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STUDY OF ULAM HYERS STABILITY OF INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITION IN BANACH SPACES

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Abstract. In this paper, Pachpatte's inequality is employed to discuss the Ulam Hyers stabilities for Volterra integrodifferential equations with nonlocal condition in Banach spaces on finite interval. Example is given to show the applicability of our obtained result.

Keywords: Ulam Hyers stability; Ulam Hyers Rassias stability; integral inequality.

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1. INTRODUCTION

In 1940, the Mathematician Ulam developed the stability problem pertaining to functional equations (see [[6],[7]]). The Ulam problem was stated as Under what conditions there exist an additive mapping near an approximately additive mapping. Initially Hyers [9] tried to find answer to

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the question of Ulam (for the additive mapping) in the case of Banach spaces. Thereafter, Rassias [11] extended Ulam–Hyers stability concept by introducing new function variables. In the literature, these concepts of stabilities are known asUlam stability, Ulam Hyers stability and Ulam Hyers Rassias stability. The basic Ulam stability problem of functional equations has been extended to different types of equations. It is observed that the Ulam stability theory plays an important role in the study of differential equations, integral equations, difference equations, fractional differential equations etc. For any kind of equations, Ulam stability problem is about (see [8, 10]) When should the solutions of an equation, differing slightly from a given one, must be close to a solution of the given equation? The notion of 'nonlocal' condition has been introduced to extend the study of the classical initial value problems. It is more precise for describing nature phenomena than the classical condition since more information is taken into account.The study of abstract nonlocal semilinear initial-value problems was initiated by L. Byszewski. We motivated by work of Kucche [2].

The purpose of this paper is to study Ulam stability problem of functional equations with nonlocal Condition of the form:

$$x'(t) = A(t)x(t) + f(t, x(t), \int_0^t g(t, s, x(s))ds), \quad t \in J = [0, b] \tag{1.1}$$

$$x(0) + H(x) = x_0 \tag{1.2}$$

where A is an infintesimal generator of strongly continuous semigroup of bounded linear operator T(t) in X with domain $D(A)$, the unknown $x(\cdot)$ takes values in the Banach space X; $f: J \times X \times X \to X$, $g: C(J \times J, X) \to X$,$H: C(J \times J, X) \to X$ are appropriate continuous functions and x_0 is given element of X.

The paper is organized as follows: We discussed the preliminaries. We dealt with study of Ulam Hyers Rassias stablity of VIE with nonlocal condition in Banach space. Finally we gave example to illustrate the application of our result.

2. PRELIMINARIES

In this section, we recall some necessary definitions and theorems which will be used in the sequel see Pazy [1] and Pachpatte[3]

Definition:- A one parameter family $T(t)_{t\geqq 0}$ of bounded linear operators from Banach space X into X is called strongly continuous semigroup (or C_0- semigroup) of operators on X if

- $T(0) = I$ the identity operator ,
- $T(t+s) = T(t)T(s) = T(s)T(t), \quad t, s \geq 0,$
- $\lim_{t\to 0} T(t)x = x \; \forall x$ in X

Definition:-The infinitesimal generator of the C_0 semigroup $T(t)_{t\geqq 0}$ is the linear operator $A: D(A) \subseteq X \to X$ defined by

$$Ax = \lim_{t\to 0^+} \frac{T(t)x - x}{t}, \text{for every } x \in D(A)$$

where

$$D(A) = \{x \in X: \lim_{t\to 0^+} \frac{T(t)x - x}{t} \text{exist in X} \}$$

Theorem 2.1 ([1]) Let $T(t)_{t\geqq 0}$ is a C_0 semigroup There exist constant $\omega \geq 0$ and $M \geq 1$ such that $\| T(t) \| \leq Me^{\omega t}$ $, 0 \leq t < \infty$

Pachpatte's inequality given below plays crucial role in our further analysis.

Theorem 2.2 ([[3], p. 39]). Let $u(t)$, $f(t)$ and $q(t)$ be nonnegative continuous functions defined on $\mathbb{R}_+$, and $n(t)$ be a positive and nondecreasing continuous function defined on $\mathbb{R}_+$ for which the inequality

$$u(t) \leq n(t) + \int_0^t f(s)[u(s) + \int_0^s q(\tau)u(\tau)d\tau]ds$$

hold for $t \in \mathbb{R}_+$ .Then

$$u(t) \leq n(t)[1 + \int_0^t f(s)\exp(\int_0^s [f(t) + q(\tau)]d\tau)\text{ds}]$$

for $t \in \mathbb{R}_+$

3. ULAM HYERS STABILITIES OF SEMILINEAR VIE

In this section, we establish Ulam Hyers stabilities of similinear VIE

$$x'(t) = Ax(t) + f\left(t, x(t), \int_0^t g\big(t, s, x(s)\big)ds\right), \quad t \in J \tag{3.1}$$

$$x(0) + H(x) = x_0, \tag{3.2}$$

in a Banach Space $(X, \|.\|)$ where

1. $J = [0, b]$
2. $A: X \to X$ is an infinitesimal generator of C_0-semigroup $T(t)_{t \geqq 0}$ in X;
3. $f: J \times X \times X \to X$ and $g: J \times J \times X \to X$,$H: C(J \times X) \to X$ are continuous

functions.

Definition 3.1 Let $\mathrm{T(t)}_{\mathrm{t} \geqq 0}$ is a $\mathrm{C_0}$-semigroup of bounded linear operators in X with infinitesimal generator A and $\mathrm{f} \in \mathrm{L^1(J, X)}$. A function $\mathrm{x} \in \mathrm{C(J, X)}$ given by

$$x(t) = T(t)[x_0 - H(x)] + \int_0^t T(t-s) f(s, x(s), \int_0^s g(s, \tau, x(\tau)) d\tau) ds,$$

is called the mild solution of initial value problem.

$$x'(t) = Ax(\mathrm{t}) + f(t, x(t), \int_0^t g(t, s, x(s))ds)$$

$$x(0) + H(x) = x_0 \tag{3.3}$$

Definition 3.2 Equation (3.1)-(3.2) is Ulam Hyers stable if there exists a real number $\mathrm{C_f} > 0$ such that for each $\varepsilon > 0$ and for each solution $\mathrm{y} \in \mathrm{C'(J, X)}$ of the inequation The function $\mathrm{x} \in \mathrm{B}$ satisfies the integral equation

$$\| y'(t) - Ay(t) - f(t, y(t), \int_0^t g(t, s, y(s))ds) \| \leq \varepsilon, t \in J \tag{3.4}$$

$\exists$ a mild solution $x: J \to X$ in $C(J, X)$ of (3.1)-(3.2) with

$$\| y(t) - x(t) \| \leq C_f \varepsilon, \quad t \in J \tag{3.5}$$

Definition 3.3 Equation (3.1)-(3.2) is Ulam Hyers Rassias stable, with respect to the positive non-decreasing continuous function $\psi: \mathrm{J} \in \mathbb{R}_+$,if there exists $\mathrm{C_{f,\psi}} > 0$ such that for each $\varepsilon > 0$ and for each solution $\mathrm{y} \in \mathrm{C_1(J, X)}$ of the inequation

$$\| y'(t) - Ay(t) - f\left(t, y(t), \int_0^t g\big(t, s, y(s)\big)ds\right) \| \leq \varepsilon\psi(t), \quad t \in J \tag{3.6}$$

there exists a mild solution $x: J \to X$ in $C(J, X)$ of (3.1)-(3.2) with

$$\| y(t) - x(t) \| \leq C_{f,\psi} \varepsilon \psi(t), \qquad t \in J$$

Definition 3.4 Equation (3.1)-(3.2) is generalized Ulam Hyers Rassias stable, with respect to the positive non-decreasing continuous function $\psi: J \in \mathbb{R}_+$,if there exists $C_{f,\psi} > 0$ such that for each solution $y \in C_1(J, X)$ of the inequation

$$\| y'(t) - Ay(t) - f\left(t, y(t), \int_0^t g\big(t, s, y(s)\big) ds\right) \| \leq \psi(t), \quad t \in J \tag{3.7}$$

there exists a mild solution $x: J \to X$ in $C(J, X)$ of (3.1)-(3.2) with

$$\| y(t) - x(t) \| \leq C_{f,\phi} \psi(t), \qquad t \in J \tag{3.8}$$

Remark 3.1

A function $y \in C^1(J, X)$ is a solution of in equation (3.4) if there exists a function $h \in C(J, X)$ (which depends on y) such that

1. $\| h(t) \| \leq \varepsilon, t \in J.$
2. $y'(t) = Ay(t) + f(t, y(t), \int_0^t g(t, s, y(s)) ds) + h(t), t \in J$

Remark 3.2

If $y \in C^1(J, X)$ satisfies inequation (3.4) then y is a solution of the following integral inequation:

$$\| y(t) - T(t)[y_0 - H(y)] + \int_0^t T(t-s) f(s, y(s), \int_0^s g(s, \tau, y(\tau)) d\tau) ds \| \leq \varepsilon \int_0^t \| T(t-s) \| dst \in J \tag{3.9}$$

Indeed, if $y \in C'(J, X)$ satisfies inequation (3.4) by Remark 3.1, we have

$$y'(t) = Ay(t) + f(t, y(t), \int_0^t g(t, s, y(s)) ds) + h(t), t \in J \tag{3.10}$$

This implies that

$$y(t) = T(t)[y_0 - H(y)] + \int_0^t T(t-s)[f(s, y(s), \int_0^s g(s, \tau, y(\tau)) d\tau) + h(s)] ds \tag{3.11}$$

Therefore

$$\| y(t) - T(t)[y_0 - H(y)] + \int_0^t T(t-s)[f(s, y(s), \int_0^s g(s, \tau, y(\tau)) d\tau)] ds \|$$
$$\leq \int_0^t \| T(t-s) \| \| h(s) \| ds \tag{3.12}$$

$$\leq \varepsilon \int_0^t \| T(t-s) \| ds \tag{3.13}$$

We list the following hypotheses for our convenience:

For Ulam Hyers stabilities of VIE on $J = [0, b]$

We need the following hypothesis to obtain Ulam Hyers stabilities of VIE

(H_1)

(a) Let $f: J \times X \times X \to X$ and there exist $L(.) \in C(J, \mathbb{R}_+)$ Let

$$\| f(t, x_1, x_2) - f(t, y_1, y_2) \| \leq L(t)(\| x_1 - y_1 \| + |x_1 - y_1 \|)$$

for all $t \in J$ and $x_1, x_2, y_1, y_2 \in X$

(b) Let $g: J \times J \times X \to X$ and $\exists G(.) \in C(J, \mathbb{R}_+)$ such that

$$\| g(t, s, x_1) - g(t, s, x_2) \| \leq G(t)(\| x_1 - y_1 \|)$$

for all $t, s \in J$ and $x_1, x_2, y_1, y_2 \in X$

(c) There exist positive constant $K_1 \in \mathbb{R}$ such that

$\| H(x) - H(y) \| \leq K_1 \| x - y \|$ for every $x, y \in X$

(H_2)

The function $\psi: [0, b] \to \mathbb{R}_+$ is positive, non-decreasing and continuous and there exists $\lambda > 0$ such that $\int_0^t \| T(t-s) \| \psi(t) ds \leq \lambda \psi(t)$.

Theorem 3.5 Let f,g,H in (3.1)-(3.2) satisfies hypotheses $(\mathrm{H}_1) - (\mathrm{H}_2)$ hold. Then the equation (3.1)-(3.2) Ulam Hyers Rassias stable with respect to ψ.

Proof: Let $y \in C'([0, b], X)$ satisfies inequation(3.6). Then as discussed in Remark 3.2 and using the hypothesis (H2), we have

$$\| y(t) - T(t)[y_0 - H(y)] + \int_0^t T(t-s)[f(s, y(s), \int_0^s g(s, \tau, y(\tau)) d\tau)] ds \|$$

$$\leq \int_0^t \| T(t-s) \| \| h(s) \| ds$$

$$\leq \varepsilon \int_0^t \| T(t-s) \| ds$$

$$\leq \varepsilon \lambda \psi(t) \tag{3.14}$$

Let $x \in C([0, b], X)$ be mild solution of ivp

$$x'(t) = Ax(t) + f(t, x(t), \int_0^t g(t, s, x(s))ds)t \in J$$

$$x_0 + H(x) = y_0 + H(y) \tag{3.15}$$

Then we have

$$x(t) = T(t)[y_0 - H(y)] + \int_0^t T(t-s)[f(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau)]ds \tag{3.16}$$

From equation (3.16) ,inequation (3.14) , and (H_1) we have

$$\| y(t) - x(t) \| \leq \| y(t) - T(t)[y(0) - H(y)] - \int_0^t T(t-s)f(s, x(s), \int_0^s g(s, \tau, x(\tau))ds \|$$

$$\leq \| y(t) - T(t)[y_0 - H(y)] - \int_0^t T(t-s)f(s, x(s), \int_0^s g(s, \tau, x(\tau))ds$$

$$+ \int_0^t T(t-s)f(s, y(s), \int_0^s g(s, \tau, y(\tau))ds$$

$$- \int_0^t T(t-s)f(s, y(s), \int_0^s g(s, \tau, y(\tau))ds \| \tag{3.17}$$

Add and subtract $\int_0^t T(t-s)f(s, y(s), \int_0^s g(s, \tau, y(\tau))ds$ in equation (3.17),we get

$$\| y(t) - x(t) \| \leq \epsilon\lambda\psi(t) + \int_0^t Me^{w(t-s)}L(s) \times (\| y(s) - x(s) \| + \int_0^s G(\tau)[\| y(\tau) - x(\tau) \|]d\tau)ds$$

$$\leq \epsilon\lambda\psi(t) + \int_0^t Me^{w(b-s)}L(s) \times (\| y(s) - x(s) \| + \int_0^s G(\tau)[\| y(\tau) - x(\tau) \|]d\tau)ds \tag{3.18}$$

And using pachpatte's inequality given in theorem (2.2) to equation (3.18) with

$u(t) = \| y(t) - x(t) \|$, $n(t) = \epsilon\lambda\psi(t)$,$f(t) = ML(t)e^{w(b-t)}$ and $q(t) = G(t)$.

we obtain

$$\| y(t) - x(t) \| \leq \epsilon\lambda\psi(t)[1 + \int_0^t M\text{exp}^{w(b-s)}L(s)exp(\int_0^s [ML(\tau)e^{w(b-\tau)} + G(\tau)]d\tau)ds] \tag{3.19}$$

$$\leq \epsilon\lambda\psi(t)[1 + \int_0^b M\text{exp}^{w(b-s)}L(s)exp(\int_0^s [ML(\tau)e^{w(b-\tau)} + G(\tau)]d\tau)ds] \tag{3.20}$$

by putting

$$C_{f,\psi} = \lambda[1 + \int_0^b M\text{exp}^{w(b-s)}L(s)exp(\int_0^s [ML(\tau)e^{w(b-\tau)} + G(\tau)]d\tau)ds]$$

we get

$$\| y(t) - x(t) \| \leq \varepsilon C_{f\psi}\psi(t) \forall t \in [0, b]$$

This proves that (3.1)-(3.2) is Ulam Hyers Rassias stable with respect to the function ψ.

Corrollary 3.4. let f, g and H in (3.1)-(3.2)satisfy the condition in hypothesis$(\mathrm{H}_1)-(\mathrm{H}_2)$.Then Equation (3.1)-(3.2)is generalized Ulam Hyers Rassias stable with respect to the function ψ.

Proof :-Taking $\varepsilon = 1$ in the proof of Main Theorem we obtain

$$\| y(t) - x(t) \| \leq C_{f,\phi}\psi(t), t \in J$$

which proves that (3.1)-(3.2) is generalized Ulam Hyers Rassias stable, with respect to function ψ.

Corrollary 3.5 If f, g in (3.1)-(3.2) satisfy the condition in hypothesis (H_1) .Then Equation (3.1)-(3.2) is Ulam Hyers Rassias stable.

Proof :- In the view of Theorem 2.1 there exists $M \geq 1$ such that $\| T(t) \| \leq M, \forall t \in [0, b]$ .Define $\psi(t) = 1 \; \forall t \in [0, b]$ Then

$$\int_0^t \| T(t-s) \| \psi(s) ds \leq Mb, \forall t \in [0, b]$$

Hence the assumption $(H2)$ holds clearly .By taking $\psi(t) = 1$ in the proof of main theorem, we obtain $\| y(t) - x(t) \| \leq \varepsilon C_f, \forall t \in [0, b]$ This proves that (3.1)-(3.2) is Ulam Hyers stable.

4. EXAMPLE

Consider the nonlinear VIEs

$$x'(t) = \frac{-3}{2} + 2\cos(x(t)) + 2\sin(x(t)) + \int_0^t \{\sin(x(s)) - \cos(x(s))\} ds, t \in [0,10] \quad (4.1)$$

$$x(0) + \frac{x}{80+x} = 0 \quad (4.2)$$

Consider the Banach space $(\mathbb{R}, \|\cdot\|)$ and the real Banach space and $C([0,10], \mathbb{R})$ with supremum norm. For each $t \geq 0$, define $T(t): \mathbb{R} \to \mathbb{R}$ by $T(t)x = \mathrm{e}^t x$, $x \in \mathbb{R}$. Then $T(t)_{t \geqq 0}$ forms the family of bounded linear operators from $\mathbb{R}$ to $\mathbb{R}$ that satisfy

$T(0) = 1$

$T(t+s) = T(t)T(s) \forall t, s \geq 0$;

$\lim_{t \to 0} T(t)x = x, \; \forall x \in \mathbb{R}$.

Therefore, $T(t)_{t \geqq 0}$ forms C_0 semigroup of bounded linear operators on $\mathbb{R}$.The infinitesimal generator of this C_0semigroup is

$$Ax = \lim_{t\to 0^+} \frac{T(t)x-x}{t} = \lim_{t\to 0^+} \frac{e^t-1}{t} x = x, x \in \mathbb{R}.$$

Thus $A = 1$.Note that Equations(4.1)-(4.2) can be written as

$$x'(t) = Ax(t) - \frac{3}{2} - x(t) + 2cos(x(t)) + 2sin(x(t))$$

$$+ \int_0^t \sin(x(s)) - \cos(x(s))ds, t \in [0,10] \quad (4.3)$$

$$= Ax(t) + f(t, x(t), \int_0^t g(t, s, x(s))ds) \quad (4.4)$$

$$x(0) + \frac{x}{80+x} = 0 \quad (4.5)$$

where $g: [0,10] \times [0,10] \times \mathbb{R} \to \mathbb{R}$

is defined as $g(t, s, x((s))) = \sin(x(s)) - \cos(x(s))$

and $f: [0,10] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

is defined as $f(t, x(t), \int_0^t g(t, s, x(s))ds)$

$$= \frac{3}{2} - (t) + 2cos(x(t)) + 2sin(x(t)) + \int_0^t \sin(x(s)) - cos(x(s))ds.$$

(i) For any $t, s \in [0,10]$and $x_1, y_1 \in \mathbb{R}$,we have

$$\| g(t, s, x_1) - g(t, s, y_1) \| \leq \| \sin(x_1) - \sin(y_1) \| + \| \cos x_1 - \cos y_1 \|$$

$$\leq \| x_1 - y_1 \| + \| \cos x_1 - \cos y_1 || \quad (4.6)$$

Let any $x, y \in \mathbb{R}$ with $x < y$. Applying mean value theorem to the function $\cos x$ on $[x, y]$, there $\sigma \in (x, y)$ such that $\frac{\cos x - \cos y}{x-y} = -sin(\sigma)$

Therefore,

$$\| \cos x - \cos y \| \leq \| x - y \|, x, y \in \mathbb{R}. \quad (4.7)$$

From (4.6) and (4.7), we obtain $\| g(t, s, x_1) - g(t, s, y_1) \| \leq 2 \| x_1 - y_1 \|$.

(ii) Let any $t \in [0,10]$ and $x_1, x_2, y_1, y_2 \in \mathbb{R}$.Then

$$\| f(t, x_1, x_2) - f(t, y_1, y_2) \| \leq \| x_1 - y_1 \| + 2 \| \cos x_1 - \cos y_1 \| + 2 \| \sin x_1 - \sin y_1 \| + \| x_2 - y_2 \|$$

$$\leq \| x_1 - y_1 \| + 2 \| x_1 - y_1 \| + 2 \| x_1 - y_1 \| + 2 \| x_1 - y_1 \| + \| x_2 - y_2 \|$$

$$\leq 5(\| x_1 - y_1 \| + \| x_2 - y_2 \|) \quad (4.8)$$

(iii) $\| H(x) - H(y) \| \leq \| \frac{x}{80+x} - \frac{x}{80+y} \|$

$\leq 80 \frac{\|x-y\|}{\|(80+x)(80+y)\|}$

$\leq \frac{80}{640} \| x - y \|$

$\leq \frac{1}{8} \| x - y \|$ (4.9)

Thus, f, g, H in Equation (4.3) with (4.5) satisfy the hypothesis (H_1) Therefore, by Corollary (3.4) , Equation (4.3) is Ulam Hyers stable. Next, we illustrate the Ulam Hyres stability of Equation (4.3) by providing the mild solution to Equation (4.3) corresponding to given values of $\varepsilon > 0$ and the given solution of the inequation

$$\| y'(t) - Ay(t) - f(t, y(t), \int_0^t g(t, s, x(s))ds) \| < \varepsilon \quad (4.10)$$

By using Definition (3.1), the mild solution of Equation (4.3) with the initial condition (4.5) is given by

$$x(t) = T(t)[0 - \frac{x}{80 + x}] + \int_0^t T(t - s)f(t, y(t), \int_0^s g(s, \tau, x(\tau))d\tau)ds$$

$$x(t) = \int_0^t \exp^{t-s}(\frac{-3}{2} - x(s) + 2\cos x(s) + 2\sin\big(x(s)\big))$$

$$+ \int_0^s (\sin(x(\tau)) - \cos(x(\tau)))d\tau)ds \quad (4.11)$$

By actual substitution we see that $x(t) = \frac{t}{2}$, $t \in [0,10]$ is solution of Equation (4.11) which is also a classical solution of Equation (4.3) with the initial condition Equation (4.5) .

Let $\varepsilon = 10$ and $y_1(t) = 0, t \in [0,10]$. Then

$\| y'_1(t) - Ay_1(t) - f(t, y_1(t), \int_0^t g(t, s, y(s))ds) \|$

$= \| y'_1(t) + \frac{3}{2} - 2\cos(y_1(t)) - 2\sin(y_1(t)) - \int_0^t [\sin(y_1(s)) - cos(y_1(s))]ds \|$

$= \frac{19}{2}$

$< \varepsilon$ (4.12)

and we have a solution $x(t) = \frac{t}{2}$, $t \in [0,10]$ of Equation (4.3) and constant $C = 1$ such that $\|$

$y_1(t) - x(t) \parallel = \parallel 0 - \frac{t}{2} \parallel \leq 5 < C_\varepsilon$

$\varepsilon = 15$ and $y_2(t) = \frac{t}{3}, t \in [0,10]$

we have

$$\parallel y'_2(t) - Ay_2(t) - f(t, y_2(t), \int_0^t g(t,s,y_2(s))ds) \parallel$$

$$= \parallel y'_2(t) + \frac{3}{2} - 2\cos(y_2(t)) - 2\sin(y_2(t)) - \int_0^t [\sin(y_2(s)) - cos(y_2(s))]ds \parallel$$

$$\leq \frac{1}{3} + \frac{3}{2} + 2 + 2 + \parallel \int_0^{10} [-\sin(\frac{s}{3}) + \cos(\frac{s}{3})]ds \parallel$$

$$= 5.833 + \parallel 3(\cos(\frac{10}{3}) - 1) + 3\sin\frac{10}{3} \parallel$$

$$< \varepsilon \qquad (4.13)$$

Corresponding to the pair $\varepsilon = 15$ and the solution $y_2(t) = \frac{t}{3}$, $t \in [0,10]$ of inequation (4.10) we have solution $x(t) = \frac{t}{2}$,$t \in [0,10]$ of Equation (4.1) and constant $C = \frac{1}{6}$

such that $\parallel y_2(t) - x(t) \parallel = \parallel \frac{t}{3} - \frac{t}{2} \parallel = \frac{t}{6} \leq 2 < C_\varepsilon$

This proves Ulam Hyres stability.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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