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PERIODIC SOLUTION OF A PERIODIC PREDATOR-PREY-MUTUALIST SYSTEM

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Abstract. In this paper, the periodic predator-prey-mutualist model of three species was discussed. Sufficient conditions for the existence of a unique globally asymptotically stable periodic solution of the system are obtained.

Keywords: Periodic solution; Predator-prey-mutualist system; Global asymptotical stability.

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1. Introduction

Mutualism is a symbiotic association between any two species, the interaction between the two species is beneficial to both of the species. For instance, ants prevent herbivores from feeding on plants (see [1]) and ants prevent predators from feeding on aphids (see [2-3]). Mutualism is one of the most important relationships in the theory of ecology. However, as was pointed out by Murray [4]: this area has not been as widely studied as the others even though its importance is comparable to that of predator-prey and competition interactions.

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Recently, Chen et al. ([5]) discussed a two species discrete model of mutualism with delays and feedback controls. They showed that feedback control variables have no influence on the persistence property of the system. Chen et al. ([6]) have considered the global asymptotical stability of the equilibria of Lotka-Volterra obligate system. Xie et al. ([7]) studied the global attractivity of an integrodifferential model of mutualism.

It brings to our attention that all the works of [5-7] are deal with the relationship between two cooperative species, while in the real world, the relationship among species is very complicated, and it needs to consider the more complicated models. To this end, Rai and Krawcewicz [8] have considered the following system:

$$\begin{aligned} \frac{dx}{dt} &= \alpha x(1-\tfrac{x}{K}) - \frac{\beta xz}{1+my}, \\ \frac{dy}{dt} &= \gamma y\Big(1-\frac{y}{lx+L_0}\Big), \\ \frac{dz}{dt} &= z\Big(-s+\frac{c\beta x}{1+my}\Big). \end{aligned} \tag{1.1}$$

They have considered the Hopf bifurcation system with diffusive migration between interacting communities.

However, due to seasonal effects of weather, temperature, food supply, mating habits, contact with predators and other resource or physical environmental quantities, we can assume temporal to be cyclic or periodic (see[9-16]). In this paper, we proposed the following system:

$$\begin{aligned} \dot{x} &= x\Big(a_1(t)-b_1(t)x-\frac{c_1(t)z}{d_1(t)+d_2(t)y}\Big), \\ \dot{y} &= y\Big(a_2(t)-\frac{y}{d_3(t)+d_4(t)x}\Big), \\ \dot{z} &= z\Big(-a_3(t)+\frac{k_1(t)c_1(t)x}{d_1(t)+d_2(t)y}-b_2(t)z\Big), \end{aligned} \tag{1.2}$$

where x is the density of the prey, y is the density of the mutualist and z is the density of the predator. The functions $a_i(t)(i=1,2,3), b_1(t), b_2(t), k_1(t), d_j(t)(j=1,2,3,4)$ are continuous, nonnegative and periodic functions with a common period $T>0$, $a_i(t)(i=1,2,3), b_1(t), b_2(t)$ are strictly positive, $a_1(t)$ is the intrinsic growth rate of prey specie x, $a_2(t)$ is the intrinsic growth rate of mutualist y, $a_3(t)$ is the death rate of the predator specie z, $c_1(t)$ is the coefficient of the functional response. The function $k_1(t)$ is called the conversion ration, which denotes the fraction of the prey biomass being converted to predator biomass. The functions d_4, d_2 are the mutualism functions. We mention here that in system (1.2), we consider the density restriction term of predator species ($b_2(t)z$), such a consideration is needed since the density of any species is restricted by the environment [17].

We arrange the rest of the paper as follows: In Section 2, we shall be interested in the existence of the positive periodic solution of (1.2). In Section 3, the sufficient conditions about the uniqueness and global attractivity of the periodic solution of the (1.2) are obtained. Finally, a suitable example is given to illustrate that the conditions of the main theorem are feasible. We end this paper by a briefly discussion.

2. Existence of a positive periodic solution

It is obvious that there exists a unique solution of the system (1.2) corresponding to any positive initial value $w_0 = (x(0), y(0), z(0))$. Such a solution is denoted by $(x(t,w_0), y(t,w_0), z(t,w_0))$.

Lemma 2.1 $R^3_+ = \Big\{(x,y,z)|x \geq 0, y \geq 0, z \geq 0\Big\}$ *is invariant with respect to* (1.2).

Proof. Since

$$
\begin{aligned}
x(t) &= x(0)\exp\int_0^t \Big(a_1(s) - b_1(s)x(s) - \frac{c_1(s)z(s)}{d_1(s)+d_2(s)y(s)}\Big)ds,\\
y(t) &= y(0)\exp\int_0^t \Big(a_2(s) - \frac{y(s)}{d_3(s)+d_4(s)x(s)}\Big)ds, \qquad (2.1)\\
z(t) &= z(0)\exp\int_0^t \Big(-a_3(s) + \frac{k_1(s)c_1(s)x(s)}{d_1(s)+d_2(s)y(s)} - b_2(s)z(s)\Big)ds,
\end{aligned}
$$

the assertion of the lemma follows immediately for all $t \in [0,+\infty)$.

We introduce the following notations. If $f(t)$ is a continuous T-periodic function defined on $[0,+\infty)$, we set

$$
f^m = \max_{t\in[0,T]} f(t),\ f^l = \min_{t\in[0,T]} f(t).
$$

Throughout this paper, we assume that

(A_1) $a_1^l(d_1^l + d_2^l\delta_2) - c_1^m B_3 > 0,$

(A_2) $\dfrac{k_1^l c_1^l \delta_1}{d_1^m + d_2^m B_2} - a_3^m > 0$

holds, where

$B_2 = a_2^m(d_3^m + d_4^m \dfrac{a_1^m}{b_1^l}),\qquad B_3 = \dfrac{k_1^m c_1^m a_1^m - a_3^l d_1^l b_1^l}{b_1^l b_2^l d_1^l},$

$\delta_1 = \dfrac{a_1^l(d_1^l + d_2^l\delta_2) - c_1^m B_3}{(d_1^l + d_2^l\delta_2) b_1^m},\qquad \delta_2 = a_2^l d_3^l.$

It follows from Lemma 2.1 that any solution of (1.2) which has a nonnegative initial condition remains nonnegative.

Lemma 2.2 *Let*

$$S = \Big\{ w = (x,y,z) \in R_+^3 | 0 < \delta_1 \le x \le B_1, 0 < \delta_2 \le y \le B_2, 0 < \delta_3 \le z \le B_3 \Big\}.$$

Then S is invariant with respect to (1.2).

Proof. From the first equation of system (1.2), we obtain

$$\dot{x} \le x(a_1^m - b_1^l x),$$

so, if

$$0 < x(0) \le \frac{a_1^m}{b_1^l} := B_1$$

holds, we have

$$x(t) \le B_1,\ t \ge 0. \tag{2.2}$$

From the second equation of (1.2), it follows that

$$\dot{y} \le y\Big(a_2^m - \frac{y}{d_3^m + d_4^m B_1}\Big),$$

if

$$0 < y(0) \le a_2^m\Big(d_3^m + d_4^m \frac{a_1^m}{b_1^l}\Big) := B_2$$

holds, we obtain

$$y(t) \le B_2,\ t \ge 0. \tag{2.3}$$

From the third equation of (1.2), it follows that

$$\dot{z} \le z\Big(- a_3^l + \frac{k_1^m c_1^m B_1}{d_1^l} - b_2^l z\Big).$$

As a direct consequence of (A_2), we know that the inequality $-a_3^l + \dfrac{k_1^m c_1^m B_1}{d_1^l} > 0$ holds. If

$$0 < z(0) \le \frac{k_1^m c_1^m a_1^m - a_3^l d_1^l b_1^l}{b_1^l b_2^l d_1^l} := B_3$$

holds, then

$$z(t) \le B_3,\ t \ge 0. \tag{2.4}$$

From the second equation of system (1.2), one has

$$\dot{y} \ge y\Big(a_2^l - \frac{y}{d_3^l}\Big),$$

it implies that if

$$y(0) \ge a_2^l d_3^l := \delta_2$$

holds, then

$$y(t) \ge \delta_2,\ t \ge 0. \tag{2.5}$$

(2.5) combining with the first equation of system (1.2) leads to

$$\dot{x} \ge\ x\Big(a_1^l - b_1^m x - \frac{c_1^m B_3}{d_1^l + d_2^l \delta_2}\Big).$$

It implies that if

$$x(0) \ge \frac{\Big(a_1^l - \dfrac{c_1^m B_3}{d_1^l + d_2^l \delta_2}\Big)}{b_1^m} = \frac{a_1^l(d_1^l + d_2^l \delta_2) - c_1^m B_3}{(d_1^l + d_2^l \delta_2) b_1^m} := \delta_1$$

holds, then

$$x(t) \ge \delta_1,\ t \ge 0. \tag{2.6}$$

From $(1.2), (2.3)$ and (2.6), we have

$$\dot{z} \ge z\Big(-a_3^m + \frac{k_1^l c_1^l \delta_1}{d_1^m + d_2^m B_2} - b_2^m z\Big).$$

If

$$z(0) \ge \frac{\dfrac{k_1^l c_1^l \delta_1}{d_1^m + d_2^m B_2} - a_3^m}{b_2^m} = \frac{k_1^l c_1^l \delta_1 - a_3^m(d_1^m + d_2^m B_2)}{(d_1^m + d_2^m B_2) b_2^m} := \delta_3$$

holds, we obtain

$$z(t) \ge \delta_3,\ t \ge 0. \tag{2.7}$$

Conditions (A_1) and (A_2) implies that $\delta_i > 0, i = 1,2,3$. Above analysis shows that

$$0 < \delta_1 \leq x(t) \leq B_1,\ 0 < \delta_2 \leq y(t) \leq B_2,\ 0 < \delta_3 \leq z(t) \leq B_3,\ \ (t \geq 0).$$

This completes the proof of Lemma 2.2.

We can define a shift operator. It is also known as a *Poincar*é map $u : R^3_+ \to R^3_+$ by

$$u(x_0) = x(T, x_0),\ \ x_0 \in\ R^3_+.$$

if u has a fixed point u^*, then it is a T-periodic solution for (1.2).

The following result is well known:

Theorem (Brouwer) *Suppose that a continuous operator u maps a closed bounded convex set* $\bar{\Omega} \subseteq R^n$ *into itself. Then* $\bar{\Omega}$ *contains at least one fixed pointed of the operator* u, *i.e., a point* x^* *such that*

$$u(x^*) = x^*,\ \ x^* = (x_1^*, ..., x_n^*).$$

Theorem 2.1 *If the coefficients of the system* (1.2) *satisfy* $(A_1), (A_2)$*, then* (1.2) *has at least one strictly positive* T*-periodic solution.*

Proof. From Lemma 2.2, the operator u defined above map S into itself, i.e., $u(S) \subset S$. Because the solution of (1.2) is continuous with respect to the initial value, the operator u is continuous. It can also be seen that S is a bounded closed convex set in R^3_+. By Brouwer's theorem, u has a fixed point in S. Consequently, there exists at least one strictly positive periodic solution.

Suppose $V(t) = (v_1(t), v_2(t), v_3(t)) \in R^3_+$ is a strictly positive T-periodic solution of the (1.2) as described in the Theorem 2.1, we have the following corollary.

Corollary 2.1. *Setting* $v_1(t), v_2(t), v_3(t), \delta_i, B_i (i = 1,2,3)$ *be defined as above, then*

$$\delta_1 \leq v_1(t) \leq B_1,\ t \geq 0;$$

$$\delta_2 \leq v_2(t) \leq B_2,\ t \geq 0;$$

$$\delta_3 \leq v_3(t) \leq B_3,\ t \geq 0.$$

3. Uniqueness and global attractivity of the periodic solution

Suppose $V(t) = (v_1(t), v_2(t), v_3(t)) \in R^3_+$ is a strictly positive periodic solution of the (1.2) as described in the Theorem 2.1.

Definition The periodic solution $V(t)$ is said to be globally attractive if every other solution $Y(t) = (y_1(t), y_2(t), y_3(t))$ of (1.2) with $Y(0) > 0$ is defined for all $t \geq 0$ and satisfies

$$\lim_{t \to +\infty} |v_i(t) - y_i(t)| = 0, (i = 1, 2, 3).$$

Theorem 3.1. *If the coefficients of* (1.2) *satisfy* $(A_1), (A_2)$ *and the following conditions*

(A_3) $b_1(t) - \dfrac{B_2}{d_4(t)} - \dfrac{c_1(t)k_1(t)}{d_1(t)} > 0,$

(A_4) $\dfrac{1}{d_3(t) + d_4(t)B_1} - \dfrac{c_1(t)B_3}{d_2(t)} - \dfrac{c_1(t)k_1(t)B_1}{d_2(t)} > 0,$

(A_5) $b_2(t) - \dfrac{c_1(t)}{d_1(t)} > 0.$

Then there exists a unique strictly positive periodic solution of the system (1.2) *which is globally attractive.*

Proof. Let $V(t) = (v_1(t), v_2(t), v_3(t)) \in R^3_+$ be a strictly positive periodic solution as described above, and let $Y(t) = (y_1(t), y_2(t), y_3(t)) \in R^3_+$ be any solution of (1.2) with $Y(0) > 0$. Since solution of (1.2) remain nonnegative, we can let

$$V_i(t) = \ln v_i(t), \;\; Y_i(t) = \ln y_i(t), \;\; (i = 1, 2, 3). \tag{3.1}$$

It follows from (3.1) and (1.2) that for $t > 0$,

$$\dot{V}_1(t) - \dot{Y}_1(t) = -b_1(t)(e^{V_1(t)} - e^{Y_1(t)}) + \frac{e^{Y_3(t)} c_1(t) d_2(t)(e^{V_2(t)} - e^{Y_2(t)})}{(d_1(t) + d_2(t)e^{Y_2(t)})(d_1(t) + d_2(t)e^{V_2(t)})}$$
$$- \frac{c_1(t)(e^{V_3(t)} - e^{Y_3(t)})}{d_1(t) + d_2(t)e^{V_2(t)}}$$

$$\dot{V}_2(t) - \dot{Y}_2(t) = \frac{d_4(t)e^{Y_2(t)}(e^{V_1(t)} - e^{Y_1(t)})}{(d_3(t) + d_4(t)e^{Y_1(t)})(d_3(t) + d_4(t)e^{V_1(t)})} - \frac{e^{V_2(t)} - e^{Y_2(t)}}{d_3(t) + d_4(t)e^{V_1(t)}}$$

$$\dot{V}_3(t)-\dot{Y}_3(t)=\frac{c_1(t)k_1(t)(e^{V_1(t)}-e^{Y_1(t)})}{d_1(t)+d_2(t)e^{V_2(t)}}-b_2(t)(e^{V_3(t)}-e^{Y_3(t)})$$

$$-\frac{e^{Y_1(t)}c_1(t)k_1(t)d_2(t)(e^{V_2(t)}-e^{Y_2(t)})}{(d_1(t)+d_2(t)e^{V_2(t)})(d_1(t)+d_2(t)e^{Y_2(t)})}.$$

Condition $(A_3)-(A_5)$ imply that

$$\alpha=\min\Big\{b_1(t)-\frac{B_2}{d_4(t)}-\frac{c_1(t)k_1(t)}{d_1(t)},b_2(t)-\frac{c_1(t)}{d_1(t)},\\ \frac{1}{d_3(t)+d_4(t)B_1}-\frac{c_1(t)B_3}{d_2(t)}-\frac{c_1(t)k_1(t)B_1}{d_2(t)}\Big\}>0.$$

Consider a Lyapunov function $V(t)$ defined by

$$V(t)=\sum_{i=1}^{3}|V_i(t)-Y_i(t)|,\quad t>0. \tag{3.2}$$

Calculating the upper right derivative $D^+V(t)$ of $V(t)$ along the positive solution of the system (1.2), we get

$$\begin{aligned}
D^+V(t)&=D^+\sum_{i=1}^{3}|V_i(t)-Y_i(t)|\\
&\le\sum_{i=1}^{3}D^+|V_i(t)-Y_i(t)|\\
&=\sum_{i=1}^{3}\frac{(V_i(t)-Y_i(t))(\dot{V}_i(t)-\dot{Y}_i(t))}{|V_i(t)-Y_i(t))|}\\
&\le\Big(-b_1(t)+\frac{B_2}{d_4(t)}+\frac{c_1(t)k_1(t)}{d_1(t)}\Big)\Big|e^{V_1(t)}-e^{Y_1(t)}\Big|\\
&+\Big(-\frac{1}{d_3(t)+d_4(t)B_1}+\frac{c_1(t)B_3}{d_2(t)}+\frac{c_1(t)k_1(t)B_1}{d_2(t)}\Big)\Big|e^{V_2(t)}-e^{Y_2(t)}\Big|\\
&+\Big(-b_2(t)+\frac{c_1(t)}{d_1(t)}\Big)\Big|e^{V_3(t)}-e^{Y_3(t)}\Big|\\
&\le-\alpha\sum_{i=1}^{3}|v_i(t)-y_i(t)|.
\end{aligned}$$

Therefore

$$D^+V(t)\le-\alpha\sum_{i=1}^{3}|v_i(t)-y_i(t)|. \tag{3.3}$$

Integrating both sides of (3.3) on interval $(0,t)$ leads to

$$\alpha\int_0^t\sum_{i=1}^{3}|v_i(s)-y_i(s)|\,ds+\sum_{i=1}^{3}|V_i(t)-Y_i(t)|\le V(0)<+\infty. \tag{3.4}$$

It follows from (3.4) that $\sum_{i=1}^{3}|V_i(t)-Y_i(t)|$ is bounded for $t \geq 0$, which implys that

$$\frac{d[V_i(t)-Y_i(t)]}{dt}\ (i=1,2,3)$$

remains bounded for $t \geq 0$; and so $\sum_{i=1}^{3}|V_i(t)-Y_i(t)|$ is uniformly continuous on $[0,\infty)$. Consequently the uniform continuity of $\sum_{i=1}^{3}|v_i(s)-y_i(s)|$ on $[0,\infty)$ will follow. Such a uniform continuity together with the integrability on $[0,\infty)$ (see(3.4)) of $\sum_{i=1}^{3}|v_i(t)-y_i(t)|$ will lead to $\lim\limits_{t\to+\infty}|v_i(t)-y_i(t)|=0,(i=1,2,3)$. This completes the proof of Theorem 3.1.

4. Examples

In this section, we shall give an example to illustrate the feasibility of main results.

Example 4.1. Considering the following predator-prey-mutualist system system:

$$\begin{aligned}
\dot{x} &= x\left(\left(0.9+0.1\cos(t)\right)-1.5x-\frac{0.1z}{3+1.4y}\right),\\
\dot{y} &= y\left(\left(0.9+0.1\sin(t)\right)-\frac{y}{0.2+2x}\right),\\
\dot{z} &= z\left(-0.03+\frac{x}{3+1.4y}-\left(0.19+0.1\cos(t)\right)z\right).
\end{aligned}\tag{4.1}$$

Corresponding to system (1.2), one has

$a_1(t)=0.9+0.1\cos(t)$, $b_1(t)=1.5$, $c_1(t)=0.1$, $d_1(t)=3$, $d_2(t)=1.4$, $a_2(t)=0.9+0.1\sin(t)$, $d_3(t)=0.2$, $d_4(t)=2$, $a_3(t)=0.03$, $b_2(t)=0.19+0.1\cos(t)$, $k_1(t)=10$. By calculating, one has $B_1\approx 0.6667$, $B_2\approx 1.5334$, $B_3\approx 2.1358$, $\delta_1\approx 0.4892$, $\delta_2=0.16$, $\delta_3\approx 0.22431$.

$$a_1^l(d_1^l+d_2^l\delta_2)-c_1^mB_3\approx 2.36562>0,\ \frac{k_1^lc_1^l\delta_1}{d_1^m+d_2^mB_2}-a_3^m\approx 0.0651>0;$$

$$b_2(t)-\frac{c_1(t)}{d_1(t)}\approx 0.8667>0,\ b_1(t)-\frac{B_2}{d_4(t)}-\frac{c_1(t)k_1(t)}{d_1(t)}\approx 0.39997>0;$$

$$\frac{1}{d_3(t)+d_4(t)B_1}-\frac{c_1(t)B_3}{d_2(t)}-\frac{c_1(t)k_1(t)B_1}{d_2(t)}\approx 0.02338>0.$$

Clearly, $0<\delta_1\leq B_1$, $0<\delta_2\leq B_2$, $0<\delta_3\leq B_3$. Condition(A_1)-(A_5) are satisfied. Thus, from Theorem 2.1 and Theorem 3.1, system (4.1) admits a unique globally asymptotically stable periodic solution.

Fig.1 shows the dynamic behaviors of the system (4.1), which strongly supports our results.

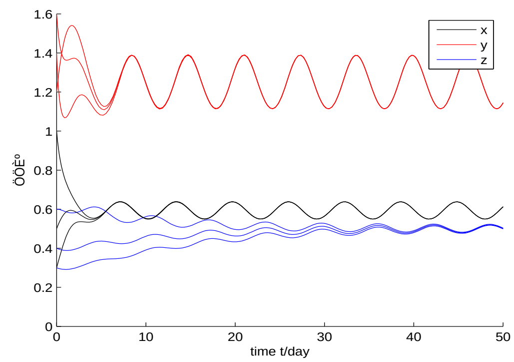

FIGURE 1. Dynamic behaviors of the solution $(x(t), y(t), z(t))$ system (4.1) with initial conditions $\big(x(0), y(0), z(0)\big) = (1.0, 1.2, 0.6), (0.5, 1.6, 0.4)$ and $(0.3, 1.4, 0.3)$, respectively.

5. Discussion

In this paper, we studied a periodic predator-prey-mutualist system. From our results we know when the death rate of the predator specie z is enough small and the density restriction of z is enough big, x and y become greater degree of cooperation, then there exists a unique strictly positive periodic solution of the system (1.2) which is globally attractive.

Conflict of Interests

The authors declare that there is no conflict of interests.

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