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Commun. Math. Biol. Neurosci. 2022, 2022:9

<https://doi.org/10.28919/cmbn/6357>

ISSN: 2052-2541

STABILITY ANALYSIS AND MODELLING THE DYNAMICS OF PSITTACOSIS IN HUMAN AND POULTRY POPULATIONS

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Abstract. Psittacosis is a disease in human beings that is commonly associated with pet birds such as cockatiels and parrots, and among poultry such as ducks and turkeys. This paper proposed and developed a deterministic epidemiological model that explains the transmission dynamics of Psittacosis infection in humans and poultry. The Psittacosis deterministic model was analyzed to determine positivity of the solution set, the invariant feasible region, the basic reproduction number, the disease free equilibrium points, the endemic equilibrium points and the corresponding stability of each of the equilibrium points. The basic reproduction number is calculated using the next generation matrix and it was found to be entirely dependent on the poultry population parameters. The study established that whenever $R_0 < 1$, Psittacosis dies out from the population and when $R_0 > 1$, Psittacosis keeps persisting in the population. Sensitivity analysis was conducted to determine the contribution of each parameter to the basic reproduction number. The more sensitive parameters were found to be responsible for the further propagation of Psittacosis while the less sensitively parameters rarely contributed to the spread. Stability analysis of both the disease free equilibrium and the Endemic equilibrium were conducted. Lastly, numerical simulation was conducted to justify quantitative analysis of the dynamics of the transmission of Psittacosis.

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Received June 15, 2021

Keywords: psittacosis; reproduction number; equilibrium points; stability; sensitivity analysis.

2010 AMS Subject Classification: 92D25.

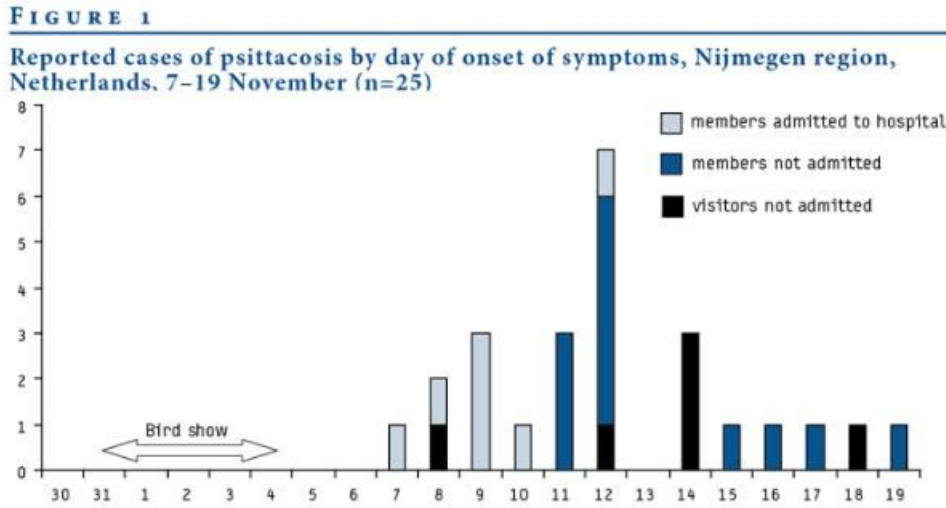
1. INTRODUCTION

Psittacosis is a zoonotic infectious disease caused by gram-negative obligate intracellular bacteria *Chlamydia psittaci*. It is also known as ornithosis or parrot fever since birds are the major epidemiological reservoirs. The disease has been documented in 467 species from 30 different orders of birds however, birds from the order of Galliformes such as turkeys, chicken and pheasants and those from the order of Psittaciformes such as budgerigars, lorries, cockatoos, parakeets, and parrots are often identified as the major epidemiological reservoirs [1].

Human infections are occasioned by contact with infected pet birds by inhaling the bacteria from contaminated dust from bird feathers, bird secretions, and dried-out droppings. Human-to-human infections are suggested and thought to be rare but there is limited documentation to back the suggestions [2]. Psittacosis has an incubation period of one to four weeks during which an individual develops influenza-like illness. Some of the common symptoms include headache, dry cough, muscle aches, myalgia, rigors, fever, and chills. In some cases, it causes systemic illness which leads to atypical pneumonia which can be fatal [3]. Mostly, antibiotics with intracellular actions are administered as a form of treatment.

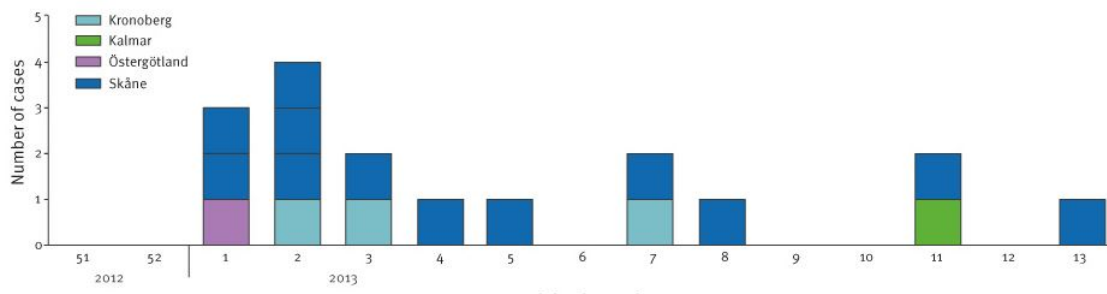
The infective dose of the bacteria is unknown since the infected birds shed the agent intermittently or continuously for weeks or months [3]. The disease infects humans across all ages and genders but is more prevalent in mid-age groups with a peak at the ages of 35 to 55. Adults in constant contact with birds have higher susceptibility such as zoo employees working with avians, poultry farm or processing plant workers, veterinary technicians, and aviary and pet shop employees [3].

Some outbreaks that have been well documented include the local cluster outbreak in the Netherlands in November 2007. The outbreak was traced to a bird show that was held in the rural town of Weurt (village of Beuningen) [4]. In this particular case 25 positive cases were recorded leading to cancellation of other bird shows as a precautionary measure. The outbreak is summarized by the graph below



Between January 12th and 9th April 2013 there was another outbreak in Southern Sweden which was attributed to free-living birds reservoirs. During this period there was a total of 25 cases reported which was a spike from the previous years mean of 3.3 cases per annum [2]. The outbreak is illustrated below.

Sporadic psittacosis cases in southern Sweden by week of symptom onset, reported between 12 January and 9 April 2013 (weeks 1–13) (n=17)^a



^a Cases from the human-to-human transmission cluster (n=8) are not shown.

There was an outbreak reported in the United States between August and September 2018 in Virginia. All the hospitalized persons tested positive for *C. psittaci* leading to the suspension of operations at the chicken slaughter house [5]. The suspension was followed by deep cleaning of the facility and inspection to ensure safety of workers. Another, outbreak was reported at in Georgia on September 12 in chicken slaughter house leading to swift measures to contain the situation. Three of the hospitalized patients of symptoms of psittacosis tested positive for *C.*

psittaci [5].

New South Wales reported an outbreak in November 2014 at a veterinary school and a local equine stud. All these cases originated from exposure to an equine fetal membrane of Mare A, which subsequently tested positive for *Chlamydia psittaci*. A cohort study of those exposed revealed that five were infected with psittacosis arising from the exposure [6]. In particular, this exposure was unique since there were no birds involved. As such, it was a clear indicator of colonization of *Chlamydia psittaci* in mammals.

Mathematical models usually explain the dynamics of the transmission of diseases and can predict the spread or die out of the infections in the system with time [7, 8, 9, 10, 11].

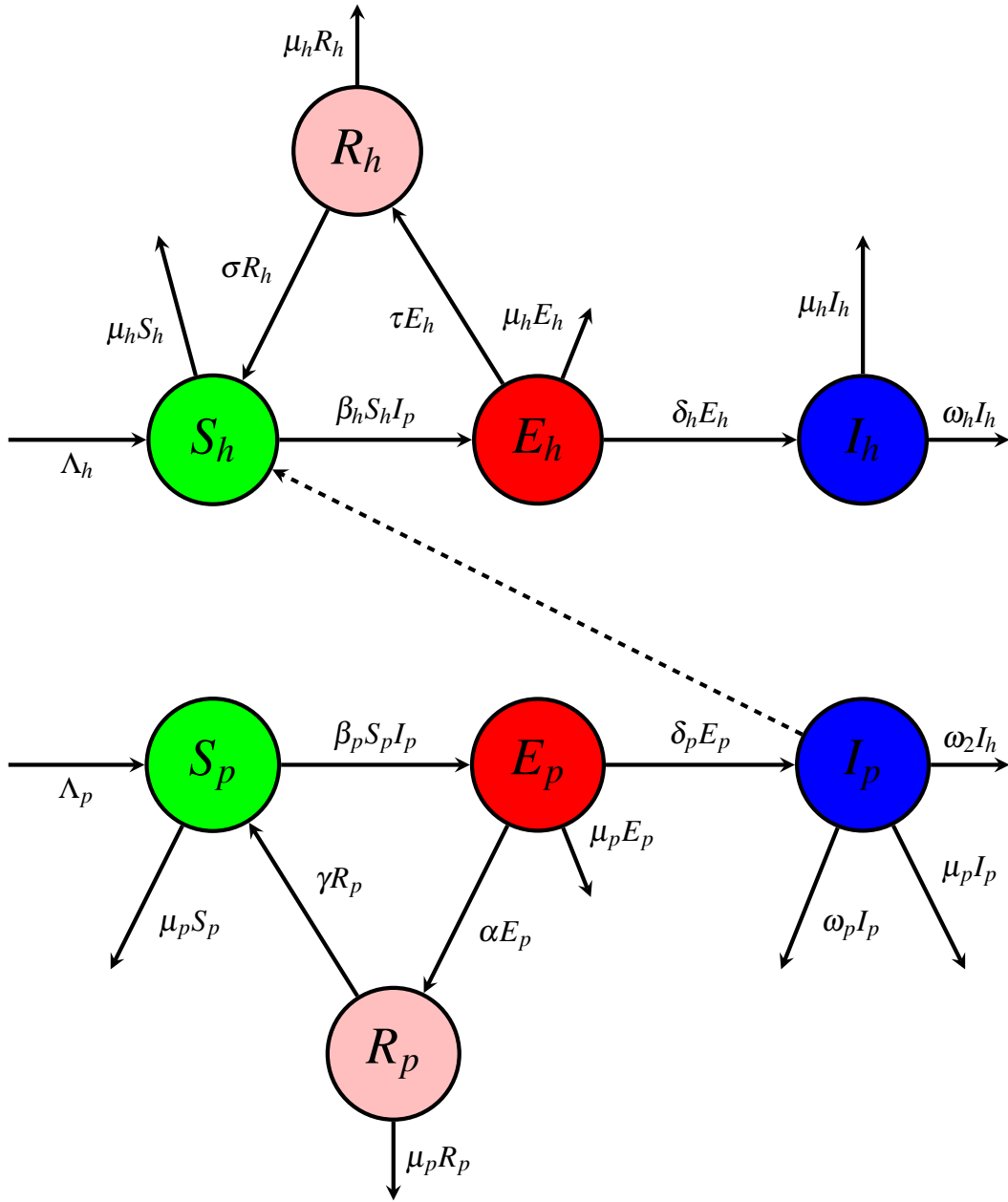
1.1. Model description and formulation.

1.1.1. Model Formation. In this model we consider population of turkeys and human. Each population is subdivided into four compartments; susceptible, exposed, infected and recovered. Susceptible humans are recruited at a rate Λ_h either by birth or immigration, and their number increase from individuals that come from sub-classes of psittacosis recovered by losing their temporary immunity with rate of σ and decrease by individuals that move to exposed compartments at a rate of β_h and natural death rate with rate μ_h . Some exposed human population move to infected compartment at the rate of δ_h and the remaining exposed human population who get the drug compartment join the recovered compartment at a rate of τ . The infected human compartment decrease both by natural and psittacosis induced death rate μ_h and ω_h respectively.

Susceptible turkeys are recruited at a rate Λ_p either by birth or immigration, and their number increases from individuals that come from sub-classes of psittacosis recovered by losing their temporary immunity with rate of γ and their number decrease by individuals that move to exposed compartments with rate of β_p and natural death rate with rate μ_p . The exposed population of turkeys which get drug go to the recovery by rate of α and the remaining which not get the drug with time go to the infected class. The infected population of turkeys reduces by natural death (μ_p), diseased induce rate (ω_p) and the removing parameter (ω_2). Total human population is given by; $N(t) = S(t) + E(t) + I(t) + R(t)$

The dynamics of psittacosis transmission in human and turkeys population is represented in the schematic diagram as shown in Fig: 1

FIGURE 1. Schematic diagram of the model



1.1.2. Model Equations. Based on the assumptions and interrelation between the variables and parameters in figure 1, the following system of ordinary differential equation generated.

$$(1) \quad \left\{ \begin{array}{l} \frac{dS_h}{dt} = \Lambda_h + \sigma R_h - (\beta_h I_p + \mu_h) S_h \\ \frac{dE_h}{dt} = \beta_h S_h I_p - (\delta_h + \tau + \mu_h) E_h \\ \frac{dI_h}{dt} = \delta_h E_h - (\mu_h + \omega_h) I_h \\ \frac{dR_h}{dt} = \tau E_h - (\sigma + \mu_h) R_h \\ \frac{dS_p}{dt} = \Lambda_p + \gamma R_p - (\beta_p I_p + \mu_p) S_p \\ \frac{dE_p}{dt} = \beta_p S_p I_p - (\delta_p + \mu_p + \alpha) E_p \\ \frac{dI_p}{dt} = \delta_p E_p - (\mu_p + \omega_2 + \omega_p) I_p \\ \frac{dR_p}{dt} = \alpha E_p - (\mu_p + \gamma) R_p \end{array} \right.$$

1.2. Model analysis.

1.2.1. Invariant region. The region in which solutions of psittacosis model system is uniformly bounded is the proper subset $\Omega \in \mathbb{R}^8$ and $\Omega = N_h \cup N_p \in \mathbb{R}^4 \times \mathbb{R}^4$.

Considering the human population at any time t ;

$$N_h = S_h + E_h + I_h + R_h.$$

The feasible solution of human population of model system in equation (1);

$$(2) \quad D_h = \left\{ (S_h, E_h, I_h, R_h) \in \mathbb{R}^4, 0 \leq N_h \leq \frac{\Lambda_h}{\mu_h} \right\}$$

Moreover, considering turkey population, denoted by N_p ;

$$N_p = S_p + E_p + I_p + R_p.$$

The feasible solution of the human population of model system in equation (1) the region

$$D_p = \left\{ (S_p, E_p, I_p, R_p) \in \mathbb{R}^4 : 0 \leq N_p \leq \frac{\Lambda_p}{\mu_p} \right\}$$

Total turkey and human population is given by; $N_M = N_p + N_h$

From equation (1.2.1) and (1.2.1)

$$N_h = S_h + E_h + I_h + R_h$$

$$N_p = S_p + E_p + I_p + R_p$$

$$N_M = S_h + E_h + I_h + R_h + S_p + E_p + I_p + R_p$$

Feasible solution of human population of model system in equation (1);

$$(3) \quad D_p = \left\{ (S_p, E_p, I_p, R_p, S_h, E_h, I_h, R_h) \in \mathbb{R}^4 \times \mathbb{R}^4 = \mathbb{R}^8 : 0 \leq N_Z \leq \frac{\Lambda_N}{\theta_Y} \right\}$$

1.2.2. Positivity of solutions. Theorem 1: Let $S_h(0) > 0$, $E_h(0) \geq 0$, $I_h(0) \geq 0$, $R_h(0) \geq 0$, $S_p(0) \geq 0$, $E_p(0) \geq 0$, $I_p(0) \geq 0$ and $R_p(0) \geq 0$ then the solution set $S_h, E_h, I_h, R_h, S_p, E_p, I_p$ and R_p of the system of the equation (1) are positive for all $t > 0$ [12, 13, 14]

Proof:

$$(4) \quad \frac{dS_h}{dt} = \Lambda_h + \sigma R_h - \beta_h S_h I_h - \mu_h S_h$$

$$\frac{dS_h}{dt} + (\beta_h I_h + \mu_h) S_h = \Lambda_h + \sigma R_h$$

Let $A(t) = e^{-\int (\beta_h I_h + \mu_h) ds}$ be the integrating factor.

$$A(t) = e^{-\int (\beta_h I_h + \mu_h) ds} \left(\int (\Lambda_h + \sigma R_h) ds + C \right) \geq 0$$

$$(5) \quad \frac{dE_h}{dt} = \beta_h S_h I_h - \delta_h E_h - \tau E_h - \mu_h E_h$$

$$(6) \quad \frac{dE_h}{dt} + (\delta_h + \tau + \mu_h) E_h = \beta_h S_h I_h$$

Let $B(t) = e^{-\int (\delta_h + \tau + \mu_h) deh}$ be the integrating factor.

$$(7) \quad \frac{dB(t)}{dt} = B(t) \beta_h S_h I_h$$

$$(8) \quad \int \frac{dB(t)}{dt} = \int (B(t) \beta_h S_h I_h) deh$$

$$(9) \quad B(t) = e^{-\int (\delta_h + \tau + \mu_h) de} \left(\int \beta_h S_h I_h deh + c \right) \geq 0$$

By applying the same approach for; $I_h(t), R_h(t), S_p(t), E_p(t), I_p(t), R_p(t)$

Hence, $S_h(t), E_h(t), I_h(t), R_h(t), S_p(t), E_p(t), I_p(t), R_p(t)$ are positive from equation (1).

1.2.3. Diseases Free Equilibrium point. Disease free equilibrium point of the system in equation (1) in the absence of rabbits infections is determined. $E_h = I_p = E_p = I_h = 0$

From equation (1)

$$(10) \quad \frac{dS_p}{dt} = \lambda_p - \beta_p S_p I_p - \mu_p S_p + \gamma R_p = 0$$

$$(11) \quad S_p^* = \frac{\lambda_p}{\mu_p}$$

$$(12) \quad \frac{dS_h}{dt} = \Lambda_h + \sigma R_h - \beta_h S_h I_p - \mu_h S_h = 0$$

$$(13) \quad S_h^* = \frac{\Lambda_h}{\mu_h}$$

$$(14) \quad DFE = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, 0, \frac{\lambda_p}{\mu_p}, 0, 0, 0 \right)$$

1.2.4. Basic reproductive number (R_0). This is a threshold value that governs the dynamics of rabbits. By employing the "Next Generation Matrix" [15, 10, 16]. Considering;

$$(15) \quad \frac{dE_p}{dt} = \beta_p S_p I_p - \delta_p E_p - \mu_p E_p - \alpha E_p$$

$$(16) \quad \frac{dI_p}{dt} = \delta_p E_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p$$

$$(17) \quad \frac{dE_h}{dt} = \beta_h S_h I_p - \delta_h E_h - \tau E_h - \mu_h E_h$$

$$(18) \quad \frac{dI_h}{dt} = \delta_h E_h - \mu_h I_h - \omega_h I_h$$

This can be written as;

$$F = \begin{bmatrix} \beta_p S_p I_p \\ 0 \\ \beta_h S_h I_p \\ 0 \end{bmatrix} \quad V = \begin{bmatrix} -\delta_p E_p - \mu_p E_p - \alpha E_p \\ \delta_p E_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p \\ -\delta_h E_h - \tau E_h - \mu_h E_h \\ \delta_h E_h - \mu_h I_h - \omega_h I_h \end{bmatrix}$$

Differentiating F and V with respect E_p, I_p, E_h and I_h .

$$F = \begin{bmatrix} 0 & \beta_p S_p & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \beta_h S_h & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \delta_p + \mu_p + \alpha & 0 & 0 & 0 \\ -\delta_p & \mu_p + \omega_2 + \omega_p & 0 & 0 \\ 0 & 0 & \delta_p + \tau + \mu_h & 0 \\ 0 & 0 & -\delta_h & \mu_h + \omega_h \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & \beta_p \frac{\lambda_p}{\mu_p} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \beta_h \frac{\lambda_h}{\mu_h} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where V^{-1} is given by;

$$(19) \quad V^{-1} = \begin{bmatrix} \frac{1}{(\delta_p + \alpha + \mu_p)} & 0 & 0 & 0 \\ \frac{\delta_p}{(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{1}{(\mu_p + \omega_2 + \omega_p)} & 0 & 0 \\ 0 & 0 & \frac{1}{(\delta_p + \tau + \mu_h)} & 0 \\ 0 & 0 & \frac{\delta_h}{(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{1}{(\mu_h + \omega_h)} \end{bmatrix}$$

Determining the product of F and V^{-1} ;

(20)

$$\begin{aligned}
 (20) \quad FV^{-1} &= \begin{bmatrix} 0 & \beta_p \frac{\lambda_p}{\mu_p} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \beta_h \frac{\Lambda_h}{\mu_h} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{(\delta_p + \alpha + \mu_p)} & 0 & 0 & 0 \\ \frac{\delta_p}{(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{1}{(\mu_p + \omega_2 + \omega_p)} & 0 & 0 \\ 0 & 0 & \frac{1}{(\delta_p + \tau + \mu_h)} & 0 \\ 0 & 0 & \frac{\delta_h}{(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{1}{(\mu_h + \omega_h)} \end{bmatrix} \\
 (21) \quad &= \begin{bmatrix} \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{\beta_p \lambda_p}{\mu_p (\mu_p + \omega_2 + \omega_p)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\beta_h \Lambda_h \delta_p}{\mu_h (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{\beta_h \Lambda_h}{\mu_h (\mu_p + \omega_2 + \omega_p)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

Let $K = FV^{-1}$, the eigenvalues of FV^{-1} can be obtained,

$$|K| = \begin{vmatrix} \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} - \lambda & \frac{\beta_p \lambda_p}{\mu_p (\mu_p + \omega_2 + \omega_p)} & 0 & 0 \\ 0 & 0 - \lambda & 0 & 0 \\ \frac{\beta_h \Lambda_h \delta_p}{\mu_h (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} & \frac{\beta_h \Lambda_h}{\mu_h (\mu_p + \omega_2 + \omega_p)} & 0 - \lambda & 0 \\ 0 & 0 & 0 & 0 - \lambda \end{vmatrix} = 0$$

$$(22) \quad \left(\frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} - \lambda \right) (\lambda)(\lambda)(\lambda) = 0$$

$$(23) \quad \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)} = 0 \quad \text{or} \quad \lambda = 0$$

Hence, eigenvalues,

$$\lambda_1 = 0 \quad \text{or} \quad \lambda_2 = \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)}$$

The dominant eigenvalue is the spectral radius (basic reproductive number).

$$(24) \quad R_O = \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)}$$

1.2.5. Endemic Equilibrium Point. Endemic equilibrium points are steady state situations where the disease persists in the population. To determine the endemic equilibrium point we put the right side of equation (1) equal to zero.

The endemic equilibrium point of the model is written below;

$$(25) \quad E^* = (S_h^*, E_h^*, I_h^*, R_h^*, S_p^*, E_p^*, I_p^*, R_p^*)$$

where;

$$S_h^* = \frac{(\delta_h + \tau + \mu_h)\Lambda_h\beta_h(\sigma + \mu_h)}{\beta_h[(\delta_h + \tau + \mu_h)(\sigma + \mu_h)(\beta_h I_p^* + \mu_h) - \sigma\tau\beta_h I_p^*]}$$

$$E_h^* = \frac{\Lambda_h\beta_h I_p^*(\sigma + \mu_h)}{[(\delta_h + \tau + \mu_h)(\sigma + \mu_h)(\beta_h I_p^* + \mu_h) - \sigma\tau\beta_h I_p^*]}$$

$$I_h^* = \frac{\delta_h\Lambda_h\beta_h I_p^*(\sigma + \mu_h)}{(\mu_h + \omega_h)[(\delta_h + \tau + \mu_h)(\sigma + \mu_h)(\beta_h I_p^* + \mu_h) - \sigma\tau\beta_h I_p^*]}$$

$$R_h^* = \frac{\tau\Lambda_h\beta_h I_p^*}{[(\delta_h + \tau + \mu_h)(\sigma + \mu_h)(\beta_h I_p^* + \mu_h) - \sigma\tau\beta_h I_p^*]}$$

$$S_p^* = \frac{(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)}{\beta_p\delta_p}$$

$$E_p^* = \frac{(\mu_p + \gamma)[\mu_p(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) - \lambda_p\beta_p\delta_p]}{(\mu_p + \omega_2 + \omega_p)\beta_p(\gamma\alpha - (\delta_p + \mu_p + \alpha)E_p)(\mu_p + \gamma)}$$

$$I_p^* = \frac{(\mu_p + \gamma)[\mu_p(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) - \lambda_p\beta_p\delta_p]}{(\mu_p + \omega_2 + \omega_p)\beta_p(\gamma\alpha - (\delta_p + \mu_p + \alpha)E_p)(\mu_p + \gamma)}$$

$$R_p^* = \frac{\alpha[\mu_p(\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) - \lambda_p\beta_p\delta_p]}{\beta_p\delta_p(\gamma\alpha - (\delta_p + \mu_p + \alpha)E_p)(\mu_p + \gamma)}$$

1.3. Stability Analysis of Disease Free Equilibrium.

1.3.1. Local Stability of Disease Free Equilibrium. Local stability of an equilibrium point means that if you put the system somewhere nearby the point then it will move itself to the equilibrium point in some time [17].

Theorem 1. The disease free equilibrium point is locally asymptotically stable if $R_0 < 1$ otherwise it is unstable.

Proof: To prove local stability of disease free equilibrium, we obtained the Jacobean's matrix of the system (1) at the disease free equilibrium (DFE).

Then the Jacobean matrix become

(26)

$$J = \begin{pmatrix} -\beta_h I_p + -\mu_h & 0 & 0 & \sigma & 0 & 0 & -\beta_h S_h & 0 \\ \beta_h I_p & -(\delta_h + \tau + \mu_h) & 0 & 0 & 0 & 0 & \beta_h S_h & 0 \\ 0 & \delta_h & -(\mu_h + \omega_h) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & -(\sigma + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_p I_p - \mu_p & 0 & -\beta_p S_p & \gamma \\ 0 & 0 & 0 & 0 & \beta_p I_p & -(\delta_p + \mu_p + \alpha) & \beta_p S_p & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & -(\mu_p + \omega_2 + \omega_p) & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -(\mu_p + \gamma) \end{pmatrix}$$

$$(27) \quad J(DFE) = \begin{pmatrix} -\mu_h & 0 & 0 & \sigma & 0 & 0 & -\beta_h \frac{\Lambda_h}{\mu_h} & 0 \\ 0 & -(\delta_h + \tau + \mu_h) & 0 & 0 & 0 & 0 & \beta_h \frac{\Lambda_h}{\mu_h} & 0 \\ 0 & \delta_h & -(\mu_h + \omega_h) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & -(\sigma + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu_p & 0 & -\beta_p \frac{\lambda_p}{\mu_p} & \gamma \\ 0 & 0 & 0 & 0 & 0 & -(\delta_p + \mu_p + \alpha) & \beta_p \frac{\lambda_p}{\mu_p} & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & -(\mu_p + \omega_2 + \omega_p) & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -(\mu_p + \gamma) \end{pmatrix}$$

$$(28) \quad J(DFE) = \begin{pmatrix} C1 & 0 & 0 & \sigma & 0 & 0 & -b3 & 0 \\ 0 & C2 & 0 & 0 & 0 & 0 & b3 & 0 \\ 0 & \delta_h & c3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & C4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c5 & 0 & -b4 & \gamma \\ 0 & 0 & 0 & 0 & 0 & C6 & b4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & b1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & b2 \end{pmatrix}$$

Where

$$(29) \quad C1 = -\mu_h \qquad C6 = -(\delta_p + \mu_p + \alpha)$$

$$(30) \quad C2 = -(\delta_h + \tau + \mu_h) \qquad b1 = -(\mu_p + \omega_2 + \omega_p)$$

$$(31) \quad C3 = -(\mu_h + \omega_h) \qquad b2 = -(\mu_p + \gamma)$$

$$(32) \quad C4 = -(\sigma + \mu_h) \qquad b3 = \beta_h \frac{\Lambda_h}{\mu_h}$$

$$(33) \quad C5 = -\mu_p \qquad b4 = \beta_p \frac{\lambda_p}{\mu_p}$$

$$(34) \quad |J(DFE) - \lambda| = \begin{vmatrix} C1 - \lambda & 0 & 0 & \sigma & 0 & 0 & -b3 & 0 \\ 0 & C2 - \lambda & 0 & 0 & 0 & 0 & b3 & 0 \\ 0 & \delta_h & C3 - \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & C4 - \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C5 - \lambda & 0 & -b4 & \gamma \\ 0 & 0 & 0 & 0 & 0 & C6 - \lambda & b4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & b1 - \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & b2 - \lambda \end{vmatrix} = 0$$

$$(35) \quad (C1 - \lambda) \begin{vmatrix} C2 - \lambda & 0 & 0 & 0 & 0 & b3 & 0 \\ \delta_h & C3 - \lambda & 0 & 0 & 0 & 0 & 0 \\ \tau & 0 & C4 - \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C5 - \lambda & 0 & -b4 & \gamma \\ 0 & 0 & 0 & 0 & C6 - \lambda & b4 & 0 \\ 0 & 0 & 0 & 0 & \delta_p & b1 - \lambda & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & b2 - \lambda \end{vmatrix} = 0$$

$$(36) \quad (C1 - \lambda)(C2 - \lambda)(C3 - \lambda)(C4 - \lambda) \begin{vmatrix} C5 - \lambda & 0 & -b4 & \gamma \\ 0 & C6 - \lambda & b4 & 0 \\ 0 & \delta_p & b1 - \lambda & 0 \\ 0 & \alpha & 0 & b2 - \lambda \end{vmatrix} = 0$$

$$(C1 - \lambda)(C2 - \lambda)(C3 - \lambda)(C4 - \lambda)(C5 - \lambda) \begin{vmatrix} C6 - \lambda & b4 & 0 \\ \delta_p & b1 - \lambda & 0 \\ \alpha & 0 & b2 - \lambda \end{vmatrix} = 0$$

$$(C1 - \lambda)(C2 - \lambda)(C3 - \lambda)(C4 - \lambda)(C5 - \lambda)(b_2 - \lambda)[(C6 - \lambda)(b_1 - \lambda) - b_4\delta_p] = 0$$

$$(C1 - \lambda)(C2 - \lambda)(C3 - \lambda)(C4 - \lambda)(C5 - \lambda)(b_2 - \lambda) = 0$$

$$\begin{aligned} \lambda_1 &= C1 & \lambda_4 &= C4 \\ \lambda_2 &= C2 & \lambda_5 &= C5 \\ \lambda_3 &= C3 & \lambda_6 &= b_2 \end{aligned}$$

Or

$$(37) \quad \begin{aligned} (C6 - \lambda)(b_1 - \lambda) - b_4\delta_p &= 0 \\ \lambda^2 - (C6 + b_1)\lambda + C6b_1 - b_4\delta_p & \end{aligned}$$

Therefore, from the Routh-Hurwitz criterion of order two, it implies that the conditions, $-(c_6 + b_1) > 0$ and $C6 b_1 - b_4\delta_p > 0$

From $C6 b_1 - b_4\delta_p > 0$

$$(38) \quad -(\delta_p + \mu_p + \alpha)(-\mu_p + \omega_2 + \omega_p) - \beta_p \frac{\lambda_p}{\mu_p} \delta_p > 0$$

$$(39) \quad (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) - \beta_p \frac{\lambda_p}{\mu_p} \delta_p > 0$$

$$(40) \quad (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) > \beta_p \frac{\lambda_p}{\mu_p} \delta_p$$

$$(41) \quad (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p) > \frac{\beta_p \lambda_p \delta_p}{\mu_p}$$

$$(42) \quad 1 > \frac{\beta_p \lambda_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha)(\mu_p + \omega_2 + \omega_p)}$$

$R_0 < 1$ therefore *DFE* is locally asymptotically stable. The prove is completed.

1.3.2. Global Stability of Disease Free Equilibrium. According to [18, 19, 20] to get the global stability of disease free equilibrium point of system (1) we write our system as follows:

$$(43) \quad \begin{cases} \frac{dX_1}{dt} = B(X_1 - X_1(E_0)) + B_1 X_2 \\ \frac{dX_2}{dt} = B_2 X_2 \end{cases}$$

$$(44) \quad X_1 = \text{transmitting compartments}$$

$$(45) \quad X_2 = \text{non - transmitting compartments}$$

$$(46) \quad X_1 = (S_h, R_h, S_p, R_p)$$

$$(47) \quad X_2 = (E_h, I_h, E_p, I_p)$$

The *DFE* is denoted by $DFE = \left\{ \frac{\Lambda_h}{\mu_h}, 0, \frac{\lambda_p}{\mu_p}, 0 \right\}$

$$(48) \quad X_1 - X_1(DFE) = \begin{cases} S_h - \frac{\Lambda_h}{\mu_h} \\ R_h \\ S_p - \frac{\lambda_p}{\mu_p} \\ R_p \end{cases}$$

For the global stability of *DFE* we need to prove the following.

- (1) B should be a matrix with real negative Eigen values.
- (2) B_2 Should be a Metzler matrix

Using system (1) together with the representation in (43) the two equations can be written as follows:

$$\begin{bmatrix} \Lambda_h + \sigma R_h - \beta_h S_h I_p - \mu_h S_h \\ \tau E_h - \sigma R_h - \mu_h R_h \\ \lambda_p - \beta_p S_p I_p - \mu_p S_p + \gamma R_p \\ \alpha E_p - \mu_p R_p - \gamma R_p \end{bmatrix} = B \begin{bmatrix} S_h - \frac{\Lambda_h}{\mu_h} \\ R_h \\ S_p - \frac{\lambda_p}{\mu_p} \\ R_p \end{bmatrix} + B_1 \begin{bmatrix} E_h \\ I_h \\ E_p \\ I_p \end{bmatrix}$$

And

$$\begin{bmatrix} \beta_h S_h I_p - \delta_h E_h - \tau E_h - \mu_h E_h \\ \delta_h E_h - \mu_h I_h - \omega_h I_h \\ \beta_p S_p I_p - \delta_p E_p - \mu_p E_p - \alpha E_p \\ \delta_p E_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p \end{bmatrix} = B_2 \begin{bmatrix} E_h \\ I_h \\ E_p \\ I_p \end{bmatrix}$$

Matrices B , B_1 and B_2 are order 4×4 matrix Using non - transmitting elements of the Jacobean matrix of system (1) and representation in (43) we get

$$(49) \quad B = \begin{bmatrix} -\mu_h & \sigma & 0 & 0 \\ 0 & -(\sigma + \mu_h) & 0 & 0 \\ 0 & 0 & -\mu_p & \gamma \\ 0 & 0 & 0 & -(\mu_p + \gamma) \end{bmatrix}$$

$$(50) \quad B_1 = \begin{bmatrix} 0 & 0 & 0 & -\beta_h S_h \\ \tau & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_p S_p \\ 0 & 0 & \alpha & 0 \end{bmatrix}$$

$$(51) \quad B_2 = \begin{bmatrix} -\delta_p - \mu_p - \alpha & \beta_p S_p & 0 & 0 \\ \delta_p & -\mu_p - \omega_2 - \omega_p & 0 & 0 \\ 0 & \beta_h S_h & -\delta_p - \tau - \mu_h & 0 \\ 0 & 0 & \delta_h & -\mu_h - \omega_h \end{bmatrix}$$

$$\begin{vmatrix} -\mu_h - \lambda & \sigma & 0 & 0 \\ 0 & -(\sigma + \mu_h) - \lambda & 0 & 0 \\ 0 & 0 & -\mu_p - \lambda & \gamma \\ 0 & 0 & 0 & -(\mu_p + \gamma) - \lambda \end{vmatrix} = 0$$

$$(52) \quad (-\mu_h - \lambda)(-\sigma - \mu_h - \lambda)(-\mu_p - \lambda)(-\mu_p - \gamma - \lambda) = 0$$

$$(53) \quad \lambda_1 = -\mu_h$$

$$(54) \quad \lambda_2 = -(\sigma + \mu_h)$$

$$(55) \quad \lambda_3 = -\mu_p$$

$$(56) \quad \lambda_4 = -(\mu_p + \gamma)$$

The Eigen value of matrix B is negative and the off diagonal elements of matrix B_2 are non - negative which is Metzler matrix. This proves that the DFE point of system (1) globally asymptotically stable in the region \mathbb{R}^8 and $R_0 < 1$.

1.4. Stability Analysis of Endemic Equilibrium point. In this section we focus on the stability of the endemic equilibrium point. The stability of endemic equilibrium point is divided into two local and global.

1.4.1. Local Stability Analysis of Endemic Equilibrium Point. Theorem 2. The endemic equilibrium E^* of model (1) is globally asymptotically stable whenever $R_0 > 1$

Proof To determine the local stability of endemic equilibrium point from the differential equation (1) first we determine the Jacobean matrix at E^* .

(57)

$$J = \begin{pmatrix} -\beta_h I_p + -\mu_h & 0 & 0 & \sigma & 0 & 0 & -\beta_h S_h & 0 \\ \beta_h I_p & -(\delta_h + \tau + \mu_h) & 0 & 0 & 0 & 0 & \beta_h S_h & 0 \\ 0 & \delta_h & -(\mu_h + \omega_h) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & -(\sigma + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_h I_p - \mu_p & 0 & -\beta_p S_p & \gamma \\ 0 & 0 & 0 & 0 & \beta_h I_p & -(\delta_p + \mu_p + \alpha) & \beta_p S_p & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & -(\mu_p + \omega_2 + \omega_p) & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -(\mu_p + \gamma) \end{pmatrix}$$

(58)

$$J(E^*) = \begin{pmatrix} C1 & 0 & 0 & \sigma & 0 & 0 & -b3 & 0 \\ b5 & C2 & 0 & 0 & 0 & 0 & b3 & 0 \\ 0 & \delta_h & C3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & C4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C5 & 0 & -b4 & \gamma \\ 0 & 0 & 0 & 0 & b6 & C6 & b4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & b1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & b2 \end{pmatrix}$$

Where

$$C1 = -\beta_h I_p^* - \mu_h$$

$$C2 = -(\delta_h + \tau + \mu_h)$$

$$C3 = -(\mu_h + \omega_h)$$

$$C4 = -(\sigma + \mu_h)$$

$$C5 = -\beta_p I_p^* - \mu_p$$

$$b6 = -\beta_p I_p^*$$

$$C6 = -(\delta_p + \mu_p + \alpha)$$

$$b1 = -(\mu_p + \omega_2 + \omega_p)$$

$$b2 = -(\mu_p + \gamma)$$

$$b3 = \beta_h S_h^*$$

$$b4 = \beta_p S_p^*$$

$$b5 = -\beta_h I_p^*$$

$$(59) \quad |J(E^*) - \lambda| = \begin{vmatrix} (C1 - \lambda) & 0 & 0 & \sigma & 0 & 0 & -b3 & 0 \\ b5 & C2 - \lambda & 0 & 0 & 0 & 0 & b3 & 0 \\ 0 & \delta_h & C3 - \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & C4 - \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C5 - \lambda & 0 & -b4 & \gamma \\ 0 & 0 & 0 & 0 & b6 & C6 - \lambda & b4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_p & b1 - \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & b2 - \lambda \end{vmatrix} = 0$$

From the above we get

$$(60) \quad \lambda^7 + m_1\lambda^6 + m_2\lambda^5 + m_3\lambda^4 + m_4\lambda^3 + m_5\lambda^2 + m_6\lambda + m_7 = 0$$

From the above $\lambda_1 = C3$ it is negative and to see the remains value of λ we use Routh Hurwitz criterion. To obtain the precise number of roots with nonnegative real part, proceed as follow arrange the coefficient of polynomial and values subsequently calculated form them as below:

$$(61) \quad m_0\lambda^n + m_1\lambda^{n-1} + \dots + m_{n-1}\lambda + m_n = 0$$

$$m_0\lambda^n + m_1\lambda^{n-1} + \dots + m_{n-1}\lambda + m_n = 0$$

$$\lambda^7 \quad 1 \quad m_2 \quad m_4 \quad m_6$$

$$\lambda^6 \quad m_1 \quad m_3 \quad m_5 \quad m_7$$

$$\lambda^5 \quad d_1 \quad d_2 \quad d_3 \quad 0$$

$$\lambda^4 \quad e_1 \quad e_2 \quad e_3$$

$$\lambda^3 \quad f_1 \quad f_2 \quad 0$$

$$\lambda^2 \quad h_1 \quad h_2$$

$$\lambda^1 \quad n_1 \quad 0$$

$$\lambda^0 \quad x_1$$

$$(62) \quad d_1 = -\frac{1}{m_1} \begin{vmatrix} -1 & m_2 \\ m_1 & m_3 \end{vmatrix} = -\frac{1}{m_1}(-m_3 + m_1 m_2) = \frac{1}{m_1}(-m_3 + m_1 m_2)$$

$$(63) \quad d_2 = -\frac{1}{m_1} \begin{vmatrix} -1 & m_4 \\ m_1 & m_5 \end{vmatrix} = -\frac{1}{m_1}(-m_5 + m_1 m_4) = \frac{1}{m_1}(-m_5 + m_1 m_4)$$

$$(64) \quad d_3 = -\frac{1}{m_1} \begin{vmatrix} -1 & m_6 \\ m_1 & m_7 \end{vmatrix} = -\frac{1}{m_1}(-m_7 + m_1 m_6) = \frac{1}{m_1}(-m_7 + m_1 m_6)$$

$$(65) \quad e_1 = -\frac{1}{d_1} \begin{vmatrix} m_1 & m_3 \\ d_1 & d_2 \end{vmatrix} = -\frac{1}{d_1}(m_1 d_2 - m_3 d_1)$$

$$(66) \quad = -\frac{1}{d_1 m_1}(m_1(-m_5 + m_1 m_4) - m_3(-m_3 + m_1 m_2))$$

$$(67) \quad = \frac{1}{d_1 m_1}[m_3^2 + m_1^2 m_4 - m_1 m_5 + m_3 m_1 m_2]$$

$$(68) \quad = \frac{1}{m_5 - m_1 m_2}[-m_3^2 - m_1^2 m_4 + m_1 m_5 + m_3 m_1 m_2]$$

$$(69) \quad e_2 = -\frac{1}{d_1} \begin{vmatrix} m_1 & m_5 \\ d_1 & d_3 \end{vmatrix} = -\frac{1}{d_1}(m_1 d_3 - m_5 d_1)$$

$$(70) \quad = -\frac{1}{d_1 m_1}(m_1(-m_7 + m_1 m_6) - m_5(-m_3 + m_1 m_2))$$

$$(71) \quad = \frac{1}{d_1 m_1}[-m_1^2 m_6 + m_1 m_7 + m_5 m_1 m_2 - m_5 m_3]$$

$$(72) \quad = \frac{1}{m_3 - m_1 m_2}[-m_1^2 m_6 + m_1 m_7 + m_5 m_1 m_2 - m_5 m_3]$$

$$(73) \quad e_3 = -\frac{1}{d_1} \begin{vmatrix} m_1 & m_7 \\ d_1 & 0 \end{vmatrix} = -\frac{1}{d_1}(-d_1 m_7) = m_7$$

$$(74) \quad f_1 = -\frac{1}{e_1} \begin{vmatrix} d_1 & d_2 \\ e_1 & e_2 \end{vmatrix} = -\frac{1}{e_1}(d_1 e_2 - d_2 e_1)$$

$$(75) \quad = -\frac{1}{e_1} \left(\frac{1}{m_1}(-m_3 + m_1 m_2) \frac{1}{m_3 - m_1 m_2} [-m_1^2 m_6 + m_1 m_7 + m_5 m_1 m_2 - m_5 m_3] - \right.$$

$$(76) \quad \left. \frac{1}{m_1}(-m_5 + m_1 m_4) \frac{1}{m_5 - m_1 m_2} [-m_3^2 - m_1^2 m_4 + m_1 m_5 + m_3 m_1 m_2] \right)$$

$$(77) \quad f_1 = \frac{1}{e_1 m_1 (m_1 m_2 - m_3)} (-m_1^2 m_2^2 m_5 - m_1^2 m_2 m_7 + m_1^3 m_2 m_6 + m_1 m_2 m_3 m_5 + m_1 m_7 m_3 -$$

$$(78) \quad 2m_3^2 m_5 - m_1^2 m_3 m_6 + m_1^2 m_2 m_3 m_4 + 2m_1^2 m_4 m_5 - m_1 m_5^2 - m_3^2 m_1 m_4 - m_1^3 m_4^2)$$

(79)

$$f_2 = -\frac{1}{e_1} \begin{vmatrix} d_1 & d_3 \\ e_1 & e_3 \end{vmatrix} = -\frac{1}{e_1} (d_1 e_3 - e_1 d_3)$$

$$(80) \quad = -\frac{1}{e_1} \left(\frac{1}{m_1} (-m_3 + m_1 m_2) m_7 - \frac{1}{m_5 - m_1 m_2} [m_3^2 + m_1^2 m_4 - m_1 m_5 - m_3 m_1 m_2] \frac{1}{m_1} (-m_7 + m_1 m_6) \right)$$

(81)

$$f_2 = \frac{1}{e_1 m_1 (m_1 m_2 - m_3)} (-m_1^2 m_2^2 m_7 + m_1 m_2 m_3 m_7 + m_1^2 m_2 m_3 m_6 + m_1^2 m_5 m_6 - m_1 m_5 m_7 -$$

$$(82) \quad m_3^2 m_1 m_6 - m_1^3 m_4 m_6 + m_1^2 m_4 m_7)$$

(83)

$$h_1 = -\frac{1}{f_1} \begin{vmatrix} e_1 & e_2 \\ f_1 & f_2 \end{vmatrix} = -\frac{1}{f_1} (e_1 f_2 - f_1 e_2)$$

$$(84) \quad = -\frac{1}{f_1} \left(\frac{1}{m_5 - m_1 m_2} [(-m_3^2 - m_1^2 m_4 + m_1 m_5 + m_3 m_1 m_2)] \frac{1}{e_1 m_1 (m_1 m_2 - m_3)} (-m_1^2 m_2^2 m_7 +$$

$$(85) \quad m_1 m_2 m_3 m_7 + m_1^2 m_2 m_3 m_6 + m_1^2 m_5 m_6 - m_1 m_5 m_7 - m_3^2 m_1 m_6 - m_1^3 m_4 m_6 + m_1^2 m_4 m_7) -$$

$$(86) \quad \frac{1}{e_1 m_1 (m_1 m_2 - m_3)} (-m_1^2 m_2^2 m_5 - m_1^2 m_2 m_7 + m_1^3 m_2 m_6 + m_1 m_2 m_3 m_5 + m_1 m_7 m_3 -$$

$$(87) \quad 2m_3^2 m_5 - m_1^2 m_3 m_6 + m_1^2 m_2 m_3 m_4 + 2m_1^2 m_4 m_5 - m_1 m_5^2 - m_3^2 m_1 m_4 - m_1^3 m_4^2)$$

$$(88) \quad \frac{1}{m_3 - m_1 m_2} [m_1^2 m_6 - m_1 m_7 + m_5 m_1 m_2 - m_5 m_3])$$

$$(89) \quad h_2 = -\frac{1}{f_1} \begin{vmatrix} e_1 & e_3 \\ f_1 & 0 \end{vmatrix}$$

$$(90) \quad h_2 = -\frac{1}{f_1} (0 - e_3 f_1) = e_3 = m_7$$

$$(91) \quad n_1 = -\frac{1}{f_1} \begin{vmatrix} f_1 & f_2 \\ h_1 & h_2 \end{vmatrix}$$

$$(92) \quad n_1 = -\frac{1}{f_1} (f_1 h_2 - h_1 f_2)$$

$$(93) \quad n_1 = \frac{1}{f_1} (h_1 f_2 - f_1 h_2)$$

Let $h_1 = z$, $f_1 = r$, $f_2 = y$ then, $m_1 = \frac{1}{f_1} (zy - rm_7)$

$$n_1 = -\frac{1}{n_1} \begin{vmatrix} h_1 & h_2 \\ n_1 & 0 \end{vmatrix} = -\frac{1}{n_1} (-n_1 h_2) = h_2 = e_3 = m_7$$

By Using the Routh Hurwitz criterion it can be seen that all the Eigen values of the characteristic equation have negative real part if and only if:

$$m_1 > 0, m_2 > 0, m_3 > 0, m_4 > 0, m_5 > 0, m_6 > 0, m_7 > 0, d_1 > 0, d_2 > 0, d_3 > 0, e_1 > 0, e_2 > 0, e_3 > 0, f_1 > 0, f_2 > 0, h_1 > 0, h_2 > 0, n_1 > 0$$

$$\text{Since } R_0 = \frac{\lambda_p \beta_p \delta_p}{\mu_p (\delta_p + \mu_p + \alpha) (\mu_p + \omega_2 + \omega_p)}$$

So, $R_0 > 1$

This implies that the endemic equilibrium point (E^*) is local asymptotical stable.

1.4.2. Global Stability Analysis of Endemic Equilibrium. (E^*)

Global stability means that the system will come to the equilibrium point from any possible starting point.

Theorem 3 If $R_0 > 1$, E^* of the model (1) is globally asymptotically stable.

Proof. Using the Lyapunov approach in [21], global asymptotic stability of E^* :

Define;

$$(94) \quad V(S_h^*, E_h^*, I_h^*, R_h^*, S_p^*, I_p^*, R_p^*)$$

$$(94) \quad = (S_h - S_h^* - S_h^* \ln \frac{S_h}{S_h^*}) + (E_h - E_h^* - E_h^* \ln \frac{E_h}{E_h^*}) + (I_h - I_h^* - I_h^* \ln \frac{I_h}{I_h^*})$$

$$(95) \quad + (R_h - R_h^* - R_h^* \ln \frac{R_h}{R_h^*}) + (S_p - S_p^* - S_p^* \ln \frac{S_p}{S_p^*}) + (E_p - E_p^* - E_p^* \ln \frac{E_p}{E_p^*})$$

$$(96) \quad + (I_p - I_p^* - I_p^* \ln \frac{I_p}{I_p^*}) + (R_p - R_p^* - R_p^* \ln \frac{R_p}{R_p^*})$$

By direct calculating the derivative of V along the solution of (1) we have;

$$(97) \quad \frac{dV}{dt} = \left(1 - \frac{S_h^*}{S_h}\right) \frac{dS_h}{dt} + \left(1 - \frac{E_h^*}{E_h}\right) \frac{dE_h}{dt} + \left(1 - \frac{I_h^*}{I_h}\right) \frac{dI_h}{dt} + \left(1 - \frac{R_h^*}{R_h}\right) \frac{dR_h}{dt}$$

$$(98) \quad + \left(1 - \frac{S_p^*}{S_p}\right) \frac{dS_p}{dt} + \left(1 - \frac{E_p^*}{E_p}\right) \frac{dE_p}{dt} + \left(1 - \frac{I_p^*}{I_p}\right) \frac{dI_p}{dt} + \left(1 - \frac{R_p^*}{R_p}\right) \frac{dR_p}{dt}$$

$$(99) \quad \frac{dV}{dt} = \left(1 - \frac{S_h^*}{S_h}\right) (\Lambda_h + \sigma R_h - \beta_h S_h I_p - \mu_h S_h) + \left(1 - \frac{E_h^*}{E_h}\right) (\beta_h S_h I_p - \delta_h E_h - \tau E_h - \mu_h E_h)$$

$$(100) \quad + \left(1 - \frac{I_h^*}{I_h}\right) (\delta_h E_h - \mu_h I_h - \omega_h I_h) + \left(1 - \frac{R_h^*}{R_h}\right) (\tau E_h - \sigma R_h - \mu_h R_h)$$

$$(101) \quad + \left(1 - \frac{S_p^*}{S_p}\right) (\lambda_p - \beta_p S_p I_p - \mu_p S_p + \gamma R_p) + \left(1 - \frac{E_p^*}{E_p}\right) (\beta_p S_p I_p - \delta_p E_p - \mu_p E_p - \alpha E_p)$$

$$(102) \quad + \left(1 - \frac{I_p^*}{I_p}\right) (\delta_p E_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p) + \left(1 - \frac{R_p^*}{R_p}\right) (\alpha E_p - \mu_p R_p - \gamma R_p)$$

$$(103) \quad \frac{dV}{dt} = \Lambda_h + \sigma R_h + \beta_h S_h^* I_d + \mu_h S_h^* + \beta_h S_h I_p + \delta_h E + \tau E^* + \mu_h E^* + \delta_h E_h + \mu_h I_h^* + \omega_h I_h^* + \tau E_h$$

$$(104) \quad + \sigma R_h^* + \mu_h R_h^* + \lambda_p + \gamma R_p + \beta_p S_p^* I_p + \mu_p S_p^* + \beta_p S_p I_p + \delta_p E_p^* + \mu_p E_p^* + \alpha E_p^* + \delta_p E_p + \mu_p I_p^*$$

$$(105) \quad + \omega_2 I_p^* + \omega_p I_p^* + \alpha E_p + \mu_p R_p^* + \gamma R_p^* - \left(-\beta_h S_h I_p - \mu_h S_h - \frac{S_h^*}{S_h} \Lambda_h - \frac{S_h^*}{S_h} \sigma R_h - \delta_h E_h - \tau E_h\right.$$

$$(106) \quad \left. - \mu_h E_h - \frac{E^*}{E_h} \beta_h S_h I_p - \mu_h I_h - \omega_h I_h - \frac{I_h^*}{I_h} \delta_h E_h - \sigma R_h - \mu_h R_h - \frac{R_h^*}{R_h} \tau E_h - \beta_p S_p I_p - \mu_p S_p\right.$$

$$(107) \quad \left. - \frac{S_p^*}{S_p} \lambda_p - \frac{S_p^*}{S_p} R_p - \delta_p E_p - \mu_p E_p - \alpha E_p - \frac{E_p^*}{E_p} \beta_p S_p I_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p - \frac{I_p^*}{I_p} \delta_p E_p\right.$$

$$(108) \quad \left. - \mu_p R_p - \gamma R_p - \frac{R_p^*}{R_p} \alpha E_p\right)$$

$$(109) \quad \frac{dV}{dt} = A - B$$

$$(110)$$

Where

$$(111) \quad A = \Lambda_h + \sigma R_h + \beta_h S_h^* I_d + \mu_h S_h^* + \beta_h S_h I_p + \delta_h E^* + \tau E^* + \mu_h E^* + \delta_h E_h + \mu_h I_h^* + \omega_h I_h^* + \tau E_h$$

$$(112) \quad + \sigma R_h^* + \mu_h R_h^* + \lambda_p + \gamma R_p + \beta_p S_p^* I_p + \mu_p S_p^* + \beta_p S_p I_p + \delta_p E_p^* + \mu_p E_p^* + \alpha E_p^* + \delta_p E_p + \mu_p I_p^*$$

$$(113) \quad + \omega_2 I_p^* + \omega_p I_p^* + \alpha E_p + \mu_p R_p^* + \gamma R_p^*$$

$$(114) \quad B = \left(-\beta_h S_h I_p - \mu_h S_h - \frac{S_h^*}{S_h} \Lambda_h - \frac{S_h^*}{S_h} \sigma R_h - \delta_h E_h - \tau E_h - \mu_h E_h - \frac{E^*}{E_h} \beta_h S_h I_p - \mu_h I_h - \omega_h I_h\right.$$

$$(115) \quad -\frac{I_h^*}{I_h} \delta_h E_h - \sigma R_h - \mu_h R_h - \frac{R_h^*}{R_h} \tau E_h - \beta_p S_p I_p - \mu_p S_p - \frac{S_p^*}{S_p} \lambda_p - \frac{S_p^*}{S_p} R_p - \delta_p E_p - \mu_p E_p$$

$$(116) \quad -\alpha E_p - \frac{E_p^*}{E_p} \beta_p S_p I_p - \mu_p I_p - \omega_2 I_p - \omega_p I_p - \frac{I_p^*}{I_p} \delta_p E_p - \mu_p R_p - \gamma R_p - \frac{R_p^*}{R_p} \alpha E_p$$

Thus if $A < B$, then $\frac{dV}{dt} \leq 0$;

Noting that:

$$\frac{dV}{dt} = 0 \text{ if and only if } S_p = S_p^*, E_p = E_p^*, I_p = I_p^*, R_p = R_p^*, S_h = S_h^*, E_h = E_h^*, I_h = I_h^*, R_h = R_h^*$$

Therefore, the largest compact invariant set in $\left\{ (S_h^*, E_h^*, I_h^*, R_h^*, S_p^*, I_p^*, R_p^*) \in \mathbb{R}^8 : \frac{dV}{dt} = 0 \right\}$ is the singleton E^* , where E^* is the endemic equilibrium of the system (1) by invariance principle (is criterion for the asymptotic stability of autonomous (possibly nonlinear) dynamical system), it implies that E^* is globally asymptotically stable in \mathbb{R}^8 if $A < B$. This implies that $R_0 > 1$ [22].

1.5. Sensitivity Analysis. The sensitivity analysis of the model parameters was carried out in order to determine parameters in the model that have a high transmission influence on the disease [23, 24]. We analysed the reproduction number to determine whether or not treatment of infective and mortality can lead to the effective elimination or control the psittacosis disease in the population.

We determine the most sensitive parameter by using the relation;

$$P_{mi} = \frac{1}{R_0} \times \frac{\partial R_0}{\partial mi}, \text{ where } mi \text{ are parameter, and } R_0 \text{ is the reproductive number.}$$

If $P_{mi} < 0$, then mi have an effect of controlling the disease

If $P_{mi} > 0$, then mi have an effect on expanding the disease

$$P_{\beta_p} = \frac{1}{R_0} \times \frac{\partial R_0}{\partial \beta_p}$$

TABLE 1. Sensitivity indices of model parameters to R_0

Sensitivity indices of R_0		
N_0	Parameter	Sensitivity index
1.	λ_p	+ve
2.	β_p	+ve
3.	δ_p	+ve
4.	μ_p	-ve
5.	α	-ve
6.	ω_2	-ve
7.	ω_p	-ve

The results of the sensitivity analysis as shown in Table 1 indicates that some parameters are more sensitive to the reproduction number than others. The following parameters; $\mu_p, \alpha, \omega_2, \omega_p$ help reduce the spread of the infection. However, the following parameters $\lambda_p, \beta_p, \delta_p$ increases the spread of the infection whenever their values increases.

2. NUMERICAL SIMULATION

Numerical simulations are required to study the behaviour of a systems whose mathematical model is too complex to provide analytical solution as in most non linear systems [19, 25, 26].

Table 2 shows the values of the parameters used in the various simulations.

TABLE 2. parameter value

Parameter	Value	Source
Λ_h	200	Assumption
μ_h	0.0008	Assumption
δ_h	0.17	[27]
ω_h	1	[28]
β_h	2.29×10^{-2}	[27]
τ	0.1	[29]
σ	1	[29]
λ_p	2×10^4	[28]
μ_p	0.083	[28]
δ_p	2.571×10^{-2}	[30]
ω_p	0.005	Assumption
β_p	3.776×10^{-6}	Assumption
α	0.005	Assumption
γ	0.5	[28]
ω_2	0.001	Assumption

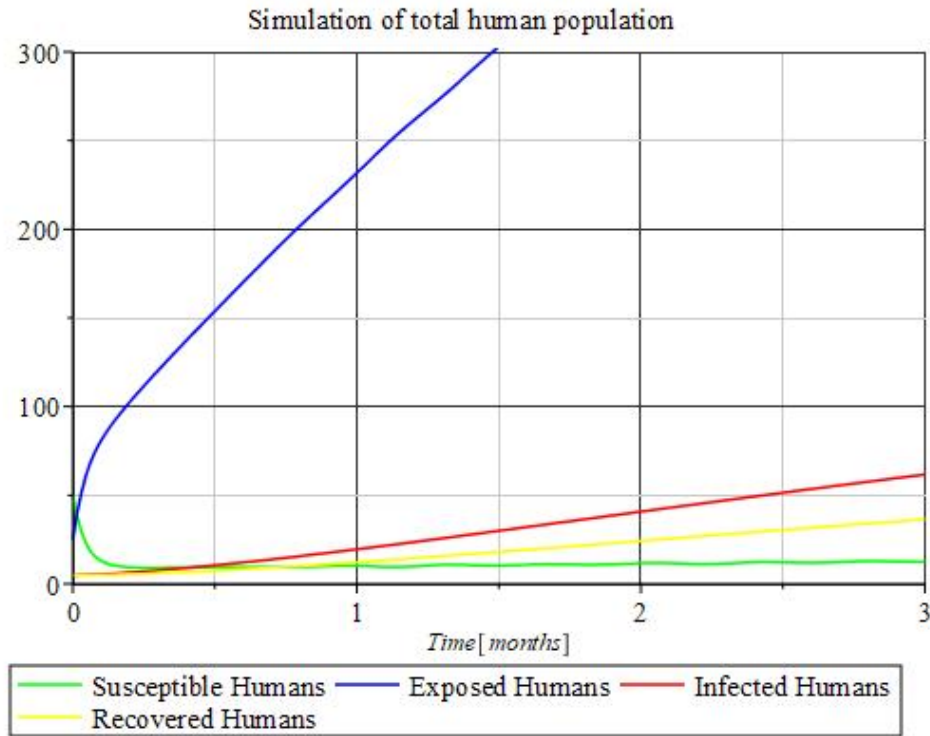


FIGURE 2. Simulation of total human population with initial value

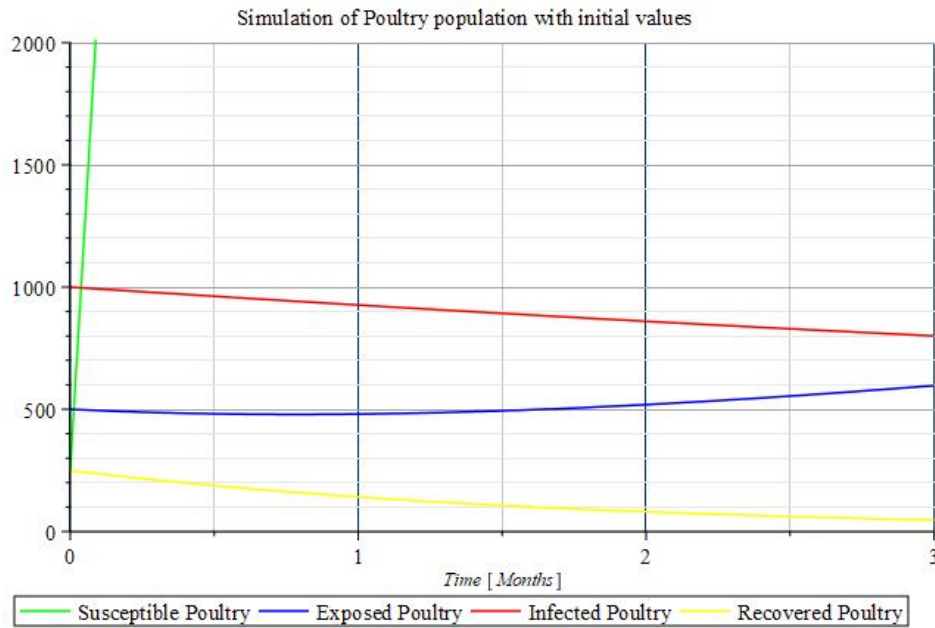


FIGURE 3. The stimulation of total population birds with initial value

2.1. Effect of Removing Rate on Psittacosis Infectious Population. In this subsection, as we see in Fig. 4, we have experimented on the effect of ω_2 in decreasing the number of psittacosis infectious population. The figure shows that when the values of ω_2 increase, the number of psittacosis infectious population is going down (decreasing).

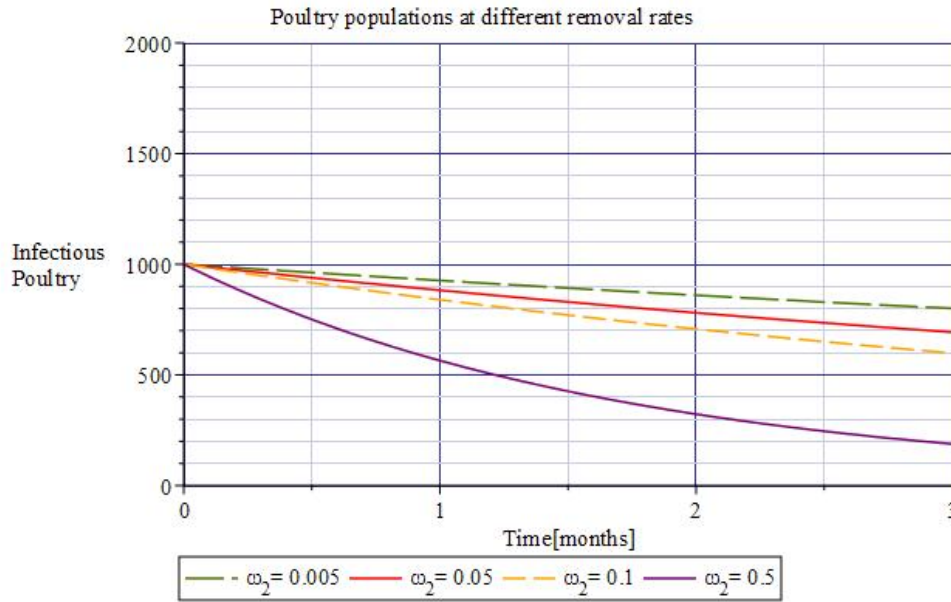


FIGURE 4. Simulation of infectious birds with different removing rates.

2.2. Effect of contact rate on exposed birds population. In this section, as we see in Fig. 5,

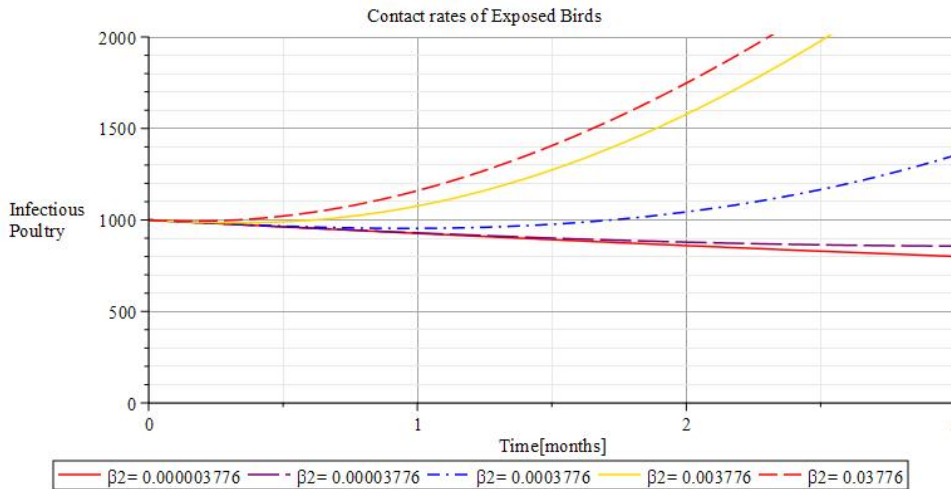


FIGURE 5. Simulation of infectious birds with different contact rates.

2.3. Effects of treatment rate on recovered birds population. In this section, we simulated the effects of treatment on recovered birds population as shown in Fig: 6.

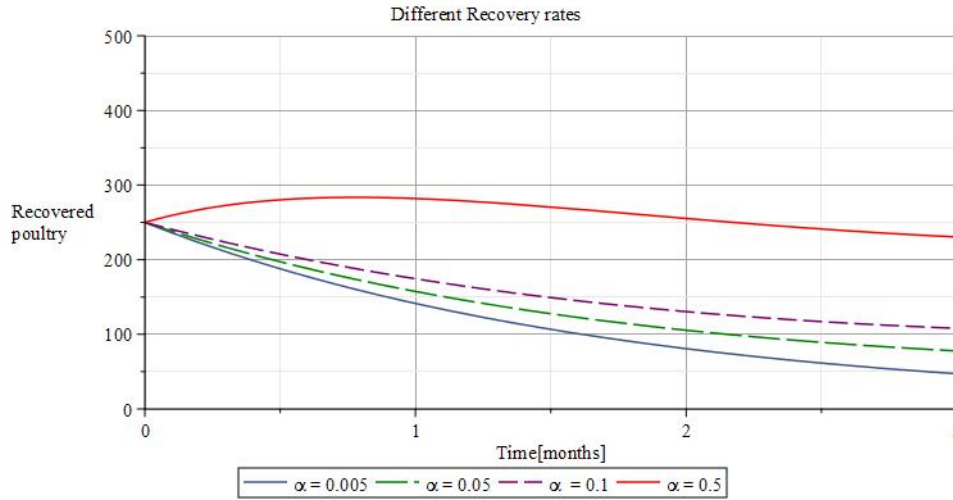


FIGURE 6. Simulation of infectious birds with different recovery rates.

2.4. Effects of rate of infection on infectious birds population. In this section, we simulated the effects of rate of infection on infectious birds population as shown in Fig 7.

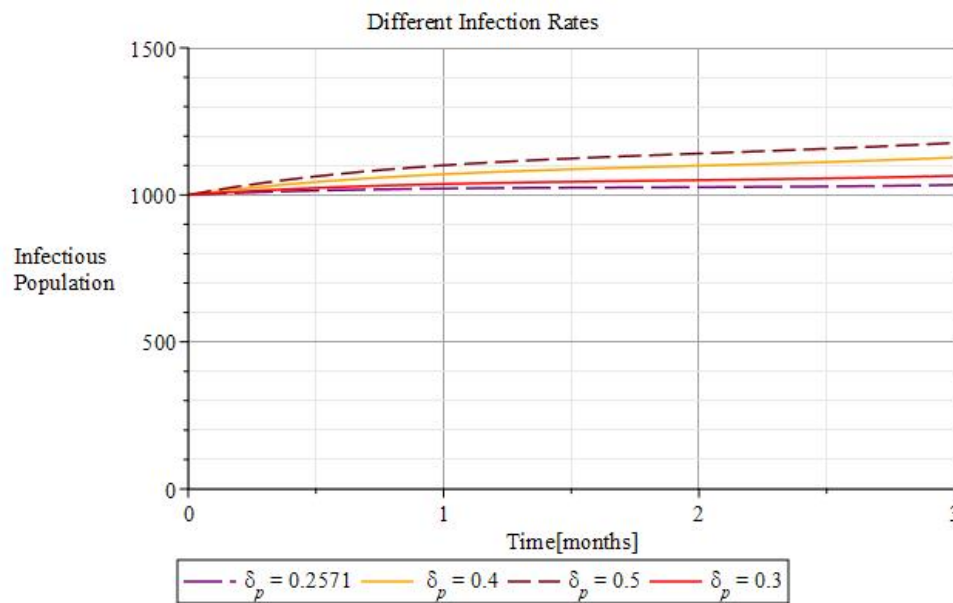


FIGURE 7. Simulation of infectious birds with different infection rates.

3. CONCLUSION

In this study we have formulated mathematical model of psittacosis. The model contains birds and human population. We defined the reproduction number in terms of the parameters and computed it by using next generation operator. The results are depending only on the parameter of birds population. It was also established that for the basic reproduction number, $R_0 < 1$, the disease free equilibrium point is asymptotically stable so that the disease dies out after some period of time and if $R_0 > 1$, the disease free equilibrium is unstable and the disease persist. We also established that when $R_0 > 1$ then the endemic equilibrium is locally asymptotically stable, and unstable if $R_0 < 1$. The local stability theorems of disease free equilibrium and endemic equilibrium points of the model are proved by using Jacobian matrix and Routh-Hurwitz criterion. Further more global stability analysis of endemic equilibrium point was computed by using invariance principle. Sensitivity analysis of basic parameters and interpretation of the sensitivity index is also computed. Depending on the value of the sensitivity analysis of parameter the natural death rate, diseases induce rate, removing parameter and vaccination rate (the rate of exposed population of birds join to the recovered population) have an effects on controlling the psittacosis disease in the community and natural birth rate, the rate susceptible population infection by infected animal and the rate of exposed birds infected (incubation period) have an effect on expansion the psittacosis disease in the community. Moreover, numerical stimulation is performed in order to check the effect of each parameter in the expansion as well as in the controlling of psittacosis. Depending on numerical stimulation the removing rate and treatment rate are reduced (decrease) the expansion the disease and the other rate like contact rate and infective rate are increase the expansion of the disease.

ACKNOWLEDGMENT

Authors profoundly acknowledged the encouragement and support from members of department.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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