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DYNAMIC BEHAVIORS OF A STAGE-STRUCTURED COOPERATION MODEL

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Abstract. In this paper, a two-species May type cooperation model with stage structure is presented and studied. Results on the global extinction, partial survival and global attractivity of the positive equilibrium are given, which generalize the well-known May's result for the two species cooperation system and, moreover, they confirm the negative effect of stage structure on the persistent of populations. Conclusions in this paper suggest that for a cooperation community, stage structure and the death rate of mature species are two of the most important reason that cause global attractivity and extinction, cooperate has no influence on the persistent property of the model, and conditions which ensure the permanence of the single species are enough to ensure the global stability of the system.

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1. Introduction

In nature, many species exhibit enormous diversity during their life histories, and they go through two or more life stages as they proceed from birth to death. Such life history diversity of species can be modelled by stage-structured models.

Aiello and Freedman (1990) proposed a single-species growth model with stage structure consisting of immature and mature stages. They showed that under suitable hypotheses there exists a globally asymptotically stable positive equilibrium; Freedman *et al.* (1994) proposed a model of a stage-structured population with fixed maturity time for the immature stage and interaction terms that may be interpreted as cooperation or cannibalism. The existence and stability of the equilibrium set were discussed. In the case of cannibalism, they found that a Hopf bifurcation could result in a stable periodic solution. Song and Chen (2002) studied the asymptotic behavior of a single-species model with stage structure and harvesting. For the constant, variable, and periodic harvesting effort, they obtained conditions for the global stability of the equilibria, permanence of the system and global attractivity of the periodic solution, respectively. Traditional two species ecosystem includes three type: predator-prey, competition and mutualism (cooperation). Several scholars had incorporated the stage-structure to a two species competition system, Chen (2006) studied a non-autonomous, almost periodic competitive two-species model with a stage structure in one species, sufficient conditions were obtained for the existence of a unique, globally attractive, strictly positive (componentwise), almost periodic solution; Liu, Chen and Li (2002) proposed a stage-structured competition system, where he used a discrete delay to denote the time taken from birth to maturity; Al-Omari and Gourley (2003), Al-Omari, Al-Omari (2011), Liu and Beretta (2006), Wang and Feng (2010) studied the competition system with stage-structured of distributed delay. Predator-prey system with stage-structured for prey or predator are also extensively investigated by many authors (Chen *et al.*, 2008, 2012, 2013a, 2013b, Cui and Song, 2004, 2007, Hu and Huang 2010, Huang *et al.*, 2010, Li *et al.*, 2009, Ma et al.,2008, Wang et al.,2001, Xu *et al.*, 2005, 2011, Zhang *et al.*, 2000, Zhang, 2005, Gui and Ge, 2005). Specially, recently, in their series works, Chen *et al.* (2012, 2013a, 2013b) investigated a stage-structured predator-prey system, their studied shows that for this kind of system, the extinction of prey may not necessarily lead to extinction of predator.

By introducing a new lemma and applying the standard comparison theorem, they investigated the persistent property of the system; By using an iterative method, the global stability of the interior equilibrium point of the system is investigated. They showed that conditions which ensure the permanence of the system are enough to ensure the global stability of the system. Their studied shows that the death rate of mature prey and predator species is one of the essential factor to determine the dynamic behaviors of the system.

As was pointed out by Murry (1998) "the mutual advantage of mutualism or symbiosis can be very important. As a topic of theoretical ecology, even for two species, this area has not been as widely studied as the others even though its importance is comparable to that of predator-prey and competition interactions." During the past ten years, such topic as existence of positive periodic solution and persistent property of the cooperation system has been extensively investigated by many scholars (Chen et al, 2006, 2007, 2008, 2009; Chen et al., 2009, Chen and Xie, 2011, Hu and Zhang, 2010, Niyaz and Muhammadhaji, 2013, Muhammadhaji and Teng, 2013, Yang and Li, 2011). However, all of the above works did not considered the influence of the stage structure of the species, to this day, only Zhang, Wu and Wang (2004) investigated the positive periodic solution of a stage-structured Lotka-Volterra type cooperation system. To the best of the author's knowledge, to this day, there are still no scholar investigate the permanence, extinction and stability property of stage-structured cooperation system.

The organization of this paper is as follows. In Section 2, we introduce some models. In Section 3, we state the main results for globally stability properties, partial survival and extinction of the system. Detailed proof of the main results are presented in Section 4. We end this paper by a detail discussion.

2. **Formulation of the models**

Traditional two species Lotka-Volterra cooperation model takes the form:

$$\begin{aligned} \dot{x}_1 &= x_1\big(a_1 - b_{11}x_1 + b_{12}x_2\big), \\ \dot{x}_2 &= x_2\big(a_2 + b_{21}x_1 - b_{22}x_2\big), \end{aligned} \tag{2.1}$$

where $a_i, b_{ij}, i, j = 1,2$ are all positive constants. Murray (1998) had pointed out that one of the drawback of above system is the sensitivity between unbounded growth and a finite positive steady state. If symbiosis of either species is too large then both populations grow unboundedly.

Base on model (2.1), Zhang, Wu and Wang (2004) proposed the following stage-structured cooperation system:

$$\begin{aligned}
\dot{x}_1(t) &= \alpha(t)x_2(t) - \gamma_1(t)x_1(t) - \beta(t)x_1(t) - \eta_1(t)x_1^2(t), \\
\dot{x}_2(t) &= \beta(t)x_1(t) - \gamma_2(t)x_2(t) - \eta_2(t)x_2^2(t) + b(t)x_2(t)y(t), \\
\dot{y}(t) &= y(t)\big(R(t) - a(t)y(t) + c(t)x_2(t)\big),
\end{aligned} \tag{2.2}$$

where $x_1(t)$ denotes the density of immaturity of species X at time t, $x_2(t)$ denotes the density of maturity of species X at time t, $y(t)$ denotes the density of species Y at time t. By using the continuation theorem of coincidence degree theory, the existence of a positive periodic solution for above system is established.

To overcome the drawbacks of system (2.1), May (1976) suggested the following set of equations to describe a pair of mutualists:

$$\begin{aligned}
\dot{u} &= r_1 u\Big[1 - \frac{u}{a_1 + b_1 v} - c_1 u\Big], \\
\dot{v} &= r_2 v\Big[1 - \frac{v}{a_2 + b_2 u} - c_2 v\Big],
\end{aligned} \tag{2.3}$$

where u, v are the densities of the species U, V at time t respectively. $r_i, a_i, b_i, i = 1,2$ are positive constants. He showed that system (2.3) has a globally asymptotically stable equilibrium point in the region $u > 0, v > 0$. The basic idea about this model is that the cooperate of two species increasing the other species carrying capacity.

Aiello and Freedman (1990) had introduced the following single-species stage-structured model:

$$\begin{aligned}
I'(t) &= aM(t) - \gamma I(t) - ae^{-\gamma\tau}M(t-\tau), \\
M'(t) &= ae^{-\gamma\tau}M(t-\tau) - bM^2(t),
\end{aligned} \tag{2.4}$$

where $I(t)$ and $M(t)$ represent the immature and mature population densities, respectively. The authors had proved the system admits a unique globally attractive positive equilibrium. Recently, in their series works, Chen et al.(2012, 2013a, 2013b) studied a predator-prey system with stage-structure for both predator and prey species, they incorporate the death rate of mature prey and predator species to their system and many new findings were obtained.

Stimulated by the works of May (1976), Aiello and Freedman (1990) and Chen *et al.* (2012, 2013a, 2013b), in this paper, we propose the following May type stage-structured cooperation model,

$$\begin{aligned}\dot{x}_1(t) &= b_1 e^{-d_{11}\tau_1}x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1x_2(t)} - a_{12}x_1^2(t),\\ \dot{y}_1(t) &= b_1x_1(t) - d_{11}y_1(t) - b_1e^{-d_{11}\tau_1}x_1(t-\tau_1),\\ \dot{x}_2(t) &= b_2 e^{-d_{22}\tau_2}x_2(t-\tau_2) - d_{21}x_2(t) - \frac{a_{22}x_2^2(t)}{c_2 + f_2x_1(t)} - a_{21}x_2^2(t),\\ \dot{y}_2(t) &= b_2x_2(t) - d_{22}y_2(t) - b_2e^{-d_{22}\tau_2}x_2(t-\tau_2),\end{aligned} \tag{2.5}$$

where $b_i, a_{ij}, d_{ij}, c_i, f_i (i, j = 1, 2)$ are all positive constants. We suppose that the system is occupied by two cooperation species denoted as species 1 and species 2. The life histories of both species are divided into two stages: the immature and the mature. Let $x_i(t)$ and $y_i(t) (i = 1, 2)$ be the density of the mature and the immature of the i-th species, respectively. We make the following assumptions for our model:

(A_1) We assume that the immature and mature individuals are divided by a fixed period, also a species needs some time to attain its level of maturity to cooperate with the other species, the cooperation benefits the other species by increasing its' carrying capacity, and the immature individual could not cooperate with each other.

(A_2) The birth rate in the immature population of i-th species is proportional to the living mature population with proportionality constant $b_i > 0$. For each species, its immature cannot give birth to babies. The death rate of the i-th immature is proportional to the existing immature population with proportionality constants $d_{ii} > 0, i = 1, 2$.

(A_3) The death rate of the i-species mature population is proportional to the existing immature population with proportionality constants $d_{ij} > 0, i, j = 1, 2, i \neq j$..

(A_4) $\tau_i > 0, i = 1, 2$ is the length of the i-immature stage, that is, those immature individuals of i-th species born at time $t - \tau_i$ and surviving to the time t leave the immature stage and enter into the mature population.

The initial conditions for system (2.5) take the form

$$x_i(\theta) = \phi_i(\theta) > 0,\ y_i(\theta) = \psi_i(\theta) > 0, -\tau \leq \theta \leq 0,\ i = 1, 2, \tag{2.6}$$

where $\tau = \max\{\tau_i, i = 1,2,3,4\}$. For the continuity of the solutions of system (1.1), in this paper, we always assume

$$y_i(0) = \psi_i(0) = \int_{-\tau_i}^{0} b_i \phi_i(s) e^{d_{ii}s} ds, \ i = 1,2. \tag{2.7}$$

Note that in the system (2.5) the equations for the variable y_1 and y_2 have a particular forms

$$y_i = -d_{ii}y_i + f_i(x_i(t), x_i(t-\tau_i)), i = 1,2,$$

where $f_i(x_i(t), x_i(t-\tau_i)) = b_i x_i(t) - b_i e^{-r_i\tau_i} x_i(t-\tau_i)$. By the well-known theory of ODE, if $x_i(t)$ is bounded then $y_i(t)$ is bounded, and if $x_i(t) \to x_i^*$ as $t \to +\infty$, then $y_i(t) \to \frac{f(x_i^*, x_i^*)}{d_i}$ as $t \to +\infty$; that is, the asymptotic behavior of $y_i(t)$ is depended on that of $x_i(t)$. Therefore, in this paper we just need to study the asymptotic behavior for the subsystem of system (2.5).

$$\begin{aligned} \dot{x}_1(t) &= b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1 x_2(t)} - a_{12}x_1^2(t), \\ \dot{x}_2(t) &= b_2 e^{-d_{22}\tau_2} x_2(t-\tau_2) - d_{21}x_2(t) - \frac{a_{22}x_2^2(t)}{c_2 + f_2 x_1(t)} - a_{21}x_2^2(t). \end{aligned} \tag{2.8}$$

3. **Main results**

Lemma 3.1 *Assume that $\lambda_1 > 0, \lambda_2 > 0$, then the following system*

$$\begin{aligned} \lambda_1 - \frac{a_{11}x_1}{c_1 + f_1 x_2} - a_{12}x_1 = 0, \\ \lambda_2 - \frac{a_{22}x_2}{c_2 + f_2 x_1} - a_{21}x_2 = 0 \end{aligned} \tag{3.1}$$

admits a unique positive solution (x_1^*, x_2^*).

Proof. Since we are focus on the positive solution of the system (3.1), it implies that we only need to consider the case $x_1 > 0, x_2 > 0$. Hence, to ensure the first equality holds, x_1 should be lies in the interval $(0, \frac{r_1}{a_{12}})$, similarly, to ensure the second equality holds, x_2 should be lies in the interval $(0, \frac{r_2}{a_{21}})$. Following we will investigate the positive solution of system (3.1) on the rectangle $(0, \frac{r_1}{a_{12}}) \times (0, \frac{r_2}{a_{21}})$.

The first equation of system (3.1) define a curve

$$l_1 : x_2 = -\frac{-\lambda_1 c_1 + a_{11}x_1 + a_{12}x_1 c_1}{f_1(-\lambda_1 + a_{12}x_1)}. \tag{3.2}$$

The second equation of system (3.1) define a curve

$$l_2 : x_2 = \frac{r_2(c_2 + f_2 x_1)}{a_2 2 + a_{21} c_2 + a_{21} f_2 x_1}. \tag{3.3}$$

Now let us consider the function

$$F := -\frac{-\lambda_1 c_1 + a_{11} x_1 + a_{12} x_1 c_1}{f_1(-\lambda_1 + a_{12} x_1)} - \frac{r_2(c_2 + f_2 x_1)}{a_2 2 + a_{21} c_2 + a_{21} f_2 x_1}. \tag{3.4}$$

Since F is a continuous function of x_1 on the interval $(0, \frac{r_1}{a_{12}})$,

$$F(0) = -\frac{r_2 f_1 c_2 + c_1 a_{22} + c_1 a_{21} c_2}{f_1(a_{22} + a_{21} c_2)} < 0, \quad \lim_{x_1 \to \frac{r_1}{a_{12}}} F = +\infty.$$

which means that F has at least one zero in the interval $(0, \frac{r_1}{a_{12}})$, that is, the curve l_1 and l_2 intersect at least one point, which is equivalent to system (3.1) admits at least one positive solution (x_1^*, x_2^*).

On the other hand, the positive solution of system (3.1) is equivalent to the positive solution of the following system

$$\begin{aligned} &-\lambda_1 c_1 - \lambda_1 f_1 x_2 + a_{11} x_1 + a_{12} x_1 c_1 + a_{12} x_1 f_1 x_2 = 0, \\ &-\lambda_2 c_2 - \lambda_2 f_2 x_1 + a_{22} x_2 + a_{21} x_2 c_2 + a_{21} x_2 f_2 x_1 = 0. \end{aligned} \tag{3.5}$$

From (3.5), by simple computation, x_1 is the solution of the following equation

$$A x_1^2 + B x_1 + C = 0, \tag{3.6}$$

where $A = a_{21} f_2 a_{11} + a_{12} a_{21} f_2 c_1 + a_{12} f_1 \lambda_2 f_2 > 0$,

$$B = -a_{21} f_2 \lambda_1 c_1 - \lambda_1 f_1 \lambda_2 f_2 + a_{11} a_{22} + a_{11} a_{21} c_2 + a_{12} c_1 a_{22} + a_{12} c_1 a_{21} c_2 + a_{12} f_1 \lambda_2 c_2,$$

and

$$C = -\lambda_1 c_1 a_{22} - \lambda_1 c_1 a_{21} c_2 - \lambda_1 f_1 \lambda_2 c_2 < 0.$$

Since $A > 0, C < 0$, one could easily see that equation (3.6) admits unique one positive solution x_1^{**}, consequently, system (3.5) admits at most one positive solution (x_1^{**}, x_2^{**}).

Above analysis shows that system (3.1) admits a unique positive solution (x_1^*, x_2^*). This ends the proof of Lemma 3.1.

For convenience, we denote

$$\lambda_1 \stackrel{\text{def}}{=} b_1 e^{-d_{11}\tau_1} - d_{12},\ \lambda_2 \stackrel{\text{def}}{=} b_2 e^{-d_{22}\tau_2} - d_{21}.$$

Let $x_1'(t) = 0, x_2'(t) = 0$ in system (2.8), we can get four equilibria as follows:

$$E_0 = (0,0),\ E_1 = (\frac{c_1\lambda_1}{a_{11} + a_{12}c_1}, 0) \stackrel{\text{def}}{=} (x_{1*}, 0),$$

$$E_2 = (0, \frac{c_2\lambda_2}{a_{22} + a_{21}c_2}) \stackrel{\text{def}}{=} (0, x_{2*}),\ E(x_1^*, x_2^*),$$

where E is the unique positive solution of system (3.1).

Concerned with the stability property of there equilibria, we have the following theorems.

Theorem 3.1 *Suppose that*

$$\lambda_1 > 0,\ \lambda_2 > 0 \tag{H_1}$$

holds, then the unique positive equilibrium E is globally attractive.

Theorem 3.2 *Suppose that*

$$\lambda_1 \le 0,\ \lambda_2 \le 0 \tag{H_2}$$

holds, then both of the predator and prey species will be driven to extinction, that is, E_0 is globally attractive.

Theorem 3.3 *Suppose that*

$$\lambda_1 > 0,\ \lambda_2 \le 0 \tag{H_3}$$

holds, then E_1 is globally attractive.

Theorem 3.4 *Suppose that*

$$\lambda_1 \le 0,\ \lambda_2 > 0 \tag{H_4}$$

holds, then E_2 is globally attractive.

Corollary 3.1. *If the parameters of system (2.5) satisfy the conditions (H_1), then $E'(x_1^*, y_1^*, x_2^*, y_2^*)$ is globally attractive, where $y_1^* = \frac{b_1 x_1^*(1-e^{-d_{11}\tau_1})}{d_{11}}, y_2^* = \frac{b_2 x_2^*(1-e^{-d_{22}\tau_2})}{d_{22}}$.*

Corollary 3.2. *If the parameters of system (2.5) satisfy the condition (H_2), then $E_0' = (0,0,0,0)$ is globally attractive.*

Corollary 3.3. *If the parameters of system (2.5) satisfy the condition* (H_3)*, then* $E_1' = (x_{1*}, y_{1*}, 0, 0)$ *is globally attractive, where* $y_{1*} = \frac{b_1 x_{1*}(1-e^{-d_{11}\tau_1})}{d_{11}}$.

Corollary 3.4. *If the parameters of system (2.5) satisfy the condition* (H_4)*, then* $E_2' = (0, 0, x_{2*}, y_{2*})$ *is globally attractive, where* $y_{2*} = \frac{b_2 x_{2*}(1-e^{-d_{22}\tau_2})}{d_{22}}$.

4. Proof of the main results

Now let us state several lemmas which will be useful in proving the main results.

Lemma 4.1. *Assume that* $x_1(\theta) \geq 0, x_2(\theta) \geq 0$ *are continuous on* $\theta \in [-\tau, 0]$*, and* $x_1(0) > 0, x_2(0) > 0$*. Let* $(x_1(t), x_2(t))^T$ *be a any solution of system (2.8), then* $x_1(t) > 0, x_2(t) > 0$ *for all* $t > 0$*.*

The proof of Lemma 3.1 is similar to the proof of Theorem 1 in [1], so we omit its proof.

Lemma 4.2 ([27]) *Consider the following equations:*

$$\begin{aligned} x'(t) &= bx(t-\delta) - a_1 x(t) - a_2 x^2(t), \\ x(t) &= \phi(t) > 0, \ -\delta \leq t \leq 0, \end{aligned}$$

and assume that $b, a_2 > 0, a_1 \geq 0$ *and* $\delta \geq 0$ *are constants, then:*
(i) If $b \geq a_1$*, then* $\lim\limits_{t\to+\infty} x(t) = \dfrac{b-a_1}{a_2}$;
(ii) If $b \leq a_1$*, then* $\lim\limits_{t\to+\infty} x(t) = 0$.

Proof of Theorem 3.1. By the first equation of system (2.8) and Lemma 4.1, we have

$$\dot{x}_1(t) \leq b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12} x_1(t) - a_{12} x_1^2(t).$$

From Lemma 4.2, it follows that

$$\limsup_{t\to+\infty} x_1(t) \leq \frac{b_1 e^{-d_{11}\tau_1} - d_{12}}{a_{12}} = \frac{\lambda_1}{a_{12}}. \tag{4.1}$$

Hence, for enough small $\varepsilon > 0 (\varepsilon < \min\{\frac{\lambda_1 c_1}{2(a_{12}c_1 + a_{11})}, \frac{\lambda_2 c_2}{2(a_{22} + a_{21}c_2)}\})$, it follows from (4.1) that there exists a $T_1' > 0$ such that

$$x_1(t) < \tfrac{\lambda_1}{a_{12}} + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)} \ \text{ for } \ t > T_1'. \tag{4.2}$$

Similarly, for above $\varepsilon > 0$, it follows from the second equation of system (2.8) that there exists a $T_1 > T_1'$ such that

$$x_2(t) < \tfrac{\lambda_2}{a_{21}} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)} \text{ for } t > T_1. \tag{4.3}$$

(4.3) together with the first equation of system (2.8) implies

$$\begin{aligned} \dot{x}_1(t) &= b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1 x_2(t)} - a_{12}x_1^2(t) \\ &\le b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1 M_2^{(1)}} - a_{12}x_1^2(t) \text{ for } t > T_1. \end{aligned} \tag{4.4}$$

Therefore, by Lemma 4.2, we have

$$\limsup_{t\to+\infty} x_1(t) \le \frac{(b_1 e^{-d_{11}\tau_1} - d_{12})(c_1 + f_1 M_2^{(1)})}{a_{12}(c_1 + f_1 M_2^{(1)}) + a_{11}} = \frac{\lambda_1(c_1 + f_1 M_2^{(1)})}{a_{12}(c_1 + f_1 M_2^{(1)}) + a_{11}}. \tag{4.5}$$

That is, for $\varepsilon > 0$ be defined by (4.2)-(4.3), there exists a $T_2' > T_1$ such that

$$x_1(t) < \frac{\lambda_1(c_1 + f_1 M_2^{(1)})}{a_{12}(c_1 + f_1 M_2^{(1)}) + a_{11}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)} > 0 \text{ for } t > T_2'. \tag{4.6}$$

It follows from (4.2) and the second equation of system (2.8) that

$$\dot{x}_2(t) \le b_2 e^{-d_{22}\tau_2} x_2(t-\tau_2) - d_{21}x_2(t) - \frac{a_{22}x_2^2(t)}{c_2 + f_2 M_1^{(1)}} - a_{21}x_2^2(t) \tag{4.7}$$

Therefore, by Lemma 4.2, we have

$$\limsup_{t\to+\infty} x_2(t) \le \frac{(b_2 e^{-r_2\tau_2} - d_{21})(c_2 + f_2 M_1^{(1)})}{a_{21}(c_2 + f_2 M_1^{(1)}) + a_{22}}. \tag{4.8}$$

That is, for $\varepsilon > 0$ be defined by (4.2) and (4.3), there exists a $T_2 > T_2'$ such that

$$x_2(t) < \frac{\lambda_2(c_2 + f_2 M_1^{(1)})}{a_{21}(c_2 + f_2 M_1^{(1)}) + a_{22}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)} > 0 \text{ for } t > T_2. \tag{4.9}$$

From the first equation of system (2.8),

$$\dot{x}_1(t) \ge b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1} - a_{12}x_1^2(t) \text{ for } t > T_2. \tag{4.10}$$

Therefore, by Lemma 4.2, we have

$$\liminf_{t\to+\infty} x_1(t) \ge \frac{\lambda_1 c_1}{a_{12}c_1 + a_{11}}. \tag{4.11}$$

Hence, for $\varepsilon > 0$ be defined by (4.2)-(4.3), there exists a $T_3' > T_2$ such that

$$x_1(t) > \frac{\lambda_1 c_1}{a_{12}c_1 + a_{11}} - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)}, \text{ for } t > T_3'. \tag{4.12}$$

Similarly, it follows from the second equation of system (2.8) that there exists a $T_3 > T_3'$ such that

$$x_2(t) > \tfrac{\lambda_2 c_2}{a_{22}+a_{21}c_2} - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)}, \text{ for } t > T_3. \tag{4.13}$$

(4.13) together with the first equation of system (2.8) implies that

$$\dot{x}_1(t) \geq b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1 m_2^{(1)}} - a_{12}x_1^2(t) \text{ for } t > T_3. \tag{4.14}$$

Therefore, by Lemma 4.2, we have

$$\liminf_{t\to+\infty} x_1(t) \geq \frac{\lambda_1(c_1 + f_1 m_2^{(1)})}{a_{12}(c_1 + f_1 m_2^{(1)}) + a_{11}}. \tag{4.15}$$

That is, for $\varepsilon > 0$ be defined by (4.2)-(4.3), there exists a $T_4' > T_3$ such that

$$x_1(t) > \frac{\lambda_1(c_1 + f_1 m_2^{(1)})}{a_{12}(c_1 + f_1 m_2^{(1)}) + a_{11}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)} > 0, \text{ for } t > T_4'. \tag{4.16}$$

Similarly, by (4.12) and the second equation of system (2.8), for $\varepsilon > 0$ be defined by (4.2)-(4.3), there exists a $T_4 > T_4'$ such that

$$x_2(t) > \frac{\lambda_2(c_2 + f_2 m_1^{(1)})}{a_{21}(c_2 + f_2 m_1^{(1)}) + a_{22}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)} > 0, \text{ for } t > T_4. \tag{4.17}$$

Note the fact that $f(x) = \dfrac{ax}{bx+c}$, $x > 0$, where a, b, c are positive constants, is a strictly increasing function. Obviously,

$$\begin{aligned}
M_1^{(2)} &= \frac{\lambda_1(c_1 + f_1 M_2^{(1)})}{a_{12}(c_1 + f_1 M_2^{(1)}) + a_{11}} + \frac{\varepsilon}{2} < \frac{\lambda_1}{a_{12}} + \varepsilon = M_1^{(1)}; \\
M_2^{(2)} &= \frac{\lambda_2(c_2 + f_2 M_1^{(1)})}{a_{21}(c_2 + f_2 M_1^{(1)}) + a_{22}} + \frac{\varepsilon}{2} < \frac{\lambda_2}{a_{21}} + \varepsilon = M_2^{(1)}; \\
m_1^{(2)} &= \frac{\lambda_1(c_1 + f_1 m_2^{(1)})}{a_{12}(c_1 + f_1 m_2^{(1)}) + a_{11}} - \frac{\varepsilon}{2} > \frac{\lambda_1 c_1}{a_{12}c_1 + a_{11}} - \varepsilon = m_1^{(1)}; \\
m_2^{(2)} &= \frac{\lambda_2(c_2 + f_2 m_1^{(1)})}{a_{21}(c_2 + f_2 m_1^{(1)}) + a_{22}} - \frac{\varepsilon}{2} > \frac{\lambda_2 c_2}{2(a_{22} + a_{21}c_2)} - \varepsilon = m_2^{(1)}.
\end{aligned} \tag{4.18}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}, i = 1,2, n = 1,2,\cdots$, such that for $n \geq 2$

$$\begin{aligned} M_1^{(n)} &= \frac{\lambda_1(c_1 + f_1 M_2^{(n-1)})}{a_{12}(c_1 + f_1 M_2^{(n-1)}) + a_{11}} + \frac{\varepsilon}{n}; \\ M_2^{(n)} &= \frac{\lambda_2(c_2 + f_2 M_1^{(n-1)})}{a_{21}(c_2 + f_2 M_1^{(n-1)}) + a_{22}} + \frac{\varepsilon}{n}; \\ m_1^{(n)} &= \frac{\lambda_1(c_1 + f_1 m_2^{(n-1)})}{a_{12}(c_1 + f_1 m_2^{(n-1)}) + a_{11}} - \frac{\varepsilon}{n}; \\ m_2^{(n)} &= \frac{\lambda_2(c_2 + f_2 m_1^{(n-1)})}{a_{21}(c_2 + f_2 m_1^{(n-1)}) + a_{22}} - \frac{\varepsilon}{n}. \end{aligned} \tag{4.19}$$

Obviously,

$$m_i^{(n)} < x_i(t) < M_i^{(n)}, \text{ for } t \geq T_{2n}, \ i = 1,2.$$

We claim that sequences $M_i^{(n)}, i = 1,2$ are non-increasing, and sequences $m_i^{(n)}, i = 1,2$ are non-decreasing. To proof this claim, we will carry out by induction. Firstly, from (4.18) we have

$$M_i^{(2)} \leq M_i^{(1)}, \ m_i^{(2)} \geq m_i^{(1)}, \ i = 1,2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} \leq M_i^{(n-1)}, \ m_i^{(n)} \geq m_i^{(n-1)}, \ i = 1,2.$$

Again from the strictly increasing of function $f(x) = \dfrac{ax}{bx+c}, \ x > 0$, where a,b,c are positive constants, we immediately obtain

$$\begin{aligned} M_1^{(n+1)} &= \frac{\lambda_1(c_1 + f_1 M_2^{(n)})}{a_{12}(c_1 + f_1 M_2^{(n)}) + a_{11}} + \frac{\varepsilon}{n+1} < \frac{\lambda_1(c_1 + f_1 M_2^{(n-1)})}{a_{12}(c_1 + f_1 M_2^{(n-1)}) + a_{11}} + \frac{\varepsilon}{n} = M_1^{(n)}; \\ M_2^{(n+1)} &= \frac{\lambda_2(c_2 + f_2 M_1^{(n)})}{a_{21}(c_2 + f_2 M_1^{(n)}) + a_{22}} + \frac{\varepsilon}{n+1} < \frac{\lambda_2(c_2 + f_2 M_1^{(n-1)})}{a_{21}(c_2 + f_2 M_1^{(n-1)}) + a_{22}} + \frac{\varepsilon}{n} = M_2^{(n)}; \\ m_1^{(n+1)} &= \frac{\lambda_1(c_1 + f_1 m_2^{(n)})}{a_{12}(c_1 + f_1 m_2^{(n)}) + a_{11}} - \frac{\varepsilon}{n+1} > \frac{\lambda_1(c_1 + f_1 m_2^{(n-1)})}{a_{12}(c_1 + f_1 m_2^{(n-1)}) + a_{11}} - \frac{\varepsilon}{n} = m_1^{(n)}; \\ m_2^{(n+1)} &= \frac{\lambda_2(c_2 + f_2 m_1^{(n)})}{a_{21}(c_2 + f_2 m_1^{(n)}) + a_{22}} - \frac{\varepsilon}{n+1} > \frac{\lambda_2(c_2 + f_2 m_1^{(n-1)})}{a_{21}(c_2 + f_2 m_1^{(n-1)}) + a_{22}} - \frac{\varepsilon}{n} = m_2^{(n)}. \end{aligned}$$

Therefore,

$$\lim_{t \to +\infty} M_i^{(n)} = \overline{x}_i, \ \lim_{t \to +\infty} m_i^{(n)} = \underline{x}_i, \ i = 1,2.$$

Letting $n \to +\infty$ in (4.19), we obtain

$$\begin{aligned}&\lambda_1 - \frac{a_{11}\overline{x}_1}{c_1 + f_1\overline{x}_2} - a_{12}\overline{x}_1 = 0, \lambda_2 - \frac{a_{22}\overline{x}_2}{c_2 + f_2\overline{x}_1} - a_{21}\overline{x}_2 = 0,\\ &\lambda_1 - \frac{a_{11}\underline{x}_1}{c_1 + f_1\underline{x}_2} - a_{12}\underline{x}_1 = 0, \lambda_2 - \frac{a_{22}\underline{x}_2}{c_2 + f_2\underline{x}_1} - a_{21}\underline{x}_2 = 0,\end{aligned} \tag{4.20}$$

(4.20) shows that $(\overline{x}_1, \overline{x}_2)$ and $(\underline{x}_1, \underline{x}_2)$ are positive solutions of (3.1). By Lemma 3.1, (3.1) has a unique positive solution $E^*(x_1^*, x_2^*)$. Hence, we conclude that

$$\overline{x}_i = \underline{x}_i = x_i^*, \;\; i = 1, 2,$$

that is

$$\lim_{t\to+\infty} x_i(t) = x_i^* \;\; i = 1, 2.$$

Thus, the unique interior equilibrium $E^*(x_1^*, x_2^*)$ is globally attractive. This completes the proof of Theorem 3.1.

Proof of Theorem 3.2. It follows from the first equation of system (2.8), we have

$$\dot{x}_1(t) < b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - a_{12}x_1^2(t).$$

According to first inequality of condition (H_2), we have $b_1 e^{-d_{11}\tau_1} - d_{12} < 0$. By applying Lemma 3.2(ii) and standard comparison theorem, we have $\limsup\limits_{t\to+\infty} x_1(t) \le 0$. That is,

$$\lim_{t\to+\infty} x_1(t) = 0.$$

Similarly, it follows from the second equation of system (2.8) and $\lambda_2 \le 0$, we have

$$\lim_{t\to+\infty} x_2(t) = 0.$$

Therefore, $E_0 = (0,0)$ is globally attractive. This completes the proof of Theorem 3.2.

Proof of Theorem 3.3. It follows from the second equation of system (2.8) and $\lambda_2 \le 0$, we have

$$\lim_{t\to+\infty} x_2(t) = 0.$$

Then for any $\varepsilon > 0$, there exists a $T > 0$ such that

$$0 < x_2(t) < \varepsilon.$$

Therefore, it follows from the first equation of system (2.8), we have

$$\dot{x}_1(t) \leq b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1 + f_1\varepsilon} - a_{12}x_1^2(t) \text{ for } t > T. \tag{4.21}$$

Applying Lemma 4.2 to (4.21) leads to

$$\lim_{t\to+\infty} x_1(t) \leq \frac{\lambda_1(c_1+f_1\varepsilon)}{a_{12}(c_1+f_1\varepsilon)+a_{11}}. \tag{4.22}$$

Setting $\varepsilon \to 0$ in above inequality, one has

$$\lim_{t\to+\infty} x_1(t) \leq \frac{\lambda_1 c_1}{a_{12}c_1+a_{11}}. \tag{4.23}$$

Again, from the first equation of system (2.8), we have

$$\dot{x}_1(t) \geq b_1 e^{-d_{11}\tau_1} x_1(t-\tau_1) - d_{12}x_1(t) - \frac{a_{11}x_1^2(t)}{c_1} - a_{12}x_1^2(t) \text{ for } t > T. \tag{4.24}$$

Applying Lemma 4.2 to (4.24) leads to

$$\lim_{t\to+\infty} x_1(t) \geq \frac{\lambda_1 c_1}{a_{12}c_1+a_{11}}. \tag{4.25}$$

(4.23) together with (4.25) implies that

$$\lim_{t\to+\infty} x_1(t) = \frac{\lambda_1 c_1}{a_{12}c_1+a_{11}}. \tag{4.26}$$

This ends the proof of Theorem 3.3.

Proof of Theorem 3.4. It follows from the second equation of system (2.8) and $\lambda_1 \leq 0$, we have

$$\lim_{t\to+\infty} x_1(t) = 0.$$

Then for any $\varepsilon > 0$, there exists a $T' > 0$ such that

$$0 < x_1(t) < \varepsilon.$$

Therefore, it follows from the second equation of system (2.8), we have

$$\dot{x}_2(t) \leq b_2 e^{-d_{22}\tau_2} x_2(t-\tau_2) - d_{21}x_2(t) - \frac{a_{22}x_2^2(t)}{c_2 + f_2\varepsilon} - a_{21}x_2^2(t) \text{ for } t > T'. \tag{4.27}$$

Applying Lemma 4.2 to (4.27) leads to

$$\lim_{t\to+\infty} x_2(t) \leq \frac{\lambda_2(c_2+f_2\varepsilon)}{a_{21}(c_2+f_2\varepsilon)+a_{22}}. \tag{4.28}$$

Setting $\varepsilon \to 0$ in above inequality, one has

$$\lim_{t\to+\infty} x_2(t) \le \frac{\lambda_2 c_2}{a_{21}c_2 + a_{22}}. \tag{4.29}$$

Again, from the second equation of system (2.8), we have

$$\dot{x}_1(t) \quad \ge \quad b_2 e^{-d_{22}\tau_2} x_2(t-\tau_2) - d_{21}x_2(t) - \frac{a_{22}x_2^2(t)}{c_2} - a_{21}x_2^2(t) \text{ for } t > T'. \tag{4.30}$$

Applying Lemma 4.2 to (4.30) leads to

$$\lim_{t\to+\infty} x_2(t) \ge \frac{\lambda_2 c_2}{a_{21}c_1 + a_{22}}. \tag{4.31}$$

(4.31) together with (4.29) implies that

$$\lim_{t\to+\infty} x_2(t) = \frac{\lambda_2 c_2}{a_{21}c_1 + a_{22}}. \tag{4.32}$$

This ends the proof of Theorem 3.4.

5. Discussion

May (1976) proposed a two species cooperation system (2.3), he showed that the system admits a unique globally asymptotically stable positive equilibrium, which means that the species are coexists in a stable state. Stimulated by a series works of Chen *et al.* (2012, 2013a, 2013b), in this paper, we further incorporate stage structure for both species to May's cooperation system, this leads to system (2.5).

By applying iterative technique and fluctuation lemma, sufficient conditions which guarantee the globally attractive of all the nonnegative equilibria are obtained. By applying the differential inequality, we also investigate the stability property of the boundary equilibrium. From the expression of $\lambda_i, i = 1,2$, one could see that the birth rate of immature species (b_i), the death rate of the immature and mature species (d_{ij}), and the time for immature to grow and into mature state (τ_i) are the factors which determine the persistent property of the system. Unlike the simple behavior of May's cooperate system, our system admits very complicate behaviors: extinction of the system, partial survival of the species and permanence of the system are all possible.

Now, we consider the effect of the stage structure on the permanence of one species. Noting that fix b_i and d_{ij}, but enlarge $\tau_i, i = 1,2$ gradually, then $\lambda_i \le 0, i = 1,2$ if τ_i enough large, which means the extinction of the i-th species. Therefore we have

Conclusion 1. In the stage-structured cooperate community, stage structure brings negative effect on permanence of one species as well as contribution to its extinction.

Our next finding is concern with the death rate of the mature species. Fix b_i and τ_i, but enlarge $d_{ij}, i, j = 1, 2$, gradually, then $\lambda_i \leq 0, i = 1, 2$ if d_{ij} enough large. Hence

Conclusion 2. The death rate of immature and mature species brings negative effect on permanence of one species as well as contribution to its extinction. Specially, if one needs to control the number of some species, increasing the death rate of the mature species is one of the effect and plausible method.

It is well known that with the cooperate of other species, traditional two species Lotka-Volterra cooperate system could admits unbounded solution. However, noting that $\lambda_i, i = 1, 2$ are independent of c_i and $f_i, i = 1, 2$, which reflect the effect of cooperation, this means that

Conclusion 3. In the stage-structured cooperate community (2.5), cooperate has no influence on the persistent property of the model.

Since cooperation has no influence on the persistent property of the system, note that $\lambda_i > 0, i = 1, 2$ is the conditions to ensure the permanence of the species i, and so

Conclusion 4. Conditions which ensure the permanence of the single species are enough to ensure the global stability of the system.

Conflict of Interests

The authors declare that there is no conflict of interests.

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