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A DYNAMIC MODEL FOR A THREE SPECIES OPEN-ACCESS FISHERY WITH TAXATION AS A CONTROL INSTRUMENT OF HARVESTING EFFORTS THE CASE OF LAKE VICTORIA

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Abstract. The Lake Victoria fishery is dominated by three commercial fish species namely Nile perch, Nile tilapia and small pelagic silver fish.The current excessive use of fishing efforts in the lake have devastating consequences to the extent of diminishing these fish species. The purpose of this study is to propose a bioeconomic mathematical model based on Lotka-Volterra dynamics by introducing taxes to the profit per unit biomass of the harvested fish of each species with the intention of controlling fishing efforts.The results of the formulated model showed that the co-existence steady state with taxation was both locally and globally assymptotically stable.The optimal harvesting policy was established using Pontryagin's maximum principle. The numerical example illustrated that imposition of optimal taxations resulted into optimal harvesting efforts and hence optimal harvesting levels which favour the sustainability of fish species.

Keywords: Lake Victoria; optimal taxation; optimal harvesting; net economic revenues; Pontryagin's maximum principle.

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1. Introduction

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Overfishing of commercial fish species in Lake Victoria namely Nile perch, Nile tilapia and small pelagic silver fish is a serious problem due to rapid growth of industrialization and population. As per 2011 Regional Catch Assessment Survey (CAS) reports conducted by Lake Victoria Fisheries Organisation (LVFO) indicated that there as been a rapid decline of these fish species. The fish stocks for the named fish species has been decreasing beyond the minimum stock required to sustain regeneration. Given the economic importance of the fishery, management measures aiming at controling fishing efforts are needed for sustainability of the species.

According to [1] fishing pressures on Lake Victoria fisheries resource has been a major concern for many researchers. Possible control instruments for regulating harvesting efforts as were pointed out by [3] could be taxation, license fees, lease of property rights, seasonal harvesting, fishing period control, creating reserve zones and many more depending on the nature of the fishery. Fishery in Lake Victoria is of an open access, in which taxation method could apply as the efficiency method. Several authors have suggested that taxation is an effective control instrument which can be used by governments or fishing regulatory agencies to regulate the extent of fishing efforts. [10] proposed a mathematical model to study the growth and exploitation of a schooling fish species by imposing a tax on the catch to control the overexploitation of fish species [2] discussed a dynamical model for a single species fishery, which depends partially on logistically growing resource with functional response and taxation as a control instrument to protect fish population from overexploitation, [9] studied a fishery model containing predator fish and prey fish in which the predator was the commercial fish by including spawning periods and taxation [5] studied a dynamic model for fishery resource with reserve area and taxation [4] further analysed a non-linear mathematical model to study the dynamics of an inshore-offshore fishery under variable harvesting by considering taxation as the control instrument. However, from the above literature survey, it may be pointed out that no attempt has been made to study the optimal taxation policy of a three species fishery in which they interact in a predator-prey manner and all species being subjected to harvesting.

2. Mathematical model

The following are variables and parameters used in developing the model.

TABLE 1. Description of variables and parameters

Variables	Descriptions
r_1	Intrinsic growth rate of Nile perch
r_2	Intrinsic growth rate of Nile tilapia
r_3	Intrinsic growth rate of small pelagic silver fish
x	Stock biomass of Nile perch
y	Stock biomass of Nile tilapia
z	Stock biomass of small pelagic silver fish
α	Predation rate of Nile tilapia to Nile perch
β	Predation rate of Nile perch to Nile tilapia
γ	Predation rate of Nile perch to small pelagic silver fish
ψ	Predation rate of Nile tilapia to small pelagic silver fish
E_1	Fishing effort for Nile perch
E_2	Fishing effort for Nile tilapia
E_3	Fishing effort for small pelagic silver fish
q_1	Catchability coefficient of Nile perch
q_2	Catchability coefficient of Nile tilapia
q_3	Catchability coefficient of small pelagic silver fish
K_1	Carrying capacity of Nile perch
K_2	Carrying capacity of Nile tilapia
K_3	Carrying capacity of small pelagic silver fish

Consider the Nile perch population in a lake growing logistically, prey to both the Nile tilapia and small pelagic silver fish and also subjected to harvesting.The dynamics of Nile perch population is governed by:

$$\frac{dx}{dt} = r_1\left(1 - \frac{x}{K_1}\right)x - \alpha yx + \beta yx + \gamma xz - E_1 q_1 x. \tag{1}$$

Similarly the Nile tilapia population in a lake grows logistically, prey to both the Nile perch and small pelagic silver fish and also subjected to harvesting.

The dynamics of Nile tilapia population is governed by:

$$\frac{dy}{dt} = r_2\left(1-\frac{y}{K_2}\right)y+\alpha yx-\beta yx+\psi yz-E_2q_2y. \tag{2}$$

The small pelagic silver fish being prey to both the Nile perch and Nile tilapia, subjected to harvesting and growing logistically, then its popualation dynamics is governed by:

$$\frac{dz}{dt} = r_3\left(1-\frac{z}{K_3}\right)z-\gamma xz-\psi yz-E_3q_3z. \tag{3}$$

In order to keep sustainable fishing of the three fish species in this lake, we take some actions to fishing efforts through taxation and thus E_1, E_2 and E_3 are dynamic variables(i.e. time dependent). Let p_1, p_2 and p_3 be the fixed selling price per unit biomass of Nile perch, Nile tilapia and small pelagic silver fish respectively and let c_1, c_2 and c_3 be the fixed cost of harvesting per unit of effort for the Nile perch, Nile tilapia, and small pelagic silver fish respectively.

Therefore, the economic revenue for the three fish species will be:

$$R_1(t) = p_1q_1E_1x-c_1E_1, \tag{4a}$$

$$R_2(t) = p_2q_2E_2y-c_2E_2, \tag{4b}$$

$$R_3(t) = p_3q_3E_3z-c_3E_3. \tag{4c}$$

Let $\tau_1 > 0$, $\tau_2 > 0$ and $\tau_3 > 0$ be the imposed taxes per unit harvested of Nile perch, Nile tilapia and small pelagic silver fish biomasses respectively. The net ecomic revenue is obtained by introducing taxes to the fixed selling price per unit biomass of fish species. Hence (4) is modified to be:

$$R_{1_{net}}(t) = (p_1-\tau_1)q_1E_1x-c_1E_1, \tag{5a}$$

$$R_{2_{net}}(t) = (p_2-\tau_2)q_2E_2y-c_2E_2, \tag{5b}$$

$$R_{3_{net}}(t) = (p_3-\tau_3)q_3E_3z-c_3E_3. \tag{5c}$$

Using (1), (2), (3) and (5) we obtain the dynamics of the system governed by the following system of first order differential equation:

$$\frac{dx}{dt} = x(r_1 - q_1E_1 - a_1x - \rho y + \gamma z), \tag{6a}$$

$$\frac{dy}{dt} = y(r_2 - q_2E_2 + \rho x - a_2y + \psi z), \tag{6b}$$

$$\frac{dz}{dt} = z(r_3 - q_3E_3 - \gamma x - \psi y - a_3z) \tag{6c}$$

$$\frac{dE_1}{dt} = \phi_1[(p_1 - \tau_1)q_1E_1x - c_1E_1], \tag{6d}$$

$$\frac{dE_2}{dt} = \phi_2[(p_2 - \tau_2)q_2E_2y - c_2E_2], \tag{6e}$$

$$\frac{dE_3}{dt} = \phi_3[(p_3 - \tau_3)q_3E_3z - c_3E_3], \tag{6f}$$

$x(0) > 0$, $y(0) > 0$, $z(0) > 0$, $E_1(0) > 0$, $E_2(0) > 0$, $E_3(0) > 0$, where $a_i = \frac{r_i}{K_i} > 0$ for $i = 1,2,3$ and ϕ_j for $j = 1,2,3$ are adjustment coefficients (stiffness parameters).

2.1. Equilibrium points of the model

The system in (6) has the following equilibrium points:

$$\bar{P}_1 \begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{r_1}{q_1} \\ \frac{r_2}{q_2} \\ \frac{r_3}{q_3} \end{pmatrix}, \qquad \bar{P}_2 \begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{P}_3 \begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \bar{P}_4 \begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{P}_5\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1+\gamma z^*) \\ \frac{1}{q_2}(r_2+\psi z^*) \\ \frac{1}{q_3}(r_3-a_3 z^*) \end{pmatrix}, \bar{P}_6\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ 0 \\ 0 \\ \frac{1}{q_1}(r_1-a_1 x^*) \\ \frac{1}{q_2}(r_2+\rho x^*) \\ \frac{1}{q_3}(r_3-\gamma x^*) \end{pmatrix}$$

$$\bar{P}_7\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ 0 \\ \frac{1}{q_1}(r_1-\rho y^*) \\ \frac{1}{q_2}(r_2-a_2 y^*) \\ \frac{1}{q_3}(r_3-\psi y^*) \end{pmatrix}, \bar{P}_8\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-\rho y^*+\gamma z^*) \\ \frac{1}{q_2}(r_2-a_2 y^*+\psi z^*) \\ \frac{1}{q_3}(r_3-\psi y^*-a_3 z^*) \end{pmatrix}$$

$$\bar{P}_9\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ 0 \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-a_1 x^*+\gamma z^*) \\ \frac{1}{q_2}(r_2+\rho x^*+\gamma z^*) \\ \frac{1}{q_3}(r_3-\gamma x^*-a_3 z^*) \end{pmatrix}, \bar{P}_{10}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ 0 \\ \frac{1}{q_1}(r_1-a_1 x^*-\rho y^*) \\ \frac{1}{q_2}(r_2+\rho x^*-a_2 y^*) \\ \frac{1}{q_3}(r_3-\gamma x^*-\psi y^*) \end{pmatrix}$$

$$\bar{P}_{11}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \bar{P}_{12}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ 0 \\ 0 \\ \frac{1}{q_3}(r_3-\gamma x^*-\psi y^*-a_3 z^*) \end{pmatrix}$$

$$\bar{P}_{13}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ 0 \\ \frac{1}{q_2}(r_2+\rho x^*-a_2y^*+\psi z^*) \\ 0 \end{pmatrix}, \bar{P}_{14}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-a_1x^*-\rho y^*+\gamma z^*) \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{P}_{15}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ 0 \\ \frac{1}{q_2}(r_2+\rho x^*-a_2y^*+\psi z^*) \\ \frac{1}{q_3}(r_3-\gamma x^*-\psi y^*-a_3z^*) \end{pmatrix}, \bar{P}_{16}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-a_1x^*-\rho y^*+\gamma z^*) \\ 0 \\ \frac{1}{q_3}(r_3-\gamma x^*-\psi y^*-a_3z^*) \end{pmatrix},$$

$$\bar{P}_{17}\begin{pmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{pmatrix} = \begin{pmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-a_1x^*-\rho y^*+\gamma z^*) \\ \frac{1}{q_2}(r_2+\rho x^*-a_2y^*+\psi z^*) \\ 0 \end{pmatrix} \text{ and, } \bar{P}_{18}\begin{bmatrix} x^* \\ y^* \\ z^* \\ E_1^* \\ E_2^* \\ E_3^* \end{bmatrix} = \begin{bmatrix} \frac{c_1}{q_1(p_1-\tau_1)} \\ \frac{c_2}{q_2(p_2-\tau_2)} \\ \frac{c_3}{q_3(p_3-\tau_3)} \\ \frac{1}{q_1}(r_1-a_1x^*-\rho y^*+\gamma z^*) \\ \frac{1}{q_2}(r_2+\rho x^*-a_2y^*+\psi z^*) \\ \frac{1}{q_3}(r_3-\gamma x^*-\psi y^*-a_3z^*) \end{bmatrix}.$$

2.2. Stability analysis of the co-existence equilibrium point

2.2.1. Local stability

The local stability of the the co-existence equilibrium point $\bar{P}_{18}$ is investigated using the trace-determinant criteria.That is, an equilibrium point is locally asymptotically stable if the Jacobian matrix evaluated at that point has a negative trace and positive determinat.

The Jacobian matrix of the system 6 evaluated at $\bar{P_{18}}$ is a 6×6 matrix given by:

$$J(\bar{P_{18}}) = \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} & 0 & 0 \\ n_{21} & n_{22} & n_{23} & 0 & n_{25} & 0 \\ n_{31} & n_{32} & n_{33} & 0 & 0 & n_{36} \\ n_{41} & 0 & 0 & 0 & 0 & 0 \\ 0 & n_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{63} & 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

where

$$\begin{aligned}
n_{11} &= \frac{-a_1 c_1}{q_1(p_1 - \tau_1)}, \\
n_{12} &= \frac{-\rho c_1}{q_1(p_1 - \tau_1)}, \\
n_{13} &= \frac{\gamma c_1}{q_1(p_1 - \tau_1)}, \\
n_{14} &= \frac{-c_1}{(p_1 - \tau_1)}, \\
n_{21} &= \frac{\rho c_2}{q_2(p_2 - \tau_2)}, \\
n_{22} &= \frac{-a_2 c_2}{q_2(p_2 - \tau_2)}, \\
n_{23} &= \frac{\psi c_2}{q_2(p_2 - \tau_2)}, \\
n_{25} &= \frac{-c_2}{(p_2 - \tau_2)}, \\
n_{31} &= \frac{-\gamma c_3}{q_3(p_3 - \tau_3)}, \\
n_{32} &= \frac{-\psi c_3}{q_3(p_3 - \tau_3)}, \\
n_{33} &= \frac{-a_3 c_3}{q_3(p_3 - \tau_3)}, \\
n_{36} &= \frac{-c_3}{(p_3 - \tau_3)}, \\
n_{41} &= \phi_1[(p_1 - \tau_1) q_1 E_1^*], \\
n_{52} &= \phi_2[(p_2 - \tau_2) q_2 E_2^*], \\
n_{63} &= \phi_3[(p_3 - \tau_3) q_2 E_3^*],
\end{aligned}$$

$$\begin{aligned}
\text{trace}[\text{J}(\bar{\text{P}_{18}})] &= n_{11}+n_{22}+n_{33}\\
&= \frac{-a_1c_1}{q_1(p_1-\tau_1)}-\frac{a_2c_2}{q_2(p_2-\tau_2)}-\frac{a_3c_3}{q_3(p_3-\tau_3)}\\
&= -\left[\frac{a_1c_1}{q_1(p_1-\tau_1)}+\frac{a_2c_2}{q_2(p_2-\tau_2)}+\frac{a_3c_3}{q_3(p_3-\tau_3)}\right]<0
\end{aligned}$$

and

$$\det[\text{J}(\bar{\text{P}_{18}})] = -n_{14}\times n_{41}\times -n_{52}\times n_{63}\times n_{25}\times n_{36}>0.$$

Hence, the co-existence equilibrium point P_{18} is locally asymptotically stable.

2.2.2. Global stability

Global stability was analysed through construction of a suitable Lyapunov function. Consider the Lyapunov function

$$\begin{aligned}
V(x,y,z,E_1,E_2,E_3) &= m_1[x-x^*-x^*\ln(\frac{x}{x^*})]+m_2[y-y^*-y^*\ln(\frac{y}{y^*})]\\
&+ m_3[z-z^*-z^*\ln(\frac{z}{z^*})]+m_4[E_1-E_1^*-E_1^*\ln(\frac{E_1}{E_1^*})]\\
&+ m_5[E_2-E_2^*-E_2^*\ln(\frac{E_2}{E_2^*})]+m_6[E_3-E_3^*-E_3^*\ln(\frac{E_3}{E_3^*})],
\end{aligned}$$

where $m_i>0$ for $i=1,2,...,6$. The time derivatives of V is given by:

$$\begin{aligned}
\frac{dV}{dt} &= m_1\left(1-\frac{x^*}{x}\right)\frac{dx}{dt}+m_2\left(1-\frac{y^*}{y}\right)\frac{dy}{dt}+m_3\left(1-\frac{z^*}{z}\right)\frac{dz}{dt}\\
&+ m_4\left(1-\frac{E_1^*}{E_1}\right)\frac{dE_1}{dt}+m_5\left(1-\frac{E_2^*}{E_2}\right)\frac{dE_2}{dt}+m_6\left(1-\frac{E_3^*}{E_3}\right)\frac{dE_3}{dt}.
\end{aligned}$$

Let

$$\frac{dV}{dt}=m_1G+m_2H+m_3I+m_4K+m_5L+m_6R, \tag{8}$$

where,

$$G=\left(1-\frac{x^*}{x}\right)\frac{dx}{dt}=(x-x^*)[-q_1(E_1-E_1^*)-a_1(x-x^*)-\rho(y-y^*)+\gamma(z-z^*)], \tag{9}$$

$$H=\left(1-\frac{y^*}{y}\right)\frac{dy}{dt}=(y-y^*)[-q_2(E_2-E_2^*)+\rho(x-x^*)-a_2(y-y^*)+\psi(z-z^*)], \tag{10}$$

$$I=\left(1-\frac{z^*}{z}\right)\frac{dz}{dt}=(z-z^*)[-q_3(E_3-E_3^*)-\gamma(x-x^*)-\psi(y-y^*)+a_3(z-z^*)], \tag{11}$$

$$K = \left(1 - \frac{E_1^*}{E_1}\right)\frac{dE_1}{dt} = \phi_1(E_1 - E_1^*)[q_1(p_1 - \tau_1)(x - x^*)], \tag{12}$$

$$L = \left(1 - \frac{E_2^*}{E_2}\right)\frac{dE_2}{dt} = \phi_2(E_2 - E_2^*)[q_2(p_2 - \tau_2)(y - y^*)], \tag{13}$$

and,

$$R = \left(1 - \frac{E_3^*}{E_3}\right)\frac{dE_3}{dt} = \phi_3(E_3 - E_3^*)[q_3(p_3 - \tau_3)(z - z^*)]. \tag{14}$$

The substitution of equations (9), (10), (11), (12), (13) and (14) into equation (8) and making necessary simplifications gives the following:

$$\begin{aligned}
\frac{dV}{dt} = &-[m_1a_1(x-x^*)^2 + m_2a_2(y-y^*)^2 + m_3a_3(z-z^*)^2 + q_1m_1(x-x^*)(E_1-E_1^*)\\
&+ q_2m_2(y-y^*)(E_2-E_2^*) + q_3m_3(z-z^*)(E_3-E_3^*)]\\
&+ [\rho(x-x^*)(y-y^*)(m_2-m_1) + \gamma(x-x^*)(z-z^*)(m_1-m_3)\\
&+ \psi(y-y^*)(z-z^*)(m_2-m_3)] + [m_4\phi_1q_1(x-x^*)(E_1-E_1^*)(p_1-\tau_1)\\
&+ m_5\phi_2q_2(y-y^*)(E_2-E_2^*)(p_2-\tau_2) + m_6\phi_3q_3(z-z^*)(E_3-E_3^*)(p_3-\tau_3)].
\end{aligned} \tag{15}$$

From (15) we have

i) $\frac{dV}{dt} = 0 \quad \forall (x,y,z,E_1,E_2,E_3) = (x^*,y^*,z^*,E_1^*,E_2^*,E_3^*)$.

ii) If we choose $m_1 = m_2 = m_3$ such that

$$\begin{aligned}
&[m_1a_1(x-x^*)^2 + m_2a_2(y-y^*)^2 + m_3a_3(z-z^*)^2 + q_1m_1(x-x^*)(E_1-E_1^*)\\
&\quad + q_2m_2(y-y^*)(E_2-E_2^*) + q_3m_3(z-z^*)(E_3-E_3^*)] >\\
&\quad [m_4\phi_1q_1(x-x^*)(E_1-E_1^*)(p_1-\tau_1) + m_5\phi_2q_2(y-y^*)(E_2-E_2^*)(p_2-\tau_2)\\
&\quad + m_6\phi_3q_3(z-z^*)(E_3-E_3^*)(p_3-\tau_3)],
\end{aligned} \tag{16}$$

then, $\frac{dV}{dt} < 0 \quad \forall (x,y,z,E_1,E_2,E_3) \neq (x^*,y^*,z^*,E_1^*,E_2^*,E_3^*)$ for which (16) holds. Therefore, the co-existence equilibrium point $\bar{P}_{18}$ is globally asymptotically stable.

3. Optimal harvesting policy

In this section we investigate the optimal harvesting policy for the dynamics of the system in (6) in order to maximize the total discounted net revenue using taxation as a control instrument on the harvested three fish species. The present value J of a continous time-stream of revenues is given by:

$$\mathrm{J} = \int_0^\infty \mathrm{e}^{-\delta \mathrm{t}} \left[(p_1 q_1 E_1 x - c_1 E_1) + (p_2 q_2 E_2 y - c_2 E_2) + (p_3 q_3 E_3 z - c_3 E_3) \right] \mathrm{dt}, \tag{17}$$

where δ is the instanteneous rate of annual discount.Thus our objective is to maximize J subject to the state equations in (6) and to the control constraints

$$\tau_{\mathrm{min_i}} < \tau < \tau_{\mathrm{max_i}} \text{ for i} = 1,2,3. \tag{18}$$

To find the optimal level of equilibrium, we use the Pontryagin's maximum principle. The associated Hamiltonian function is given by:

$$\begin{aligned}
\mathrm{H} &= \mathrm{e}^{-\delta t} \left[(p_1 q_1 E_1 x - c_1 E_1) + (p_2 q_2 E_2 y - c_2 E_2) + (p_3 q_3 E_3 z - c_3 E_3) \right] \\
&+ \lambda_1 \left[x(r_1 - q_1 E_1 - a_1 x - \rho y + \gamma z) \right] \\
&+ \lambda_2 \left[y(r_2 - q_2 E_2 + \rho x - a_2 y + \psi z) \right] \\
&+ \lambda_3 \left[z(r_3 - q_3 E_3 - \gamma x - \psi y - a_3 z) \right] \\
&+ \lambda_4 \left[\phi_1 (p_1 - \tau_1) q_1 E_1 x - c_1 E_1 \right] \\
&+ \lambda_5 \left[\phi_2 (p_2 - \tau_2) q_2 E_2 y - c_2 E_2 \right] \\
&+ \lambda_6 \left[\phi_3 (p_3 - \tau_3) q_3 E_3 z - c_3 E_3 \right],
\end{aligned} \tag{19}$$

where λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6 are adjoint variables interms of time(t).Hamiltonian H must be maximized for $\tau(\mathrm{t}) \in \left[\tau_{\mathrm{min_i}}, \tau_{\mathrm{max_i}}\right]$ where $\mathrm{i} = 1,2,3$. Assuming that the control constraints are not binding (that is, the optimal solution does not occur at $\tau(\mathrm{t}) = \tau_{\mathrm{min_i}}$ or $\tau_{\mathrm{max_i}}$ for $\mathrm{i} = 1,2,3$). Hence we have singular control given by (20) and (21) below:

$$\frac{\partial \mathrm{H}}{\partial \tau_1} = 0, \ \frac{\partial \mathrm{H}}{\partial \tau_2} = 0 \quad \text{and} \quad \frac{\partial \mathrm{H}}{\partial \tau_3} = 0, \tag{20}$$

$$\frac{\partial \mathrm{H}}{\partial \mathrm{E}_1} = 0, \ \frac{\partial \mathrm{H}}{\partial \mathrm{E}_2} = 0 \quad \text{and} \quad \frac{\partial \mathrm{H}}{\partial \mathrm{E}_3} = 0. \tag{21}$$

Applying (20), we obtain

$$\lambda_4 = \lambda_5 = \lambda_6 = 0. \tag{22}$$

Applying (21), we obtain (23)

$$\lambda_1 = \lambda_1(\mathrm{t}) = \mathrm{e}^{-\delta \mathrm{t}}\left(\mathrm{p}_1 - \frac{\mathrm{c}_1}{\mathrm{q}_1 x}\right), \tag{23a}$$

$$\lambda_2 = \lambda_2(\mathrm{t}) = \mathrm{e}^{-\delta \mathrm{t}}\left(\mathrm{p}_2 - \frac{\mathrm{c}_2}{\mathrm{q}_2 y}\right), \tag{23b}$$

$$\lambda_3 = \lambda_3(\mathrm{t}) = \mathrm{e}^{-\delta \mathrm{t}}\left(\mathrm{p}_3 - \frac{\mathrm{c}_3}{\mathrm{q}_3 z}\right). \tag{23c}$$

Again, by Pontryagin's maximum principle we have

$$\frac{\mathrm{d}\lambda_1}{\mathrm{dt}} = -\frac{\partial \mathrm{H}}{\partial x}, \tag{24a}$$

$$\frac{\mathrm{d}\lambda_2}{\mathrm{dt}} = -\frac{\partial \mathrm{H}}{\partial y}, \tag{24b}$$

$$\frac{\mathrm{d}\lambda_3}{\mathrm{dt}} = -\frac{\partial \mathrm{H}}{\partial z}. \tag{24c}$$

Considering (24a), we obtain

$$\frac{\mathrm{d}\lambda_1}{\mathrm{dt}} = -[\mathrm{e}^{-\delta t} p_1 q_1 E_1 + \lambda_1(r_1 - q_1 E_1 - 2a_1 x - \rho y + \gamma z) + \lambda_2 \rho y - \lambda_3 \gamma z] \tag{25}$$

Substituting (23b) and (23c) into (25) and making necessary simplifications, we obtain

$$\frac{\mathrm{d}\lambda_1}{\mathrm{dt}} - \mathrm{A}_1\lambda_1 = -\mathrm{A}_2\mathrm{e}^{-\delta \mathrm{t}}, \tag{26}$$

where

$$\mathrm{A}_1 = -(r_1 - q_1 E_1 - 2a_1 x - \rho y + \gamma z), \tag{27a}$$

$$\mathrm{A}_2 = p_1 q_1 E_1 + \rho y\left(p_2 - \frac{c_2}{q_2 y}\right) - \gamma z\left(p_3 - \frac{c_3}{q_3 z}\right). \tag{27b}$$

Employing an integrating factor $\mathrm{I.F} = \mathrm{e}^{-\mathrm{A}_1 t}$ to solve (26) resulted into

$$\lambda_1 = \lambda_1(t) = \frac{\mathrm{A}_2}{\mathrm{A}_1 + \delta}\mathrm{e}^{-\delta \mathrm{t}} + \mathrm{T}_0\mathrm{e}^{\mathrm{A}_1 \mathrm{t}}, \tag{28}$$

where T_0 is a constant of integration. Let $\mu_0(t) = \lambda_1 \mathrm{e}^{\delta t} = \left(p_1 - \frac{c_1}{q_1 x}\right)$, the shadow price per unit biomass of harvested Nile perch. When $t \to \infty$ then $\mu_0(t)$ is bounded if and only if $\mathrm{T}_0 = 0$ Hence, (28) can be re-written as

$$\lambda_1 = \lambda_1(t) = \frac{\mathrm{A}_2}{\mathrm{A}_1 + \delta}\mathrm{e}^{-\delta \mathrm{t}}. \tag{29}$$

Using (24b), we obtain the following equation

$$\frac{\mathrm{d}\lambda_2}{\mathrm{dt}} = -\mathrm{e}^{-\delta \mathrm{t}} p_2 q_2 E_2 - \lambda_2 (r_2 - q_2 E_2 + \rho x - 2a_2 y + \psi z) + \lambda_1 \rho x + \lambda_3 \psi z. \tag{30}$$

Substituting (23a) and (23c) into (30) and making necessary simplifications we obtain

$$\frac{\mathrm{d}\lambda_2}{\mathrm{dt}} - \mathrm{B}_1 \lambda_2 = -\mathrm{B}_2 \mathrm{e}^{-\delta \mathrm{t}}, \tag{31}$$

where

$$\mathrm{B}_1 = -(r_2 - q_2 E_2 + \rho x - 2a_2 y + \psi z), \tag{32a}$$

$$\mathrm{B}_2 = p_2 q_2 E_2 - \rho x \left(p_1 - \frac{c_1}{q_1 x} \right) - \psi z \left(p_3 - \frac{c_3}{q_3 z} \right). \tag{32b}$$

Applying an integrating factor $\mathrm{I.F} = \mathrm{e}^{-\mathrm{B}_1 \mathrm{t}}$ on attempt to solve (31) resulted into the following:

$$\lambda_2 = \lambda_2(t) = \frac{\mathrm{B}_2}{\mathrm{B}_1 + \delta} \mathrm{e}^{-\delta \mathrm{t}} + \mathrm{T}_1 \mathrm{e}^{\mathrm{B}_1 \mathrm{t}}, \tag{33}$$

where T_1 is a constant of integration. Let $\mu_1(t) = \lambda_2 \mathrm{e}^{\delta t} = \left(p_2 - \frac{c_2}{q_2 y} \right)$, the shadow price per unit biomass of harvested Nile tilapia. When $t \to \infty$ then $\mu_1(t)$ is bounded if and only if $\mathrm{T}_1 = 0$ Hence, (33) can be re-written as

$$\lambda_2 = \lambda_2(t) = \frac{\mathrm{B}_2}{\mathrm{B}_1 + \delta} \mathrm{e}^{-\delta \mathrm{t}}. \tag{34}$$

Using (24c), we obtain

$$\frac{\mathrm{d}\lambda_3}{\mathrm{dt}} = -\mathrm{e}^{-\delta \mathrm{t}} p_3 q_3 E_3 - \lambda_3 (r_3 - q_3 E_3 - \gamma x - \psi y - 2a_3 z) - \lambda_1 \gamma x - \lambda_2 \psi y. \tag{35}$$

Substituting (23a) and (23b) into (35) and making necessary simplifications we obtain

$$\frac{\mathrm{d}\lambda_3}{\mathrm{dt}} - \mathrm{D}_1 \lambda_3 = -\mathrm{D}_2 \mathrm{e}^{-\delta \mathrm{t}}, \tag{36}$$

where

$$\mathrm{D}_1 = -(r_3 - q_3 E_3 - \gamma x - \psi y - 2a_3 z), \tag{37a}$$

$$\mathrm{D}_2 = p_3 q_3 E_3 - \gamma x \left(p_1 - \frac{c_1}{q_1 x} \right) - \psi y \left(p_2 - \frac{c_2}{q_2 y} \right). \tag{37b}$$

Applying an integrating factor $\mathrm{I.F} = \mathrm{e}^{-\mathrm{D}_1 \mathrm{t}}$ on attempt to solve (36) resulted into the following

$$\lambda_3 = \lambda_3(t) = \frac{\mathrm{D}_2}{\mathrm{D}_1 + \delta} \mathrm{e}^{-\delta \mathrm{t}} + \mathrm{T}_2 \mathrm{e}^{\mathrm{D}_1 \mathrm{t}}, \tag{38}$$

where T_2 is a constant of integration. Let $\mu_2(t) = \lambda_3 e^{\delta t} = \left(p_3 - \frac{c_3}{q_3 z}\right)$, the shadow price per unit biomass of harvested small pelagic silver fish. When $t \to \infty$ then $\mu_2(t)$ is bounded if and only if $T_2 = 0$ Hence, (38) can be re-written as

$$\lambda_3 = \lambda_3(t) = \frac{D_2}{D_1 + \delta} e^{-\delta t}. \tag{39}$$

Equating (23a) with (29) resulted into the following equation

$$\frac{p_1 q_1 x - c_1}{q_1 x} = \frac{A_2}{A_1 + \delta}. \tag{40}$$

Upon substituting (27) into (40) and making algebraic simplifications resulted into the following

$$\begin{aligned} E_{1\delta} &= \frac{1}{q_1 c_1}[(c_1 - p_1 q_1 x_\delta)(r_1 - 2a_1 x_\delta - \rho y_\delta + \gamma z_\delta) + \delta(p_1 q_1 x_\delta - c_1) \\ &- q_1 x_\delta\{\rho y_\delta(p_2 - \frac{c_2}{q_2 y_\delta}) - \gamma z_\delta(p_3 - \frac{c_3}{q_3 z_\delta})\}]. \end{aligned} \tag{41}$$

Equating(23b) with (34) resulted into the following equation

$$\frac{B_1 + \delta}{B_2} = \frac{q_2 y}{p_2 q_2 y - c_2}. \tag{42}$$

Upon substituting (32) into (42) and making algebraic simplifications resulted into the following

$$\begin{aligned} E_{2\delta} &= \frac{1}{q_2 c_2}[(c_2 - p_2 q_2 y_\delta)(r_2 + \rho x_\delta - 2a_2 y_\delta + \psi z_\delta) + \delta(p_2 q_2 y_\delta - c_2) \\ &- q_2 y_\delta\{-\rho x_\delta(p_1 - \frac{c_1}{q_1 x_\delta}) - \psi z_\delta(p_3 - \frac{c_3}{q_3 z_\delta})\}]. \end{aligned} \tag{43}$$

Equating (23c) with (39) resulted into the following equation

$$\frac{D_1 + \delta}{D_2} = \frac{q_3 z}{p_3 q_3 z - c_3}. \tag{44}$$

The substitution of (37) into (44) resulted into the following equation

$$\begin{aligned} E_{3\delta} &= \frac{1}{q_3 c_3}[(c_3 - p_3 q_3 z_\delta)(r_3 - \gamma x_\delta - \psi y_\delta - 2a_3 z_\delta) + \delta(p_3 q_3 z_\delta - c_3) \\ &- q_3 z_\delta\{-\gamma x_\delta(p_1 - \frac{c_1}{q_1 x_\delta}) - \psi y_\delta(p_2 - \frac{c_2}{q_2 y_\delta})\}]. \end{aligned} \tag{45}$$

At the optimal level, equations (6a), (6b) and (6c) became

$$r_1 - q_1 E_{1\delta} - a_1 x_\delta - \rho y_\delta + \gamma z_\delta = 0, \tag{46a}$$

$$r_2 - q_2 E_{2\delta} + \rho x_\delta - a_2 y_\delta + \psi z_\delta = 0, \tag{46b}$$

$$r_3 - q_3 E_{3\delta} - \gamma x_\delta - \psi y_\delta - a_3 z_\delta = 0. \tag{46c}$$

Therefore the optimal values x_δ, y_δ, z_δ, $E_{1\delta}$, $E_{2\delta}$ and $E_{3\delta}$ are computed using (41), (43), (45) and (46) where as the optimal taxations, $\tau_{1\delta}$, $\tau_{2\delta}$ and $\tau_{3\delta}$ are computed using (47) below:

$$\tau_{1\delta} = p_1 - \frac{c_1}{q_1 x_\delta}, \tag{47a}$$

$$\tau_{2\delta} = p_2 - \frac{c_2}{q_2 y_\delta}, \tag{47b}$$

$$\tau_{3\delta} = p_3 - \frac{c_3}{q_3 z_\delta}. \tag{47c}$$

4. Numerical simulation

The following paratemeters summarized in Table 2 gives the equilibrium point at

$$(x^*, y^*, z^*, E_1^*, E_2^*, E_3^*) = (20, 20, 20, 20, 30, 10).$$

Our main task is to determine the optimal solutions at this particular equilibrium point.

TABLE 2. Parameters for the equilibrium point $(x^*, y^*, z^*, E_1^*, E_2^*, E_3^*) = (20, 20, 20, 20, 30, 10)$.

$\rho = 0.04$	$\gamma = 0.005$	$\psi = 0.005$
$a_1 = 0.0125$	$a_2 = 0.07$	$a_3 = 0.01$
$r_1 = 0.99$	$r_2 = 0.80$	$r_3 = 0.70$
$q_1 = 0.002$	$q_2 = 0.01$	$q_3 = 0.03$
$c_1 = 100$	$c_2 = 800$	$c_3 = 780$
$\phi_1 = 0.10$	$\phi_2 = 0.10$	$\phi_3 = 0.10$
$p_1 = 2800$	$p_2 = 4500$	$p_3 = 1500$
$\tau_1 = 300$	$\tau_2 = 500$	$\tau_3 = 200$

With the choice of $\delta = 5$ and substitutions of parameters in Table 2 into equations (41),(43),(45) and (46) resulted into equations (48) and (49) below:

$$-0.0014x_\delta^2 - 0.27346x_\delta + 0.00136x_\delta y_\delta + 0.00013x_\delta z_\delta + 5 = 0, \tag{48a}$$

$$-0.007875y_\delta^2 - 0.139625y_\delta + 0.00085x_\delta y_\delta + 0.0001875y_\delta z_\delta + 5 = 0, \tag{48b}$$

$$\begin{aligned} &-0.001153846z_\delta^2 - 0.213076919z_\delta - 0.0008269230627x_\delta z_\delta \\ &\qquad - 0.001153846134y_\delta z_\delta + 4.999999926 = 0, \end{aligned} \tag{48c}$$

$$-0.0125x_\delta - 0.04y_\delta + 0.005z_\delta - 0.002\mathrm{E}_{1\delta} + 0.99 = 0, \tag{49a}$$

$$0.04x_\delta - 0.07y_\delta + 0.005z_\delta - 0.01\mathrm{E}_{2\delta} + 0.80 = 0, \tag{49b}$$

$$-0.005x_\delta - 0.005y_\delta - 0.01z_\delta - 0.03\mathrm{E}_{3\delta} + 0.70 = 0. \tag{49c}$$

Solving equations (48) and (49), and utilizing (47) we obtain the optimal solutions as summarized in Table 3 below:

TABLE 3. Optimal solutions

Optimal values
$x_\delta = 18.42007701$
$y_\delta = 18.68227930$
$z_\delta = 18.44046469$
$\mathrm{E}_{1\delta} = 52.33009441$
$\mathrm{E}_{2\delta} = 32.12458529$
$\mathrm{E}_{3\delta} = 11.00278572$
$\tau_{1\delta} = 85.57052325$
$\tau_{2\delta} = 217.8672519$
$\tau_{3\delta} = 90.05722268$

5. Discussion and Conclusions

We conclude that τ_1, τ_2 and τ_3 are important parameters which governs the dynamics of the system in (6). The behaviour of (x with E_1), (y with E_2) and (z with E_3) with respect to time t for different values of τ_1, τ_2 and τ_3 are shown in figure 2, 4 and 6 respectively. From these figure, we observe that the population densities for the Nile perch (x), Nile tilapia (y) and small pelagic silver fish (z) increased as the tax rates increased, where as the densities (magnitudes) of harvesting efforts (E_1, E_2 and E_3) decreased as the tax rates increased. Moreover at the optimal tax rates, the population of fish species and their corresponding harvesting efforts settled down at their respective optimal level (as illustrated in figure 3, 5 and 7). We also observed that as the harvesting efforts increased, the population of fish species decreased (as illustrated in figure 8, 9 and 10) and that applying harvesting efforts above the optimal harvesting efforts levels leads to overfishing of fish species. Suitable tax policies are proper measures to manage fishery, however we noted that implementations of tax policies has to be done with great care inorder to attain the bioeconomic equilibrium. Low taxes rates will provide higher net revenues to fishers [refer (5)] and hence encouraging higher harvesting efforts-this may lead to extinction of fish species, where as higher taxes rates will results into lower net revenues to fishers which lead to extrem reduction of fishing efforts and hence abundant of fish species population-this does not favour the ecosystem. Hence the bioeconomic equilibrium (the balance) is attained at the optimal taxes rates and at the optimal harvesting efforts levels.

Conflict of Interests

The authors declare that there is no conflict of interests.

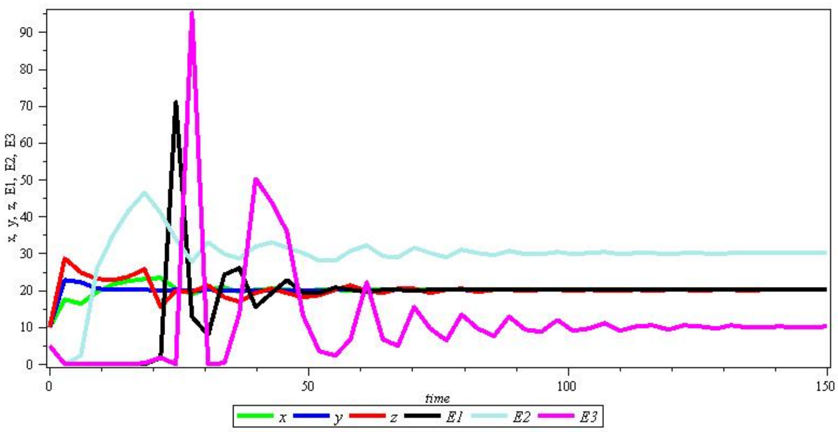
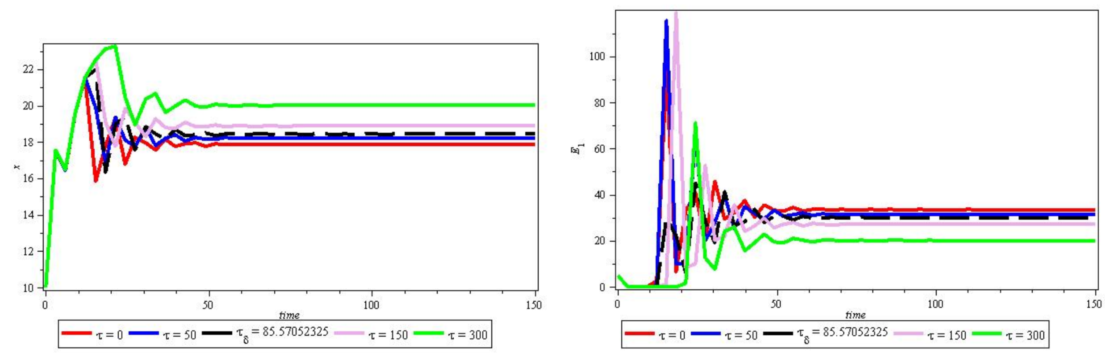


FIGURE 1. Plot of x, y, z, E_1, E_2 and E_3 verses time(t) for the parameters in Table 2



(a) Variation of **x** population against time for different tax levels of τ_1

(b) Variation of harvesting effort $\mathbf{E}_1$ against time for different tax levels of τ_1

FIGURE 2. Effects of taxation rates to the population **x(t)** and the harvesting effort $E_1(t)$

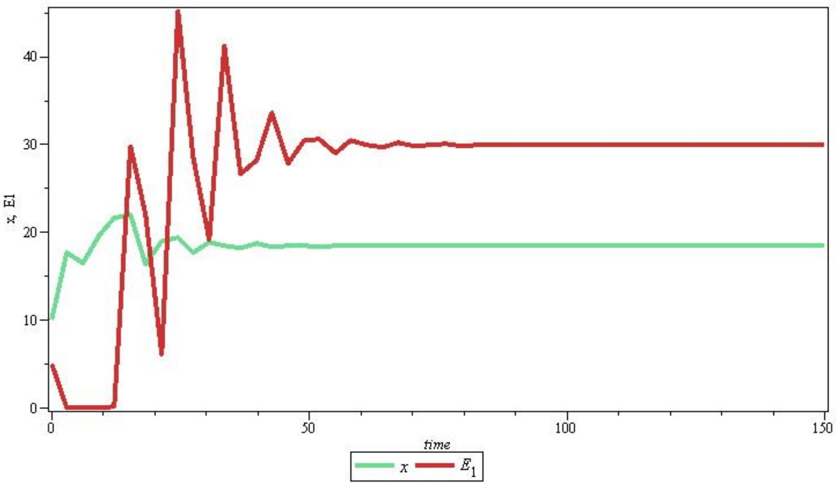
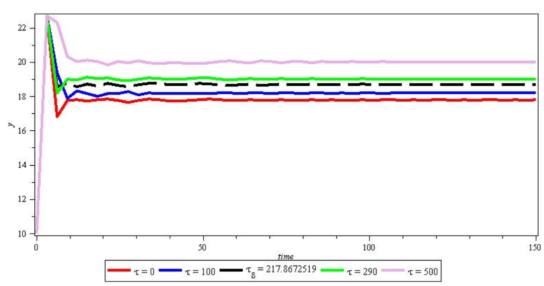
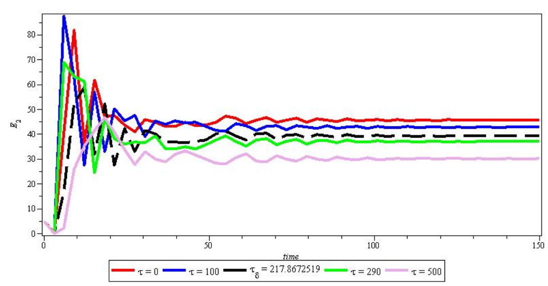


FIGURE 3. The trend of population $x(t)$ and $E_1(t)$ at the optimal tax $\tau_{1\delta} = 85.57052325$



(a) Variation of $\mathbf{y}$ population against time for different tax levels of τ_2



(b) Variation of harvesting effort $\mathbf{E}_2$ against time for different tax levels of τ_2

FIGURE 4. Effects of taxation rates to the population $\mathbf{y}(t)$ and the harvesting effort $\mathbf{E}_2(t)$

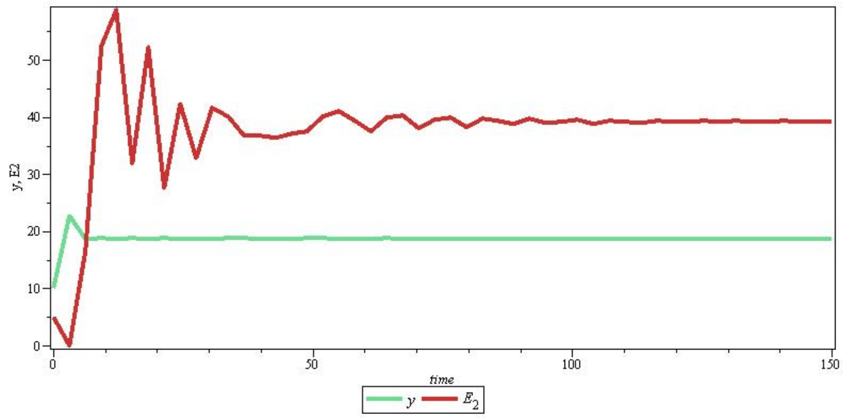
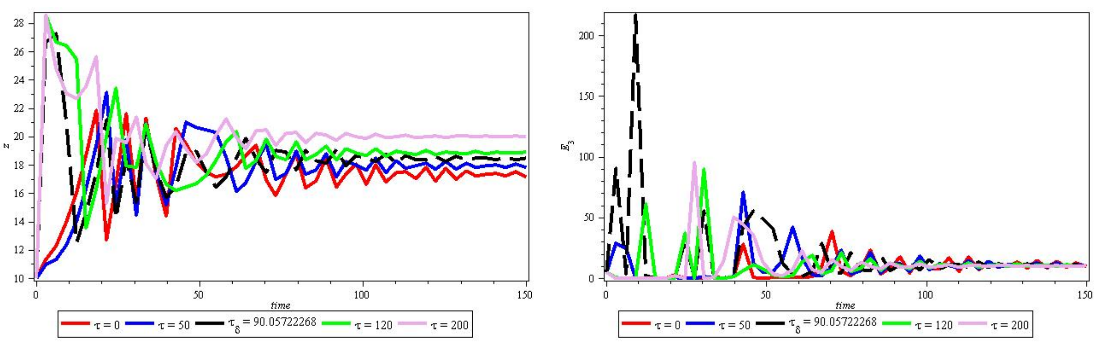


FIGURE 5. The trend of population $y(t)$ and $E_2(t)$ at the optimal tax $\tau_{2\delta} = 217.8672519$



(a) Variation of **z** population against time for different tax levels of τ_3

(b) Variation of harvesting effort $\mathbf{E}_3$ against time for different tax levels of τ_3

FIGURE 6. Effects of taxation rates to the population $\mathbf{z}(t)$ and the harvesting effort $\mathbf{E}_3(t)$

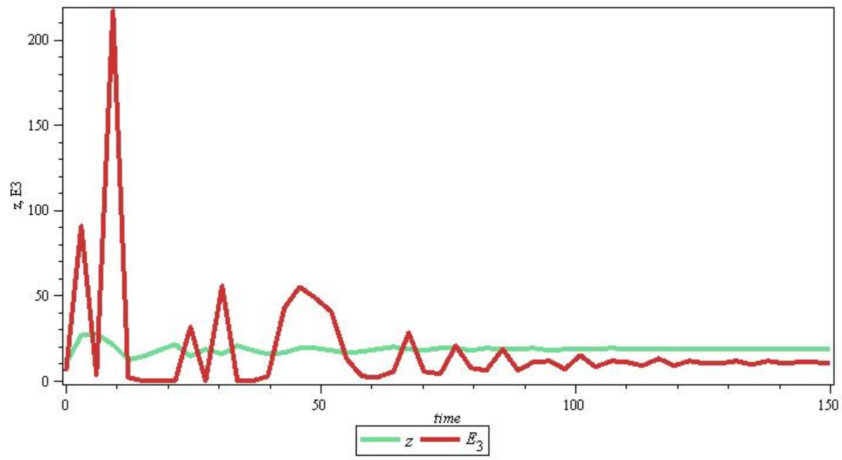


FIGURE 7. The trend of population $z(t)$ and $E_3(t)$ at the optimal tax $\tau_{3\delta} = 90.05722268$

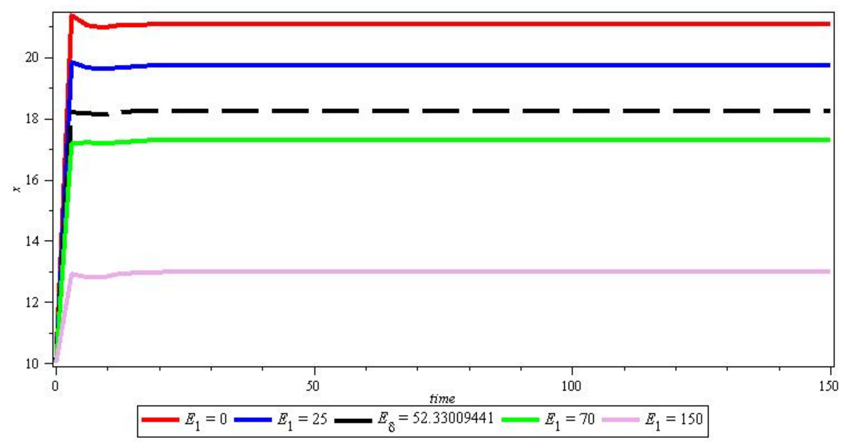


FIGURE 8. Variation of **x** population against time for different values of harvesting effort $\mathbf{E}_1$

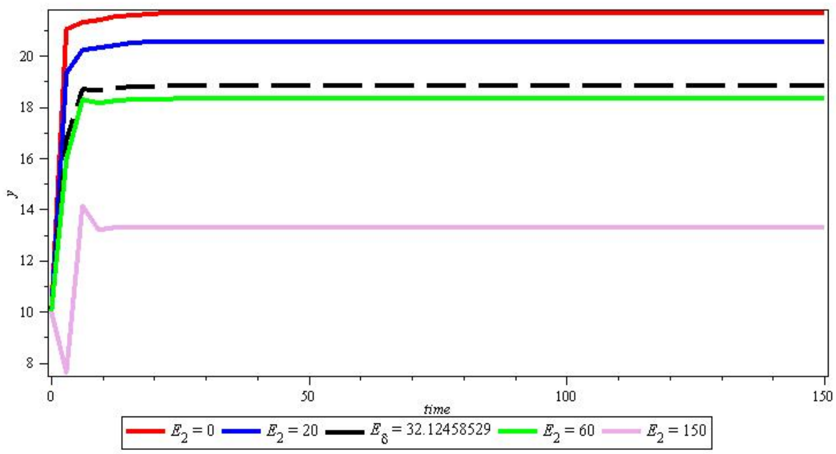


FIGURE 9. Variation of **y** population against time for different values of harvesting effort $\mathbf{E}_2$

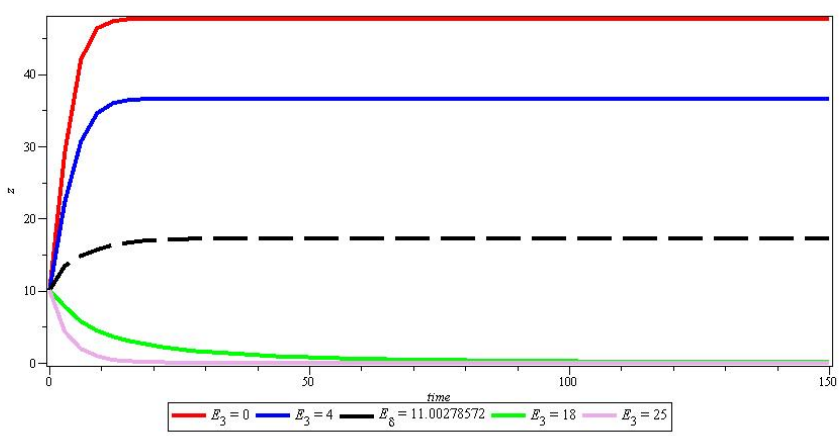


FIGURE 10. Variation of **z** population against time for different values of fishing effort $\mathbf{E}_3$

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