NOTE ON SOFT FRACTIONAL IDEAL OF RING

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Abstract. In this note we introduce soft fractional ideal of soft rings. Then, we study fractional ideal by applying few basic soft operations.

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1. Introduction and Preliminaries

Theory of probability, theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [4], theory of vague sets [7], theory of interval mathematics [8], and theory of rough sets [11] which were considered best mathematical tools for dealing with uncertainties. In [10], Molodtsov showed that to fix uncertainties soft set theory works more efficient than any other tool. In [2] authors discussed soft groups, soft subgroups. In [1] soft rings, soft ideals of soft rings have been introduced, furthermore the authors also introduced idealistic soft rings. For basic terminologies

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of soft set one may consult [10] and, for soft rings and soft ideals we refer [1]. In the beginning we recall few useful definitions and terminologies.

Let $R$ be an integral domain, and $K$ be its field of fractions. $R$-submodule $I$ of $K$ such that there exists a non-zero $r \in R$ such that $rI \subseteq R$ is said to be a fractional ideal of $R$. Every integral ideal is a fractional ideal of ring $R$. This type of ideal has its own importance while study Dedekind domains, valuation, domains etc.

Following [10, definition 2.1] pair $(F, E)$ is called a soft set (over $U$) if and only if $F$ is a mapping on $E$ into the set of subsets of the set $U$. Assume that $(F, A)$ and $(H, B)$ are two soft sets over a common universe $U$. We say that $(F, A)$ is a soft subset of $(H, B)$, if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $H(x)$ are identical approximations for all $x \in A$[10]. In [1, definition 3.1] authors introduced soft rings i.e., Let $(F, A)$ be a non-null soft set over a ring $R$. Then $(F, A)$ is called a soft ring over $R$ if $F(x)$ is a subring of $R$ for all $x \in A$. Further in [1, definition 4.1] introduce soft ideal of a soft ring i.e., Let $(F, A)$ be a soft ring over $R$. A non-null soft set $(\gamma, I)$ over $R$ is called soft ideal of $(F, A)$, if it satisfies: (1) $I \subset A$ and (2) $\gamma(x)$ is an ideal of $F(x)$ for all $x \in \text{Supp}(\gamma, I)$. Throughout this paper $E$ is a set of parameters, $P(R)$ is the power set of $R$, $\mathbb{Z}$ is the ring of integer numbers.

**Definition 1.** Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$, we say that $(F, A)$ is a soft subset of $(G, B)$, if it satisfies: (1) $A \subset B$ and (2) $F(x)$ and $G(x)$ are identical approximations for all $x \in A$ [9, definition 2.3]. We write it $(F, A) \subset (G, B)$

**Definition 2.** Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$. The intersection of $(F, A)$ and $(G, B)$ is defined as the soft set $(H, C)$ satisfying the following conditions:

(i) $C = A \cap B$

(ii) For all $x \in C$, $H(x) = F(x)$ or $G(x)$ (while the two sets are the same).

In this case we write $F(A) \cap G(B)$[9, definition 2.12].

**Definition 3.** Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$. The bi-intersection of $(F, A)$ and $(G, B)$ is defined as the soft set $(H, C)$ satisfying the following conditions:

(i) $C = A \cap B$
(ii) For all \( x \in C, H(x) = F(x) \cap G(x) \)

In this case we write \( H(C) = F(A) \cap G(B) \) [9].

**Definition 4.** Let \((F, A)\) and \((G, B)\) be two soft sets over a common universe \(U\). The union of \((F, A)\) and \((G, B)\) is defined as the soft set \((H, C)\) satisfying the following conditions:

1. \( C = A \cup B \)
2. For all \( x \in C, \)

\[
H(x) = \begin{cases} 
F(x), & \text{if } x \in A-B \\
G(x), & \text{if } x \in B-A \\
F(x) \cup G(x), & \text{if } x \in A \cap B.
\end{cases}
\]

In this case we write \( H(x) = F(A) \cup G(B) \) [9, definition 2.11].

**Definition 5.** If \((F, A)\) and \((G, B)\) be two soft sets over a common universe \(U\). Then “\((F, A)\) AND \((G, B)\)” denoted by \( F(A) \wedge G(B) \) is defined as \( F(A) \wedge G(B) = (H, C) \), where \( C = A \times B \) and \( H(x, y) = F(x) \cap G(y) \) for all \((x, y) \in C\) [9, definition 2.9].

**Definition 6.** If \((F, A)\) and \((G, B)\) be two soft sets over a common universe \(U\). Then “\((F, A)\) OR \((G, B)\)” denoted by \( F(A) \vee G(B) \) is defined as \( F(A) \vee G(B) = (H, C) \), where \( C = A \times B \) and \( H(x, y) = F(x) \cup G(y) \) for all \((x, y) \in C\) [9, definition 2.10].

**Definition 7.** Let \((F, A)\) be a soft set. The support of \((F,A)\) i.e., \( Supp(F,A) = \{ x \in A | F(x) \neq \emptyset \} \). A soft set \((F, A)\) is said to be non-null if its support is not equal to empty set [6].

**Definition 8.** Let \((F,A)\) be a non-null soft set over a ring \(R\). Then \((F,A)\) is called a soft ring over \(R\) if \( F(x) \) is a subring of \(R\) for all \( x \in A \) [1, definition 3.1].

**Definition 9.** Let \((F, A)\) is a soft ring over \(R\), a non-null soft set \((\gamma, I)\) over \(R\) is called soft ideal of \((F, A)\), and is denoted by \((\gamma, I) \triangleleft (F, A)\) if it satisfies:

1. \( I \subset A \)
2. \( \gamma(x) \) is an ideal of \( F(x) \) for all \( x \in Supp(\gamma, I) \) [1, definition 4.1].

**Definition 10.** Let \((F,A)\) and \((G, B)\) be non-null soft sets over a ring \(R\). Then \((G, B)\) is called a soft subring of \((F, A)\) if it satisfy the following
(1) \( A \subset B \)

(2) \( G(x) \) is a subring of \( F(x) \), for all \( x \in \text{Supp}(G,B) \) [1, definition 4.1].

**Definition 11.** Let \((F,A)\) be a non-null soft sets over a ring \( R \). Then \((F,A)\) is called an idealistic soft ring over \( R \), if \( F(x) \) is an ideal of \( R \) for all \( x \in \text{Supp}(F,A) \) [1, definition 5.1].

**Definition 12.** Let \( M \) be a left \( R \)-module, \( A \) be any nonempty set \( F : A \to P(M) \) refers to a set-valued function and the pair \((F,A)\) is a soft set over \( M \). Then, \((F,A)\) is said to be a soft module over \( M \) if and only if \( F(x) < M \) for all \( x \in A \) [12].

### 2. Soft fractional ideal of rings

Fuzzy fractionary ideal has been introduced and discussed in the literature (see [5]). Different types of soft ideals have been also introduced in the literature. Soft substructures of rings, fields and modules have been discussed in the literature [3]. Soft module and submodules have been introduced in the literature [12]. In this section we introduce and discuss about soft fractional ideals of soft rings. Throughout by \( R \) we mean an integral domain and \( K \) be its field of fraction. We begin with the definition.

**Definition 13.** Let \( \mu \) be a soft set over the field \( K \) and \( \mu_\alpha = \{ x \in K : \mu(x) \supseteq \alpha \} \) be a level set for every \( \alpha \in P(K) \).

We let \( \chi_A^\alpha \) the characteristic function for a subset \( A \) of a ring \( R \subseteq K \). Let \( \chi_A^\alpha \) be a soft subset of \( K \) such that \( \chi_A^\alpha(x) = U, \) if \( x \in R, \) and \( \chi_A^\alpha(x) = \alpha \) if \( x \in K - R, \) where \( \alpha \in P(K) \).

A soft subset \( \mu \) is said to be a soft ideal of a ring \( R \) if \( \mu(x - y) \supseteq \mu(x) \cap \mu(y) \) and \( \mu(xy) \supseteq \mu(x) \cup \mu(y) \). A soft subset of \( R \) is said to be an ideal iff \( \mu(0) \supseteq \mu(x) \) for every \( x \in R \) and \( \mu_\alpha \) is an ideal for every \( \alpha \in P(K) \).

**Definition 14.** Let \( R \) be a ring contained in a field \( K \), and \((\beta, K)\) be a soft subset over the field \( K \). Then \( \beta \) is said to be soft \( R \)-submodule of \( K \) if:

\[
\begin{align*}
(i) \quad & \beta(x - y) \supseteq \beta(x) \cap \beta(y) \\
(ii) \quad & \beta(rx) \supseteq \beta(x) \\
(iii) \quad & \beta(0) = R, \text{ for every } x, y \in K, r \in R.
\end{align*}
\]
For $d \in K$ and $\alpha \in P(K)$, we let $d\alpha$ denote the soft subset of $K$, defined by: for every $x \in K$, $d\alpha(x) = \alpha$ if $x = d$ and $d\alpha(x) = 0$, otherwise. We call $d\alpha(x)$ a soft singleton.

**Definition 15.** A soft $R$-submodule of $K$ is called a fractionary soft ideal of $R$ if there exists $d \in R; d \neq 0$, such that $dR \circ \beta \subseteq \chi^\alpha_R$ for some $\alpha \in K - R$.

**Theorem 1.** Let $\alpha, \beta$ be fractional soft ideals of $R$. Then $\alpha + \beta$ and $\alpha \circ \beta$ are fractional soft ideals of $R$.

**Proof.** Since $\alpha, \beta$ are fractional soft ideals of $R$ there exist $0 \neq d, d' \in R$ such that $dR \circ \alpha \subseteq \chi^\alpha_R; d'_R \circ \beta \subseteq \chi^\beta_R$ for some $\alpha, \beta \in R$. Thus $(d'd)_R \circ \alpha = d'_R \circ d \circ \alpha \subseteq d'_R \circ \chi^\beta_R$. Similarly, $(dd')_R \subseteq \chi^\alpha_R$.

Hence $(d'd)_R \circ (\alpha + \beta) = (d'd)_R \circ \alpha + (d'd)_R \circ \beta \subseteq \chi^\alpha_R + \chi^\beta_R$. And $(dd')_R \circ (\alpha \circ \beta) \subseteq \chi^\alpha_R \circ \chi^\beta_R$.

Hence, $\alpha + \beta$ and $\alpha \circ \beta$ are fractional soft ideals of $R$. □

**REFERENCES**


