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LOGARITHMIC AND IDENTRIC MEAN LABELINGS OF GRAPHS

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Abstract. Graph labeling was first introduced by Rosa in 1966. Labeling of graphs is an assignment of non-negative integers to vertices, edges or both according to some specified conditions. Mean labeling of graphs was introduced by Somasundaram and Ponraj in 2003. Subsequently, labelings of graphs were done with geometric mean, harmonic mean etc. In this paper we introduce two concepts called 'Logarithmic mean labeling', 'Identric mean labeling' and acquire their mean labelings for some standard graphs.

Keywords: logarithmic mean; identric mean; labeling; mean graphs; comb; crown; path; cycle.

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1. Introduction

Labeled graphs provide mathematical models for a broad range of applications. Labelings of graph elements have been used in diverse fields such as Radar location codes, Coding theory, Circuit design, Communication network etc.,.

In Molecular Chemistry (or Cell Biology), different molecules (or cells) bear different numbers for chemical (or biological) characteristics. The byproducts of a chemical reation (or a biological process) are supposed to inherit some type of average (equivalently mean) as its characteristic number. What type of mean depends on the particular characteristic and it is determined by

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observations. Also different graphs represent different molecular structure. This motivates the study of various mean labelings for graphs. Here we come up with new types of mean labeling called Logarithmic and Identric mean labeling.

First we give the definitions of Logarithmic mean labeling and Identric mean labeling and make some observations on the ranges of both the means for any two positive integers upto a distance of 7 which will be helpful while labeling graphs. We collect all the relevant definitions needed in the sequel.

Definition 1.1. *A **graph** is an ordered pair $G = (V,E)$ where V is a set of vertices and E is a set of edges which are unordered pair of vertices from V. This type of graph is called undirected and simple. In addition, if the number of vertices is finite, then the graph is called finite too.*

Definition 1.2. *Let $V = \{x_1,x_2,\ldots x_n\}$ and $E = \{(x_i,x_{i+1})\,/0 \le i \le n-1\}$. The graph (V,E) is called a **path** on n vertices and is denoted by P_n.*

Definition 1.3. *Let $V = \{x_1,x_2,\ldots x_n\}$ and $E = \{(x_i,x_{i+1})\,/0 \le i \le n-1\} \cup \{(x_1,x_n)\}$. The graph (V,E) is called a **cycle** on n vertices and is denoted by C_n.*

Definition 1.4. *Any cycle with a pendent edge attached to each vertex is called a **crown***

Definition 1.5. *The graph obtained by joining a single pendent edge to each vertex of a path is called a **comb**.*

Definition 1.6. *A graph on n vertices in which any two vertices are adjacent is called a **complete** graph and is denoted byK_n.*

Definition 1.7. *A graph is **bipartite** if the set of vertices $V(G)$ can be partitioned into two nonempty subsets X with m vertices and Y with n vertices such that each edge has one end in X and the other end in Y. It is denoted by$K_{m,n}$.*

Definition 1.8. *The complete bipartite graph $K_{1,n}$ is called a **star**.*

Definition 1.9. *The **bistar** $B_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end of K_2 and n pendent edges to the other end of K_2.*

Definition 1.10. *A **Triangular snake** denoted by T_n is obtained from a path $v_1v_2.....v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \le i \le n-1$. It is denoted by T_n*

Definition 1.11. *A **Quadrilateral snake** is obtained from a path $u_1u_2...u_n$ by joining u_i, u_{i+1} to a new pair of vertices v_i, w_i respectively and joining v_i and w_i. It is denoted byQ_n.*

Definition 1.12. *A **Ladder** graph, denoted by L_n is a graph obtained from the cartesian product of P_n and P_1.*

Definition 1.13. *A tree which yields a path on removing its pendent vertices is called **caterpillar**.*

Definition 1.14. *A **dragon** is formed by joining the end of a path to a cycle.*

Definition 1.15. *Let a and b be two positive integers. The Logarithmic mean of a and b, denoted as $L(a,b)$, is defined by $\frac{b-a}{log\ b\ -\ log\ a}$.*

Definition 1.16. *Let a and b be two positive integers. The Identric mean of a and b, denoted as $I(a,b)$, is defined by $\frac{1}{e}\left(\frac{a^a}{b^b}\right)^{\frac{1}{a-b}}$.*

Fact 1. [3]. *For distinct postive numbers a and b, $G(a,b) < L(a,b) < I(a,b) < A(a,b), where\ G(a,b)$ and $A(a,b)$ denote respectively, the geometric and arithemtic means of a, b.*

2. Main Results

Definition 2.1. *Let $G = (V, E)$ be a graph with p vertices and q edges. G is said to be a Logarithmic mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, ...q+1$ in such a way that on labeling each edge $e = xy$ with $\left\lfloor \frac{y-x}{log\ y\ -\ log\ x} \right\rfloor$ or $\left\lceil \frac{y-x}{log\ y\ -\ log\ x} \right\rceil$, the resulting edge labels are distinct and are from $1, 2, ...q$. In this case f is called a Logarithmic mean labeling.*

Definition 2.2. *Let $G = (V, E)$ be a graph with p vertices and q edges. G is said to be an Identric mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, ...q+1$ in such a way that on labeling each edge $e = xy$ with $\left\lfloor \frac{1}{e}\left(\frac{x^x}{y^y}\right)^{\frac{1}{x-y}} \right\rfloor$ or $\left\lceil \frac{1}{e}\left(\frac{x^x}{y^y}\right)^{\frac{1}{x-y}} \right\rceil$, the resulting edge labels are distinct and are from $1, 2, ...q$. In this case f is called an Identric mean labeling.*

Remark 2.3. *For G to be a Logarithmic/Identric mean graph, (1,2) or (1,3) must be the labels for some pair of vertices, since with respect to Logarithmic/Identric mean labeling, only the labels (1,2) and (1,3) for vertices allow us to give the label 1 for an edge.*

Remark 2.4. *Clearly a (p,q) graph cannot admit Logarithmic and Identric mean labeling if $p > q+1$.*

Remark 2.5. *For G to be a Logarithmic/Identric mean graph, q must be a label for one of the edges. To obtain q as one of the edge labels, some pair of vertices must receive one of the following set of labels : $(q-1,q)$, $(q-1,q+1)$, $(q,q+1)$, $(q-2,q+1)$. This claim can be substantiated using the following two propositions.*

Proposition 2.6. *Let a, b be positive integers with $a < b$. Then*
1.$a < L(a,b) < b$.
2.$a < I(a,b) < b$.

Proof. 1.To prove : $a < L(a,b) < b$
That is to prove : $a < \frac{b-a}{log\ b\ -\ log\ a} < b$
To prove : (i) $a < \frac{b-a}{log\ b\ -\ log\ a}$ (ii)$b > \frac{b-a}{log\ b\ -\ log\ a}$
Proof of (i) : Enough to prove (henceforth abbreviated as ETP) : $a < \frac{b-a}{log\frac{b}{a}}$
ETP : $a\ log\frac{b}{a} < b-a$
ETP : $a\ log\left(1+\frac{h}{a}\right) < h$ where $b = a+h$; $h > 0$
ETP : $log(1+x) < x$ for $x > 0$
ETP : $f(x) = x - log(1+x) > 0$, for $x > 0$
Now $f'(x) = 1 - \frac{1}{1+x} > 0$ for $x > 0$.
Therefore, f is strictly increasing in ∞. Also $f(0) = 0$.
Therefore, $f(x) > f(0) = 0\ \forall x > 0$.
Hence (i) - the left side of the inequality of (1) is proved.

Proof of (ii) : ETP : $\frac{b-a}{log\frac{b}{a}} < b$
ETP : $b-a < b\ log\frac{b}{a}$
ETP : $h < b\ log\left(1+\frac{h}{a}\right)$ where $b = a+h$; $h > 0$

$$Note\ that\ \frac{h}{b} = \frac{h}{a+h}$$
$$= 1 - \frac{a}{a+h}$$
$$= 1 - \frac{1}{1+\frac{h}{a}}$$

ETP : $1-\frac{1}{1+\frac{h}{a}} \ < \ log\left(1+\frac{h}{a}\right)$

ETP : $1-\frac{1}{1+x} \ < \ log\,(1+x)$ for $x>0$.

ETP : $g\,(x) = log\,(1+x)-1+\frac{1}{1+x} \ > \ 0$ for $x>0$.

Now $g'\,(x) = \frac{1}{1+x}-\frac{1}{(1+x)^2} > 0$ for $x>0$.

Therefore, g is strictly increasing in ∞. Also $g\,(0)=0$.

Therefore, $g\,(x) > g\,(0) = 0 \ \forall x>0$.

Hence (ii) - the right side of the inequality of (1) is proved.

2.To prove : $a < I\,(a,b) < b$

That is to prove : $a < \frac{1}{e}\left(\frac{a^a}{b^b}\right)^{\frac{1}{a-b}} < b$

It is the same as proving $a < \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} < b$

Now to prove : (i)$a < \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}}$ (ii)$\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} < b$

Proof of (i) :

$$Now\ a < \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} \Leftrightarrow ae < \left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}}$$

$$\Leftrightarrow log\ a\ +1 < \frac{1}{b-a}\ log\frac{b^b}{a^a} = \frac{1}{b-a}\,(b\ logb - a\ loga)$$

$$\Leftrightarrow \ (b-a)\,loga + b - a < b\ logb - a\ loga$$

$$\Leftrightarrow b-a < b\ logb - b\ loga = b\ log\frac{b}{a}$$

$$\Leftrightarrow 1-\frac{a}{b} < log\frac{b}{a}$$

$$\Leftrightarrow 1-\frac{a}{a+h} < log\frac{a+h}{a},\ where\ b=a+h;\ h>0$$

$$\Leftrightarrow 1-\frac{1}{1+x} < log\,(1+x)\,,\ where\ x=\frac{h}{a}$$

$$\Leftrightarrow \frac{x}{1+x} < log\,(1+x)$$

$$\Leftrightarrow x < (1+x)\ \ log\,(1+x)$$

To prove that $x < (1+x)\ \ log\,(1+x)$, it is enough to prove that

$f\,(x) = (1+x)\ \ log\,(1+x) - x > 0$ for $x \geq 0$.

$f'\,(x) = log\,(1+x)$

$f'\,(x) > 0$ for $x>0$.

f is strictly increasing in $[0,\infty)$. Also $f\,(0)=0$.

Therefore $f(x) > f(0) = 0$

Therefore $f(x) > 0$ for $x \geq 0$.

Proof of (ii) : To prove : $\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} < b$

$$
\begin{aligned}
Now\ \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} < b &\Leftrightarrow \left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}} < be \\
&\Leftrightarrow \frac{1}{b-a}\ log\frac{b^b}{a^a} < logb\ +1 \\
&\Leftrightarrow b\ logb - a\ loga < (b-a)(logb+1) \\
&\Leftrightarrow a\ log\frac{b}{a} < b-a \\
&\Leftrightarrow log\frac{b}{a} < \frac{b-a}{a} = \frac{b}{a} - 1 \\
&\Leftrightarrow log\frac{a+h}{a} < \frac{a+h}{a} - 1\ , where\ b = a+h;\ h > 0 \\
&\Leftrightarrow log\left(1+\frac{h}{a}\right) < \frac{h}{a} \\
&\Leftrightarrow log(1+x) < x\ [where\ x = \frac{h}{a}]. \\
&\Leftrightarrow 1+x < e^x
\end{aligned}
$$

For $x > 0$, $1+x < e^x$ is always true. Hence proved. □

Proposition 2.7. *For $k > n(n+2)$, $i+n < L(i,i+k) < I(i,i+k) < i+\frac{k}{2}$.*

Proof. $L(i,i+k) < I(i,i+k)$ is always true, by Fact 1. Again by Fact 1, $L(i,i+k) < I(i,i+k) < A(i,i+k) = i+\frac{k}{2}$. Now to prove that $i+n < L(i,i+k)$, it is enough if we prove that $i+n <$

$G(i, i+k)$ for $k > n(n+2)$.

$$\begin{aligned}
i+n < G(i,i+k) &\Leftrightarrow (i+n)^2 < i(i+k) \\
&\Leftrightarrow i^2+2ni+n^2 < i^2+ki \\
&\Leftrightarrow n^2 < (k-2n)\,i \\
&\Leftrightarrow 1 < \left(\frac{k}{n^2}-\frac{2}{n}\right) i \\
&\Leftrightarrow \frac{k}{n^2}-\frac{2}{n} > 1 \\
&\Leftrightarrow \frac{k-2n}{n^2} > 1 \\
&\Leftrightarrow k-2n > n^2 \\
&\Leftrightarrow k > n^2+2n \\
&\Leftrightarrow k > n(n+2)
\end{aligned}$$

Hence the proposition. □

Observation 1. *From the above Proposition, we note that* $i+1 < L(i,i+k) < I(i,i+k) < i+\frac{k}{2}$ *for* $k > 3$

Observation 2. *If the vertices u and v have labels i and* $i+1$ *respectively, then by* (1) *and* (2) *of Proposition 2.6,* $i < L(i,i+1) < I(i,i+1) < i+1$ *and hence the edge uv can be given the label* i *or* $i+1$.

Observation 3. *If the vertices u and v are labeled with i and* $i+2$ *respectively, then* $i < L(i,i+2) < I(i,i+2) < A(i,i+2) = i+1$. *So the edge uv can be labeled with i or* $i+1$.

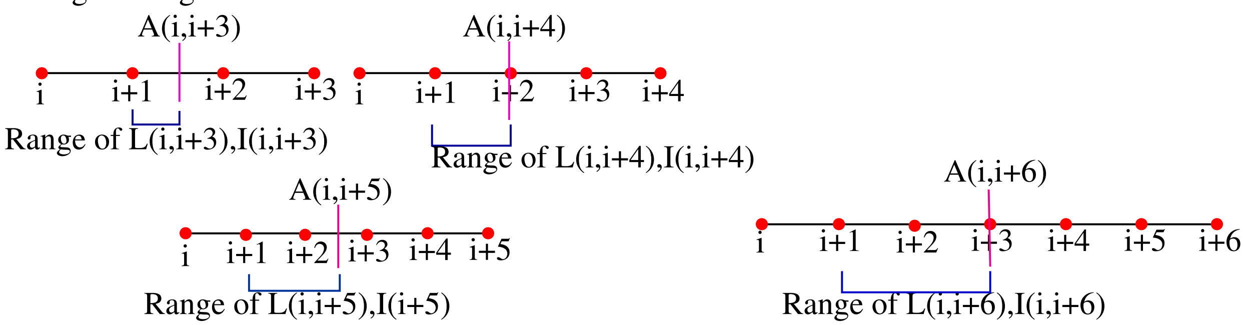
Observation 4. *We have* $(i+1)^2 = i^2+2i+1 \leq i^2+3i = i(i+3)$. *Hence* $i+1 \leq G(i,i+3)$ *and so* $i+1 < L(i,i+3) < I(i,i+3) < A(i,i+3) = i+\frac{3}{2}$. *If the vertices u and v have i and* $i+3$ *respectively as labels, the edge uv can be given the label* $i+1$ *or* $i+2$.

Observation 5. *If the vertices u and v are labeled with i and* $i+4$ *respectively, then by Observation 1, taking* $k=4$, $i+1 < L(i,i+4) < I(i,i+4) < i+2$ *and hence the edge uv can be labeled with* $i+1$ *or* $i+2$.

Observation 6. *By Observation 1, we have*
$i+1 < L(i,i+5) < I(i,i+5) < i+\frac{5}{2}$. *Hence on labeling the vertices u and v with i and $i+5$ respectively, the edge uv can be guarenteed the label $i+2$.*

Observation 7. *By Observation 1, we have*
$i+1 < L(i,i+6) < I(i,i+6) < i+3$. *Hence on labeling the vertices u and v with i and $i+6$ respectively, the edge uv can be given the label $i+2$.*

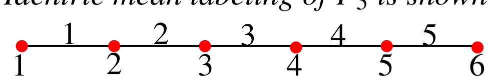
Considering all the above observations, we now give the following figures which depict the range of Logarithmic/Identric mean.



Theorem 2.8. *Any path is both a Logarithmic and an Identric mean graph.*

Proof. Let P_n be a path on n vertices given by $u_1, u_2, u_3, ..., u_n$ with $n-1$ edges.
Let $f : V(P_n) \to \{1, 2, ...q+1\}$ be defined by
$f(u_i) = i,\ 1 \le i \le n$. Now each edge $u_i u_{i+1}$ can be labeled as i, $1 \le i \le n$. Thus f becomes both Logarithmic and Identric mean labeling and hence paths are Logarithmic and Identric mean graphs. □

Example 1. *A Logarithmic/Identric mean labeling of P_5 is shown below.*

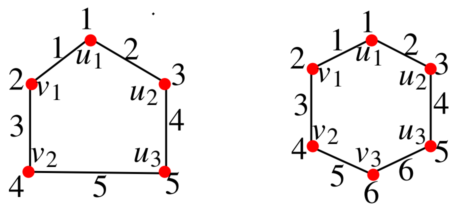


P_5

Theorem 2.9. *Any cycle is both a Logarithmic and an Identric mean graph.*

Proof. Let C_n be a cycle on n vertices $u_1, u_2, ...u_n$. Rename the vertices
$u_{\lceil \frac{n}{2} \rceil+1}, u_{\lceil \frac{n}{2} \rceil+2}, ...u_n$ as $v_m, v_{m-1}, ..., v_1$ respectively, where $m + \lceil \frac{n}{2} \rceil = n$.
Define $f : V(C_n) \to \{1, 2, ...q+1\}$ by $f(u_i) = 2i-1$ and $f(v_i) = 2i$. The edges $u_i u_{i+1}$ get labels $\lceil \frac{2i+1}{2} \rceil$ and the edges $v_i v_{i+1}$ get labels $2i+1$. $u_1 v_1$ gets the label 1 and $u_{\lceil \frac{n}{2} \rceil} v_m$ gets the label n. □

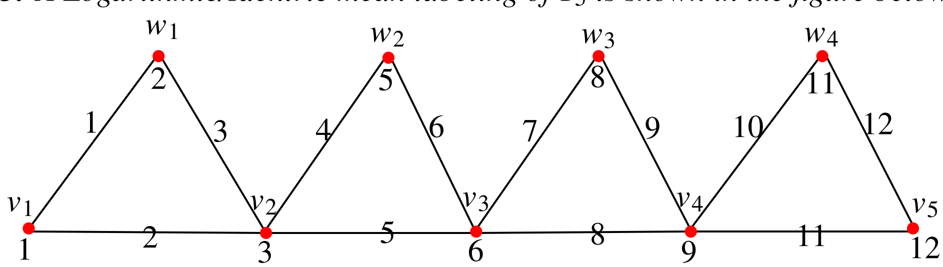
Example 2. *A Logarithmic/Identric mean labeling of C_5 and C_6 are shown below.*



Theorem 2.10. *Triangular snake T_n is both a Logarithmic and an Identric mean graph.*

Proof. Let T_n be a traingular snake with $v_i, 1 \leq i \leq n$ being the vertices of the path and $w_i, 1 \leq i \leq n$ being the vertices that are joined to $v_i v_{i+1}$. Let $f : V(T_n) \to \{1, 2, \ldots . q+1\}$ be defined by $f(v_1) = 1$, $f(v_i) = 3i-3$, $2 \leq i \leq n$ and $f(w_i) = 3i-1, 1 \leq i \leq n-1$. With respect to this type of labeling for the vertices, for $1 \leq i \leq n-1$ the edges $v_i v_{i+1}$, $v_i w_i$ and $v_{i+1} w_i$ get the labels $3i-1$, $3i-2$ and $3i$ respectively. Thus f becomes both Logarithmic and Identric mean labeling and hence T_n is both Logarithmic and Identric mean graphs. □

Example 3. *A Logarithmic/Identric mean labeling of T_5 is shown in the figure below.*

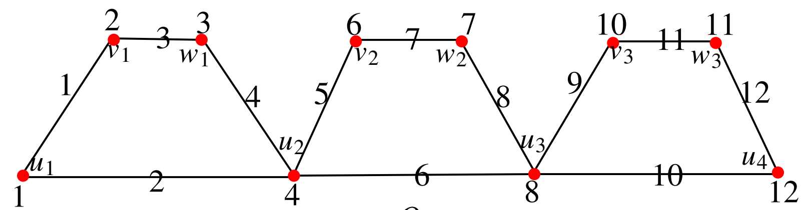


T_5

Theorem 2.11. *Quadrilateral snake Q_n admits Logarithmic/Identric mean labeling of graphs.*

Proof. Let Q_n be a quadrilateral snake as given in definition.
Define $f : V(Q_n) \to \{1, 2, 3, \ldots .. q+1\}$ as follows. $f(u_1) = 1$, $f(u_i) = 4i-4$ for $2 \leq i \leq n$, $f(v_i) = 4i-2$ for $1 \leq i \leq n$, $f(w_i) = 4i-1$ for $1 \leq i \leq n$. For$1 \leq i \leq n-1$ Edges $u_i v_i$, $v_i w_i$, $u_i u_{i+1}$, $u_{i+1} w_i$ get the labels $4i-3, 4i-1, 4i-2, 4i$ respectively. This results in distinct edge labels. □

Example 4. *A Logarithmic/Identric mean labeling of Q_3.*

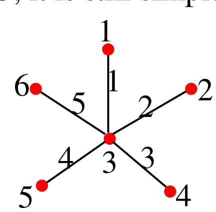


Q_3

Theorem 2.12. *The Star graph $K_{1,n}, n > 5$ is not a Logarithmic/Identric mean graph.*

Proof. Let $K_{1,n}, n > 5$ be a star graph where the vertex u is attached to $v_1, v_2, ...v_n$. By Remark 2.3, to get the label 1, 1 and 2 must be adjacent or 1 and 3 must be adjacent. The central vertex u should receive one of the labels : 1, 2, 3. Consider the case when the central vertex is given 1 or 2. None of $L(1,n)$, $I(1,n)$, $L(2,n)$, $I(2,n)$ is n for $n > 4$, since $A(1,n)$ and $A(2,n)$ are less than $n-1$ for $n > 4$. Hence the central vertex is labeled with 3. In this case too, none of $L(3,n)$, $I(3,n)$ is n for $n > 5$, since $A(3,n)$ is less than $n-1$ for $n > 5$. □

$K_{1,n}$ is both Logarithmic and Identric mean graph for $n \leq 5$. Logarithmic/Identric mean labeling of $K_{1,5}$ is as follows. For $n < 5$, it is still simpler.



$K_{1,5}$

Theorem 2.13. *The following standard graphs admit both Logarithmic and Identric mean labeling :*

1.Crown

2.Ladder

3.Comb

4.Caterpillar

5.Dragon

Proof. 1. Let G be a crown graph, where $u_1, u_2, ...u_n$ are the verices of the cycle C_n and $v_1, v_2, ...v_n$ are the pendent vertices such that each v_i is attached to u_i.
Define $f : V(G) \to \{1, 2, 3, ...q+1\}$ by $f(u_i) = 2i - 1$ for $i = 1, 2$, $f(u_i) = 2i + 1$ for $3 \leq i \leq n$ and $f(v_i) = 2i$ for $1 \leq i \leq n$.

2. Let L_n be a ladder graph with $u_1, u_2, ...u_n$ as vertices of one side of the ladder and $v_1, v_2, ...v_n$ as vetices on the other side of the ladder such that u_i is adjacent to v_i. Define $f : V(L_n) \rightarrow \{1, 2, 3, ...q+1\}$ by $f(u_i) = 1$, $f(u_i) = 3i-3$ for $2 \leq i \leq n$ and $f(v_i)$ and $f(v_i) = f(u_i)+1$ for $1 \leq i \leq n$. Edges u_iu_{i+1}, u_iv_i, v_iv_{i+1} receive the labels $3i-1, 3i-2, 3i$ respectively.

3. Let G be a comb graph such that for $1 \leq i \leq n$, v_i are the vertices of the path and u_i are the pendent vertices with every u_i attached to v_i.
Define $f : v(G) \rightarrow \{1, 2, 3, ...q+1\}$ by $f(v_i) = 2i-1$ and $f(u_i) = 2i$ for $1 \leq i \leq n$. Edges v_iu_i and v_iv_{i+1} get $2i-1, 2i$ as labels respectively.

4. Let G be a caterpillar graph. For $1 \leq i \leq n$, let v_i be the vertices of the path P_n, u_i, w_i be the vertices attcahed to v_i. Thus G has $3n$ vertices and $3n-1$ edges. $f : V(G) \rightarrow \{1, 2, 3, ...q+1\}$ is defined by $f(v_i) = 3i-2$, $f(u_i) = 3i-1$ and $f(w_i) = 3i$, $1 \leq i \leq n$. Edges u_iv_i, v_iw_i, v_iv_{i+1} are labeled with $3i-2, 3i-1, 3i$ respectively.

5. Let G be a dragon graph with $u_i, 1 \leq i \leq n$ as the vertices of the cycle C_n and $v_i, 1 \leq i \leq m$ as the vertices of the path P_m. Note that $u_n = v_1$. Define $f : V(G) \rightarrow \{1, 2, 3, ...q+1\}$ by $f(u_i) = i$ for $1 \leq i \leq n$, $f\left(v_j\right) = n+j-1$ for $2 \leq j \leq m$. Edges u_iu_{i+1} and v_jv_{j+1} receive $i \; and \; n+j$ as labels respectively.

With respect to the labelings given above all the graphs $(1)-(5)$ become both Logarithmic and Identric mean graphs. □

Throughout we note that the same labeling works for both the means.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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