Available online at http://scik.org
Commun. Math. Biol. Neurosci. 2015, 2015:18
ISSN: 2052-2541

PERMANENCE OF A LOTKA-VOLTERRA COOPERATIVE SYSTEM WITH TIME DELAYS AND FEEDBACK CONTROLS

JUNYAN XU*, FENGDE CHEN

College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian, 350002, China

Abstract. In this paper, we investigate a Lotka-Volterra cooperative system with time delays and feedback controls. By applying new inequalities, we obtain some new sufficient conditions which ensure the system to be permanent. Our results show that feedback control variables have no influence on the permanence of the system, which enrich the previous corresponding research results.

Keywords: Lotka-Volterra system; Cooperative; Discrete delay; Feedback control; Permanence

2010 AMS Subject Classification: 92B20, 34K20, 45J05.

1. Introduction

In virtue of their significance in theory and practice, the Lotka-Volterra systems with time delays have been extensively studied. As one of the important interactions among species, cooperative behavior is commonly seen in animal society and human society. The study of Lotka-Volterra cooperative models with delays have attracted the interest of many researchers(see for example, [1-7] and references cited therein). In [1], Lu, Lu and Lian studied the permanence of

*Corresponding author

E-mail address: xujunyan159@163.com

Received February 26, 2015

the following Lotka-Volterra system with discrete delays

$$\dot{x}_1(t) = x_1(t)\big[r_1 - a_1x_1(t) - a_{11}x_1(t-\tau_{11}) + a_{12}x_2(t-\tau_{12})\big],$$

$$\dot{x}_2(t) = x_2(t)\big[r_2 - a_2x_2(t) + a_{21}x_1(t) - a_{22}x_2(t-\tau_{22})\big], \tag{1.1}$$

with initial conditions

$$x_i(t) = \phi_i(t) \geq 0,\ \ t \in [-\tau_0, 0];\ \ \phi_i(0) > 0\ \ (i = 1, 2)$$

where r_i, a_i, a_{ij} and τ_{ij} are constants with $a_i > 0, \tau_{ij} \geq 0 (i, j = 1, 2)$ and $\tau_0 = \max\{\tau_{ij} : i, j = 1, 2\}$, $\phi_i(t)$ is continuous on $[-\tau_0, 0]$. In the cooperative case $(a_{12} > 0, a_{21} > 0)$, they obtained the main result.

Theorem A. *If $\tau_{11} = \tau_{12} = \tau_{22} = \tau$, $a_i = a_{ji} < a_{ii} (i \neq j,\ i, j = 1, 2)$ and $(a_1 + a_{11})(a_2 + a_{22}) - a_{12}a_{21} > 0$, then system (1.1) is permanent.*

Moreover, they gave an example (see example 4.1 in [1]) to show that the size of time delays can destroy the permanence of system (1.1). To eliminate the influence of the size of delays on the permanence, Nakata and Muroya [2] considered the following nonautonomous two species Lotka-Volterra cooperative population systems

$$\dot{x}_1(t) = x_1(t)\big[r_1(t) - a^1_{11}(t)x_1(t-\tau) - a^2_{11}(t)x_1(t-2\tau) + a^1_{12}(t)x_2(t-\tau)\big],$$

$$\dot{x}_2(t) = x_2(t)\big[r_2(t) + a^0_{21}(t)x_1(t) + a^1_{21}(t)x_1(t-\tau) - a^0_{22}(t)x_2(t) - a^1_{22}(t)x_2(t-\tau)\big], \tag{1.2}$$

with initial conditions

$$x_1(t) = \Phi_1(t) \geq 0,\ \ t \in [-2\tau, 0);\ \ \Phi_1(0) > 0,$$

$$x_2(t) = \Phi_2(t) \geq 0,\ \ t \in [-\tau, 0);\ \ \Phi_2(0) > 0.$$

Through establishing new inequalities (one can see Lemma 2.2 - Lemma 2.4 in [2]), they obtained weaker conditions which is not dependent on the size of time delays to prove that the cooperative system (1.2) is permanent. For more works on the permanence of cooperative system with delays, one could refer to [1-3,6,14] and the references cited therein.

In another aspect, ecosystems in the real world are often distributed by unpredictable forces which can result in changes in biological parameters such as survival rates, so it is necessary to study models with control variables which are so-called disturbance functions [8-15]. Recently, Fan and Wang [10], Nie, Peng and Teng [11] , Chen [12,13] and Chen [14] investigated the persistent property of discrete or continuous feedback control systems with delays. It is interesting that all of their research results show that feedback controls have no influence on the persistent property of these systems.

However, to the best of the authors' knowledge, there are few scholars who study the Lotka-Volterra cooperative system with feedback control. Whether the feedback control variables play an essential role on the persistent property of Lotka-Volterra cooperative system or not? To find an answer to this question, we investigate the following Lotka-Volterra cooperative system with time delays and feedback controls

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[r_1(t)-a_1(t)x_1(t)-a_{11}(t)x_1(t-\tau)+a_{12}(t)x_2(t-\tau)-b_1(t)u_1(t-\sigma_1)\right],\\
\dot{x}_2(t) &= x_2(t)\left[r_2(t)-a_2(t)x_2(t)+a_{21}(t)x_1(t)-a_{22}(t)x_2(t-\tau)-b_2(t)u_2(t-\sigma_2)\right],\\
\dot{u}_1(t) &= -c_1(t)u_1(t)+d_1(t)x_1(t-\eta_1),\\
\dot{u}_2(t) &= -c_2(t)u_2(t)+d_2(t)x_2(t-\eta_2),
\end{aligned} \tag{1.3}$$

where $x_i(t)$ denotes the density of ith cooperative species X_i, $u_i(t)$ is the control variable, $i=1,2$. $r_i(t), a_i(t), b_i(t), c_i(t), d_i(t), a_{ij}(t) (i,j=1,2)$ are all continuous, real-valued functions which are bounded above and below by positive constants. $\tau, \sigma_i, \eta_i (i=1,2)$ are positive constants and $\delta=\max\{\tau,\sigma_1,\sigma_2,\eta_1,\eta_2\}$.

We consider system (1.3) with the following initial conditions

$$\begin{aligned}
x_i(s) &= \varphi_i(s), \;\; s\in[-\delta,0], \;\; \varphi_i(0)>0,\\
u_i(s) &= \psi_i(s), \;\; s\in[-\delta,0], \;\; \psi_i(0)>0,
\end{aligned} \tag{1.4}$$

where $\varphi_i(s)$ and $\psi_i(s)$ are continuous on $[-\delta,0]$. It is not difficult to see that solutions of (1.3) and (1.4) are well defined for all $t\geq 0$ and satisfy

$$x_i(t)>0, u_i(t)>0 \;\text{ for }\; t\geq 0, \;\; i=1,2.$$

For a continuous bounded function $f(t)$ defined on $[0,+\infty)$, we set

$$f^{\mu} = \sup_{0\leq t<+\infty} f(t) \;\text{ and }\; f^{l} = \inf_{0\leq t<+\infty} f(t).$$

More precisely, in this paper, we will prove the following results.

Theorem 1.1. *Assume that* $a_{11}^l > a_{21}^\mu, a_1^l > a_{21}^\mu$ *and* $a_{22}^l > a_{12}^\mu, a_2^l > a_{12}^\mu$. *Then system (1.3) is permanent.*

As a direct consequence, the following conditions for autonomous case to (1.3) are obtained. Now we set $a_i(t) = a_i, a_{ij}(t) = a_{ij}$ for $i, j = 1, 2$.

Corollary 1.2. *Assume that* $a_{11} > a_{21}, a_1 > a_{21}$ *and* $a_{22} > a_{12}, a_2 > a_{12}$. *Then system (1.3) is permanent.*

Remark 1.3. Theorem 1.1 and Corollary 1.2 show that feedback control variables have no influence on the persistent property of the system (1.3). Obviously, compared with Theorem A of Lu, Lu and Lian [1], Corollary 1.2 not only is founded in weaker conditions, but also is suitable for more extensive application. Therefore, our results give some more deeply insight on the dynamic behaviors of the Lotka-Volterra cooperative system with feedback controls.

2. Lemmas and proof of theorem

At first, let us introduce some basic and important lemmas.

Lemma 2.1. [9] *If* $a > 0, b > 0$ *and* $\frac{dx}{dt} \geq b - ax$, *when* $t \geq 0$ *and* $x(0) > 0$, *we have* $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$. *If* $a > 0, b > 0$ *and* $\frac{dx}{dt} \leq b - ax$, *when* $t \geq 0$ *and* $x(0) > 0$, *we have* $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$.

Lemma 2.2.[9] *If* $a > 0, b > 0$ *and* $\frac{dx}{dt} \geq x(b - ax)$, *when* $t \geq 0$ *and* $x(0) > 0$, *we have* $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$. *If* $a > 0, b > 0$ *and* $\frac{dx}{dt} \leq x(b - ax)$, *when* $t \geq 0$ *and* $x(0) > 0$, *we have* $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$.

Lemma 2.3.(see [13, Lemma 2.2]) *Assume that* $a > 0, b(t) > 0$ *is a boundedness continuous function and* $x(0) > 0$. *Further suppose that*

$$\frac{dx(t)}{dt} \leq -ax(t) + b(t),$$

then for all $t \geq s \geq 0$,

$$x(t) \leq x(t-s)\exp\{-as\} + \int_{t-s}^{t} b(\tau)\exp\{a(\tau - t)\}d\tau. \tag{2.1}$$

Especially, if $b(t)$ is bounded above with respect to M, then

$$\limsup_{t\to+\infty} x(t) \le \frac{M}{a}. \tag{2.2}$$

Lemma 2.4. (see [2, Lemma 2.2]) *Assume that for $y(t)>0$, it holds that*

$$\frac{dy(t)}{dt} \le y(t)\big(\lambda - \sum_{l=0}^{m} \mu^l y(t-l\tau)\big) + D,$$

with initial conditions $y(t)=\phi(t)\ge 0$ for $t\in[-m\tau,0)$ and $\phi(0)>0$, where

$$\lambda>0,\ \ \mu^l \ge 0 (l=0,1,2,\cdots,m),\ \ \mu=\sum_{l=0}^{m}\mu^l>0 \ \text{ and } \ D\ge 0$$

are constants. Then there exists a positive constant $M_y<+\infty$ such that

$$\limsup_{t\to+\infty} y(t) \le M_y = -\frac{D}{\lambda} + \big(\frac{D}{\lambda}+y^*\big)\exp(\lambda m\tau) < +\infty,$$

where $y=y^*$ is the unique solution of $y(\lambda-\mu y)+D=0$.

From now on, we will prove the boundedness of $x_i(t)$ and $u_i(t)$, for $i=1,2$.

Lemma 2.5. *For system (1.3) with initial conditions (1.4), suppose that $a_{11}^l > a_{21}^\mu, a_1^l > a_{21}^\mu$ and $a_{22}^l > a_{12}^\mu, a_2^l > a_{12}^\mu$ hold, then there exist positive constants P_1 and P_2 such that*

$$\limsup_{t\to+\infty} x_1(t)x_2(t) \le P_1 = \frac{(r_1^\mu + r_2^\mu)^2}{a_{11}^l(a_{22}^l - a_{12}^\mu)} \exp\big((r_1^\mu+r_2^\mu)\tau\big) < +\infty, \tag{2.3}$$

$$\limsup_{t\to+\infty} x_1(t)x_2(t-\tau) \le P_2 = \frac{(r_1^\mu + r_2^\mu)^2}{a_{22}^l(a_{11}^l - a_{21}^\mu)} \exp\big((r_1^\mu+r_2^\mu)\tau\big) < +\infty. \tag{2.4}$$

Proof. Firstly, suppose that $\limsup_{t\to+\infty} x_1(t)x_2(t) = +\infty$. Then there exists a subsequence $\{t_k^{(1)}\}_{k=1}^{+\infty}$ such that

$$\lim_{k\to+\infty} x_1(t_k^{(1)})x_2(t_k^{(1)}) = +\infty \ \text{ and } \ \frac{d}{dt}\big(x_1(t)x_2(t)\big)\big|_{t=t_k^{(1)}} \ge 0,\ \ k=1,2,\cdots. \tag{2.5}$$

From (1.3), we obtain the following equation

$$\begin{aligned}\frac{d}{dt}\big(x_1(t)x_2(t)\big) &= x_1(t)x_2(t)\big[r_1(t)+r_2(t)-(a_1(t)-a_{21}(t))x_1(t)-a_{11}(t)x_1(t-\tau)-a_2(t)x_2(t)\\ &\quad -(a_{22}(t)-a_{12}(t))x_2(t-\tau)-b_1(t)u_1(t-\sigma_1)-b_2(t)u_2(t-\sigma_2)\big]\\ &\le x_1(t)x_2(t)\big[r_1^\mu+r_2^\mu-(a_1^l-a_{21}^\mu)x_1(t)-a_{11}^l x_1(t-\tau)-(a_{22}^l-a_{12}^\mu)x_2(t-\tau)\big]\end{aligned} \tag{2.6}$$

From (2.5), (2.6) and the assumption of the lemma, it holds that

$$r_1^\mu + r_2^\mu \geq (a_1^l - a_{21}^\mu)x_1(t_k^{(1)}) + a_{11}^l x_1(t_k^{(1)} - \tau) + (a_{22}^l - a_{12}^\mu)x_2(t_k^{(1)} - \tau).$$

It implies that $x_1(t_k^{(1)} - \tau) \leq \dfrac{r_1^\mu + r_2^\mu}{a_{11}^l}$ and $x_2(t_k^{(1)} - \tau) \leq \dfrac{r_1^\mu + r_2^\mu}{a_{22}^l - a_{12}^\mu}$. Moreover, integrating both sides of (2.6) from $t_k^{(1)} - \tau$ to $t_k^{(1)}$, we have

$$\begin{aligned} x_1(t_k^{(1)})x_2(t_k^{(1)}) &\leq x_1(t_k^{(1)} - \tau)x_2(t_k^{(1)} - \tau)\exp\big((r_1^\mu + r_2^\mu)\tau\big) \\ &\leq \frac{(r_1^\mu + r_2^\mu)^2}{a_{11}^l(a_{22}^l - a_{12}^\mu)}\exp\big((r_1^\mu + r_2^\mu)\tau\big) < +\infty. \end{aligned}$$

It leads to a contradiction to our consumption. Thus, we obtain that

$$\limsup_{t\to+\infty} x_1(t)x_2(t) < +\infty.$$

Furthermore, similar to the above discussion, we get that

$$\limsup_{t\to+\infty} x_1(t)x_2(t) \leq \frac{(r_1^\mu + r_2^\mu)^2}{a_{11}^l(a_{22}^l - a_{12}^\mu)}\exp\big((r_1^\mu + r_2^\mu)\tau\big) = P_1 < +\infty.$$

Secondly, suppose that $\limsup_{t\to+\infty} x_1(t)x_2(t-\tau) = +\infty$. Then there exists a subsequence $\{t_k^{(2)}\}_{k=1}^{+\infty}$ such that

$$t_k^{(2)} > \tau, \ \lim_{k\to+\infty} x_1(t_k^{(2)})x_2(t_k^{(2)} - \tau) = +\infty \ \text{ and } \ \frac{d}{dt}\big(x_1(t)x_2(t-\tau)\big)\big|_{t=t_k^{(2)}} \geq 0, \ \ k = 1, 2, \cdots. \tag{2.7}$$

From (1.3), we obtain the following equation

$$\begin{aligned} \frac{d}{dt}\big(x_1(t)x_2(t-\tau)\big) = {} & x_1(t)x_2(t-\tau)\big[r_1(t) + r_2(t-\tau) - a_1(t)x_1(t) - (a_{11}(t) - a_{21}(t-\tau))x_1(t-\tau) \\ & -(a_2(t-\tau) - a_{12}(t))x_2(t-\tau) - a_{22}(t-\tau)x_2(t-2\tau) - b_1(t)u_1(t-\sigma_1) \\ & -b_2(t-\tau)u_2(t-\tau-\sigma_2)\big] \\ \leq {} & x_1(t)x_2(t-\tau)\big[r_1^\mu + r_2^\mu - (a_{11}^l - a_{21}^\mu)x_1(t-\tau) - (a_2^l - a_{12}^\mu)x_2(t-\tau) \\ & -a_{22}^l x_2(t-2\tau)\big]. \end{aligned} \tag{2.8}$$

From (2.7), (2.8) and our assumption, it holds that

$$r_1^\mu + r_2^\mu \geq (a_{11}^l - a_{21}^\mu)x_1(t_k^{(2)} - \tau) + (a_2^l - a_{12}^\mu)x_2(t_k^{(2)} - \tau) + a_{22}^l x_2(t_k^{(2)} - 2\tau).$$

It implies that $x_1(t_k^{(2)}-\tau)\le \dfrac{r_1^\mu+r_2^\mu}{a_{11}^l-a_{21}^\mu}$ and $x_2(t_k^{(2)}-2\tau)\le \dfrac{r_1^\mu+r_2^\mu}{a_{22}^l}$. Moreover, integrating both sides of (2.8) from $t_k^{(2)}-\tau$ to $t_k^{(2)}$, we have

$$\begin{aligned} x_1(t_k^{(2)})x_2(t_k^{(2)}-\tau) &\le x_1(t_k^{(2)}-\tau)x_2(t_k^{(2)}-2\tau)\exp\big((r_1^\mu+r_2^\mu)\tau\big)\\ &\le \frac{(r_1^\mu+r_2^\mu)^2}{a_{22}^l(a_{11}^l-a_{21}^\mu)}\exp\big((r_1^\mu+r_2^\mu)\tau\big)<+\infty. \end{aligned}$$

It leads to a contradiction to our consumption. Thus, we obtain that $\limsup\limits_{t\to+\infty} x_1(t)x_2(t-\tau)<+\infty$. Furthermore, similar to the above discussion, we get that

$$\limsup_{t\to+\infty} x_1(t)x_2(t-\tau)\le \frac{(r_1^\mu+r_2^\mu)^2}{a_{22}^l(a_{11}^l-a_{21}^\mu)}\exp\big((r_1^\mu+r_2^\mu)\tau\big)=P_2<+\infty.$$

The proof is complete.

Lemma 2.6 *Assume that $a_{11}^l>a_{21}^\mu, a_1^l>a_{21}^\mu$ and $a_{22}^l>a_{12}^\mu, a_2^l>a_{12}^\mu$, then for any positive solution $(x_1(t),x_2(t),u_1(t),u_2(t))^T$ of system (1.3), there exists a positive constant M such that*

$$\limsup_{t\to+\infty} x_i(t)\le M,\ \ \limsup_{t\to+\infty} u_i(t)\le M,\ \ i=1,2.$$

Proof. Let $(x_1(t),x_2(t),u_1(t),u_2(t))^T$ be a solution of system (1.3) satisfies the initial conditions (1.4). From Lemma 2.5, there exists a large enough $T_1>0$ such that $x_1(t)x_2(t)\le 2P_1$ and $x_1(t)x_2(t-\tau)\le 2P_2$, for all $t\ge T_1$. And according to the first equation of system (1.3), for $t\ge T_1$,

$$\begin{aligned} \frac{dx_1(t)}{dt} &\le x_1(t)\big[r_1^\mu-a_1^l x_1(t)-a_{11}^l x_1(t-\tau)+a_{12}^\mu x_2(t-\tau)-b_1^l u_1(t-\sigma_1)\big]\\ &\le x_1(t)\big[r_1^\mu-a_1^l x_1(t)-a_{11}^l x_1(t-\tau)\big]+2a_{12}^\mu P_2. \end{aligned}$$

By applying Lemma 2.4 to above inequality, we have

$$\limsup_{t\to+\infty} x_1(t)\le -\frac{2a_{12}^\mu P_2}{r_1^\mu}+\Big(\frac{2a_{12}^\mu P_2}{r_1^\mu}+y_1^*\Big)\exp(r_1^\mu\tau)\triangleq M_{x_1}<+\infty, \tag{2.9}$$

where y_1^* is the unique positive solution of $y[r_1^\mu-(a_1^l+a_{11}^l)y]+2a_{12}^\mu P_2=0$.

Similarly, for $t\ge T_1$,

$$\begin{aligned} \frac{dx_2(t)}{dt} &\le x_2(t)\big[r_2^\mu-a_2^l x_2(t)+a_{21}^\mu x_1(t)-a_{22}^l x_2(t-\tau)-b_2^l u_2(t-\sigma_2)\big]\\ &\le x_2(t)\big[r_2^\mu-a_2^l x_2(t)-a_{22}^l x_2(t-\tau)\big]+2a_{21}^\mu P_1. \end{aligned}$$

Again from Lemma 2.4, we have

$$\limsup_{t\to+\infty} x_2(t) \le -\frac{2a_{21}^{\mu}P_1}{r_2^{\mu}} + \Big(\frac{2a_{21}^{\mu}P_1}{r_2^{\mu}} + y_2^*\Big)\exp(r_2^{\mu}\tau) \triangleq M_{x_2} < +\infty, \tag{2.10}$$

where y_2^* is the unique positive solution of $y[r_2^{\mu} - (a_2^l + a_{22}^l)y] + 2a_{21}^{\mu}P_1 = 0$.

From (2.9) and (2.10), there exists a $T_2 \ge T_1$, such that $x_i(t) \le 2M_{x_i}$ $(i = 1,2)$ for all $t \ge T_2$. According to the latter two equations of system (1.3), for all $t \ge T_2 + \delta$

$$\frac{du_i(t)}{dt} \le -c_i^l u_i(t) + 2d_i^{\mu} M_{x_i}, \quad i = 1,2.$$

By Lemma 2.1, we obtain

$$\limsup_{t\to+\infty} u_i(t) \le \frac{2d_i^{\mu} M_{x_i}}{c_i^l} \triangleq M_{u_i} < +\infty, \quad i = 1,2.$$

Set $M = \max\{M_{x_1}, M_{x_2}, M_{u_1}, M_{u_2}\}$, obviously, for $i = 1,2$, we have

$$\limsup_{t\to+\infty} x_i(t) \le M, \quad \limsup_{t\to+\infty} u_i(t) \le M.$$

This completes the proof.

Lemma 2.7 *Assume that $a_{11}^l > a_{21}^{\mu}, a_1^l > a_{21}^{\mu}$ and $a_{22}^l > a_{12}^{\mu}, a_2^l > a_{12}^{\mu}$, then for any positive solution $(x_1(t), x_2(t), u_1(t), u_2(t))^T$ of system (1.3), there exists a positive constant m such that*

$$\liminf_{t\to+\infty} x_i(t) \ge m, \quad \liminf_{t\to+\infty} u_i(t) \ge m, \quad i = 1,2.$$

Proof. Let $i \in \{1,2\}$ and $(x_1(t), x_2(t), u_1(t), u_2(t))^T$ be a solution of system (1.3) satisfies the initial conditions (1.4). From Lemma 2.6, there exists a $T_3 > T_2 + \delta$ such that $x_i(t) \le 2M, u_i(t) \le 2M$ for all $t \ge T_3$. And so, for $t \ge T_3 + \delta$, from the former two equations of system (1.3), it follows that

$$\begin{aligned}\frac{dx_i(t)}{dt} &\ge x_i(t)\big[r_i^l - a_i^{\mu}x_i(t) - a_{ii}^{\mu}x_i(t-\tau) - b_i^{\mu}u_i(t-\sigma_i)\big]\\ &\ge x_i(t)\big[-\big(a_i^{\mu} + a_{ii}^{\mu} + b_i^{\mu}\big)2M\big] \triangleq \xi_i x_i(t) \end{aligned}\tag{2.11}$$

where $\xi_i = -2\big(a_i^{\mu} + a_{ii}^{\mu} + b_i^{\mu}\big)M < 0$. Integrating both sides of (2.11) from $s(s \le t)$ to t, it leads to

$$x_i(s) \le x_i(t)\exp\{-\xi_i(t-s)\}. \tag{2.12}$$

Take $s = t-\tau, t-\eta_i$ and $t-\sigma_i$, respectively, one has

$$x_i(t-\tau) \le x_i(t)\exp(-\xi_i\delta), \tag{2.13}$$

$$x_i(t-\eta_i) \le x_i(t)\exp(-\xi_i\delta), \tag{2.14}$$

$$x_i(t-\sigma_i) \le x_i(t)\exp(-\xi_i\delta). \tag{2.15}$$

Substituting (2.14) into the latter two equations of system (1.3) leads to

$$\frac{du_i(t)}{dt} \le -c_i^l u_i(t) + d_i^\mu x_i(t)\exp(-\xi_i\delta).$$

Applying Lemma 2.3 and (2.12), for all $t \ge s \ge 0$,

$$\begin{aligned} u_i(t) &\le u_i(t-s)\exp(-c_i^l s) + \int_{t-s}^{t} d_i^\mu x_i(\upsilon)\exp(-\xi_i\delta)\exp\{c_i^l(\upsilon-t)\}d\upsilon \\ &\le u_i(t-s)\exp(-c_i^l s) + \int_{t-s}^{t} d_i^\mu x_i(t)\exp\{-\xi_i(t-\upsilon)\}\exp(-\xi_i\delta)d\upsilon \\ &\le u_i(t-s)\exp(-c_i^l s) + d_i^\mu x_i(t)\frac{1}{\xi_i}[1-\exp(-\xi_i s)]\exp(-\xi_i\delta). \end{aligned} \tag{2.16}$$

Note that for large t, θ and $t-\theta \ge T_3+\delta > T_3$, then $u_i(t-s) \le 2M$. Thus, for $t, \theta \to +\infty$ and $t-\theta \ge T_3+\delta > T_3$, $0 \le u_i(t-\theta)\exp(-c_i^l\theta) \le 2M\exp(-c_i^l\theta) \to 0$. So, there exists a $\Gamma_i > 0$, such that

$$r_i^l - 2b_i^\mu M\exp(-c_i^l\theta) \ge \frac{1}{2}r_i^l, \ \text{ as } \ \theta \ge \Gamma_i. \tag{2.17}$$

Then fix Γ_i, for all $t \ge T_3+\delta+\Gamma_i$, (2.16) can be expressed in the following form

$$\begin{aligned} u_i(t) &\le u_i(t-\Gamma_i)\exp(-c_i^l\Gamma_i) + d_i^\mu x_i(t)\frac{1}{\xi_i}(1-\exp(-\xi_i\Gamma_i))\exp(-\xi_i\delta) \\ &\le 2M\exp(-c_i^l\Gamma_i) + A_i x_i(t), \end{aligned}$$

where $A_i = d_i^\mu \dfrac{1}{\xi_i}(1-\exp(-\xi_i\Gamma_i))\exp(-\xi_i\delta) > 0$. Then, for all $t \ge T_3+2\delta+\Gamma_i$, we have

$$u_i(t-\sigma_i) \le 2M\exp(-c_i^l\Gamma_i) + A_i x_i(t-\sigma_i). \tag{2.18}$$

Substituting (2.13)-(2.18) into the former two equations of system (1.3), for $t \ge T_3+2\delta+\Gamma_i$, we obtain

$$\begin{aligned} \frac{dx_i(t)}{dt} &\ge x_i(t)\left[r_i^l - a_i^\mu x_i(t) - a_{ii}^\mu x_i(t)\exp(-\xi_i\delta) - b_i^\mu 2M\exp(-c_i^l\Gamma_i) - b_i^\mu A_i x_i(t-\sigma_i)\right] \\ &\ge x_i(t)\left\{r_i^l - b_i^\mu 2M\exp(-c_i^l\Gamma_i) - [a_i^\mu + a_{ii}^\mu\exp(-\xi_i\delta) + b_i^\mu A_i\exp(-\xi_i\delta)]x_i(t)\right\} \end{aligned}$$

$$\geq x_i(t)\big[\frac{1}{2}r_i^l - B_i x_i(t)\big],$$

where $B_i = a_i^\mu + a_{ii}^\mu \exp(-\xi_i\delta) + b_i^\mu A_i \exp(-\xi_i\delta) > 0$. Using the first part of Lemma 2.2, we get

$$\liminf_{t\to+\infty} x_i(t) \geq \frac{r_i^l}{2B_i} \triangleq m_{x_i}. \tag{2.19}$$

From (2.19), there exists a $T_4 > \max_{i=1,2}\{T_3 + 2\delta + \Gamma_i\}$ such that

$$x_i(t) \geq \frac{1}{2}m_{x_i} \quad \text{for all} \quad t \geq T_4.$$

Together with the latter two equations of system (1.3), it is easy to see that

$$\frac{du_i(t)}{dt} \geq -c_i^\mu u_i(t) + d_i^l \frac{1}{2}m_{x_i} \quad \text{for all} \quad t \geq T_4 + \delta.$$

Thus, by applying Lemma 2.1 to the above differential inequality, we have

$$\liminf_{t\to+\infty} u_i(t) \geq \frac{d_i^l m_{x_i}}{2c_i^\mu} \triangleq m_{u_i}.$$

Set $m = \min\{m_{x_1}, m_{x_2}, m_{u_1}, m_{u_2}\}$, obviously, for $i = 1,2$, we have

$$\liminf_{t\to+\infty} x_i(t) \geq m, \quad \liminf_{t\to+\infty} u_i(t) \geq m.$$

This completes the proof.

Proof of the theorem 1.1. From Lemmas 2.6 and 2.7 and the definition of permanence, the conclusion is obvious.

3. Conclusion

With the help of a series of inequalities, some new sufficient conditions of the permanence of system (1.3) are obtained in this paper. More meaningfully, our results show that feedback control variables have no influence on the persistent property of the system (1.3). As for Lotka-Volterra cooperative systems with more general delays, we will carry out the research works in the future.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgements

The authors would like to express their gratitude to the reviewers and editor for their valuable comments and suggestions. Without the expert comments made by the reviewers, the paper would not be of this quality. This work was supported by the Program for Science and Technology Development Foundation of Fuzhou University (NO. 2011-XY-18).

References

[1] G. Lu, Z. Lu and X. Lian, Delay effect on the permanence for Lotka-Volterra cooperative systems, Nonlinear Anal. Real World Appl. 11 (2010), 2810-2816.

[2] Y. Nakata and Y. Muroya, Permanence for nonautonomous Lotka-Volterra cooperative systems with delays, Nonlinear Anal. Real World Appl. 11 (2010), 528-534.

[3] G. Lu, Z. Lu, Y. Enatsu, Permanence for Lotka-Volterra systems with multiple delays Nonlinear Anal. Real World Appl. 12 (2011), 2552-2560.

[4] I. M. Stamova, Impulsive control for stability of n-species Lotka-Volterra cooperation models with finite delays, Appl. Math. Lett. 23 (2010), 1003-1007.

[5] F. Wei, K. Wang, Global stability and asymptotically periodic solution for nonautonomous cooperative Lotka-Volterra diffusion system, Appl. Math. Comput. 182 (2006), 161-165.

[6] F. Chen and X. Xie, Study on the dynamic behaviors of cooperative system, Science Press, 2014.

[7] L. Hung, Traveling wave solutions of competitive-cooperative Lotka-Volterra systems of three species, Nonlinear Anal. Real World Appl. 12 (2011), 3691-3700.

[8] K. Gopalsamy and P. Weng, Feedback regulation of logistic growth, Internat. J. Math. Math. Sci. 16 (1993), 177-192.

[9] F. Chen, Z. Li and Y. Huang, Note on the permanence of a competitive system with infinite delays and feedback controls, Nonlinear Anal. Real World Appl. 8 (2007), 680-687.

[10] Y. Fan and L. Wang, Permanence for a discrete model with feedback control and delay, Discrete Dyn. Nat. Soc. 2008 (2008), Article ID 945109.

[11] L. Nie, J. Peng and Z. Teng, Permanence and stability in multi-species non-autonomous Lotka-Volterra competitive systems with delays and feedback controls, Math. Comput. Modelling 49 (2009), 295-306.

[12] F. Chen, J. Yang, L. Chen and X. Xie, On a mutualism model with feedback controls, Appl. Math. Comput. 214 (2009), 581-587.

[13] F. Chen, J. Yang and L. Chen, Note on the persistent property of a feedback control system with delays, Nonlinear Anal. 11 (2010), 1061-1066.

[14] L. Chen and X. Xie, Permanence of an N-species cooperation system with continuous time delays and feedback controls, Nonlinear Anal. 12 (2011), 34-38.

[15] Y. Muroya, Global stability of a delayed nonlinear Lotka-Volterra system with feedback controls and patch structure, Appl. Math. Comput. 239 (2014), 60-73.