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DYNAMIC BEHAVIORS OF A PERIODIC LOTKA-VOLTERRA COMMENSAL SYMBIOSIS MODEL WITH IMPULSIVE

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Abstract. In this paper, the dynamic behaviors of an impulsive periodic Lotka-Volterra commensal symbiosis model is studied in this paper. Firstly, by constructing a suitable Lyapunov function and using the comparison theorem of impulsive differential equation, some sufficient conditions which ensure the permanence and global attractivity of the system are obtained; Secondly, conditions which guarantee that one species in the system are permanent while the remaining species is driven to extinction is obtained. Thirdly, conditions which ensure the extinction of the system are also obtained. Our results show that, for Lotka-Volterra commensal symbiosis model, impulsive is one of the important reasons that can change the long time behaviors of species.

Keywords: Lotka-Volterra commensal symbiosis model; Impulsive; Extinction; Lyapunov function; Permanence; Global attractivity.

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1. Introduction

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During the past decade, the dynamic behaviors of the mutualism model has been extensively investigated [1-12] and many excellent works concerned with the persistence, existence of positive periodic solution, and stability of the system were obtained. However, there are still few study on the commensal symbiosis model.

To describe the intraspecific commensal relationship, Sun and Wei[13] proposed the following model:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(\frac{k_1 - x + ay}{k_1}\Big), \\ \frac{dy}{dt} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big). \end{aligned} \tag{1.1}$$

The authors investigated the local stability of all equilibrium points. They showed that there is only one local stable equilibrium point in the system.

As was pointed out by He and Chen [14], the ecological system is often deeply perturbed by human exploit activities such as planting and harvesting etc., which are not suitable to be considered continually. To accurate describe the system, one needs to use the impulsive differential equations.

Stimulated by the works of Sun and Wei [13] and He and Chen [14], in this paper, we study the dynamic behaviors of the following periodic Lotka-Volterra commensal symbiosis model with impulsive.

$$\begin{cases} \dfrac{dx_1}{dt} = x_1\Big(a_1(t) - b_1(t)x_1 + c_1(t)x_2\Big), \\ \dfrac{dx_2}{dt} = x_2\Big(a_2(t) - b_2(t)x_2\Big), \qquad t \neq \tau_k, \\ x_i(\tau_k^+) = (1 + h_{ik})x_i(\tau_k), \qquad t = \tau_k,\ k = 1, 2, \cdots \end{cases} \tag{1.2}$$

where $i = 1, 2$; $x_i(t)$ denotes the density of the i-th species X_i at time t; $a_i(t)$, $b_i(t)$ and $c(t)$ are continuous strictly positive periodic functions defined on $[0, +\infty)$ with a common period $T > 0$; $\tau_k \to +\infty\,(t \to +\infty)$ and $0 = \tau_0 < \tau_1 < \tau_2 < \tau_3 < \cdots < \tau_k < \tau_{k+1} < \cdots$. Assume that h_{ik}, $i = 1, 2, \cdots, n$, $k = 1, 2, \cdots$, are constants and there exists an integer $q > 0$ such that

$$h_{i(k+q)} = h_{ik},\ \tau_{k+q} = \tau_k + T.$$

It is natural for biological meanings:

$$1 + h_{ik} > 0 (i = 1, 2).$$

Definition 1.1. The system (1.2) is called permanent, if for any positive solution $F(t) = (x_1(t), x_2(t))^T$ of (1.2), there exist positive constants λ_i and θ_i such that

$$\lambda_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le \theta_i,\ (i=1,2).$$

Definition 1.2. The system (1.2) is said to be globally attractive if any two positive solutions $F(t) = (x_1(t), x_2(t))^T$ and $W(t) = (u_1(t), u_2(t))^T$ of system (1.2) satisfy

$$\lim_{t\to+\infty} |x_i(t) - u_i(t)| = 0,\ i = 1,2.$$

Definition 1.3. For any $(t, F(t)) \in [\tau_{k-1}, \tau_k) \times \Re^2_+$, the right-hand derivative $D^+V(t, F(t))$ along the solution $F(t, F_0)$ of system (1.2) is defined by

$$D^+V(t,F(t)) = \liminf_{h\to 0^+} \frac{1}{h}[V(t+h, F(t+h)) - V(t, F(t))].$$

By the basic theories of impulsive differential equations in [15, 16], system (1.2) has a unique solution $F(t) = F(t, F_0) \in PC([0,+\infty), R^2)$ and $PC([0,+\infty), R^2)$ $= \{\phi : [0,+\infty) \to R^2, \phi$ is continuous for $t \ne \tau_k$, $\phi(\tau_k^-)$ and $\phi(\tau_k^+)$ exist and $\phi(\tau_k^-) = \phi(\tau_k)$, $k = 1,2,\cdots\}$ for each initial value $F(0) = F_0 \in R^2$.

Throughout this paper, for a continuous T-periodic function $f(t)$, we set

$$m[f] = \frac{1}{T}\int_0^T f(t)dt,\ f^l = \min_{t\in[0,T]}\{f(t)\},\ f^u = \max_{t\in[0,T]}\{f(t)\}.$$

The organization of this paper is as follows: In Section 2, necessary preliminaries are presented. The dynamic behaviors such as the permanence, extinction and the globally attractivity of the system are investigated in Section 3, and we end this paper by a briefly discussion.

2. Preliminaries

Now let us state several lemmas which will be useful in the proving of the main theorems.

Firstly, we introduce an important comparison theorem on impulsive differential equation [16].

Lemma 2.1. *Assume that $m \in PC[R_+, R]$ with points of discontinuity at $t = t_k$ and is left continuous at $t = t_k$, $k = 1, 2, \cdots$, and*

$$\begin{cases} Dm(t) \le g(t, m(t)), & t \ne t_k, \ \ k = 1, 2, \cdots, \\ m(t_k^+) \le \phi_k(m(t_k)), & t = t_k, \ \ k = 1, 2, \cdots, \end{cases} \tag{2.1}$$

where $g \in C[R_+ \times R_+, R]$, $\phi_k \in C[R, R]$ and $\phi_k(u)$ is nondecreasing in u for each $k = 1, 2, \cdots$. Let $r(t)$ be the maximal solution of the scalar impulsive differential equation

$$\begin{cases} \dot{u} = g(t, u), & t \ne t_k, \ \ k = 1, 2, \cdots, \\ u(t_k^+) = \phi_k(u(t_k)) \ge 0, & t = t_k, \ \ t_k > t_0, \ k = 1, 2, \cdots, \\ u(t_0^+) = u_0, & \end{cases} \tag{2.2}$$

existing on $[t_0, \infty)$, then $m(t_0^+) \le u_0$ implies $m(t) \le r(t)$, $t \ge t_0$.

Remark 2.1. *In Lemma 2.1, assume the inequalities (2.1) reversed. Let $p(t)$ be the minimal solution of (2.2) existing on $(t_0, +\infty)$. Then, $m(t_0^+) \ge u_0$ implies $m(t) \ge p(t)$, $t \ge t_0$.*

Lemma 2.2. *Let $F(t) = (x_1(t), x_2(t))^T$ be any solution of system (1.2) such that $x_i(0^+) > 0$, $(i = 1, 2)$, then $x_i(t) > 0$ for all $t \ge 0$.*

Proof. From the i-th equation of (1.2), one has

$$x_i'(t) = P_i(t)x_i(t), \ \ t \ne \tau_k, \ \ i = 1, 2,$$

where

$$P_1(t) = a_1(t) - b_1(t)x_1 + c_1(t)x_2; \ P_2(t) = a_2(t) - b_2(t)x_2.$$

Thus we have

$$x_i(t) = \prod_{0 < \tau_k < t} (1 + h_{ik}) x_i(0^+) \exp\left(\int_0^t P_i(s) ds \right) > 0,$$

because of $x_i(0^+) > 0$. Consider the periodic logistic equation with impulsive

$$\begin{cases} x'(t) = x(t)\left(b(t) - a(t)(x(t))\right), & t \ne \tau_k, \\ x(\tau_k^+) = (1 + h_k)x(\tau_k), & t = \tau_k, \ k = 1, 2, \cdots, \end{cases} \tag{2.3}$$

where $a(t),\ b(t)$ are continuous T-periodic function with $a(t)>0,\ m[b]>0$ and $h_{k+q}=h_k,\ \tau_{k+q}=\tau_k+T$.

Lemma 2.3. [17] (1) *If*

$$\sum_{k=1}^{q}\ln(1+h_k)+Tm[b]>0, \tag{2.4}$$

then system (2.3) has a unique T-periodic solution $x^*(t)$, and $x^*(t)$ is global asymptotically stable in the sense that

$$\lim_{t\to+\infty}|x(t)-x^*(t)|=0,$$

where $x(t)$ is any solution of system (2.3) with initial value $x(0^+)>0$.

(2) *If*

$$\sum_{k=1}^{q}\ln(1+h_k)+Tm[b]<0, \tag{2.5}$$

then

$$\lim_{t\to\infty}x(t)=0,$$

where $x(t)$ is any solution of system (2.3) with the initial value $x(0^+)>0$.

From Theorem 3.1 in [18], we can get the following lemma.

Lemma 2.4. *If $x^*(t)$ is the unique T-periodic positive solution of system (2.3), and $\tilde{x^*}(t)$ is the unique T-periodic positive solution of the following system*

$$\begin{cases} x'(t)=x(t)\left(\tilde{b}(t)-a(t)(x(t))\right), & t\neq\tau_k,\\ x(\tau_k^+)=(1+h_k)x(\tau_k), & t=\tau_k,\ k=1,2,\cdots, \end{cases}$$

where $\tilde{b}$ is the continuous T-periodic function with $m[\tilde{b}]>0$ and $\tilde{b}$ is a function asymptotic to b. Then

$$\lim_{t\to+\infty}|\tilde{x^*}(t)-x^*(t)|=0.$$

(I) Consider the periodic logistic equation with impulsive

$$\begin{cases} x_i'(t)=x_i(t)\left(a_i(t)-b_i(t)x_i(t)\right), & t\neq\tau_k,\\ x_i(\tau_k^+)=(1+h_{ik})x_i(\tau_k), & t=\tau_k,\ \ k=1,2,\cdots \end{cases} \tag{2.6}$$

where $i = 1,2$.

From Lemma 2.3, we notice that if $\sum_{k=1}^{q} \ln(1+h_{ik}) + Tm[a_i] > 0$, then system (2.6) has a unique T-periodic solution $X_i^*(t)$, and $X_i^*(t)$ is global asymptotically stable in the sense that

$$\lim_{t\to+\infty} |x_i(t) - X_i^*(t)| = 0,$$

where $x_i(t)$ is any solution of system (2.6) with initial value $x_i(0^+) > 0$.

(II) Consider the periodic logistic equation with impulsive

$$\begin{cases} \dfrac{dx_1}{dt} &= x_1\Big(a_1(t) - b_1(t)x_1 + c_1(t)X_2^*(t)\Big), \quad t \neq \tau_k, \\ x_1(\tau_k^+) &= (1+h_{1k})x_1(\tau_k), \qquad t = \tau_k, \;\; k = 1,2,\cdots \end{cases} \tag{2.7}$$

From Lemma 2.3, we notice that if

$$\sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)X_2^*(t)] > 0,$$

then system (2.7) has a unique T-periodic solution $X_{1*}(t)$, and $X_{1*}(t)$ is global asymptotically stable in the sense that

$$\lim_{t\to+\infty} |x_1(t) - X_{1*}(t)| = 0,$$

where $x_1(t)$ is any solution of system (2.7) with initial value $x_1(0^+) > 0$.

3. Main results

In this section, we present out our main results for system (1.2).

Let

$$\alpha_i = \sum_{k=1}^{q} \ln(1+h_{ik}) + Tm[a_i], \qquad i = 1,2,$$

$$\beta_1 = \sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)X_2^*(t)].$$

Assume that

$$\alpha_2 > 0,\ \beta_1 > 0; \tag{H_1}$$

$$\alpha_2 > 0,\ \beta_1 < 0; \tag{H_2}$$

$$\alpha_2 < 0,\ \alpha_1 > 0. \tag{H_3}$$

$$\alpha_2 < 0,\ \alpha_1 < 0. \tag{H_4}$$

Under the condition (H_1), we study the permanence and global attractivity of system (1.2).

Theorem 3.1. *Assume that (H_1) holds, let $F(t) = (x_1(t), x_2(t))^T$ be any solution of system (1.2) with $x_i(0^+) > 0$ $(i = 1,2)$, then there exist constants $\theta_i > 0, \lambda_i > 0 (i = 1,2)$ such that*

$$\lambda_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le \theta_i, \quad i = 1,2.$$

Proof. From the second equation of system (1.2), we obtain

$$\begin{cases} x_2'(t) = x_2(t)\left(a_2(t) - b_2(t)x_2(t)\right), & t \ne \tau_k, \\ x_2(\tau_k^+) = (1+h_{2k})x_2(\tau_k), & t = \tau_k, \quad k = 1,2,\cdots \end{cases} \tag{3.1}$$

By $\alpha_2 > 0$, from Lemma 2.3, system (3.1) has a unique T-periodic solution $X_2^*(t)$ which is global asymptotically stable. Let $\theta_2 = \sup\left\{X_2^*(t)\middle| t \in [0,T]\right\}$, $\lambda_2 = \inf\left\{X_2^*(t)\middle| t\in[0,T]\right\}$, let $\varepsilon > 0$ small enough such that $\varepsilon < \frac{1}{2}\lambda_2$ and $\sum_{k=1}^{q}\ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)(X_2^*(t) - \varepsilon)] > 0$, there exists a $T_1 > 0$ such that

$$\lambda_2 - \varepsilon \le X_2^*(t) - \varepsilon < x_2(t) < X_2^*(t) + \varepsilon \le \theta_2 + \varepsilon \quad \text{for all } t > T_1. \tag{3.2}$$

Setting $\varepsilon \to 0$, we obtain

$$\lambda_2 < \liminf_{t\to+\infty} x_2(t) \le \limsup_{t\to+\infty} x_2(t) \le \theta_2.$$

From the first equation of system (1.2), for $t > T_1$ and $t \ne \tau_k\,(k = 1,2,\cdots)$, we obtain

$$x_1'(t) \le x_1(t)\left(a_1(t) + c_1(t)(X_2^*(t)+\varepsilon) - b_1(t)x_1(t)\right). \tag{3.3}$$

Consider the following system

$$\begin{cases} w_1'(t) = w_1(t)x_1(t)\left(a_1(t) + c_1(t)(X_2^*(t)+\varepsilon) - b_1(t)w_1(t)\right), & t \ne \tau_k, \\ w_1(\tau_k^+) = (1+h_{1k})w_1(\tau_k), & t = \tau_k, \quad k = 1,2,\cdots \end{cases} \tag{3.4}$$

By Lemma 2.1, we have $x_1(t) \le w_1(t)$, where $w_1(t)$ is any solution of (3.4) with $w_1(T_1^+) = x_1(T_1^+)$. By (H_1) and Lemma 2.3, system (3.4) has a unique T-periodic solution $X_{1\varepsilon}^*(t)$ which is global asymptotically stable. Setting $\varepsilon \to 0$, from Lemma 2.4, we have

$$X_{1\varepsilon}^*(t) \to X_{1*}(t).$$

Let $\theta_1 = \sup\left\{X_{1*}(t)\Big| t \in [0,T]\right\}$, and for any positive constant $\varepsilon > 0$, there exists a $T_2 > T_1$ such that

$$x_1(t) \le w_1(t) < X_{1*}(t) + \varepsilon_0 \le \theta_1 + \varepsilon_0, \quad \text{for all } t > T_2.$$

Setting $\varepsilon_0 \to 0$, we obtain

$$\limsup_{t\to+\infty} x_1(t) \le \theta_1.$$

From the first equation of system (1.2), for $t > T_1$ and $t \ne \tau_k\,(k = 1,2,\cdots)$, we also have

$$x_1'(t) \ge x_1(t)\left(a_1(t) + c_1(t)(X_2^*(t) - \varepsilon) - b_1(t)x_1(t)\right). \tag{3.5}$$

Consider the following system

$$\begin{cases} w_2'(t) = w_2(t)\left(a_1(t) + c_1(t)(X_2^*(t) - \varepsilon) - b_1(t)w_2(t)\right), & t \ne \tau_k, \\ w_2(\tau_k^+) = (1 + h_{1k})w_2(\tau_k), \qquad t = \tau_k, & k = 1,2,\cdots \end{cases} \tag{3.6}$$

By Lemma 2.1, we have $x_1(t) \ge w_2(t)$, where $w_2(t)$ is any solution of (3.6) with $w_2(T_1^+) = x_2(T_1^+)$. Since $\sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)(X_2^*(t) - \varepsilon)] > 0$, by Lemma 2.3, system (3.6) has a unique T-periodic solution $X_{1*\varepsilon}(t)$, which is global asymptotically stable. Setting $\varepsilon \to 0$, from Lemma 2.4, we have

$$X_{1*\varepsilon}(t) \to X_{1*}(t).$$

Let $\lambda_1 = \inf\left\{X_{1*}(t)\Big| t \in [0,T]\right\}$, and for any positive constant $\varepsilon_0 > 0(\varepsilon_0 < \frac{1}{2}\lambda_1)$, there exists a $T_3 > T_2$ such that

$$x_1(t) \ge w_1(t) > X_{1*}(t) - \varepsilon_0 \ge \lambda_1 - \varepsilon_0 \quad \text{for all } t > T_3.$$

Setting $\varepsilon_0 \to 0$, we obtain

$$\liminf_{t\to+\infty} x_1(t) \ge \lambda_1.$$

The proof is completed.

Now let us consider the global attractivity of $x_i(t)(i = 1,2)$ of system (1.2), we can obtain the following result.

Theorem 3.2. *Suppose that (H_1) holds. Assume further that*

$$b_2^l > c_1^u, \tag{H_5}$$

then the species x_i $(i=1,2)$ are globally attractive, i.e., for any positive solutions $F(t)=(x_1(t),x_2(t))^T$ and $W(t)=(u_1(t),u_2(t))^T$ of (1.2), one has

$$\lim_{t\to+\infty}|x_i(t)-u_i(t)|=0,\ i=1,2.$$

Proof. Let $F(t)=(x_1(t),x_2(t))^T$ and $W(t)=(u_1(t),u_2(t))^T$ be any solutions of system (1.2). For any positive constant $\varepsilon<\min\{\lambda_i,\big|i=1,2\}$, from Theorem 3.1, it immediately follows that there exists an enough large $T_0>0$ such that for all $t>T_0$, one has

$$\lambda_i-\varepsilon\le x_i(t),u_i(t)\le\theta_i+\varepsilon,\ i=1,2. \tag{3.7}$$

Set

$$V(t)=\sum_{i=1}^{2}|\ln u_i(t)-\ln x_i(t)|.$$

For $t\ge0$, and $t\ne\tau_k,\ k=1,2,\cdots$, calculating the upper right derivative of $V(t)$, we have

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^{2}\left(\frac{u_i'(t)}{u_i(t)}-\frac{x_i'(t)}{x_i(t)}\right)\operatorname{sgn}(u_i(t)-x_i(t)) \\ &\le -b_1^l|x_1(t)-u_1(t)|-(b_2^l-c_1^u)|x_2(t)-u_2(t)|\Big\}, \end{aligned}$$

By using the Mean Value Theorem and (3.7), for any closed interval contained in $t\in(\tau_k,\tau_{k+1}],\ k=p,p+1,\cdots$ and $\tau_p>T_0$, it follows that

$$\frac{1}{\theta_i+\varepsilon}|x_i(t)-u_i(t)|\le|\ln x_i(t)-\ln u_i(t)|\le\frac{1}{\lambda_i-\varepsilon}|x_i(t)-u_i(t)|,\ i=1,2. \tag{3.8}$$

Therefore, for $t\in(\tau_k,\tau_{k+1}],\ k=p,p+1,\cdots$ and $\tau_p>T_0$, it follows from conditions (H_5) and (3.8) that

$$\begin{aligned} D^+V(t) &\le -\min\{b_1^l,b_2^l-c_1^u\}\sum_{i=1}^{2}|x_i(t)-u_i(t)| \\ &\le -\min\{b_1^l,b_2^l-c_1^u\}\sum_{i=1}^{2}(\lambda_i-\varepsilon)|\ln x_i(t)-\ln u_i(t)| \\ &\le -\phi_\varepsilon V(t), \end{aligned} \tag{3.9}$$

where

$$\phi_\varepsilon=\min\{b_1^l,b_2^l-c_1^u\}\cdot\min\Big\{\lambda_1-\varepsilon,\lambda_2-\varepsilon\Big\}.$$

For $t=\tau_k,\ \ k=1,2,\cdots$, we have

$$\begin{aligned} V(\tau_k^+) &= \sum_{i=1}^{2}|\ln u_i(\tau_k^+)-\ln x_i(\tau_k^+)| \\ &= \sum_{i=1}^{2}|\ln[(1+h_{ik})u_i(\tau_k)]-\ln[(1+h_{ik})x_i(\tau_k)]| = V(\tau_k). \end{aligned}$$

Above analysis show that, for all $t>\tau_p>T_0$,

$$D^+V(t) < -\phi_\varepsilon V(t). \tag{3.10}$$

Applying the differential inequality theorem and the variation of constants formula of solutions of first-order linear differential equation, we have

$$V(t) \le V(\tau_p)\exp(-\phi_\varepsilon(t-\tau_p)). \tag{3.11}$$

It obvious that $V(t)\to 0$ as $t\to+\infty$, that is

$$\sum_{i=1}^{2}|\ln u_i(t)-\ln x_i(t)| \to 0 \ \text{ as } \ t\to+\infty. \tag{3.12}$$

From (3.12) and (3.8), one could easily see that

$$\sum_{i=1}^{2}\frac{1}{\theta_i+\varepsilon}|u_i(t)-x_i(t)| \to 0 \ \text{ as } \ t\to+\infty.$$

And so

$$\lim_{t\to+\infty}|x_i(t)-u_i(t)|=0,\ i=1,2.$$

The proof is completed.

From Theorem 3.1 and 3.2, we consider the permanence and stability of system (1.2) with $\alpha_2>0,\ \beta_1>0$. But with the impulsive perturbations in system (1.2), the property of the system (1.2) will be changed with $\alpha_2<0$ or $\beta_1<0$, then we have the following results.

Theorem 3.3. *Assume that (H_2) holds, let $F(t)=(x_1(t),x_2(t))^T$ be any solution of system (1.2) with $x_i(0^+)>0\ (i=1,2)$, then there exist constants $\theta_i>0,\lambda_i>0(i=1,2)$ such that*

$$\lambda_2 \le \liminf_{t\to+\infty} x_2(t) \le \limsup_{t\to+\infty} x_2(t) \le \theta_2.$$

$$\lim_{t\to+\infty} x_1(t)=0.$$

Proof. From $\beta_1 < 0$, for positive constant $\varepsilon > 0$ small enough,

$$\beta_{1\varepsilon} = \sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)(X_2^*(t)+\varepsilon)] < 0. \tag{3.13}$$

By $\alpha_2 > 0$, similarly to the analysis of (3.1)-(3.2), we have

$$\lambda_2 < \liminf_{t\to+\infty} x_2(t) \le \limsup_{t\to+\infty} x_2(t) \le \theta_2. \tag{3.14}$$

From the first equation of system (1.2), for $t > T_1$ and $t \neq \tau_k\,(k = 1,2,\cdots)$, we obtain

$$x_1'(t) \le x_1(t)\left(a_1(t) + c_1(t)(X_2^*(t)+\varepsilon) - b_1(t)x_1(t)\right). \tag{3.15}$$

Consider the following system

$$\begin{cases} w_3'(t) = w_3(t)\left(a_1(t) + c_1(t)(X_2^*(t)+\varepsilon) - b_1(t)w_3(t)\right), & t \neq \tau_k, \\ w_3(\tau_k^+) = (1+h_{1k})w_3(\tau_k), & t = \tau_k,\ \ k = 1,2,\cdots \end{cases} \tag{3.16}$$

By Lemma 2.1, we have $x_1(t) \le w_3(t)$, where $w_3(t)$ is any solution of (3.16) with $w_3(T_1^+) = x_1(T_1^+)$. By (3.13) and Lemma 2.3, we find that $\lim_{t\to+\infty} w_3(t) = 0$. This combine with the positivity of $x_1(t)$, implies that $\lim_{t\to+\infty} x_1(t) = 0$. The proof is completed.

Theorem 3.4. *Assume that (H_3) holds and let $F(t) = (x_1(t), x_2(t))^T$ be any solution of system (1.2) with $x_i(0^+) > 0\ (i = 1,2)$, then there exist constants $\theta_{11} > 0, \lambda_{11} > 0$ such that*

$$\lambda_{11} \le \liminf_{t\to+\infty} x_1(t) \le \limsup_{t\to+\infty} x_1(t) \le \theta_{11}.$$

$$\lim_{t\to+\infty} x_2(t) = 0.$$

Proof. Since $\alpha_2 < 0$, by Lemma 2.3 (2),

$$\lim_{t\to+\infty} x_2(t) = 0. \tag{3.17}$$

Also, $\alpha_1 > 0$ implies that for positive constant ε small enough,

$$\alpha_{1\varepsilon} = \sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)\varepsilon] > 0. \tag{3.18}$$

It follows from (3.17) that there exists a $T_1 > 0$ such that

$$x_2(t) < \varepsilon \ \text{ for all } \ t \ge T_1. \tag{3.19}$$

From the first equation of system (1.2), for $t > T_1$ and $t \neq \tau_k\,(k = 1,2,\cdots)$, we obtain

$$x_1'(t) \leq x_1(t)\left(a_1(t) + c_1(t)\varepsilon - b_1(t)x_1(t)\right). \tag{3.20}$$

Consider the following system

$$\begin{cases} w_4'(t) = w_4(t)\left(a_1(t) + c_1(t)\varepsilon - b_1(t)w_4(t)\right), & t \neq \tau_k, \\ w_4(\tau_k^+) = (1 + h_{1k})w_4(\tau_k), & t = \tau_k,\ \ k = 1,2,\cdots \end{cases} \tag{3.21}$$

By Lemma 2.1, we have $x_1(t) \leq w_4(t)$, where $w_4(t)$ is any solution of (3.21) with $w_4(T_1^+) = x_1(T_1^+)$. By (3.18) and Lemma 2.3, system (3.21) has a unique T-periodic solution $X_{1\varepsilon}^*(t)$, which is global asymptotically stable. Setting $\varepsilon \to 0$, from Lemma 2.4, we have

$$X_{1\varepsilon}^*(t) \to X_1^*(t).$$

Let $\theta_{11} = \sup\left\{X_1^*(t)\Big|t \in [0,T]\right\}$, and for any positive constant $\varepsilon_0 > 0$, there exists a $T_2 > T_1$ such that

$$x_1(t) \leq w_1(t) < X_1^*(t) + \varepsilon_0 \leq \theta_{11} + \varepsilon_0 \ \text{ for all } t > T_2.$$

Setting $\varepsilon_0 \to 0$, we obtain

$$\limsup_{t\to+\infty} x_1(t) \leq \theta_{11}. \tag{3.22}$$

From the first equation of system (1.2) and the positivity of $x_2(t)$, for $t > T_1$ and $t \neq \tau_k\,(k = 1,2,\cdots)$, we also have

$$x_1'(t) \geq x_1(t)\left(a_1(t) - b_1(t)x_1(t)\right). \tag{3.23}$$

Consider the following system

$$\begin{cases} w_5'(t) = w_5(t)\left(a_1(t) - b_1(t)w_5(t)\right), & t \neq \tau_k, \\ w_5(\tau_k^+) = (1 + h_{1k})w_5(\tau_k), & t = \tau_k,\ \ k = 1,2,\cdots \end{cases} \tag{3.24}$$

By Lemma 2.1, we have $x_1(t) \geq w_5(t)$, where $w_5(t)$ is any solution of (3.24) with $w_5(T_1^+) = x_2(T_1^+)$. By (H_3) and Lemma 2.3, system (3.24) has a unique T-periodic solution $X_1^*(t)$, which is global asymptotically stable. Let $\lambda_{11} = \inf\left\{X_1^*(t)\Big|t \in [0,T]\right\}$, and for any positive constant $\varepsilon_0 > 0(\varepsilon_0 < \frac{1}{2}\lambda_{11})$, there exists a $T_3 > T_2$ such that

$$x_1(t) \geq w_5(t) > X_1^*(t) - \varepsilon_0 \geq \lambda_{11} - \varepsilon_0 \ \text{ for all } t > T_3.$$

Setting $\varepsilon_0 \to 0$, we obtain

$$\liminf_{t\to+\infty} x_1(t) \geq \lambda_{11}.$$

The proof is completed.

Theorem 3.5. *Assume that (H_4) holds, let $F(t) = (x_1(t), x_2(t))^T$ be any solution of system (1.2) with $x_i(0^+) > 0$ $(i = 1,2)$, then*

$$\lim_{t\to+\infty} x_i(t) = 0, \ \ i = 1,2.$$

Proof. Since $\alpha_2 < 0$, by Lemma 2.3 (2),

$$\lim_{t\to+\infty} x_2(t) = 0. \tag{3.25}$$

Also, $\alpha_1 < 0$ implies that for positive constant ε small enough,

$$\alpha_{1\varepsilon} = \sum_{k=1}^{q} \ln(1+h_{1k}) + Tm[a_1(t) + c_1(t)\varepsilon] < 0. \tag{3.26}$$

It follows from (3.25) that there exists a $T_1 > 0$ such that

$$x_2(t) < \varepsilon \ \text{ for all } \ t \geq T_1. \tag{3.27}$$

From the first equation of system (1.2), for $t > T_1$ and $t \neq \tau_k\,(k = 1,2,\cdots)$, we obtain

$$x_1'(t) \leq x_1(t)\left(a_1(t) + c_1(t)\varepsilon - b_1(t)x_1(t)\right). \tag{3.28}$$

Consider the following system

$$\begin{cases} w_6'(t) = w_6(t)\left(a_1(t) + c_1(t)\varepsilon - b_1(t)w_6(t)\right), & t \neq \tau_k, \\ w_6(\tau_k^+) = (1+h_{1k})w_6(\tau_k), & t = \tau_k, \ \ k = 1,2,\cdots \end{cases} \tag{3.29}$$

By Lemma 2.1, we have $x_1(t) \leq w_6(t)$, where $w_6(t)$ is any solution of (3.29) with $w_6(T_1^+) = x_3(T_1^+)$. By (3.26) and Lemma 2.3,

$$\lim_{t\to+\infty} w_6(t) = 0.$$

This combine with the positivity of $x_1(t)$, implies that

$$\lim_{t\to+\infty} x_1(t) = 0.$$

The proof is completed.

4. Discussion

Sun and Wei [13] proposed the commensal symbiosis model (1.1). They showed that the positive equilibrium of the system is locally stable. In this paper, we further incorporate the impulsive to the system (1.1) and propose the system (1.2). We found that depending on the choice of impulsive, the system could still be permanence and globally attractivity, or some of the species be extinct while the other one still permanent, or all the species will be driven to extinction. That is, impulsive is one of the important reasons that can change the long time behaviors of species.

Conflict of Interests

The author declares that there is no conflict of interests.

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