) \in \tilde{M}(.,z_n(t))$, $\bar{w}'_n(t) \in \tilde{S}(.,z_n(t)), \bar{q}'_n\,(t) \in \tilde{T}(.,z_n(t)), \bar{u}_n(t) \in \tilde{M}(.,y_n(t)), \bar{w}_n(t) \in \tilde{S}(.,y_n(t)), \bar{q}_n(t) \in \tilde{T}(.,y_n(t))$; $\{s_n(t)\}_{n\geq 0}, \{f_n(t)\}_{n\geq 0}, \{\chi_n(t)\}_{n\geq 0}$ are bounded sequences in $H$ and $\{\alpha_n(t)\}_{n\geq 0}, \{\beta_n(t)\}_{n\geq 0}$, $\{\gamma_n(t)\}_{n\geq 0}, \{\alpha'_n(t)\}_{n\geq 0}, \{\beta'_n(t)\}_{n\geq 0}, \{\gamma'_n(t)\}_{n\geq 0}, \{\alpha''_n(t)\}_{n\geq 0}, \{\beta''_n(t)\}_{n\geq 0}, \{\gamma''_n(t)\}_{n\geq 0}$ are sequences in [0,1] satisfying suitable conditions and

$$\begin{aligned}
u_n(t) \in \tilde{M}(t,x_n(t)),\ \|u_n(t) - u_{n+1}(t)\| &\leq H(\tilde{M}(t,x_n(t)), \tilde{M}(t,x_{n+1}(t))) \\
w_n(t) \in \tilde{S}(t,x_n(t)),\ \|w_n(t) - w_{n+1}(t)\| &\leq H(\tilde{S}(t,x_n(t)), \tilde{S}(t,x_{n+1}(t))) \\
q_n(t) \in \tilde{T}(t,x_n(t)),\ \|z_n(t) - z_{n+1}(t)\| &\leq H(\tilde{T}(t,x_n(t)), \tilde{T}(t,x_{n+1}(t)))
\end{aligned} \tag{2.3}$$

for any $t \in \Omega$ and $n = 0,1,2,\cdots$.

**Lemma 2.4.** *Let $\eta : \Omega \times H \times H \to H$ be strongly monotone and Lipschitz continuous random map with constant $q(t) > 0$ and $z(t) > 0$, respectively and satisfy Assumption 1.1. Then*

$$\|J^{\phi_t}_{\rho(t)} x(t) - J^{\phi_t}_{\rho(t)} y(t)\| \leq \tau(t)\|x(t) - y(t)\|;\ x(t), y(t) \in H,$$

*where $\tau(t) = \frac{z(t)}{q(t)}$.*

**Proof.** For the proof, see Lemma 3 of [18].

## 3. Existence and convergence theorems

In this section, we establish an existence result for solutions for problem (1.1) and the convergence of the iterative sequences generated by Algorithm 2.1.

**Theorem 3.1.** *Let $\eta : \Omega \times H \times H \to H$ be strongly monotone and Lipschitz continuous random map with constant $q(t) > 0$ and $z(t) > 0$ respectively. Let $g : \Omega \times H \to H$ be $r$ -strongly monotone and s-Lipschitz continuous with corresponding constants $r(t)$ and $s(t)$, respectively, and $h, F, G, P : \Omega \times H \to H$ be Lipschitz continuous random operator with corresponding constants $\alpha(t), \xi(t), \lambda_G(t)$ and $\sigma(t)$, respectively. Let $M, S, T : \Omega \times H \to \mathscr{F}(H)$ be three random fuzzy mappings satisfying the condition $(*)$. Let $\tilde{M}, \tilde{S}, \tilde{T} : \Omega \times H \to CB(H)$ be three random multivalued mappings induced by $M, S$ and $T$ respectively. Suppose that $\tilde{M}, \tilde{S}$ and $\tilde{T}$ are $\mathscr{H}$-Lipschitz* *continuous with corresponding constants $\gamma(t), h(t)$ and $d(t)$, respectively, and $\tilde{S}$ be relaxed Lipschitz with respect to $F$ with constant $k(t)$ and $\tilde{T}$ be relaxed monotone with respect to $G$ with constant $c(t)$. Let $\phi : \Omega \times H \times H \to R \cup \{+\infty\}$ be such that for each $y(t) \in H$, $\phi(t,.,y(t))$ is a proper, convex and lower semicontinuous function on $H$, $g(H) \cap dom\, \partial_\eta \phi(t,.,y(t)) \neq \phi$, and for each $x(t), y(t), z(t) \in H$, $\mu(t) > 0$ and*

$$\|J_{\rho(t)}^{\partial_\eta \phi(t,.,x(t))}(z) - J_{\rho(t)}^{\partial_\eta \phi(t,.,y(t))}(z)\| \;=\; 0. \tag{3.1}$$

*Suppose that there exists a constant $\rho(t) > 0$, such that*

$$0 < \chi(t) + \tau(t)[\chi(t) + (1 - 2\rho(t)(k(t) - c(t)) + \rho^2(t)(\xi(t)h(t) + \lambda_G(t)d^2(t)))^{1/2} + \sigma(t)\alpha(t)\gamma(t)) < 1.$$

*where*

$$\chi(t) = (1 - 2r(t) + s^2(t))^{1/2} < 1.$$

*Then problem (1.1) has a solution $(x^\star(t), u^\star(t), w^\star(t), q^\star(t))$.*

We require the following definitions to achieve the results of this paper.

**Definition 3.1.** A random operator $g : \Omega \times H \to H$ is said to be

(i) *r-Strongly monotone* if there exists a measurable function $r : \Omega \to (0,\infty)$ such that

$$\langle g(t,x_1(t)) - g(t,x_2(t)), x_1(t) - x_2(t)\rangle \;\geq\; r(t)\|x_1(t) - x_2(t)\|^2,\ \forall\, x_1(t), x_2(t) \in H,\ t \in \Omega.$$

(ii) *s-Lipschitz continuous* if there exists a measurable function $s : \Omega \to (0,\infty)$ such that

$$\|g(t,x_1(t)) - g(t,x_2(t))\| \;\leq\; s(t)\|x_1(t) - x_2(t)\|,\ \forall\, x_1(t), x_2(t) \in H,\ t \in \Omega.$$

**Definition 3.2.** A random multivalued mapping $S : \Omega \times H \to 2^H$ is said to be

(i) *$\mathscr{H}$-Lipschitz continuous* if there exists a measurable function $h : \Omega \to (0,\infty)$ such that

$$H(S(t,x_1(t)), S(t,x_2(t))) \;\leq\; h(t)\|x_1(t) - x_2(t)\|,\ \forall\, x_1(t), x_2(t) \in H,\ t \in \Omega.$$

(ii) *relaxed Lipschitz* with respect to a random operator $F : \Omega \times H \to H$, if there exists a measurable function $k : \Omega \to (0,\infty)$ such that

$$\langle F(t,w_1(t)) - F(t,w_2(t)), x_1(t) - x_2(t)\rangle \;\leq\; -k(t)\|x_1(t) - x_2(t)\|^2,$$

$$\forall\, x_1(t), x_2(t) \in H, w_1(t) \in S(t,x_1(t)), w_2(t) \in S(t,x_2(t)),\ t \in \Omega.$$

(iii) *relaxed monotone* with respect to a random operator $G : \Omega \times H \to H$, if there exists a measurable function $c : \Omega \to (0, \infty)$ such that
$\langle G(t,w_1(t)) - G(t,w_2(t)), x_1(t) - x_2(t)\rangle \geq -c(t)\|x_1(t) - x_2(t)\|^2,$
$\forall\, x_1(t), x_2(t) \in H, w_1(t) \in S(t,x_1(t)), w_2(t) \in S(t,x_2(t)),\ t \in \Omega.$

**Proof of Theorem 3.1.** By Lemma 2.3, it is sufficient to prove that there exists $x^\star(t) \in H, u^\star(t) \in \tilde{M}(t,x^\star(t)), w^\star(t) \in \tilde{S}(t,x^\star(t))$ and $q^\star(t) \in \tilde{T}(t,x^\star(t)),\ \forall t \in \Omega$ satisfying (2.1). Define a random operator $A : \Omega \times H \to H$ by
$A(t,x(t)) = x(t) - g(t,x(t)) + J_{\rho(t)}^{\partial\phi(t,.,x(t))}[g(t,x(t)) - \rho(t)(P(t,(h(t,u(t))) - (F(t,w(t)) - G(t,q(t))))]$
$\forall x(t) \in H,\ u(t) \in \tilde{M}(t,x(t)),\ w(t) \in \tilde{S}(t,x(t)$ and $q(t) \in \tilde{T}(t,x(t))$ and
$A(t,y(t)) = y(t) - g(t,y(t)) + J_{\rho(t)}^{\partial_\eta\phi(t,.,y(t))}[g(t,y(t)) - \rho(t)(P(t,h(t,\bar{u}(t))) - (F(t,\bar{w}(t)) - G(t,\bar{q}(t))))]$
$\forall\, \bar{x}(t) \in H,\ \bar{u}(t) \in \tilde{M}(t,y(t)), \bar{w}(t) \in \tilde{S}(t,y(t))$ and $\bar{q}(t) \in \tilde{T}(t,y(t))$.
It follows that

$$\begin{aligned}
\|A(t,x(t)) - A(t,y(t))\| \leq\ & \|x(t) - y(t) - (g(t,x(t)) - g(t,y(t)))\| \\
& + \|J_{\rho(t)}^{\partial_\eta\phi(t,.,x(t))}[g(t,x(t)) - \rho(t)(P(t,h(t,u(t))) \\
& -(F(t,w(t)) - G(t,q(t)))] - J_{\rho(t)}^{\partial_\eta\phi(t,.,y(t))}[g(t,y(t)) \\
& -\rho(t)(P(t,h(t,\bar{u}(t))) - (F(t,\bar{w}(t)) - G(t,\bar{q}(t))))]\|. (3.3)
\end{aligned}$$

Since $g$ is $r$-strongly monotone and $s$-Lipschitz continuous, it follows that

$$\begin{aligned}
\|x(t) - y(t) - (g(t,x(t)) - g(t,y(t))\|^2 =\ & \|x(t) - y(t)\|^2 - 2\langle x(t) - y(t), g(t,x(t)) - g(t,y(t))\rangle \\
& + \|g(t,x(t)) - g(t,y(t))\|^2 \\
\leq\ & (1 - 2r(t) + s^2(t))\|x(t) - y(t)\|^2. (3.4)
\end{aligned}$$

Since the operator $J_{\rho(t)}^{\partial_\eta\phi}$ is nonexpansive and using (3.1), we have
$\|J_{\rho(t)}^{\partial_\eta\phi(t,.,x(t))}[g(t,x(t)) - \rho(t)(P(t,h(t,u(t))) - (F(t,w(t)) - G(t,q(t))))]$

$$-J_{\rho(t)}^{\partial_\eta \phi(t,.,y(t))}[g(t,y(t))-\rho(t)(P(t,h(t,\bar{u}(t)))-(F(t,\bar{w}(t))-G(t,\bar{q}(t))))]\|$$

$$\begin{aligned}
&\leq \|J_{\rho(t)}^{\partial_\eta \phi(t,.,x(t))}[g(t,x(t))-\rho(t)(P(t,h(t,u(t)))-(F(t,w(t))-G(t,q(t))))]\\
&\quad -J_{\rho(t)}^{\partial_\eta \phi(t,.,x(t))}[g(t,y(t))-\rho(t)(P(t,h(t,\bar{u}(t)))-(F(t,\bar{w}(t))-G(t,\bar{q}(t))))]\|\\
&\quad +\|J_{\rho(t)}^{\partial_\eta \phi(t,.,x(t))}[g(t,y(t))-\rho(t)(P(t,h(t,\bar{u}(t)))-(F(t,\bar{w}(t))-G(t,\bar{q}(t))))]\\
&\quad -J_{\rho(t)}^{\partial_\eta \phi(t,.,y(t))}[g(t,y(t))-\rho(t)(P(t,h(t,\bar{u}(t)))-(F(t,\bar{w}(t))-G(t,\bar{q}(t))))]\|\\
&\leq \tau(t)[\|x(t)-y(t)-(g(t,x(t))-g(t,y(t)))\|+\|x(t)-y(t)\\
&\quad +\rho(t)(F(t,w(t))-F(t,\bar{w}(t)))-\rho(t)(G(t,q(t))-G(t,\bar{q}(t)))\|\\
&\quad +\rho(t)\|P(t,h(t,u(t)))-P(t,h(t,\bar{u}(t)))\|].(3.5)
\end{aligned}$$

Since $\tilde{M},\tilde{S}$ and $\tilde{T}$ are $\mathscr{H}$-Lipschitz continuous and $h,F,G$ and $P$ are Lipschitz continuous, we get

$$\begin{aligned}
\|P(t,h(t,u(t)))-P(t,h(t,\bar{u}(t)))\| &\leq \sigma(t)\alpha(t)\|u(t)-\bar{u}(t)\| \leq \sigma(t)\alpha(t)\gamma(t)\|x(t)-y(t)\|\\
\|F(t,w(t))-F(t,\bar{w}(t))\| &\leq \xi(t)\|w(t)-\bar{w}(t)\| \leq \xi(t)h(t)\|x(t)-y(t)\|\\
\|G(t,q(t))-G(t,\bar{q}(t))\| &\leq \lambda_G(t)\|q(t)-\bar{q}(t)\| \leq \lambda_G(t)d(t)\|x(t)-y(t)\|. \qquad (3.6)
\end{aligned}$$

Further, since $\tilde{S}$ is relaxed Lipschitz and $\tilde{T}$ is relaxed monotone, we have

$$\|x(t)-y(t)+\rho(t)(F(t,w(t))-F(t,\bar{w}(t)))-\rho(t)(G(t,q(t))-G(t,\bar{q}(t)))\|^2$$

$$\begin{aligned}
&= \|x(t)-y(t)\|^2+2\rho(t)\langle F(t,w(t))-F(t,\bar{w}(t)),x(t)-y(t)\rangle-2\rho(t)\langle G(t,q(t))\\
&\quad -G(t,\bar{q}(t)),x(t)-y(t)\rangle+\rho(t)^2\|F(t,w(t))-F(t,\bar{w}(t))-(G(t,q(t))-G(t,\bar{q}(t)))\|^2\\
&\leq [1-2\rho(t)(k(t)-c(t))+\rho(t)^2(1+1/n)^2(\xi(t)h(t)+\lambda_G(t)d(t))^2]\|x(t)-y(t)\|^2. \qquad (3.7)
\end{aligned}$$

Using (3.4)-(3.7), (3.3) becomes

$$\|A(t,x(t))-A(t,y(t))\| \leq \theta(t)\|x(t)-y(t)\|, \qquad (3.8)$$

where

$$\theta(t)=\chi(t)+\tau(t)[\chi(t)+(1-2\rho(t)(k(t)-c(t))+\rho^2(t)(\xi(t)h(t)+\lambda_G(t)d^2(t)))^{1/2}+\sigma(t)\alpha(t)\gamma(t))$$

$$\theta(t)=\rho(t)\sigma(t)\alpha(t)\gamma(t)+\mu(t)\chi(t)[(1-2\rho(t)(k(t)-c(t))+\rho^2(t)(\xi(t)h(t)+\eta(t)d(t))^2]^{1/2}$$

$$\text{and } \chi(t)=[1-2r(t)+s^2(t)]^{1/2}. \tag{3.9}$$

It follows from (3.2) that $\theta(t)<1$. Hence $A$ is a contraction mapping and it has a fixed point $x^\star(t)\in H$, it follows by the definition of $A$ that $u^\star(t)\in\tilde{M}(t,x^\star(t)), w^\star(t)\in\tilde{S}(t,x^\star(t))$ and $q^\star(t)\in\tilde{T}(t,x^\star(t))$, $\forall t\in\Omega$ such that $(x^\star(t),u^\star(t),w^\star(t),q^\star(t))$ is a solution of (1.1). This completes the proof.

**Theorem 3.2.** *Let $g,h,\eta,F,G,P,\tilde{M},\tilde{S},\tilde{T}$ and $\phi$ be same as in Theorem 3.1 and for each $n\geq 0$, $\phi_n:\Omega\times H\times H\to R\cup\{+\infty\}$ be such that for each fixed $y(t)\in H$, $\phi_n(t,.,y(t))$ is a proper convex lower-semicontinuous functional on H and $g(H)\cap dom\,\partial\phi_n(t,.,y(t))\neq\phi$. Assume that*

$$\lim_{n\to\infty}\|J_{\rho(t)}^{\partial_\eta\phi_n(t,.,y(t))}(z)-J_{\rho(t)}^{\partial\phi(t,.,y(t))}(z)\|=0,\ \forall\, y(t),z(t)\in H,\ t\in\Omega.$$

*Suppose that $\{\chi_n(t)\}_{n\geq0}$, $\{f_n(t)\}_{n\geq0}$ and $\{s_n(t)\}_{n\geq0}$ are any bounded sequences in $H$ and $\{\alpha_n(t)\}_{n\geq0}$, $\{\beta_n(t)\}_{n\geq0}$, $\{\gamma_n(t)\}_{n\geq0}$, $\{\alpha'_n(t)\}_{n\geq0}$, $\{\beta'_n(t)\}_{n\geq0}$, $\{\gamma'_n(t)\}_{n\geq0}$, $\{\alpha''_n(t)\}_{n\geq0}$, $\{\beta''_n(t)\}_{n\geq0}$ and $\{\gamma''_n(t)\}_{n\geq0}$ are sequences in [0,1] satisfying the following conditions:*

$$\begin{aligned}\alpha_n(t)+\beta_n(t)+\gamma_n(t)&=\alpha'_n(t)+\beta'_n(t)+\gamma'_n(t)\\&=\alpha''_n(t)+\beta''_n(t)+\gamma''_n(t)=1,\\ \gamma_n(t)&=\alpha_n(t)\beta_n(t),\ \textit{for all } n\geq 0\end{aligned}\tag{3.10}$$

$$\lim_{n\to\infty}\gamma'_n(t)\ =\ \lim_{n\to\infty}\gamma''_n(t)\ =\ \lim_{n\to\infty}a_n(t)\ =\ 0 \tag{3.11}$$

$$\sum_{n=0}^{\infty}\beta_n(t)\ =\ \infty. \tag{3.12}$$

*If there exists a positive constant $\rho(t)$ satisfying (3.2), then problem (1.1) has a solution $(x^\star(t),u^\star(t),w^\star(t),q^\star(t))$ and the sequences $\{x_n(t)\}_{n\geq0}$, $\{u_n(t)\}_{n\geq0}$, $\{w_n(t)\}_{n\geq0}$ and $\{q_n(t)\}_{n\geq0}$ defined by Algorithm 2.1 converges strongly to $x^\star(t),u^\star(t),w^\star(t)$ and $q^\star(t)$, respectively.*

**Proof.** It follows that Theorem 3.1 that (1.1) has a solution $(x^\star(t), u^\star(t), w^\star(t), q^\star(t))$ and for all $n \geq 0$

$$\begin{aligned} x^\star(t) &= \alpha_n''(t)x^\star(t) + \beta_n''(t)[x^\star(t) - g(t,x^\star(t)) + J_{\rho(t)}^{\partial_\eta \phi(t,.,x^\star(t))}\{g(t,x^\star(t)) \\ &\quad -\rho(t)(P(t,h(t,u^\star(t))) - (F(t,w^\star(t)) - G(t,q^\star(t))))\}] + \gamma_n''(t)x^\star(t) \\ &= \alpha_n(t)x^\star(t) + \beta_n(t)[x^\star(t) - g(t,x^\star(t)) + J_{\rho(t)}^{\partial_\eta \phi(t,.,x^\star(t))}\{g(t,x^\star(t)) \\ &\quad -\rho(t)(P(t,h(t,u^\star(t))) - (F(t,w^\star(t)) - G(t,q^\star(t))))\}] + \gamma_n(t)x^\star(t). \end{aligned}$$

Using the same arguments as in the proof of Theorem 3.1, we have

$$\|x_n(t) - x^\star(t) - (g(t,x_n(t)) - g(t,x^\star(t))\| \leq \sqrt{1 - 2r(t) + s^2(t))}\|x_n(t) - x^\star(t)\|.$$

By Algorithm 2.1, assumptions on $\phi, \phi_n$ and the non-expansiveness of the resolvant operator $J_{\rho(t)}^{\partial_\eta \phi}$, we have

$$\begin{aligned} \|z_n(t) - x^\star(t)\| &= \|\alpha_n''(t)x_n(t) + \beta_n''(t)[x_n(t) - g(t,x_n(t)) + J_{\rho(t)}^{\partial_\eta \phi_n(t,.,x_n(t))}\{g(t,x_n(t)) \\ &\quad -\rho(t)(P(t,h(t,u_n(t))) - (F(t,w_n(t)) - G(t,q_n(t))))\}] + \gamma_n''(t)\chi_n(t) \\ &\quad -\{\alpha_n''(t)x^\star(t) + \beta_n''(t)[x^\star(t) - g(t,x^\star(t)) - J_{\rho(t)}^{\partial_\eta \phi(t,.,x^\star(t))}\{g(t,x^\star(t)) \\ &\quad -\rho(t)(P(t,h(t,u^\star(t))) - (F(t,w^\star(t)) - G(t,q^\star(t))))\}] + \gamma_n''(t)x^\star(t)\| \\ &\leq \alpha_n''(t)\|x_n(t) - x^\star(t)\| + \beta_n''(t)\|x_n(t) - x^\star(t) - (g(t,x_n(t)) - g(t,x^\star(t)))\| \\ &\quad +\beta_n''(t)\|J_{\rho(t)}^{\partial_\eta \phi_n(t,.,x_n(t))}\{g(t,x_n(t)) - \rho(t)(P(t,h(t,u_n(t))) \\ &\quad -(F(t,w_n(t)) - G(t,q_n(t))))\} - J_{\rho(t)}^{\partial_\eta \phi_n(t,.,x^\star(t))}\{g(t,x_n^\star(t)) \\ &\quad -\rho(t)(P(t,h(t,u_n^\star(t)) - (F(t,w_n^\star(t)) - G(t,q_n^\star(t))))\}\| \\ &\quad +\|J_{\rho(t)}^{\partial_\eta \phi_n(t,.,x_n^\star(t))}\{g(t,x_n^\star(t)) - \rho(t)(P(t,h(t,u_n^\star(t))) \\ &\quad -(F(t,w_n^\star(t)) - G(t,q_n^\star(t))) \\ &\quad -J_{\rho(t)}^{\partial_\eta \phi(t,.,x^\star(t))}\{g(t,x^\star(t)) - \rho(t)(P(t,h(t,u^\star(t))) \\ &\quad -(F(t,w^\star(t)) - G(t,q^\star(t))))\}\| + \gamma_n''(t)\|\chi_n(t) - x^\star(t)\| \\ &\leq \alpha_n''(t)\|x_n(t) - x^\star(t)\| + \beta_n''(t)\theta(t)\|x_n(t) - x^\star(t)\| \\ &\quad +\beta_n''(t)M_n(t) + M\gamma_n''(t), \end{aligned} \tag{3.13}$$

where

$$M(t) \;=\; \sup\{\|\chi_n(t)-x^\star(t)\|,\ \|f_n(t)-x^\star(t)\|,\ \|s_n(t)-x^\star(t)\|,\ n\geq 0\}$$

and

$$M_n(t) = \|J_{\rho(t)}^{\partial\phi_n(t,.,x^\star(t))}\{g(t,x^\star(t))-\rho(t)(P(t,h(t,u^\star(t)))-(F(t,w^\star(t))-G(t,q^\star(t))))\}$$
$$-J_{\rho(t)}^{\partial\phi(t,.,x^\star(t))}\{g(t,x^\star(t))-\rho(t)(P(t,h(t,u^\star(t)))-(F(t,w^\star(t)-G(t,q^\star(t))))\}\|$$

for all $n\geq 0$ and $\theta(t)$ is defined by (3.9). Similarly we can obtain

$$\|y_n(t)-x^\star(t)\| \;\leq\; \alpha_n'(t)\|x_n(t)-x^\star(t)\|+\beta_n'(t)\theta(t)\|z_n(t)-x^\star(t)\|+\beta_n'(t)M_n(t)+M(t)\gamma_n'(t) \tag{3.14}$$

and

$$\|x_{n+1}(t)-x^\star(t)\| \;\leq\; \alpha_n(t)\|x_n(t)-x^\star(t)\|+\beta_n(t)\theta(t)\|y_n(t)-x^\star(t)\|+\beta_n(t)M_n(t)+M(t)\gamma_n(t) \tag{3.15}$$

for all $n\geq 0$. Using (3.13) and (3.14), (3.15) becomes

$$\begin{aligned}\|x_{n+1}(t)-x^\star(t)\| \;&\leq\; [\alpha_n(t)+\theta(t)\beta_n(t)(\alpha_n'(t)+\theta(t)\beta_n'(t)(\alpha_n''(t)+\theta(t)\beta_n''(t))]\|x_n(t)-x^\star(t)\|\\ &\quad +M_n(t)\beta_n(t)(1+\theta(t)\beta_n'(t)+\theta^2(t)\beta_n'(t)\beta_n''(t))\\ &\quad +M(t)\theta(t)\beta_n(t)(\gamma_n'(t)+\theta(t)\beta_n'(t)\gamma_n''(t))+M(t)\gamma_n(t)\\ &\leq\; (1-(1-\theta(t))\beta_n(t))\|x_n(t)-x^\star(t)\|\\ &\quad +[3M_n(t)+M(t)(\gamma_n(t)+\gamma_n''(t)+a_n)(t)]\beta_n(t)\end{aligned}$$

for all $n\geq 0$. It follows from Lemma 1.1 and (3.9)-(3.11) that $x_n(t)\to x^\star(t)$ as $n\to\infty$. By the Lipschitz continuity of $\tilde{M},\tilde{S}$ and $\tilde{T}$, we have $u_n(t)\to u^\star(t)$, $w_n(t)\to w^\star(t)$ and $q_n(t)\to q^\star(t)$. This completes the proof.

## 4. Conclusion

We introduced completely generalized random variational inclusion problem for random fuzzy mappings. By using the resolvant operator technique, we constructed an iterative algorithm and prove existence and convergence result for completely generalized random variational inclusion problem for random fuzzy mappings.

To deal with the problems in mathematical sciences a further attention is needed for the study of relationship between fuzzy sets, fuzzy variational inclusions and random sets. It may provide an useful mathematical tools.

## Conflict of Interests

The authors declare that there is no conflict of interests.

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