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ITERATIVE SOLUTIONS OF A SYSTEM OF EQUILIBRIUM PROBLEMS IN HILBERT SPACES

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Abstract. In this paper, a system of equilibrium problems is investigated based on an implicit iterative algorithms with errors. The theorem of weak convergence for solutions of the system of equilibrium problems is established in the framework of Hilbert spaces.

Keywords: equilibrium problem; fixed point; nonexpansive mapping; solution; variation inequality.

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1. Introduction

Equilibrium problems which were introduced by Blum and Oettli [1] in 1994 have had a great impact and influence in the development of several branches of pure and applied sciences. It has been shown that the equilibrium problem theory provides a novel and unified treatment of a wide class of problems which arise in economics, Finance, image reconstruction, ecology, transportation, network, elasticity and optimization. It has been shown that equilibrium problems include variational inequalities, fixed point, Nash equilibrium and game theory as special cases. Hence collectively, equilibrium problems cover a vast range of applications. To study solution problem of equilibrium problem,

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iterative methods are efficient. Indeed, many well known problems arising in various branches of science can be studied by using algorithms which are iterative in their nature. For iterative algorithms, the oldest and simplest one is Picard iterative algorithm. For a contraction mapping T, it is known that T enjoys a unique fixed point and the sequence generated in Picard iterative algorithm can converge to the unique fixed point. However, for more general nonexpansive mappings, Picard iterative algorithm fails to convergence to fixed points of nonexpansive even that it enjoys a fixed point. Recently, implicit or explicit type iterative algorithms have been considered for the approximation of fixed points of nonexpansive mappings and solutions of equilibrium problem; see, for example [2-19]. For implicit iterative algorithms, classical weak convergence theorems of implicit iterative algorithms were established in Xu and Ori [15]. In this paper, we study a system of equilibrium problems based on an implicit iterative algorithm. Convergence theorems for solutions of equilibrium problems are established in Hilbert spaces.

2. Preliminaries

Throughout this paper, we assume that H is a real Hilbert space, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$. Let C be a nonempty closed and convex subset of H and $A : C \to H$ a nonlinear mapping.

Recall that the classical variational inequality problem is to find $x \in C$ such that

$$\langle Ax, y - x\rangle \geq 0, \quad \forall y \in C. \tag{2.1}$$

It is known that $x \in C$ is a solution to the variational inequality (2.1) if and only if x is a fixed point of the mapping $P_C(I - \rho A)$, where P_C is the metric projection from H onto C, $\rho > 0$ is a constant and I is identity mapping. This implies that the variational inequality (2.1) is equivalent to a fixed point problem. This alternative formula is very important form the numerical analysis point of view.

Let F_1 and F_2 be bifunctions of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. In this paper, we consider the following problem based on an implicit iterative algorithm:

$$\text{Find } x \in C \text{ such that } F_i(x, y) \geq 0, \quad \forall y \in C, \tag{2.2}$$

where $i \in \{1, 2\}$. To study bifunctions equilibrium problem, we may assume that the bifunction $F : C \times C \to \mathbb{R}$ satisfies the following conditions:

(A1) $F(x, x) = 0$ for all $x \in C$;

(A2) F is monotone, i.e., $F(x, y) + F(y, x) \leq 0$ for all $x, y \in C$;

(A3) for each $x, y, z \in C$, $\limsup_{t \downarrow 0} F(tz + (1 - t)x, y) \leq F(x, y)$;

(A4) for each $x \in C$, $y \mapsto F(x, y)$ is convex and lower semi-continuous.

Let $T : C \to C$ be a mapping. Recall that the mapping T is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C.$$

In 2001, Xu and Ori [15] introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_1, T_2, \ldots, T_N\}$, with $\{\alpha_n\}$ a real sequence in $(0, 1)$, and an initial point $x_0 \in C$:

$$\begin{aligned}
&x_1 = \alpha_1 x_0 + (1 - \alpha_1) T_1 x_1, \\
&x_2 = \alpha_2 x_1 + (1 - \alpha_2) T_2 x_2, \\
&\vdots \\
&x_N = \alpha_N x_{N-1} + (1 - \alpha_N) T_N x_N, \\
&x_{N+1} = \alpha_{N+1} x_N + (1 - \alpha_{N+1}) T_1 x_{N+1}, \\
&\vdots
\end{aligned}$$

which can be re-written in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n) T_n x_n, \quad \forall n \geq 1, \tag{2.3}$$

where $T_n = T_{n(modN)}$ (here the mod N function takes values in $\{1, 2, \ldots, N\}$). Xu and Ori [15] obtained the following results in a Hilbert space.

Theorem XO. *Let H be a real Hilbert space, C a nonempty closed convex subset of H, and $T : C \to C$ be a finite family of nonexpansive self-mappings on C such that $F = \cap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{x_n\}$ be defined by (2.3). If $\{\alpha_n\}$ is chosen so that $\alpha_n \to 0$, as $n \to \infty$, then $\{x_n\}$ converges weakly to a common fixed point of the family of $\{T_i\}_{i=1}^N$.*

We remark that, from the view of computation, the implicit iterative scheme (2.3) is often impractical since, for each step, we must solve a nonlinear operator equation. Therefore, one of the interesting and important problems in the theory of implicit iterative algorithm is to consider the iterative algorithm with errors. That is an efficient iterative algorithm to compute approximately fixed point of nonlinear mappings and solutions of equilibrium problems.

In this paper, motivated by Xu and Ori [15], we introduce a two-step implicit iterative algorithm with errors for the problem (2.2). Weak convergence theorems of purposed implicit iterative schemes are established in the framework of Hilbert spaces.

In order to prove our main results, we need the following concepts and lemmas.

Recall that a space X is said to satisfy *Opial* condition [17], if for each sequence $\{x_n\}$ in X, the condition that the sequence $x_n \to x$ weakly implies that

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|$$

for all $y \in X$ and $y \neq x$.

Lemma 2.1 ([18]). *Suppose that H is a real Hilbert space and $0 < p \leq t_n \leq q < 1$ for all $n \geq 1$. Suppose further that $\{x_n\}$ and $\{y_n\}$ are sequences of H such that*

$$\limsup_{n\to\infty} \|x_n\| \leq r, \quad \limsup_{n\to\infty} \|y_n\| \leq r$$

and

$$\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = r$$

hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

The following lemma can be found in [1] and [16].

Lemma 2.2. *Let C be a nonempty closed convex subset of H ad let $F : C \times C \to \mathbb{R}$ be a bifunction satisfying (A1)-(A4). Then, for any $r > 0$ and $x \in H$, there exists $z \in C$ such that $F(z, y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0$, $\forall y \in C$. Further, define a mapping*

$$T_r x = \{z \in C : F(z, y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \quad \forall y \in C\}$$

for all $r > 0$ and $x \in H$. Then, the following hold:

(1) T_r *is single-valued;*

(2) T_r *is firmly nonexpansive, i.e., for any* $x, y \in H$,

$$\|T_r x - T_r y\|^2 \leq \langle T_r x - T_r y, x - y\rangle;$$

(3) $F(T_r) = EP(F)$;

(4) $EP(F)$ *is closed and convex.*

Lemma 2.3 ([19]). *Let* $\{a_n\}$, $\{b_n\}$ *and* $\{c_n\}$ *be three nonnegative sequences satisfying the following condition:*

$$a_{n+1} \leq (1 + b_n)a_n + c_n, \quad \forall n \geq n_0.$$

where n_0 *is some nonnegative integer,* $\sum_{n=0}^{\infty} c_n < \infty$ *and* $\sum_{n=0}^{\infty} b_n < \infty$. *Then* $\lim_{n\to\infty} a_n$ *exists.*

3. Main results

Theorem 3.1. *Let* C *be a nonempty closed and convex subset of* H. *Let* F_1 *and* F_2 *be two bifunctions from* $C \times C$ *to* $\mathbb{R}$ *satisfying (A1)-(A4) such that* $EP = EP(F_1) \cap EP(F_2) \neq \emptyset$. *Let* $\{x_n\}$ *be a sequence generated by the following manner:*

$$\begin{cases} x_0 \in H, \text{ chosen arbitrily} \\ y_n = \beta_n x_n + (1 - \beta_n)\theta_n + v_n, \\ x_n = \alpha_n x_{n-1} + (1 - \alpha_n)\eta_n + u_n, \quad \forall n \geq 1, \end{cases} \tag{3.1}$$

where θ_n, *and* η_n *are such that*

$$F_1(\theta_n, \mu) + \frac{1}{\lambda_n}\langle \mu - \theta_n, \theta_n - x_n\rangle \geq 0, \quad \forall \mu \in C,$$

and

$$F_2(\eta_n, \nu) + \frac{1}{\rho_n}\langle \nu - \eta_n, \eta_n - y_n\rangle \geq 0, \quad \forall \nu \in C,$$

$\{\lambda_n\} \subset (0, \infty)$, $\{\rho_n\} \subset (0, \infty)$, $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset (0, 1)$ *and* $\{u_n\}$, $\{v_n\}$ *are bounded sequences. Assume that the following conditions are satisfied*

(1) $a \leq \alpha_n, \beta_n \leq b$, *where* $0 < a < b < 1$;

(2) $\liminf_{n\to\infty} \lambda_n > 0$ *and* $\liminf_{n\to\infty} \rho_n > 0$;

(3) $\sum_{n=1}^{\infty}(\|u_n\|+\|v_n\|)<\infty$.

Then the sequence $\{x_n\}$ *converges weakly to some point in* EP.

Proof. In view of Lemma 2.2, we see that $\eta_n = S_{\rho_n}y_n$ and $\theta_n = T_{\lambda_n}x_n$. Whenever needed, we shall equivalently write the implicit iteration (3.1) as

$$\begin{cases} x_0 \in H, \ \textit{chosen arbitrily} \\ y_n = \beta_n x_n + (1-\beta_n)T_{\lambda_n}x_n + v_n, \\ x_n = \alpha_n x_{n-1} + (1-\alpha_n)S_{\rho_n}y_n + u_n. \end{cases} \tag{3.2}$$

Fixing $p \in EP$, we obtain that

$$\begin{aligned} \|y_n - p\| &\le \beta_n\|x_n - p\| + (1-\beta_n)\|T_{\lambda_n}x_n - p\| + \|v_n\| \\ &\le \|x_n - p\| + \|u_n\|. \end{aligned} \tag{3.3}$$

This implies that

$$\begin{aligned} \|x_n - p\| &\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)\|S_{\rho_n}y_n - p\| + \|u_n\| \\ &\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)\|y_n - p\| + \|u_n\| \\ &\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)\|x_n - p\| + \|u_n\| + \|u_n\|. \end{aligned}$$

This in turn implies that

$$\|x_n - p\| \le \|x_{n-1} - p\| + \frac{\|u_n\| + \|u_n\|}{a}.$$

In view of Lemma 2.3, we obtain that $\lim_{n\to\infty}\|x_n - p\|$ exits. Next, we assume that $\lim_{n\to\infty}\|x_n - p\| = d > 0$. Note that

$$\limsup_{n\to\infty}\|x_{n-1} - p + u_n\| \le d. \tag{3.4}$$

In view of (3.3), we see that

$$\limsup_{n\to\infty}\|S_{\rho_n}y_n - p + u_n\| \le \limsup_{n\to\infty}(\|y_n - p\| + \|u_n\|) \le d. \tag{3.5}$$

On the other hand, we have

$$\lim_{n\to\infty}\|x_n - p\| = \lim_{n\to\infty}\|\alpha_n(x_{n-1} - p + u_n) + (1-\alpha_n)(S_{\rho_n}y_n - p + u_n)\| = d. \tag{3.6}$$

Combining (3.4), (3.5) with (3.6), we obtain from Lemma 2.1 that

$$\lim_{n\to\infty} \|x_{n-1} - S_{\rho_n} y_n\| = 0. \tag{3.7}$$

It follows from (3.2) that

$$x_n - x_{n-1} = (1-\alpha_n)(S_{\rho_n} y_n - x_{n-1}) + u_n.$$

In view of (3.7), we see from the condition (3) that

$$\lim_{n\to\infty} \|x_{n-1} - x_n\| = 0. \tag{3.8}$$

Note that

$$\begin{aligned} \|x_n - p\| &\le \alpha_n \|x_{n-1} - p\| + (1-\alpha_n)\|S_{\rho_n} y_n - p\| + u_n \\ &\le \alpha_n \|x_{n-1} - S_{\rho_n} y_n\| + \|S_{\rho_n} y_n - p\| + u_n \\ &\le \alpha_n \|x_{n-1} - S_{\rho_n} y_n\| + \|y_n - p\| + u_n, \end{aligned}$$

from which it follows that $\liminf_{n\to\infty} \|y_n - p\| \ge d$. In view of (3.3), we also have $\limsup_{n\to\infty} \|y_n - p\| \le d$. It follows that

$$\lim_{n\to\infty} \|y_n - p\| = \lim_{n\to\infty} \|\beta_n(x_n - p + v_n) + (1-\beta_n)(T_{\lambda_n} x_n - p + v_n)\| = d.$$

On the other hand, we have

$$\limsup_{n\to\infty} \|x_n - p + v_n\| \le d$$

and

$$\limsup_{n\to\infty} \|T_{\lambda_n} x_n - p + v_n\| \le \limsup_{n\to\infty} (\|x_n - p\| + \|v_n\|) \le d.$$

By virtue of Lemma 2.1, we obtain that

$$\lim_{n\to\infty} \|T_{\lambda_n} x_n - x_n\| = 0. \tag{3.9}$$

Note that

$$\begin{aligned} \|S_{\rho_n} x_n - x_n\| &\le \|S_{\rho_n} x_n - S_{\rho_n} x_{n-1}\| + \|S_{\rho_n} x_{n-1} - S_{\rho_n} y_n\| + \|S_{\rho_n} y_n - x_n\| \\ &\le 2\|x_n - x_{n-1}\| + \|x_{n-1} - y_n\| + \|S_{\rho_n} y_n - x_{n-1}\| \\ &\le 3\|x_n - x_{n-1}\| + \|T_{\lambda_n} x_n - x_n\| + v_n + \|S_{\rho_n} y_n - x_{n-1}\|. \end{aligned}$$

In view of (3.7), (3.8) and (3.9), we arrive at

$$\lim_{n\to\infty} \|S_{\rho_n} x_n - x_n\| = 0. \tag{3.10}$$

Since the sequence $\{x_n\}$ is a bounded, we see that there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \rightharpoonup q$. It follows that $\theta_{n_i} \rightharpoonup q$.

Next, we show that $q \in EP$. First, we prove $q \in FP(F_1)$. Since $\theta_n = T_{\lambda_n} x_n$, we have

$$F_1(\theta_n, \mu) + \frac{1}{\lambda_n}\langle \mu - \theta_n, \theta_n - x_n\rangle \geq 0, \quad \forall \mu \in C.$$

It follows from (A2) that

$$\langle \mu - \theta_n, \frac{\theta_n - x_n}{\lambda_n}\rangle \geq F_1(\mu, \theta_n)$$

and hence

$$\langle \mu - \theta_{n_i}, \frac{\theta_{n_i} - x_{n_i}}{\lambda_{n_i}}\rangle \geq F_1(\mu, \theta_{n_i}).$$

Since $\frac{\theta_{n_i} - x_{n_i}}{\lambda_{n_i}} \to 0$, $\theta_{n_i} \rightharpoonup q$ and (A4), we have $F_1(\mu, q) \leq 0$ for all $\mu \in C$. For t with $0 < t \leq 1$ and $\mu \in C$, let $\mu_t = t\mu + (1-t)q$. Since $\mu \in C$ and $q \in C$, we have $\mu_t \in C$ and hence $F_1(\mu_t, q) \leq 0$. So, we obtain from (A1) and (A4) that

$$0 = F_1(\mu_t, \mu_t) \leq tF_1(\mu_t, \mu) + (1-t)F_1(\mu_t, q) \leq tF_1(\mu_t, \mu).$$

That is, $F_1(\mu_t, \mu) \geq 0$. It follows from (A3) that $F_1(q, \mu) \geq 0$ for all $\mu \in C$ and hence $q \in EP(F_1)$. Note that

$$\begin{aligned}
\|y_n - S_{\rho_n} y_n\| &\leq \|y_n - x_{n-1}\| + \|x_{n-1} - S_{\rho_n} y_n\| \\
&\leq \beta_n \|x_n - x_{n-1}\| + (1-\beta_n)\|T_{\lambda_n} x_n - x_{n-1}\| + \|x_{n-1} - S_{\rho_n} y_n\| + \|v_n\| \\
&\leq \|x_n - x_{n-1}\| + (1-\beta_n)\|T_{\lambda_n} x_n - x_n\| + \|x_{n-1} - S_{\rho_n} y_n\| + \|v_n\|.
\end{aligned}$$

From (3.7), (3.8) and (3.9), we see that

$$\lim_{n\to\infty} \|y_n - S_{\rho_n} y_n\| = 0.$$

On the other hand, we have

$$\|S_{\rho_n} y_n - x_n\| \leq \|S_{\rho_n} y_n - x_{n-1}\| + \|x_{n-1} - x_n\|.$$

In view of (3.7) and (3.8), we arrive at

$$\lim_{n\to\infty} \|S_{\rho_n} y_n - x_n\| = 0.$$

This in turn implies that $\eta_{n_i} \rightharpoonup q$. In a similar way, we can obtain that $q \in EP(F_2)$. This proves that $q \in EP = EP(F_1) \cap FP(F_2)$.

Finally, we show that the sequence $\{x_n\}$ converges weakly to q. Suppose the contrary holds. It follows that there exists some subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $x_{n_j} \rightharpoonup \bar{q}$ and $q \neq \bar{q}$. By the same method as given above, we can prove that $\bar{q} \in EP$. Put

$$\lim_{n\to\infty} \|x_n - q\| = d_1 \quad \text{and} \quad \lim_{n\to\infty} \|x_n - \bar{q}\| = d_2,$$

where d_1 and d_2 are two nonnegative numbers. In view of Opial's condition, we see that

$$d_1 = \liminf_{i\to\infty} \|x_{n_i} - q\| < \liminf_{j\to\infty} \|x_{n_i} - \bar{q}\| = \liminf_{j\to\infty} \|x_{n_j} - \bar{q}\| < \liminf_{j\to\infty} \|x_{n_j} - q\| = d_1.$$

This is a contradiction. Hence $\bar{q} = q$. This shows that the sequence $\{x_n\}$ converges weakly to q. The proof is completed.

As corollaries of Theorem 3.1, we have the following results.

Corollary 3.2. *Let C be a nonempty closed and convex subset of H. Let F be a bifunction from $C \times C$ to $\mathbb{R}$ satisfying (A1)-(A4) such that $EP(F) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by the following manner:*

$$\begin{cases} x_0 \in H, \ \textit{chosen arbitrily} \\ y_n = \beta_n x_n + (1-\beta_n)\theta_n + v_n, \\ x_n = \alpha_n x_{n-1} + (1-\alpha_n) P_C y_n + u_n, \quad \forall n \geq 1, \end{cases}$$

where θ_n is such that

$$F(\theta_n, \mu) + \frac{1}{\lambda_n}\langle \mu - \theta_n, \theta_n - x_n\rangle \geq 0, \quad \forall \mu \in C,$$

$\{\lambda_n\} \subset (0,\infty)$, $\{\alpha_n\} \subset (0,1)$, $\{\beta_n\} \subset (0,1)$ and $\{u_n\}$, $\{v_n\}$ are bounded sequences. Assume that the following conditions are satisfied

(1) *$a \leq \alpha_n, \beta_n \leq b$, where $0 < a < b < 1$;*

(2) *$\liminf_{n\to\infty} \lambda_n > 0$;*

(3) $\sum_{n=1}^{\infty}(\|u_n\| + \|v_n\|) < \infty$.

Then the sequence $\{x_n\}$ *weakly converges to some point in* $EP(F)$.

Proof. Put $F_2(x, y) = 0$ for all $x, y \in C$ and $\rho_n = 1$ for all $n \geq 1$. It follows that $\eta_n = P_C y_n$. We can conclude the desired conclusion easily from Theorem 2.1.

If we put $F_1(x, y) = 0$ for all $x, y \in C$ and $\lambda_n = 1$ for all $n \geq 1$, then we have the following result.

Corollary 3.3. *Let* C *be a nonempty closed and convex subset of a real Hilbert space* H. *Let* F *be a bifunction from* $C \times C$ *to* $\mathbb{R}$ *satisfying* (A1)-(A4) *such that* $EP(F) \neq \emptyset$. *Let* $\{x_n\}$ *be a sequence generated by the following manner:*

$$\begin{cases} x_0 \in C, \ \textit{chosen arbitrily} \\ x_n = \alpha_n x_{n-1} + (1 - \alpha_n)\eta_n, \quad \forall n \geq 1, \end{cases}$$

where η_n *is such that*

$$F(\eta_n, \nu) + \frac{1}{\rho_n}\langle \nu - \eta_n, \eta_n - x_n \rangle \geq 0, \quad \forall \nu \in C,$$

$\{\rho_n\} \subset (0, \infty)$, $\{\alpha_n\} \subset (0, 1)$ *and* $\{\beta_n\} \subset (0, 1)$. *Assume that the following conditions are satisfied*

(1) $a \leq \alpha_n, \beta_n \leq b$, *where* $0 < a < b < 1$;

(2) $\liminf_{n \to \infty} \rho_n > 0$.

Then the sequence $\{x_n\}$ *weakly converges to some point in* $EP(F)$.

Remark 3.4. In this paper, we study a system of bifunction equilibrium problems based on an implicit iterative algorithm. Weak convergence theorems of solutions are established. However, we do no know what restrictions imposed on the parameters or the subset C can let strong convergence theorems be guaranteed. It is of interest to improve the paper on the framework from Hilbert spaces to Banach spaces.

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