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A RESEARCH OF PEST MANAGEMENT SI STOCHASTIC MODEL CONCERNING SPRAYING PESTICIDE AND RELEASING NATURAL ENEMIES

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Abstract. In this paper, we study the dynamics of the stochastic SI epidemic model for pest management concerning spraying pesticide and releasing natural enemies. Existence of a unique global positive solution is proved firstly. And we show that the positive solution to the stochastic system is stochastically bounded. Third, by using Khasminshii's method and Lyapunov function, we derive the sufficient conditions for the existence of the nontrivial stochastically positive T-periodic solution. Then, by comparison theorem for stochastic differential equation, the sufficient conditions for existence and global attraction of the boundary periodic solution are obtained. Finally, Numerical simulations are carried out to substantiate the analytical results.

Keywords: pest management; SI model; pest extinction periodic solution; environmental fluctuations.

2010 AMS Subject Classification: 60H10.

1. Introduction

Since the beginning of recorded history, outbreaks of pests have plagued humanity, coming in direct competition with people for life-sustaining food. Reportedly, an estimated 67,000

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different pest species attack agricultural crops, and about 35% of the yearly agricultural crop production is lost to pests worldwide [1]. That problem is one of how to control or suppress damaging populations of pests over widespread areas. As we know, the most effective strategy for controlling pests may be to combine methods in an approach known as integrated pest management (IPM) that emphasizes preventing pest damage. In IPM, information about pests and available pest-control methods (including biological and chemical) is used to manage pest damage by the most economical means and with the least possible hazard to people, property, and environment [1,2].

Chemical control is the approach of controlling pests through the spraying pesticide which is liable to reduce the pest populations considerably and which is indispensable when there are not enough natural enemies to decrease pest populations. In most cropping systems, insecticides are still the principal means of controlling pests once the economic threshold (ET) has been reached. They are easy to apply, fast-acting, and in most instances can be relied on to control the pests [3]. However, excessive use of chemicals has led to environmental contamination, created pesticide residues and acts on non-targets especially on soil microorganisms. It is reported that there are about 545 million kg of pesticides are applied to US crops each year: 20% insecticides; 68% herbicides; and 12% fungicides for pest control. Despite this heavy and costly application of pesticides, pests destroy an estimated 37% of all US crops [4]. On the other hand, the chemical pesticide kills not only pests but also their natural enemies. Therefore, insect pests are rampant again. Furthermore, several studies have estimated that less than 0.3% of the pesticide reaches its target pest; the remaining 99.7% is released to the environment, representing a potential hazard for non-target organisms including humans [5].

Biological control is defined as the reduction of pest populations by natural enemies. This method not only reduces the cost of pest control but also protects the environment. There are many examples of successful classical biological control programs. One of the earliest successes was with the cottony cushion scale, a pest that was devastating the California citrus industry in the late 1800s [6]. A small wasp, Trichogramma ostriniae, introduced from China to help control the European corn borer, is a recent example of a long history of classical biological control efforts for this major pest [7]. Biological control is not a quick fix for most pest problems. Natural enemies usually take longer to suppress a pest population than other forms of pest-control, and farmers often regard this as a disadvantage.

One of the most important questions in IPM is how many natural enemies should be released and what fraction of the pest population should be killed to avoid economic damage and reduce the pesticide applications when the pest population reaches or exceeds the ET level. In many cases, the most effective release rate or spraying rate has not been identified as it will vary depending on crop type and target host density. To reduce the pesticide applications, the pesticide is sprayed only when it is necessary, i.e. when the pest population density reaches the ET. With this in mind and inspired by Liu's [9] pollutant emission model, we will establish a kind of integrated pest management SI model with impulse control in this paper.

In fact, population dynamics is inevitably affected by environmental fluctuations which is an important component in an ecosystem. However, the parameters in the deterministic model are assumed all deterministic irrespective environmental fluctuations. Hence they have some limitations in mathematical modelling of ecological systems, in addition to, they are quite difficult to fitting data entirely and to predict the future dynamics of the system accurately [10]. Stochastic differential equation play an important role in many kinds of applied sciences, including in the management of pests, since they can provide an additional degree of realism in comparison with their deterministic system. May [11] also pointed out that the birth rates, carrying capacity, competition coefficients and other parameters involved in the system can be affect by random fluctuation. Therefore, a number of authors introduce stochastic perturbation into deterministic models to reveal the effect of environmental variability on the population dynamics in mathematical ecology [8,12,13,18,22]. In the real world, insecticides is inevitably affected by environmental fluctuations owing to pesticides are directly sprayed in the environment, such as volatilization, photosynthesis and so on.

In addition, due to the seasonal variation, weather changes, food supplies, and mating habits and so on, the birth rate, the death rate of the population and other parameters will not remain constant, but exhibit a periodicity. Therefore, the periodic solution problem of non-autonomous systems as a new research direction of biomathematics become one of the topics of current popular. And to our knowledge, only a few authors [8,12,15,17] studied the existence of periodic solution of a non-autonomous systems with stochastic disturbance.

The rest of the paper is organized as follows. In Section 2, we formulate our model. In Section 3, we give some notations, definitions and lemmas which are useful for our main results. In Section 4, our main results and their proofs are given respectively. Finally, the conclusions are given and numerical simulations are carried out to substantiate the analytical results.

2. Model Formulation

Motivated by the above discussion, we assume that the prey is a pest with a prevalence of infectious disease, and that the predator is introduced to suppress its density. We also assume that natural enemies only predate susceptible pests and the effects of random interference will manifest on the insecticide function of susceptible pests, infected pests and natural enemies. Then, the following stochastic SI epidemic model for pest management concerning spraying pesticide and releasing natural enemies are considered:

$$\begin{cases} \left.\begin{aligned} dS(t) &= S(t)[r_1 - a_1 S(t) - \tfrac{b_1 I(t)}{S(t)+I(t)} - \tfrac{\alpha_1 y(t)}{\gamma+\alpha S(t)+\beta y(t)} - c_1 g_1(t)]dt + \sigma_1 g_1(t) S(t) dB_1(t), \\ dI(t) &= I(t)[-d_2 + \tfrac{b_1 S(t)}{S(t)+I(t)} - c_2 g_2(t)]dt + \sigma_2 g_2(t) I(t) dB_2(t), \\ dy(t) &= y(t)[r_0 - \tfrac{\alpha_3 y(t)}{k_3+S(t)} - c_3 g_3(t)]dt + \sigma_3 g_3(t) y(t) dB_3(t), \end{aligned}\right\} t \neq t_k \\ \left.\begin{aligned} \Delta S(t) &= 0, \\ \Delta I(t) &= 0, \\ \Delta y(t) &= \delta_k y(t), \end{aligned}\right\} t = t_k \end{cases} \tag{2.1}$$

where $S(t)$, $I(t)$, $y(t)$ are the densities of susceptible pest, infected pest and natural enemy at time t, respectively. All parameters involved with the model are positive. The parameters have the following biological meanings: r_1, r_0 are the intrinsic growth rates of the susceptible pest and natural enemy, respectively. a_1 denotes the density-dependent coefficient of the susceptible pest. α_1 is the maximum value at which per capita reduction rate of susceptible pest can attain; α_3 has the similar meaning as α_1; α denotes the effect of handling time for natural enemy; β measures the magnitude of interference among natural enemy. γ measures the extent to which environment provides protection to susceptible; d_2 denotes the death rates of infected pest; k_3 measures the extent to which the environment provides protection to natural enemy; $\frac{b_1 SI}{S+I}$ the transmission of the infection or the incidence rate [19]; $B_1(t)$, $B_2(t)$, $B_3(t)$ are mutually independent Brownian motions defined on the complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains all P-null sets); σ_1^2, σ_2^2, σ_3^2 represent the intensities of the white noise; $\delta_k(\delta_k > 0)$ is the proportion of released natural enemy; in addition, $0 < t_1 < t_2 < \ldots < t_k < \ldots$ and $\lim_{k\to\infty} t_k = +\infty$, we assume there exists a positive integer q such that $t_{k+q} = t_k + T$, $k \in Z$; $c_i(i = 1,2,3)$ represent the decreasing rate of the intrinsic growth rate associated with the uptake of the pesticide in the organism for the susceptible pest, infected pest and natural enemy, respectively; g_1, g_2, g_3 represent the concentration of pesticide in the organism for the susceptible pest, infected pest and natural enemy at time t, respectively, where $g_i(t)(i = 1,2,3)$ satisfy the following model

$$\begin{cases} \left.\begin{aligned} &dg_i = [l_i c_e(t) - m_i g_i(t) - n_i g_i(t)]dt, \\ &dc_e(t) = -pc_e(t)dt, \end{aligned}\right\} t \neq nT \\ \Delta c_e(t) = \mu. \quad t = nT \end{cases} \tag{2.2}$$

where $c_e(t)$ represent the concentration of pesticide in the environment at time t. All parameters involved with the model are positive. $l_i c_e(t)$ represents the organism's net uptake of pesticide from the environment; and $-m_i g_i(t)$ and $-n_i g_i(t)$ represent the egestion and depuration rates of the pesticide in the organism, respectively; $-pc_e(t)$ represents the pesticide loss from the environment itself by volatilization and so on; μ is the pesticide input amount at every time and $n \in Z^+$.

Lemma 2.1.([9]) *System (2.2) has a unique positive T-periodic solution $(g_i^*(t), c_e^*(t))$ which is globally asymptotically stable, where*

$$g_i^*(t) = g_i^*(0)e^{-(m_i+n_i)(t-nT)} + \frac{l_i\mu(e^{-(m_i+n_i)(t-nT)} - e^{-p(t-nT)})}{(p-m_i-n_i)(1-e^{-pT})},\ c_e^*(t) = \frac{\mu e^{-p(t-nT)}}{1-e^{-pT}}$$

and

$$g_i^*(0) = \frac{l_i\mu(e^{-(m_i+n_i)T} - e^{-pT})}{(p-m_i-n_i)(1-e^{-(m_i+n_i)T})(1-e^{-pT})},\ c_e^*(0) = \frac{\mu}{1-e^{-pT}},$$

for $t \in [nT, (n+1)T)$.

By the system (2.1) and (2.2), we can obtain the limitation system (2.3)

$$\begin{cases} \left.\begin{aligned} &dS(t) = S(t)[r_1 - a_1 S(t) - \tfrac{b_1 I(t)}{S(t)+I(t)} - \tfrac{\alpha_1 y(t)}{\gamma+\alpha S(t)+\beta y(t)} - c_1 g_1^*(t)]dt + \sigma_1 g_1^*(t) S(t) dB_1(t),\\ &dI(t) = I(t)[-d_2 + \tfrac{b_1 S(t)}{S(t)+I(t)} - c_2 g_2^*(t)]dt + \sigma_2 g_2^*(t) I(t) dB_2(t),\\ &dy(t) = y(t)[r_0 - \tfrac{\alpha_3 y(t)}{k_3+S(t)} - c_3 g_3^*(t)]dt + \sigma_3 g_3^*(t) y(t) dB_3(t), \end{aligned}\right\} t \neq t_k \\ \left.\begin{aligned} &\Delta S(t) = 0,\\ &\Delta I(t) = 0,\\ &\Delta y(t) = \delta_k y(t). \end{aligned}\right\} t = t_k \end{cases} \tag{2.3}$$

Moreover, due to the individual lifecycle and seasonal variation and so on, the birth rate, the death rate and the carrying capacity of the species and other parameters all exhibit cycle changes. In the article, we also consider the corresponding periodic system of (2.4):

$$\begin{cases} \left.\begin{aligned} &dS(t) = S(t)[r_1(t) - a_1(t) S(t) - \tfrac{b_1(t) I(t)}{S(t)+I(t)} - \tfrac{\alpha_1(t) y(t)}{\gamma(t)+\alpha(t) S(t)+\beta(t) y(t)} - c_1(t) g_1^*(t)]dt\\ &\qquad + \sigma_1(t) g_1^*(t) S(t) dB_1(t),\\ &dI(t) = I(t)[-d_2(t) + \tfrac{b_1(t) S(t)}{S(t)+I(t)} - c_2(t) g_2^*(t)]dt + \sigma_2(t) g_2^*(t) I(t) dB_2(t),\\ &dy(t) = y(t)[r_0(t) - \tfrac{\alpha_3(t) y(t)}{k_3(t)+S(t)} - c_3(t) g_3^*(t)]dt + \sigma_3(t) g_3^*(t) y(t) dB_3(t), \end{aligned}\right\} t \neq t_k \\ \left.\begin{aligned} &\Delta S(t) = 0,\\ &\Delta I(t) = 0,\\ &\Delta y(t) = \delta_k y(t), \end{aligned}\right\} t = t_k \end{cases} \tag{2.4}$$

where $r_1(t)$, $r_0(t)$, $a_1(t)$, $b_1(t)$, $\alpha_1(t)$, $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $d_2(t)$,$\alpha_3(t)$, $k_3(t)$, $c_1(t)$, $c_2(t)$, $c_3(t)$, $\sigma_1(t)$, $\sigma_2(t)$, $\sigma_3(t)$ are all positive T-periodic continuous functions, we will obtain the existence of the periodic Markov process of the system (2.4) by the method of Khasminskii[16].

Now we consider the non-impulsive system

$$\begin{cases} dx_1 = x_1(r_1 - a_1 x_1 - \frac{b_1 x_2}{x_1+x_2} - \frac{\alpha_1 A(t) x_3}{\gamma+\alpha x_1+\beta A(t) x_3} - c_1 g_1^*)dt + \sigma_1 g_1^* x_1 dB_1,\\ dx_2 = x_2(-d_2 + \frac{b_1 x_1}{x_1+x_2} - c_2 g_2^*)dt + \sigma_2 g_2^* x_2 dB_2,\\ dx_3 = x_3(\frac{\ln \prod_{j=1}^{p}(1+\delta_j)}{T} + r_0 - \frac{\alpha_3 A(t) x_3}{k_3+x_1} - c_3 g_3^*)dt + \sigma_3 g_3^* x_3 dB_3. \end{cases} \tag{2.5}$$

where

$$A(t) = \Big(\prod_{j=1}^{q}(1+\delta_j)\Big)^{-\frac{t}{T}} \prod_{0 \le t_k < t}(1+\delta_k).$$

By the [12], we can obtain that $A(t)$ is positive T-periodic continuous functions.

By the method of [13] the following result were obtained.

Lemma 2.2.

(1) If (x_1,x_2,x_3) is a solution of system (2.5), then (x_1,x_2,Ax_3) is a solution of system (2.3).

(2) If (S,I,y) is a solution of system (2.3), then $(S,I,A^{-1}y)$ is a solution of system (2.5).

Analogously, for the non-impulsive system

$$\begin{cases} dx_1 = x_1\big(r_1(t) - a_1(t)x_1 - \frac{b_1(t)x_2}{x_1+x_2} - \frac{\alpha_1(t)A(t)x_3}{\gamma(t)+\alpha(t)x_1+\beta(t)A(t)x_3} - c_1(t)g_1^*(t)\big)dt + \sigma_1(t)g_1^*(t)x_1dB_1, \\ dx_2 = x_2\big(-d_2(t) + \frac{b_1(t)x_1}{x_1+x_2} - c_2(t)g_2^*(t)\big)dt + \sigma_2(t)g_2^*(t)x_2dB_2, \\ dx_3 = x_3\Big(\frac{\ln\prod_{j=1}^{p}(1+\delta_j)}{T} + r_0(t) - \frac{\alpha_3(t)A(t)x_3}{k_3(t)+x_1} - c_3(t)g_3^*(t)\Big)dt + \sigma_3(t)g_3^*(t)x_3dB_3. \end{cases} \tag{2.6}$$

Lemma 2.3.

(1) If (x_1,x_2,x_3) is a solution of system (2.6), then (x_1,x_2,Ax_3) is a solution of system (2.4).

(2) If (S,I,y) is a solution of system (2.4), then $(S,I,A^{-1}y)$ is a solution of system (2.6).

3. Preliminaries

For convenience, we introduce several notations and recall some basic definitions.

Let $R_+^n = \{X(t) = (x_1,x_2...x_n) \in R^n : x_i > 0,\ \forall\ 1 \le i \le n\}$ and $|X(t)| = \sqrt{\sum_{i=1}^{n} x_i^2}$. For a bounded function $f(t)$ on $[0,\infty)$, define $f^u = \sup_{t\in[0,\infty)} f(t)$, $f^l = \inf_{t\in[0,\infty)} f(t)$, $\langle f(t)\rangle_T = \frac{1}{T}\int_0^T f(t)dt$. In addition, we assumed $r_3 = \frac{\ln\prod_{j=1}^{p}(1+\delta_j)}{T} + r_0$.

Definition 3.1.([15]) *The solution $X(t) = (x_1,x_2,x_3)$ of equation (2.5) is said to be stochastically ultimately bounded, if for any $\varepsilon \in (0,1)$ there is a positive constant $\delta = \delta(\varepsilon)$, such that for any initial value $X(0) \in R_+^3$, the solution X(t) to (2.5) has the property that*

$$\limsup_{t\to\infty} P\{|X(t)| > \delta\} < \varepsilon.$$

Definition 3.2.([16]) *A stochastic process $\xi(t) = \xi(t,\omega)(-\infty < t < +\infty)$ is said to be T-periodic if for every finite sequence of numbers $t_1,t_2,\cdots,t_n$, the joint distribution of random variables $\xi(t_1+h),\xi(t_2+h),\cdots,\xi(t_n+h)$ is independent of h, where $h = kT,(k = 1,2,\cdots)$.*

Now, we consider the integral equation

$$X(t)=X(t_0)+\int_{t_0}^{t}b\big(s,X(s)\big)ds+\sum_{r=1}^{k}\int_{t_0}^{t}\sigma_r\big(s,X(s)\big)d\xi_r(s). \tag{3.1}$$

where $b(s,x),\sigma_i(s,x)(i=1,2,\cdots,k)(s\in[t_0,T],x\in R^l)$ are continuous functions of (s,x) and for some constant B, the following conditions hold.

$$\begin{array}{l}|b(s,x)-b(s,y)|+\sum_{r=1}^{k}|\sigma_r(s,x),\sigma_r(s,y)|\leq B|x-y|,\\ |b(s,x)|+\sum_{r=1}^{k}|\sigma_r(s,x)|\leq B(1+|x|).\end{array}. \tag{3.2}$$

Lemma 3.1.([16]) *Suppose that the coefficients of* (3.1) *are T-periodic in t and satisfy the conditions* (3.2) *in every cylinder $I\times U$, and assume further there exists a function $V(t,x)\in C^2$, which is T-periodic in t and satisfies,*

$(Q1)$ $\inf_{|x|>R}V(t,x)\to\infty$,

$(Q2)$ $LV(t,x)\leq -1$ *outside some compact set. Then the system* (3.1) *exists at least a T-periodic Markov process.*

To further our study, we consider one dimensional stochastic differential equation

$$dN(t)=N(t)[a(t)-b(t)N(t)]dt+\sigma(t)N(t)dB(t), \tag{3.3}$$

on $t\geq 0$ with initial value $N(0)=N_0$, and $a(t),b(t),\sigma(t)$ are nonnegative, continuous functions. $B(t)$ is a Brownian motion defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq0},P)$.

Lemma 3.2.([17]) *Assume*

(H1) There exist constants σ_1, σ_2, a_1, a_2, $b_1>0$, $b_2>0$ and continuous bounded function $h(t)\geq 0$ such that $\sigma_1 h(t)\leq\sigma^2(t)\leq\sigma_2 h(t)$, $a_1h(t)\leq a(t)\leq a_2h(t)$, $b_1h(t)\leq b(t)\leq b_2h(t)$.

(H2) $B=\int_0^T[a(s)-\frac{1}{2}\sigma^2(s)]ds>0$ hold. Then equation (3.3) *has a positive T-periodic solution. Moreover, if*

(H3) $\inf\limits_{t\geq0}\int_t^{t+\varepsilon}h(s)ds>0$, for any $\varepsilon>0$ hold. Then equation (3.3) has a positive T-periodic solution, which attracts all other positive solutions of equation (3.3).

Next, consider the following stochastic equation

$$\begin{cases}dz(t)=z(t)\big(r_3-\dfrac{\alpha_3A(t)z(t)}{k_3}-c_3g_3^*(t)\big)dt+\sigma_3g_3^*(t)z(t)dB_3(t),\\ z(0)=x_3(0),\end{cases} \tag{3.4}$$

where $z(t)>0,x_3(0)>0$ and $r_3,\alpha_3,A(t),k_3,c_3,g_3^*(t),\sigma_3,B_3(t)$ are given in model (2.5).

Remark 3.1. *If* $\int_0^T r_3 - c_3 g_3^*(s) - \frac{1}{2}\sigma_3^2(g_3^*(s))^2 ds > 0$*, then equation (3.4) has a unique positive* T*-periodic solutions* $z^T(t)$ *and* $\lim_{t\to\infty}|z(t) - z^T(t)| = 0$ *a.s., where* $z(t)$ *is any positive solution of equation (3.4).*

4. Main results

The first result is concerned with the existence and uniqueness of positive solution, which is a prerequisite for analyzing the long-term behavior of model (2.5).

Theorem 4.1. *Let* $(x_1(t), x_2(t), x_3(t))$ *be a solution of (2.5) with positive initial value* $(x_1(0), x_2(0), x_3(0)) \in R_+^3$*. Then there exists a unique global positive solution to equation (2.5), i.e.* $\big(x_1(t), x_2(t), x_3(t)\big)$ *exists on* R_+^3 *for all* $t \geq 0$ *with probability 1.*

Proof. First consider system

$$\begin{cases} du(t) = [r_1 - a_1 e^{u(t)} - \frac{b_1 e^{v(t)}}{e^{u(t)}+e^{v(t)}} - \frac{\alpha_1 A e^{e^{w(t)}}}{\gamma+\alpha e^{u(t)}+\beta A e^{w(t)}} - c_1 g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2]dt + \sigma_1 g_1^* dB_1(t), \\ dv(t) = [-d_2 + \frac{b_1 e^{u(t)}}{e^{u(t)}+e^{v(t)}} - c_2 g_2^* - \frac{1}{2}\sigma_2^2(g_2^*)^2]dt + \sigma_2 g_2^* dB_2(t), \\ dw(t) = [r_3 - \frac{\alpha_3 A e^w}{k_3+e^u} - c_3 g_3^* - \frac{1}{2}\sigma_3^2(g_3^*)^2]dt + \sigma_3 g_3^* dB_3(t), \end{cases} \tag{4.1}$$

with initial value $u(0) = \ln x_1(0)$, $v(0) = \ln x_2(0)$, $w(0) = \ln x_3(0)$. Since the coefficients of system (4.1) satisfy the local Lipschitz condition, then system (4.1) has a unique local solution $\big(u(t), v(t), w(t)\big)$ on $t \in [0, \tau_e)$ where τ_e is the explosion time. Therefore, by Itô's formula, it is easy to see $(e^{u(t)}, e^{v(t)}, e^{w(t)})$ is the unique positive local solution to system (2.5) with initial value $\big(x_1(0), x_2(0), x_3(0)\big) \in R_+^3$.

Next, we will prove that this solution is global, i.e. $\tau_e = +\infty$ a.s.. To this end, we will use the similar approach to Theorem 2.1 of Mao et al. [15]. The key step is to construct a nonnegative C^2-function $V(x_1, x_2, x_3) : R_+^3 \to R_+$ such that

$$\liminf_{k\to\infty,\ (x_1,x_2,x_3)\in R_+^3\setminus D_k} V(x_1, x_2, x_3) = +\infty \ \text{ and } \ LV(x_1, x_2, x_3) \leq M,$$

where $D_k = (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k)$ and M is some positive constant. Now we define

$$V = x_1 + x_2 + x_3 - (\ln x_1 + \ln x_2 + \ln x_3) + (x_1 + k_3)x_3^2 := V_1 + V_2 + V_3,$$

where

$$V_1 = x_1 + x_2 + x_3,\ \ V_2 = \ln x_1 + \ln x_2 + \ln x_3,\ \ V_3 = (x_1 + k_3)x_3^2.$$

An application of Itô's formula to V_1, V_2, V_3 respectively, yields

$$\begin{aligned}
dV_1 =&[x_1(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^*) + x_2(-d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^*)\\
&+ x_3(r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^*)]dt + \sigma_1g_1^*x_1dB_1 + \sigma_2g_2^*x_2dB_2 + \sigma_3g_3^*x_3dB_3,\\
dV_2 =&\big(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2\big)dt\\
&+ \big(-d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^* - \frac{1}{2}\sigma_2^2(g_2^*)^2\big)dt + \sigma_1g_1^*dB_1 + \sigma_2g_2^*dB_2\\
&+ \big(r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^* - \frac{1}{2}\sigma_3^2(g_3^*)^2\big)dt + \sigma_3g_3^*dB_3\\
=&\big(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^* - d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^* + r_3 - \frac{\alpha_3Ax_3}{k_3+x_1}\\
&- c_3g_3^* - \frac{1}{2}\sigma_1^2(g_1^*)^2 - \frac{1}{2}\sigma_2^2(g_2^*)^2 - \frac{1}{2}\sigma_3^2(g_3^*)^2\big)dt + \sigma_1g_1^*dB_1 + \sigma_2g_2^*dB_2 + \sigma_3g_3^*dB_3,\\
dV_3 =&x_1x_3^2(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^*)dt + \sigma_1g_1^*x_1x_3^2dB_1\\
&+ [2(k_3+x_1)x_3^2(r_3 - c_3g_3^* - \frac{\alpha_3Ax_3}{k_3+x_1}) + (x_1+k_3)\sigma_3^2(g_3^*)^2x_3^2]dt + 2\sigma_3g_3^*(k_3+x_1)x_3^2dB_3.
\end{aligned}$$

Hence

$$\begin{aligned}
LV_1 :=&[x_1(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^*) + x_2(-d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^*)\\
&+ x_3(r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^*)]\\
\le& -a_1x_1^2 + (r_1 - c_1g_1^*)x_1 - (d_2 + c_2g_2^*)x_2 + (r_3 - c_3g_3^*)x_3,\\
-LV_2 :=& -[r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^* - d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^*\\
&+ r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^* - \frac{1}{2}(\sigma_1^2(g_1^*)^2 + \sigma_2^2(g_2^*)^2 + \sigma_3^2(g_3^*)^2)]\\
\le& -r_1 + a_1x_1 + b_1 + \frac{\alpha_1}{\beta} + c_1g_1^* + d_2 + c_2g_2^* - r_3 + \frac{\alpha_3A}{k_3}x_3 + c_3g_3^*\\
&+ \frac{1}{2}(\sigma_1^2(g_1^*)^2 + \sigma_2^2(g_2^*)^2 + \sigma_3^2(g_3^*)^2)]\\
\le& a_1x_1 + \frac{\alpha_3A}{k_3}x_3 + M_1,
\end{aligned}$$

where $-r_1+b_1+\frac{\alpha_1}{\beta}+c_1g_1^*+d_2+c_2g_2^*-r_3+c_3g_3^*+\frac{1}{2}(\sigma_1^2(g_1^*)^2+\sigma_2^2(g_2^*)^2+\sigma_3^2(g_3^*)^2)\le M_1$. and

$$\begin{aligned}
LV_3 :=& x_1x_3^2(r_1-a_1x_1-\frac{b_1x_2}{x_1+x_2}-\frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3}-c_1g_1^*)\\
&+[2(k_3+x_1)x_3^2(r_3-c_3g_3^*-\frac{\alpha_3Ax_3}{k_3+x_1})+(x_1+k_3)\sigma_3^2(g_3^*)^2x_3^2]\\
\le& -2\alpha_3Ax_3^3-a_1x_1^2x_3^2+(r_1-c_1g_1^*)x_1x_3^2+2k_3(r_3-c_3g_3^*)x_3^2\\
&+2(r_3-c_3g_3^*)x_1x_3^2+k_3\sigma_3^2(g_3^*)^2x_3^2+\sigma_3^2(g_3^*)^2x_1x_3^2\\
=& -2\alpha_3Ax_3^3-a_1x_1^2x_3^2+(r_1-c_1g_1^*+2r_3-2c_3g_3^*+\sigma_3^2(g_3^*)^2)x_1x_3^2\\
&+(2k_3(r_3-c_3g_3^*)+k_3\sigma_3^2(g_3^*)^2)x_3^2.
\end{aligned}$$

Therefore, one can see that

$$\begin{aligned}
LV =& LV_1-LV_2+LV_3\\
\le& -a_1x_1^2+(r_1-c_1g_1^*)x_1-(d_2+c_2g_2^*)x_2+(r_3-c_3g_3^*)x_3+a_1x_1+\frac{\alpha_3A}{k_3}x_3+M_1\\
&-2\alpha_3Ax_3^3-a_1x_1^2x_3^2+(r_1-c_1g_1^*+2r_3-2c_3g_3^*+\sigma_3^2(g_3^*)^2)x_1x_3^2\\
&+(2k_3(r_3-c_3g_3^*)+k_3\sigma_3^2(g_3^*)^2)x_3^2\\
\le& M.
\end{aligned} \tag{4.2}$$

In fact, if $x_1\ge\frac{r_1}{a_1}$, then $-a_1x_1^2-2\alpha_3Ax_3^3-a_1x_1^2x_3^2+r_1x_1x_3^2=-a_1x_1^2-2\alpha_3Ax_3^3-x_1^2x_3^2(a_1x_1-r_1)\le 0$, else $x_1<\frac{r_1}{a_1}$, then $-a_1x_1^2-2\alpha_3Ax_3^3-a_1x_1^2x_3^2+r_1x_1x_3^2\le -a_1x_1^2-2\alpha_3Ax_3^3+\frac{r_1^2}{a_1}x_3^2$ have supper bound.

Hence, one can see that $-a_1x_1^2-2\alpha_3Ax_3^3-a_1x_1^2x_3^2+r_1x_1x_3^2$ have supper bound. Similarly, we derive LV also have supper bound M(i.e. $LV\le M$).

The rest of the proof is very similar to Theorem 2.1 of Mao et al. [15], and is omitted here.

Theorem 4.2. *The solutions $X(t)=(x_1(t),x_2(t),x_3(t))$ of system (2.5) are stochastically ultimately bounded for any initial value $X(0)=(x_1(0),x_2(0),x_3(0))\in R_+^3$.*

Proof. Define the function

$$V=(x_1+x_2+x_3)+(k_3+x_1)x_3.$$

Making use of Itô's formula, we have

$$
\begin{aligned}
dV =&[x_1(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^*) + x_2(-d_2 + \frac{b_1x_1}{x_1+x_2} - c_2g_2^*) \\
&+ x_3(r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^*) + x_1x_3(r_1 - a_1x_1 - \frac{b_1x_2}{x_1+x_2} - \frac{\alpha_1Ax_3}{\gamma+\alpha x_1+\beta Ax_3} - c_1g_1^*) \\
&+ (k_3+x_1)x_3(r_3 - \frac{\alpha_3Ax_3}{k_3+x_1} - c_3g_3^*)]dt + \sigma_1g_1^*x_1dB_1 + \sigma_2g_2^*x_2dB_2 + \sigma_3g_3^*x_3dB_3 \\
&+ \sigma_1g_1^*x_1x_3dB_1 + (k_3+x_1)\sigma_3g_3^*x_3dB_3 \\
\leq&(-a_1x_1^2 + r_1x_1 - (d_2+c_2g_2^*)x_2 + r_3x_3 - \alpha_3Ax_3^2 - a_1x_1^2x_3 + (r_1+r_3)x_1x_3 + r_3k_3x_3)dt \\
&+ \sigma_1g_1^*x_1dB_1 + \sigma_2g_2^*x_2dB_2 + \sigma_3g_3^*x_3dB_3 + \sigma_1g_1^*x_1x_3dB_1 + (k_3+x_1)\sigma_3g_3^*x_3dB_3.
\end{aligned}
$$

Define the function again $W = e^{d_2t}V$. By Itô's formula one may calculate the operator LW

$$
\begin{aligned}
LW =&e^{d_2t}(d_2V + LV) \\
\leq&e^{d_2t}[d_2(x_1+x_2+x_3+(k_3+x_1)x_3) \\
&- a_1x_1^2 + r_1x_1 - (d_2+c_2g_2^*)x_2 + r_3x_3 - \alpha_3Ax_3^2 - a_1x_1^2x_3 + (r_1+r_3)x_1x_3 + r_3k_3x_3] \\
=&e^{d_2t}\big(-a_1x_1^2 - c_2g_2^*x_2 - \alpha_3Ax_3^2 - a_1x_1^2x_3 + (r_1+r_3+d_2)x_1x_3 + (d_2+r_1)x_1 \\
&+ (d_2+r_3+r_3k_3+d_2k_3)x_3\big),
\end{aligned}
$$

obviously, there exists a constant M_3 such that

$$
-a_1x_1^2 - c_2g_2^*x_2 - \alpha_3Ax_3^2 - a_1x_1^2x_3 + (r_1+r_3+d_2)x_1x_3 + (d_2+r_1)x_1 + (d_2+r_3+r_3k_3+d_2k_3)x_3 \leq M_3.
$$

Therefore

$$
dW \leq M_3e^{d_2t}dt + e^{d_2t}(\sigma_1g_1^*x_1dB_1 + \sigma_2g_2^*x_2dB_2 + \sigma_3g_3^*x_3dB_3 + \sigma_1g_1^*x_1x_3dB_1 + (k_3+x_1)\sigma_3g_3^*x_3dB_3),
$$

integrating both sides from 0 to t and taking expectation, we derive

$$
E(e^{d_2t}V) \leq W(0) + M_3(e^{d_2t} - 1), \text{ i.e.}
$$

$$
E(x_1+x_2+x_3+(x_1+k_3)x_3) \leq W(0)e^{-d_2t} + M_3(1 - e^{-d_2t}).
$$

Consequently

$$
E(|X|) = E(\sqrt{x_1^2+x_2^2+x_3^2}) \leq E(x_1+x_2+x_3) \leq W(0)e^{-d_2t} + M_3(1 - e^{-d_2t}),
$$

i.e. $E(|X|)$ have supper bound. To proceed, applying the Chebyshev inequality yields the required assertion.

Next, we give the existence of a periodic Markov process of the system (2.6). For convenience, we introduce the notations

$$r_3(t) = r_0(t) + \frac{\ln \prod_{j=1}^{p}(1+\delta_j)}{T},\ f_1(t) = r_1(t) - c_1(t)g_1^*(t) - \sigma_1^2(t)(g_1^*(t))^2 - b_1(t) - \frac{\alpha_1(t)}{\beta(t)},$$

$$f_2(t) = b_1(t) - d_2(t) - c_2(t)g_2^*(t) - \sigma_2(t)^2(g_2^*)^2(t),\ f_3(t) = r_3(t) - c_3(t)g_3^*(t) - \sigma_3^2(t)(g_3^*(t))^2.$$

Theorem 4.3. *If $\langle f_1(t)\rangle_T > 0,\ \langle f_2(t)\rangle_T > 0,\ \langle f_3(t)\rangle_T > 0$, then the system (2.6) exists one positive T-periodic solution.*

Proof. By the same way as in Theorem 4.1, one can see that, for any initial $\big(x_1(0), x_2(0), x_3(0)\big) \in R_+^3$, the system (2.6) has a unique global positive solution $\big(x_1(t), x_2(t), x_3(t)\big) \in R_+^3$, we only need to verify the conditions $(Q1)$, $(Q2)$ of Lemma 3.1.

Define a $C^{2,1}$-function $V(x_1, x_2, x_3, t) : R_+^3 \times R_+ \to R_+$ as follows

$$\begin{aligned} V(x_1,x_2,x_3,t) =& \frac{e^{w_1(t)}}{x_1} + \frac{\ell_2 e^{w_2(t)}}{x_2} + \frac{e^{w_3(t)}}{x_3} + (x_1+x_2) + (x_1 + k_3(t))x_3^2 \\ =& V_1 + \ell_2 V_2 + V_3 + V_4 + V_5, \end{aligned}$$

where

$$\begin{aligned} w_1'(t) =& f_1(t) - \langle f_1(t)\rangle_T,\ w_2'(t) = f_2(t) - \langle f_2(t)\rangle_T,\ w_3'(t) = f_3(t) - \langle f_3(t)\rangle_T, \\ V_1 =& \frac{e^{w_1(t)}}{x_1},\ V_2 = \frac{e^{w_2(t)}}{x_2},\ V_3 = \frac{e^{w_3(t)}}{x_3},\ V_4 = x_1 + x_2,\ V_5 = (x_1 + k_3(t))x_3^2, \end{aligned}$$

and $0 < \ell_2 < (\frac{e^{w_1(t)}\langle f_1(t)\rangle_T}{b_1 e^{w_2(t)}})^l$.

Obviously, $V(x_1, x_2, x_3, t)$ is T-periodic in t and satisfies

$$\liminf_{k\to\infty,\ (x_1,x_2,x_3)\in R_+^3 \setminus D_k} V(x_1,x_2,x_3,t) \to +\infty,$$

where $D_k = (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k)$, which shows that $(Q1)$ in Lemma 3.1 holds.

By It$\hat{o}$'s formula, one can get that

$$
\begin{aligned}
LV_1 =& \frac{e^{w_1(t)}}{x_1} w_1'(t) + e^{w_1(t)} \sigma_1^2(t)(g_1^*)^2(t) x_1^2 \\
& - \frac{e^{w_1(t)}}{x_1} \big(r_1(t) - a_1(t)x_1 - \frac{b_1(t)x_2}{x_1+x_2} - \frac{\alpha_1(t)A(t)x_3}{\gamma(t)+\alpha(t)x_1+\beta(t)A(t)x_3} - c_1(t)g_1^*(t) \big) \\
\leq & \frac{e^{w_1(t)}}{x_1} (w_1'(t) - r_1(t) + c_1(t)g_1^*(t) + \sigma_1^2(t)(g_1^*)^2(t)) + a_1(t)e^{w_1(t)} + \frac{b_1(t)e^{w_1(t)}}{x_1} + \frac{\alpha_1(t)}{\beta(t)x_1} \\
=& \frac{e^{w_1(t)}}{x_1} (w_1'(t) - r_1(t) + c_1(t)g_1^*(t) + \sigma_1^2(t)(g_1^*)^2(t) + b_1(t) + \frac{\alpha_1(t)}{\beta(t)}) + a_1(t)e^{w_1(t)} \\
=& - \frac{e^{w_1(t)} \langle f_1(t) \rangle_T}{x_1} + a_1(t)e^{w_1(t)},
\end{aligned}
$$

similarly

$$
\begin{aligned}
LV_2 =& \frac{e^{w_2(t)}}{x_2} w_2'(t) - \frac{e^{w_2(t)}}{x_2} (-d_2(t) - c_2(t)g_2^*(t) + \frac{b_1(t)x_1}{x_1+x_2}) + \frac{e^{w_2(t)}\sigma_2^2(t)(g_2^*)^2(t)}{x_2} \\
=& \frac{e^{w_2(t)}}{x_2} (w_2'(t) + d_2(t) + c_2(t)g_2^*(t) + \sigma_2^2(t)(g_2^*)^2(t)) - \frac{b_1(t)x_1}{(x_1+x_2)x_2} e^{w_2(t)} \\
=& \frac{e^{w_2(t)}}{x_2} (w_2'(t) + d_2(t) + c_2(t)g_2^*(t) + \sigma_2^2(t)(g_2^*)^2(t)) - (\frac{b_1(t)}{x_2} - \frac{b_1(t)}{x_1+x_2}) e^{w_2(t)} \\
\leq & \frac{e^{w_2(t)}}{x_2} (w_2'(t) + d_2(t) + c_2(t)g_2^*(t) + \sigma_2^2(t)(g_2^*)^2(t) - b_1(t)) + \frac{b_1(t)}{x_1} e^{w_2(t)} \\
=& - \frac{e^{w_2(t)} \langle f_2(t) \rangle_T}{x_2} + \frac{b_1(t)}{x_1} e^{w_2(t)},
\end{aligned}
$$

and

$$
\begin{aligned}
LV_3 =& \frac{e^{w_3(t)}}{x_2} w_3'(t) - \frac{e^{w_3(t)}}{x_3} (r_3(t) - \frac{\alpha_3(t)A(t)x_3}{k_3(t)+x_1} - c_3(t)g_3^*(t)) + \frac{e^{w_3(t)}\sigma_3^2(t)(g_3^*)^2(t)}{x_3} \\
\leq & \frac{e^{w_3}(t)}{x_3} (w_3'(t) - r_3(t) + c_3(t)g_3^*(t) + \sigma_3^2(t)(g_3^*)^2(t)) + \frac{\alpha_3(t)A(t)e^{w_3(t)}}{k_3(t)} \\
=& - \frac{e^{w_3(t)} \langle f_3(t) \rangle_T}{x_3} + \frac{\alpha_3(t)A(t)e^{w_3(t)}}{k_3(t)},
\end{aligned}
$$

and

$$
LV_4 \leq -a_1(t)x_1^2 + (r_1(t) - c_1(t)g_1^*(t))x_1 - (d_2(t) + c_2(t)g_2^*(t))x_2,
$$

moreover, we also have

$$\begin{aligned}
LV_5 =& x_1x_3^2(r_1(t)-a_1(t)x_1-\frac{b_1(t)x_2}{x_1+x_2}-\frac{\alpha_1(t)A(t)x_3}{\gamma(t)+\alpha(t)x_1+\beta(t)A(t)x_3}-c_1(t)g_1^*(t)) \\
&+2(k_3(t)+x_1)x_3^2(r_3(t)-c_3(t)g_3(t)^*(t)-\frac{\alpha_3(t)A(t)x_3}{k_3(t)+x_1})+(x_1+k_3(t))\sigma_3^2(t)(g_3^*)^2(t)x_3^2 \\
\leq& -2\alpha_3(t)A(t)x_3^3-a_1(t)x_1^2x_3^2+(r_1(t)-c_1(t)g_1^*(t))x_1x_3^2+2k_3(t)(r_3(t)-c_3(t)g_3^*(t))x_3^2 \\
&+2(r_3(t)-c_3(t)g_3^*(t))x_1x_3^2+k_3(t)\sigma_3^2(t)(g_3^*)^2(t)x_3^2 \\
=& -2\alpha_3(t)A(t)x_3^3-a_1(t)x_1^2x_3^2+(r_1(t)-c_1(t)g_1^*(t)+2r_3(t)-2c_3(t)g_3^*(t) \\
&+\sigma_3^2(t)(g_3^*)^2(t))x_1x_3^2+(2k_3(t)(r_3(t)-c_3(t)g_3^*(t))+k_3(t)\sigma_3^2(t)(g_3^*)^2(t))x_3^2.
\end{aligned}$$

Similar to the proof of inequation (4.2), one can see that

$$\begin{aligned}
&-\frac{1}{2}a_1(t)x_1^2+\big(r_1(t)-c_1(t)g_1^*(t)\big)x_1-\alpha_3(t)A(t)x_3^3-a_1(t)x_1^2x_3^2+\big(r_1(t)-c_1(t)g_1^*(t)+2r_3(t) \\
&-2c_3(t)g_3^*(t)+\sigma_3^2(t)(g_3^*)^2(t)\big)x_1x_3^2+\big(2k_3(t)(r_3(t)-c_3(t)g_3^*(t))+k_3(t)\sigma_3^2(t)(g_3^*)^2(t)\big)x_3^2 \\
&+a_1(t)+\frac{\alpha_3(t)A(t)}{k_3(t)}
\end{aligned}$$

also have supper bound. Let

$$\begin{aligned}
M_6 = \max\{&-\frac{1}{2}a_1(t)x_1^2+\big(r_1(t)-c_1(t)g_1^*(t)\big)x_1-\alpha_3(t)A(t)x_3^3-a_1(t)x_1^2x_3^2 \\
&+\big(r_1(t)-c_1(t)g_1^*(t)+2r_3(t)-2c_3(t)g_3^*(t)+\sigma_3^2(t)(g_3^*)^2(t)\big)x_1x_3^2 \\
&+\big(2k_3(t)(r_3(t)-c_3(t)g_3^*(t))+k_3(t)\sigma_3^2(t)(g_3^*)^2(t)\big)x_3^2+a_1(t)+\frac{\alpha_3(t)A(t)}{k_3(t)},0\},
\end{aligned}$$

one can get that

$$\begin{aligned}
LV \leq& -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T-\ell_2b_1(t)e^{w_2(t)}}{x_1}-\frac{\ell_2e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2}-\frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3} \\
&-\frac{1}{2}a_1x_1^2-(d_2+c_2g_2^*)x_2-\alpha_3Ax_3^3+M_6.
\end{aligned}$$

Consider the bounded open subset

$$D_{\varepsilon_{1,2,3}} = \{(x_1,x_2,x_3)|\varepsilon_1<x_1<\frac{1}{\varepsilon_1},\varepsilon_2<x_2<\frac{1}{\varepsilon_2},\varepsilon_3<x_3<\frac{1}{\varepsilon_3}\},$$

where $0 < \varepsilon_i < 1$ is a sufficiently small number. In the set $D^C_{\varepsilon_{1,2,3}} = R^3_+ \setminus D_{\varepsilon_{1,2,3}}$, let us choose sufficiently small ε_i such that

$$\varepsilon_1 \leq \min\{\frac{(e^{w_1}\langle f_1(t)\rangle_T - \ell_2 b_1 e^{w_2})^l}{M_6+1}, \frac{\sqrt{\frac{1}{2}a_1^l}}{\sqrt{M_6+1}}\},\ \varepsilon_2 \leq \min\{\frac{\ell_2 e^{w_2}\langle f_2(t)\rangle_T^l}{M_6+1}, \frac{(d_2+c_2(g_2^*))^l}{M_6+1}\},$$
$$\varepsilon_3 \leq \min\{\frac{e^{w_3}\langle f_3(t)\rangle_T^l}{M_6+1}, \frac{\sqrt[3]{\alpha_3^l A^l}}{\sqrt[3]{M_6+1}}\}.$$

For convenience, we divide $D^C_{\varepsilon_{1,2,3}}$ into six domains,

$$\begin{aligned}
&D_1 = \{(x_1,x_2,x_3) \in R^3_+ | 0 < x_1 < \varepsilon_1\}, \quad D_2 = \{(x_1,x_2,x_3) \in R^3_+ | 0 < x_2 < \varepsilon_2\},\\
&D_3 = \{(x_1,x_2,x_3) \in R^3_+ | x_1 \geq \varepsilon_1, 0 < x_3 < \varepsilon_3\}, \quad D_4 = \{(x_1,x_2,x_3) \in R^3_+ | x_1 > \tfrac{1}{\varepsilon_1}\},\\
&D_5 = \{(x_1,x_2,x_3) \in R^3_+ | x_2 > \tfrac{1}{\varepsilon_2}\}, \quad D_6 = \{(x_1,x_2,x_3) \in R^3_+ | x_1 > \tfrac{1}{\varepsilon_3}\},
\end{aligned}$$

clearly, $D^C_{\varepsilon_{1,2,3}} = D_1 \cup D_2 \cup D_3 \cup D_4 \cup D_5 \cup D_6$. Now we prove $LV(x_1,x_2,x_3,t) \leq -1$ on $D^C_{\varepsilon_{1,2,3}} \times R$, which is equivalent to showing it on the above six domains.

Case 1. On domain D_1, we get

$$\begin{aligned}
LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t) e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3}\\
& -\frac{1}{2}a_1 x_1^2 - (d_2 + c_2 g_2^*)x_2 - \alpha_3 A x_3^3 + M_6\\
\leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t) e^{w_2(t)}}{\varepsilon_1} + M_6\\
\leq & -1.
\end{aligned}$$

Case 2. Similarly, for any $(x_1,x_2,x_3) \in D_2$, one can see that

$$\begin{aligned}
LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t) e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3}\\
& -\frac{1}{2}a_1 x_1^2 - (d_2 + c_2 g_2^*)x_2 - \alpha_3 A x_3^3 + M_6\\
\leq & -\frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{\varepsilon_2} + M_6\\
\leq & -1.
\end{aligned}$$

Case 3. on D_3 we derive

$$\begin{aligned}LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t)e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3} \\ & -\frac{1}{2}a_1x_1^2 - (d_2 + c_2g_2^*)x_2 - \alpha_3Ax_3^3 + M_6 \\ \leq & -\frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{\varepsilon_3} + M_6 \\ \leq & -1.\end{aligned}$$

Case 4. on D_4 one can get that

$$\begin{aligned}LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t)e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3} \\ & -\frac{1}{2}a_1x_1^2 - (d_2 + c_2g_2^*)x_2 - \alpha_3Ax_3^3 + M_6 \\ \leq & -\frac{1}{2}a_1\varepsilon_1^{-2} + M_6 \\ \leq & -1.\end{aligned}$$

Case 5. on D_5 yields

$$\begin{aligned}LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t)e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3} \\ & -\frac{1}{2}a_1x_1^2 - (d_2 + c_2g_2^*)x_2 - \alpha_3Ax_3^3 + M_6 \\ \leq & -(d_2 + c_2g_2^*)\varepsilon_2^{-1} + M_6 \\ \leq & -1.\end{aligned}$$

Case 6. on D_6 we obtain

$$\begin{aligned}LV \leq & -\frac{e^{w_1(t)}\langle f_1(t)\rangle_T - \ell_2 b_1(t)e^{w_2(t)}}{x_1} - \frac{\ell_2 e^{w_2(t)}\langle f_2(t)\rangle_T}{x_2} - \frac{e^{w_3(t)}\langle f_3(t)\rangle_T}{x_3} \\ & -\frac{1}{2}a_1x_1^2 - (d_2 + c_2g_2^*)x_2 - \alpha_3Ax_3^3 + M_6 \\ \leq & -\alpha_3A\varepsilon_3^{-3} + M_6 \\ \leq & -1.\end{aligned}$$

Consequently

$$LV(x_1,x_2,x_3,t) \leq -1, \quad for\ \forall\ (x_1,x_2,x_3,t) \in D^C_{\varepsilon_{1,2,3}} \times R.$$

That is, the condition $(Q2)$ in Lemma 3.1 holds. Hence the proof of this theorem is completed.

Corollary 4.1. *If* $\langle r_1 - c_1 g_1^*(t) - \sigma_1^2\big(g_1^*(t)\big)^2 - b_1 - \frac{\alpha_1}{\beta}\rangle_T > 0$, $\langle b_1 - d_2 - c_2 g_2^*(t) - \sigma_2^2\big(g_2^*(t)\big)^2\rangle_T > 0$, $\langle r_3 - c_3 g_3^*(t) - \sigma_3^2\big(g_3^*(t)\big)^2\rangle_T > 0$, *then the system (2.5) exists one positive T-periodic solution.*

Theorem 4.4. *Let* $(x_1(t), x_2(t), x_3(t))$ *be a positive solution of system (2.5) with initial value* $(x_1(0), x_2(0), x_3(0))$. *Then if* $r_3 > c_3 g_3^*(t)$, $r_3 > \langle c_3 g_3^*(t) + \frac{1}{2}\sigma_3^2(g_3^*(t))^2\rangle_T$, $a_1\beta\gamma > \alpha\alpha_1$, $b_1 < d_2 + \langle c_2 g_2^*(t) + \frac{1}{2}\sigma_2^2(g_2^*(t))^2\rangle_T$ *and* $r_1 < \frac{\alpha_1}{\beta} + \langle c_1 g_1^*(t) + \frac{1}{2}\sigma_1^2(g_1^*(t))^2 - \frac{\alpha_1\gamma}{\beta(\gamma+\beta A z^T(t))}\rangle_T$ *then the system (2.5) has a boundary T-periodic solution* $(0, 0, z^T(t))$, *which is globally attractive.*

Proof. By the third function of (2.5), yields

$$\begin{aligned}
dx_3 =& x_3(r_3 - \frac{\alpha_3 A x_3}{k_3 + x_1} - c_3 g_3^*)dt + \sigma_3 g_3^* x_3 dB_3 \\
\geq& x_3(r_3 - \frac{\alpha_3 A x_3}{k_3} - c_3 g_3^*)dt + \sigma_3 g_3^* x_3 dB_3.
\end{aligned}$$

By Remark 3.1 and comparison theorem for stochastic differential equations, one can get that

$$x_3(t) \geq z(t) \quad \text{a.s.},$$

moreover for arbitrary small $\varepsilon > 0$, there exist t_0 and a set Ω_ε such that $P(\Omega_\varepsilon) \geq 1 - \varepsilon$ and $z(t) > z^T(t) - \varepsilon$ almost surely. Hence

$$x_3(t) \geq z^T(t) - \varepsilon \quad \text{a.s.}.$$

By the Itô's formula, one can get that

$$\begin{aligned}
d\ln x_1 =& (r_1 - a_1 x_1 - \frac{b_1 x_2}{x_1 + x_2} - \frac{\alpha_1 A x_3}{\gamma + \alpha x_1 + \beta A x_3} - c_1 g_1^*)dt + \sigma_1 g_1^* dB_1 \\
\leq& (r_1 - a_1 x_1 - \frac{\alpha_1}{\beta} + \frac{\alpha_1(\alpha x_1 + \gamma)}{\beta(\gamma + \alpha x_1 + \beta A x_3)} - c_1 g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2)dt + \sigma_1 g_1^* dB_1 \\
=& (r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2 - \frac{a_1\beta x_1(\gamma + \alpha x_1 + \beta A x_3) - \alpha_1(\alpha x_1 + \gamma)}{\beta(\gamma + \alpha x_1 + \beta A x_3)})dt + \sigma_1 g_1^* dB_1 \\
\leq& (r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma + \beta A(z^T(t) - \varepsilon))})dt + \sigma_1 g_1^* dB_1.
\end{aligned}$$

Integrating both sides from 0 to t and dividing by t, yields

$$\frac{\ln x_1(t) - \ln x_1(0)}{t} \leq \frac{\int_0^t (r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2(g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma+\beta A(z^T(t)-\varepsilon))})dt + \int_0^t \sigma_1 g_1^* dB_1}{t}.$$

In view of the strong law of large numbers for martingales, we obtain $\limsup_{t\to\infty} \frac{\int_0^t \sigma_1 g_1^* dB_1}{t} = 0$, and then

$$\lim_{t\to\infty} \frac{\ln x_1(t)}{t} \leq \lim_{t\to\infty} \frac{\int_0^t [r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2 (g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma+\beta A(z^T(t)-\varepsilon))}]dt}{t}. \tag{4.3}$$

On the other hand, since r_1, α_1, β, c_1, σ_1, γ are all positive constant and $g_1^*(t)$, $A(t)$ are both positive T-periodic continuous functions, therefore

$$\begin{aligned}&\lim_{t\to\infty} \frac{\int_0^t [r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2 (g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma+\beta A(z^T(t)-\varepsilon))}]dt}{t} \\ &= \frac{\int_0^T [r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2 (g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma+\beta A(z^T(t)-\varepsilon))}]dt}{T}.\end{aligned}$$

For the arbitrary of ε, we must have

$$\lim_{t\to\infty} \frac{\ln x_1(t)}{t} \leq \frac{\int_0^T [r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^* - \frac{1}{2}\sigma_1^2 (g_1^*)^2 + \frac{\alpha_1\gamma}{\beta(\gamma+\beta A z^T(t))}]dt}{T} < 0 \quad \text{almost surely, i.e.}$$

$$\lim_{t\to\infty} x_1(t) = 0 \quad a.s..$$

In view of Itô's formula, one can get that

$$\begin{aligned} d\ln x_2(t) =& (-d_2 - c_2 g_2^* - \frac{1}{2}\sigma_2^2 (g_2^*)^2 + \frac{b_1 x_1}{x_1 + x_2})dt + \sigma_2 g_2^* dB_2 \\ \leq& (-d_2 - c_2 g_2^* - \frac{1}{2}\sigma_2^2 (g_2^*)^2 + b_1)dt + \sigma_2 g_2^* dB_2, \end{aligned}$$

Similarly, one can see that

$$\lim_{t\to\infty} \frac{\ln x_2(t)}{t} \leq \frac{\int_0^T (-d_2 - c_2 g_2^* - \frac{1}{2}\sigma_2^2 (g_2^*)^2 + b_1)dt}{T} < 0 \quad \text{almost surely, i.e.}$$

$$\lim_{t\to\infty} x_2(t) = 0 \quad \text{a.s..}$$

On the other hand, for arbitrary small $\varepsilon_3 > 0$, there exists $t_2 > t_0$ and a set Ω_{ε_3} such that $P(\Omega_{\varepsilon_3}) \geq 1 - \varepsilon_3$ and $x_1 < \varepsilon_3$ for $t > t_2$ and $\omega \in \Omega$.

Consider the periodic Logistic equation

$$\begin{cases} d\phi = \phi(r_3 - \frac{\alpha_3 A \phi}{k_3 + \varepsilon_3} - c_3 g_3^*)dt + \sigma_3 g_3^* \phi dB_3, \\ \phi(0) = x_3(0), \end{cases} \tag{4.4}$$

obviously,

$$\phi(t) = \frac{e^{\int_0^t (r_3 - c_3 g_3^* - \frac{1}{2}\sigma_3^2 (g_3^*)^2)ds + \int_0^t \sigma_3 g_3^* dB_1(s)}}{\frac{1}{x_3(0)} + \int_0^t \frac{\alpha_3 A}{k_3+\varepsilon_3} e^{\int_0^\tau (r_3 - c_3 g_3^* - \frac{1}{2}\sigma_3^2 (g_3^*)^2)ds + \int_0^\tau \sigma_3 g_3^* dB_1(s)} d\tau},$$

is the solution to the equation (5.4). By the comparison theorem for stochastic equations, yields

$$z(t) \leq x_3(t) \leq \phi(t) \quad \text{a.s..}$$

For the arbitrary of ε_3, we must have

$$\lim_{t\to\infty} \phi(t) = z(t) \quad \text{a.s..}$$

The result is confirmed.

5. Numerical simulations and conclusion

In this paper, in order to investigate the consequences of periodically spraying pesticides and releasing natural enemies at different fixed moment in pest-natural enemy system, a stochastic SI epidemic model for pest management concerning spraying pesticide and releasing natural enemies is proposed. For the stochastic system (2.5), the existence and uniqueness of the positive global solution are obtained. Moreover, the positive solution is stochastically ultimately bounded is proved, it is known to all that the stochastic boundedness is one of most important topics because boundedness of a system guarantees its validity. we still established the sufficient conditions for global attractiveness of the pest-extinction periodic solution have been obtained, which shows that there exists a globally asymptotically stable pest-eradication periodic solution under certain parametric restrictions.

In addition, in view of the factors, such as seasonal variation, weather changes, food supplies and so on, we investigate a periodic system (2.6) with stochastic disturbance. The result shows that, the system (2.6) has at least one positive periodic system solution under a certain condition. Thus, the difference and connection in dealing with the system (2.5) and system (2.6) are obtained by comparison.

The numerically simulate the solution of stochastic model by the Milstein's Higher Order Method proposed by Higham [21].

Numerical simulations are carried out to investigate effects of impulsive period varying on dynamical behaviors of system (2.1) and (2.2) as well as to illustrate our theoretical results, we choose the parameters $r_1 = 0.55$, $r_0 = 0.4$, $c_1 = 0.465$, $c_2 = 0.5$, $c_3 = 0.3$, $a_1 = 0.1$, $b_1 = 0.3$,

$\alpha_1 = 1$, $\alpha = 4$, $\beta = 10$, $\gamma = 6$, $d_2 = 0.2$, $\alpha_3 = 0.5$, $k_3 = 1$, $\sigma_1 = 0.02$, $\sigma_2 = 0.02$, $\sigma_3 = 0.02$, $p = 0.5$, $l_1 = 0.3$, $m_1 = 0.1$, $n_1 = 0.1$, $l_2 = 0.3$, $m_2 = 0.1$, $n_2 = 0.1$, $l_3 = 0.3$, $m_3 = 0.1$, $n_3 = 0.1$, $q = 1$, $\delta_k = 0.8$, and $\mu = 0.4$.

We start our numerical simulation with $T = 10$ and starting from the initial point $(S(0), I(0), y(0)) = (3.5, 0.55, 6)$, $\left(g_i^*(0), c_e^*(0)\right) = (0.0599, 0.4027)$. By the Lemma 2.1, one can see that

$$g_i^*(t) \approx 0.4626e^{-0.2t} - 0.4027e^{-0.5t},\ \int_0^{10} g_i^*(t)dt \approx 1.2000\ (i = 1,2,3),$$

$$\int_0^{10} r_1 - c_1 g_1^*(t) - \sigma_1^2 (g_1^*(t))^2 - b_1 - \frac{\alpha_1}{\beta} dt > 0,\ \int_0^{10} b_1 - d_2 - c_2 g_2^*(t) - \sigma_2^2 (g_2^*(t))^2 dt > 0,$$

$$\int_0^{10} r_0 + \frac{\ln \prod_{j=1}^{1} (1+\delta_j)}{10} - c_3 g_3^*(t) - \sigma_3^2 (g_3^*(t))^2 dt > 0.$$

Obviously, parameter values chosen above and the choice of T are consistent with the conditions required for the existence of periodic solution (see Corollary 4.1). Results of two simulation run are reported in Fig 1 and Fig 2.

Next we decrease impulsive period to $T = 1$. By computation, we get

$$g_i^*(0) \approx 1.1901,\ g_i^*(t) \approx 2.2067e^{-0.2t} - 1.0166e^{-0.5t}, \int_0^1 g_i^*(t)dt \approx 1.1996\ (i = 1,2,3),$$

$$r_3 > c_3 g_3^*(t),\ r_3 > \langle c_3 g_3^*(t) + \frac{1}{2}\sigma_3^2 (g_3^*(t))^2 \rangle_T,\ a_1 \beta\gamma > \alpha\alpha_1,$$

$$\langle r_1 - \frac{\alpha_1}{\beta} - c_1 g_1^*(t) - \frac{1}{2}\sigma_1^2 (g_1^*)^2 + \frac{\alpha_1 \gamma}{\beta[\gamma + \beta A(t) z^T(t)]} \rangle_T < 0.$$

Obviously, parameter values chosen above are consistent with the conditions required for existence and global attractivity of the boundary periodic solutions (see Theorem 4.4). Result of one simulation run is reported in Fig 3.

Remark 5.1: From Figs. 1, 2 and 3, we see that, if the impulsive period is larger than some critical value, the concentration of pestcide will not be sufficiently high to kill pest and the pest will be permanent and tends to the unique positive T-periodic solution of system (2.1).

In the following, we give an example to illustrate our result of Theorem 4.3.

Choose parameters $T = 10$, $r_1 = 2 + 0.01\sin(\frac{\pi t}{5})$, $r_0 = 0.94 + 0.01\sin(\frac{\pi t}{5})$, $c_1 = 0.2 + 0.01\sin(\frac{\pi t}{5})$, $c_2 = 0.2 + 0.01\sin(\frac{\pi t}{5})$, $c_3 = 0.1 + 0.01\sin(\frac{\pi t}{5})$, $a_1 = 0.1 + 0.01\sin(\frac{\pi t}{5})$, $b_1 = 0.5 + 0.01\sin(\frac{\pi t}{5})$, $\alpha_1 = 1 + 0.01\sin(\frac{\pi t}{5})$, $\alpha = 4 + 0.01\sin(\frac{\pi t}{5})$, $\beta = 2 + 0.01\sin(\frac{\pi t}{5})$, $\gamma = 6 + 0.01\sin(\frac{\pi t}{5})$, $d_2 =$

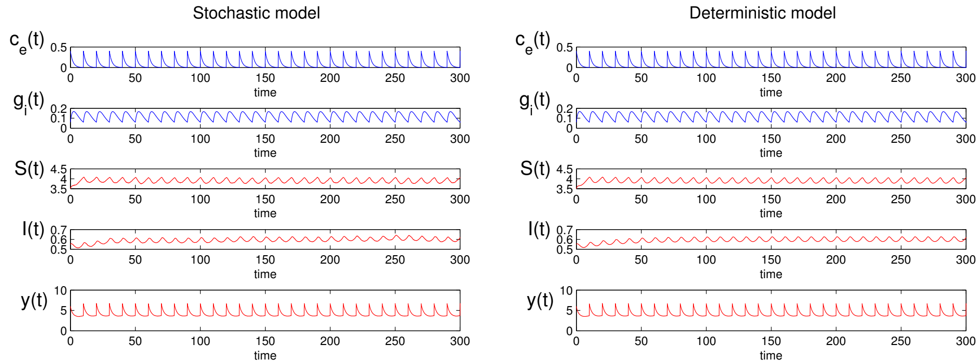

FIGURE 1. The solutions of system (2.1), system (2.2) and their corresponding deterministic system with initial conditions $(S(0),I(0),y(0)) = (3.5,0.55,6)$, $\big(g_i(0),c_e(0)\big) = (0.0599,0.4027)$.

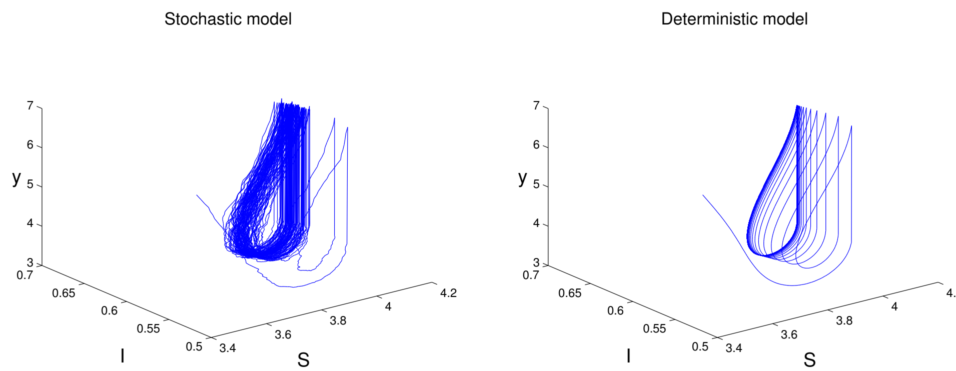

FIGURE 2. Sample phase portrait of system (2.1), system (2.1) and their corresponding deterministic system with initial conditions $(S(0),I(0),y(0)) = (3.5,0.55,6)$, $\big(g_i(0),c_e(0)\big) = (0.0599,0.4027)$.

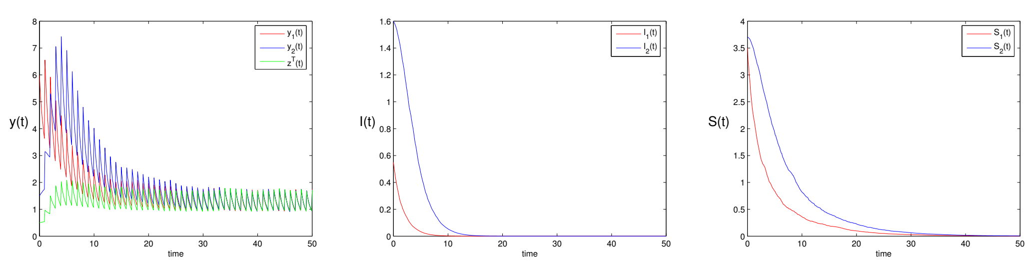

FIGURE 3. The solutions $X_1(t) = (S_1(t),I_1(t),y_1(t))$, $X_2(t) = (S_2(t),I_2(t),y_2(t))$ and $X_3(t) = (0,0,z^T(t))$ of system (2.1) with the initial value $(3.5,0.55,6)$, $(3.7,1.6,1.5)$ and $(0,0,0.5)$, respectively. $X_1(t)$ and $X_2(t)$ both are attracted the pest-extinction periodic solution $(0,0,z^T(t))$.

$0.3+0.01\sin(\frac{\pi t}{5})$, $\alpha_3 = 0.5+0.01\sin(\frac{\pi t}{5})$, $k_3 = 1+\sin(\frac{\pi t}{5})$, $\sigma_1 = 0.3+0.001\sin(\frac{\pi t}{5})$, $\sigma_2 = 0.3+0.001\sin(\frac{\pi t}{5})$, $\sigma_3 = 0.3+0.001\sin(\frac{\pi t}{5})$, $p=1$, $\mu = 0.01$, $l_1 = 0.6$, $m_1 = 0.01$, $n_1 = 0.01$, $l_2 = 0.6$, $m_2 = 0.01$, $n_2 = 0.01$, $l_3 = 0.6$, $m_3 = 0.01$, $n_3 = 0.01$, $q = 1$ and $\delta_k = 0.01$.

By the Lemma 2.1, one can see that

$$g_i^*(0) = \frac{l_i\mu(e^{-(m_i+n_i)T} - e^{-pT})}{(p-m_i-n_i)(1-e^{-(m_i+n_i)T})(1-e^{-pT})} \approx 0.0277,$$

therefore

$$g_i^*(t) = g_i^*(0)e^{-(m_i+n_i)(t-nT)} + \frac{l_i\mu(e^{-(m_i+n_i)(t-nT)} - e^{-p(t-nT)})}{(p-m_i-n_i)(1-e^{-pT})} \leq 0.0388.$$

By computation, one can get that

$\langle f_1(t)\rangle_T = \langle r_1(t) - c_1(t)g_1^*(t) - \sigma_1^2(t)(g_1^*(t))^2 - b_1(t) - \frac{\alpha_1(t)}{\beta(t)}\rangle_T > 0$,

$\langle f_2(t)\rangle_T = \langle b_1(t) - d_2(t) - c_2(t)g_2^*(t) - \sigma_2(t)^2(g_2^*)^2(t)\rangle_T > 0$,

$\langle f_3(t)\rangle_T = \langle r_0(t) + \frac{\ln\prod_{j=1}^{q}(1+\delta_j)}{T} - c_3(t)g_3^*(t) - \sigma_3^2(t)(g_3^*(t))^2\rangle_T > 0$, by Theorem 4.3, we know that the system (2.6) exists one positive T-periodic solution. (see Figs 4 and 5.)

Remark 5.2: From Figs. 1,2 and 4,5, we see that, for any positive initial value, the solution of the deterministic system will enter the periodic orbit after a period of time, and the solution of the stochastic system is fluctuating in a small neighborhood of the periodic orbit when the noise intensity is small.

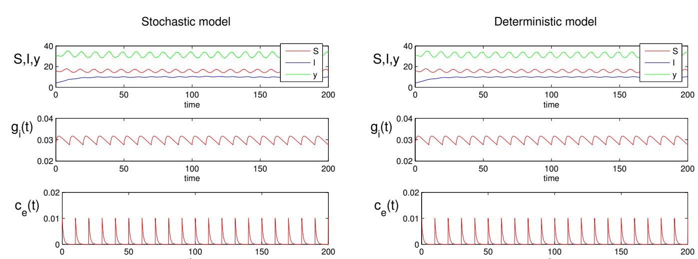

FIGURE 4. The solutions of system (2.3), system (2.2) and their corresponding deterministic system with initial conditions $(S(0), I(0), y(0)) = (15, 4, 30)$, $\big(g_i(0), c_e(0)\big) = (0.0277, 0.0100)$.

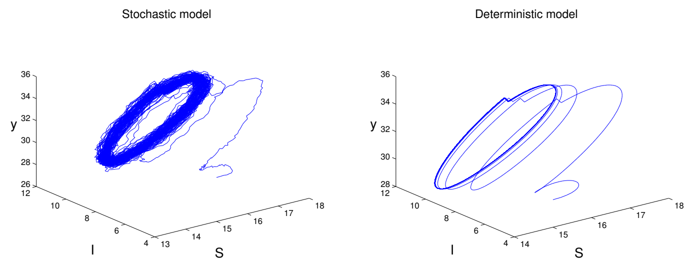

FIGURE 5. Sample phase portrait of system (2.1), system (2.1) and their corresponding deterministic system with initial conditions $(S(0), I(0), y(0)) = (15, 4, 30)$, $\left(g_i(0), c_e(0)\right) = (0.0277, 0.0100)$.

Conflict of Interests

The authors declare that there is no conflict of interests.

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