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K - PARITY MEAN CORDIAL LABELING OF GRAPHS

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Abstract: Cordial labeling was first introduced by I. Cahit in 1987 [1]. Consequently difference cordial, product cordial labeling, prime cordial, mean cordial labeling etc., were introduced and studied by R. Ponraj et. al. [4,7,8]. In this paper we introduce a new notion called 'k-parity Mean Cordial Labeling of Graphs' and we investigate its behavior for some standard graphs.

Keywords: cordial labeling; mean labeling; k – parity.

2010 AMS Subject Classification: 05C78, 05C38.

1. INTRODUCTION

Mean Labeling of graphs was first introduced and studied by S.Somasundaram and R.Ponraj[5,6]. Then several types of mean labelings (geometric, harmonic etc.,.) were studied by various authors. Mean Cordial labeling was introduced by R.Ponraj et al. For various mean labelings, we have a mathematical modeling. In molecular chemistry or cell biology, different molecules or cells bear different numbers for chemical or biological characteristics. The by products are supposed to possess some type of average (mean). What type of mean depends on the particular

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characteristic and it is determined by observations. Also different graphs represent different molecular structure.

All the graphs considered in this paper are simple and undirected. We now introduce and study k-parity Mean Cordial Labeling.

Definition 1.1: Let G be a (p,q) graph. Let $f: V(G) \rightarrow \{1,2,3 \dots k\}\,; 2 \leq k \leq |V(G)|$. For each edge uv, assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. The function f is called a k – parity mean cordial labeling of G if $\left|v_f(i) - v_f(\mathrm{j})\right| \leq 1$ $(\, i,j \in \{1,2,3 \dots k\})$ and $|\eta_e - \eta_o| \leq 1$, where $v_f(i)$ is the number of vertices labeled with i. η_e , η_o denote the number of edges labeled with even integers and odd integers respectively. A graph with a k – parity mean cordial labeling is called a **k – parity mean cordial graph**.

Remark: 2 – parity mean cordial graphs are product cordial graphs.

We collect some more definitions needed in the sequel.[2]

Definition 1.2. Let $V = \{x_1\,,\ x_2\,, \dots x_n\}$ and $E = \{(x_i\,,\ x_{i+1}) : 0 \leq i \leq n-1\}$. The graph $G = (V,E)$ is a path on n vertices and is denoted by P_n.

Definition 1.3. Let $V = \{x_1\,,\ x_2\,, \dots x_n\}$ and $E = \{(x_i\,,\ x_{i+1}) : 0 \leq i \leq n-1\} \cup \{(x_1\,, x_n)\}$. The graph $G = (V,E)$ is a cycle on n vertices and is denoted by C_n.

Definition 1.4. A graph on n vertices in which any two vertices are adjacent is called a complete graph and is denoted by K_n.

Definition 1.5. A Ladder graph, denoted by L_n is obtained from the Cartesian product of P_n and P_2.

Definition 1.6. The complete bipartite graph $K_{1,n}$ is called a star.

Definition 1.7. Wheel graph, denoted by W_n, is a graph formed by connecting a single universal vertex (also called central vertex) to all vertices of a cycle C_n .

2. MAIN RESULTS

Theorem 2.1: Every graph is a subgraph of a connected k – parity mean cordial graph.

Proof:

Let G be a (p,q) graph. Take k – copies of the complete graph K_p. That is, the graph kK_p.

Case (i): k is even

Let $u_1^{(i)}, u_2^{(i)}, \dots, u_p^{(i)}$ be the vertices of the ith copy of K_p. Let G^* be the graph with $V(G^*) = V(\mathrm{k}K_p)$ and $E(G^*) = E(\mathrm{k}K_p) \cup \left\{u_1^{(i)}\, u_1^{(i+1)} : 1 \le i \le k+1\right\}$. Clearly G^* is a super graph of G. We now assign labels to the vertices of G^*. Assign the label i to all the vertices of the ith copy of $K_p : u_1^{(i)}, u_2^{(i)}, \dots, u_p^{(i)}, 1 \le i \le k$. Note that $v_f(1) = v_f(2) = \cdots = V_f(k) = p$ and

$\eta_e = \frac{k}{2}\, pC_2 + 2$ and $\eta_o = \frac{k}{2}\, pC_2 + 1$.

Case (ii): $k \equiv 1 \ (mod\ 4)$

Let k=4t+1. Construct the graph G^{**} with $V(G^{**}) = V(G^*)$ and $E(G^{**}) = E(G^*) \cup \left\{u_1^{(1)} v_i : 1 \le i \le 2t\right\} \cup \left\{u_1^{(3)} w_i : 1 \le i \le 2t+1\right\}$. Assign the labels 2, 3, 6, 7 …, 4t-2, 4t-1 respectively to the vertices $v_1, v_2, \dots, v_{2t}$ and the labels 4, 5, 8, 9,.., 4t, 4t+1 to the vertices $w_1, w_2, .., w_{2t}$. Finally assign the label 1 to the vertex w_{2t+1}. Since $v_f(1) = v_f(2) = \cdots = v_f(k) = p$ and $\eta_e = \frac{k+1}{2}\, pC_2 + 2$ and $\eta_o = \frac{k+1}{2}\, pC_2 + 1$, this labeling pattern is a k – parity mean cordial labeling of G^{**}.

Case (iii): $k \equiv 3 \ (mod\ 4)$

In this case, we construct G^{**} as follows. Let $V(G^{**}) = V(G^*)$ and $E(G^{**}) = E(G^*) \cup \left\{u_1^{(1)} v_i : 1 \le i \le 2t+1\right\} \cup \left\{u_1^{(2)} w_1\right\} \cup \left\{u_1^{(3)} x_i : 1 \le i \le 2t+1\right\}$. For this case, assign the labels 2, 3, 6, 7,…, 4t-2, 4t-1 to the vertices $v_1, v_2, \dots, v_{2t}$ respectively and assign 1 to the vertex v_{2t+1}. Next assign the label 2 to the vertex w_1. Finally assign the labels 4, 5, 8, 9,…4t, 4t+1 respectively to the vertices $x_1, x, \dots, x_{2t}$ and assign the label 1 to the vertex x_{2t+1}. Clearly $v_f(1) = v_f(2) = p+1\,; v_f(3) = v_f(4) = \cdots = v_f(k) = p$ and $\eta_e = \frac{k+1}{2}\, pC_2 + 2$ and $\eta_o = \frac{k+1}{2}\, pC_2 + 1$. Hence the theorem.

Theorem 2.2: If k is even, then the Star graph $K_{1,n}$ is a k – parity mean cordial graph.

Proof:

Let $n = kt + r, 0 \leq r < n$. Let $V(K_{1,n}) = \{u\} \cup \{u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Assign the label 1 to the central vertex u. Next assign the label 1 to the vertices $u_1, u_{k+1}, u_{2k+1}, \dots, u_{(t-1)k+1}$. We now assign the label 2 to the vertices $u_2, u_{k+2}, \dots, u_{(t-1)k+2}$. In general, assign the label i to the vertices $u_i, u_{k+i}, u_{2k+i} \dots, u_{(t-1)k+i}$, $1 \leq i \leq k$. Now we consider the vertices $u_{kt+1}, u_{kt+2}, \dots, u_{kt+r}$. When r =1, assign the label 2 to the vertex u_{kt+1}. When r =2, assign the labels 3, 4 to the vertices u_{kt+1}, u_{kt+2} respectively. For the case $r > 2$, assign the labels r+1, 2, 3…, r-1, r respectively to the vertices $u_{kt+1}, u_{kt+2}, \dots, u_{kt+r}$. Clearly $v_f(1) = v_f(2) = \cdots = v_f(r-1) = v_f(r) = t+1$; $v_f(r+1) = v_f(r+2) = \cdots = v_f(k) = t$. Also the edge conditions are given in the table below.

n	η_o	η_e
Even	$\frac{n}{2}$	$\frac{n}{2}$
Odd	$\frac{n-1}{2}$	$\frac{n+1}{2}$

Remark: Hereafter we investigate 3 – parity mean cordial labeling for some standard graphs.

Theorem 2.3: Paths and Cycles are 3-parity mean cordial graphs.

Proof:

Let $u_1\, u_2 \dots u_n$ be the path.

Case (i): n = 3t

Each of the labels 1, 2, 3 must be given to k vertices.

Subcase (i): n is odd

Then P_n has even number of edges. Vertices have to be labeled in such a way that $\eta_e = \eta_o = \frac{n-1}{2}$. Label $u_1, u_2, \dots, u_t$ with the label 2 and u_{t+1} by 1. u_1u_2 , u_2u_3 , ... $u_{t-1}u_t$, u_tu_{t+1} receive 2 as edge label. t number of even edge labels have been obtained so far. $\frac{t-1}{2}\left(=\frac{n-1}{2}-t\right)$ number of even edge labels are yet to be attained. Since the label 2 for vertices has been exhausted, we are left with 1 and 3. (1,1) and (3,3) are the only possibilities to generate odd edge labels. Our plan is to label the vertices with 1s and 3s in a stretch continuously. But the point where the stretch of 1s changes to 3 or vice versa, we will get an even edge label. Therefore we produce $\frac{t-1}{2}-1$ even edge labels using 1 and 3 alternatively. Label u_{t+2} by 3, u_{t+3} by 1 and so on. This kind of alternative labeling must be stopped as soon as we get $\frac{n-1}{2}-1$ even edge labels. Let u_l be the vertex with which this alternative labeling ends. If u_l has the label 1(3), then starting from u_{l+1} give the label 1(3) until it gets exhausted. Let u_m be the vertex at which the change of stretches 1s and 3s takes place. Note that the edge $u_{m-1}u_m$ receives the edge label 2. Thus $\frac{n-1}{2}$ even edge labels are obtained. The remaining last stretch of vertices must be given the label 3(1). This type of labeling satisfies all the conditions for 3-parity mean cordial labeling.

Subcase (ii): n is even

Then P_n has odd number of edges. So we label the vertices in such a way that $\eta_e=\frac{n}{2}$ & $\eta_o=\frac{n-1}{2}$. Let $u_1,u_2,\dots u_n$ be the path. Label $u_1,u_2,\dots,u_t$ with the label 2 and $u_{t+1}$by 1, u_{t+2} by 3, u_{t+3} by 1 and so on. This kind of alternative labeling must be stopped as soon as $\frac{n}{2}-1$ even edge labels are obtained. By the same argument as in Subcase (i), we see that this is an ideal way to obtain a 3-parity labeling for path on 3t vertices with 3t even.

Case (ii): n = 3t+1

Follow the previous procedure and label $u_1,u_2,\dots u_{n-1}$. Note that u_{n-1} has the label either 1 or 3. If 3t is odd, then $\eta_e=\eta_o$ for P_{n-1}. Therefore, the vertex u_n can be given any of the labels 1, 2, 3. If 3t is even, then $\eta_e=\eta_o+1$ in P_{n-1}. So u_n must be labeled in such a way that the edge $u_{n-1}u_n$ receives an odd label. That is, if u_{n-1} has the label 1(3), label u_n with 1(3).

Case (iii): n = 3t+2

Label P_{n-2} with the procedure as described in Case(i). Note that u_{n-2} has the label either 1 or 3. If 3t is odd, then $\eta_e = \eta_o$. Therefore u_{n-1} and u_n have to be labeled so that one of $u_{n-2}u_{n-1}$, $u_{n-1}u_n$ receives an odd edge label and the other receives an even edge label. Also u_{n-1}, u_n can't receive same labels. If u_{n-2} has the label 1, then (1,2), (1,3), (2,3), (3,2) are the possibilities to label (u_{n-1}, u_n) to obtain 3-parity cordial mean labeling. If u_{n-2} has the label 3, then (3,1), (2,1) are the possibilities to label (u_{n-1}, u_n). Let 3t be even. If u_{n-2} has 3 as the label, then (2,3), (3,2) for (u_{n-1}, u_n) gives $\eta_o = \eta_e + 1$ and (2,1), (3,1) for (u_{n-1}, u_n) gives $\eta_e = \eta_o + 1$. If u_{n-2} has the label 1, then (1,2), (1,3), (2,3), (3,2) for (u_{n-1}, u_n) gives $\eta_e = \eta_o + 1$.

Hence Paths are 3- parity mean cordial graphs. Similar procedure of labeling can be followed for Cycles also.

Theorem 2.4: Complete graphs K_n $(n > 3)$, are not 3 – parity mean cordial graphs.

Proof:

Let K_n $(n > 3)$ be a complete graph on n vertices.

Case (i) : $n \equiv 0 \ (mod\ 3)$

Since n = 3t, each t set of vertices must receive the labels 1, 2, 3.

$\eta_o = tC_2 + tC_2 + t^2 = 2t^2 - t$

$\eta_e = tC_2 + t^2 + t^2 = \frac{5t^2 - t}{2}$

$\eta_e - \eta_o = \frac{5t^2 - t}{2} - (2t^2 - t) = \frac{t^2 + t}{2}$

$|\eta_e - \eta_o| > 1$ for $t > 1$

Case (ii): $n \equiv 1 \ (mod\ 3)$

Subcase (i): t+1 vertices receive the label 1, t vertices receive 2 and the remaining t vertices receive 3 as the label.

$\eta_o = (t+1)C_2 + tC_2 + t^2 = 2t^2$

$\eta_e = (3t+1)C_2 - 2t^2$

$\eta_e - \eta_o = (3t+1)C_2 - 4t^2 = \frac{t^2 + 3t}{2} > 1$, for $t > 1$

Subcase (ii): t vertices receive 1 as the label, t+1 vertices receive 2 as the label and t vertices receive the label 3.

$\eta_o = tC_2 + \ tC_2 + (t+1)t = 2t^2$

$\eta_e = (3t{+}1)C_2 - 2t^2$

$\eta_e - \eta_o = (3t{+}1)C_2 - 4t^2 = \frac{t^2+3t}{2} > 1$, for $t > 1$

Subcase(iii): t vertices receive the label 1, t vertices receive the label 2, t+1 vertices receive the label 3.

$\eta_o = tC_2 + \ (t+1)C_2 + t(t+1) = 2t^2 + t$

$\eta_e = (3t{+}1)C_2 - (2t^2 + t)$

$\eta_e - \eta_o = (3t{+}1)C_2 - 2(2t^2 + t) = \ tC_2 > 1$, for $t > 1$

Case (iii): $n \ \equiv 2 \ (mod \ 3)$

Subcase(i):t+1 vertices receive the label 1, t+1 vertices receive the label 2 and t vertices receive 3.

$\eta_o = (t{+}1)C_2 + \ tC_2 + t(t+1) = 2t^2 + t$

$\eta_e = (3t{+}2)C_2 - (2t^2 + t)$

$\eta_e - \eta_o = (3t{+}2)C_2 - 2(2t^2 + t) = \frac{t^2+5t+2}{2} > 1$, for $t > 1$

Subcase (ii): t+1 vertices receive the label 1, t vertices receive 2 and the remaining t+1 vertices receive 3 as the label.

$\eta_o = 2[(t{+}1)C_2] + t(t+1) = 2t(t+1)$

$\eta_e = (3t{+}2)C_2 - 2t(t+1)$

$\eta_e - \eta_o = (3t{+}2)C_2 - 4t(t+1) = \frac{t^2+t+2}{2} > 1$, for $t > 1$

Subcase (iii) : t vertices receive the label 1, t+1 vertices receive the label 2, t+1 vertices receive 3.

$\eta_o = tC_2 + \ (t+1)C_2 + (t+1)^2 = 2t^2 + 2t + 1$

$\eta_e = (3t{+}2)C_2 - (2t^2 + 2t + 1)$

$\eta_e - \eta_o = (3t{+}2)C_2 - 2(2t^2 + 2t + 1) = \frac{t^2+t-2}{2} > 1$, for $t > 1$

Hence Complete graphs do not admit 3 – parity mean cordial labeling.

Theorem 2.5: Wheel graphs are 3 – parity mean cordial labeling graphs.

Proof:

Let W_n be a wheel graph and let u be the central vertex of degree n. Label the vertex u with 1. Let $u_1, u_2, \dots u_n$ be the vertices of the cycle in W_n . For n=3t, label $u_1, u_2, \dots u_t$ by 1, $u_{t+1}, u_{t+2}, \dots u_{2t-1}, u_n(= u_{3t})$ by 2 and $u_{2t}, u_{2t+1}, \dots u_{3t-1}$ by 3. We get t even edge labels and 2t odd edge labels on the cycle and 2t even edge labels and t odd edge labels on the spikes of the wheel. Thus $\eta_e = \eta_o$ and $|v_f(i) - v_f(\text{j})| \leq 1$ $(i, j \in \{1,2,3\})$. For n=3t+1, label $u_1, u_2, \dots u_t$ by 1, $u_{t+2}, u_{t+3}, \dots u_{2t}, u_{3t+1}$ by 2 and $u_{t+1}, u_{2t+1}, u_{2t+2}, u_{2t+3}, \dots u_{3t}$ by 3. For n = 3t+2, label $u_1, u_2, \dots u_t$ by 1, $u_{t+2}, u_{t+3}, \dots u_{2t}, u_{2t+2}, u_{3t+2}$ by 2, $u_{t+1}, u_{2t+1}, u_{2t+3}, u_{2t+4}, \dots u_{3t}, u_{3t+1}$ by 3. Hence for all the cases, the labeling prescribed above serve as a 3-parity mean cordial labeling.

In connection with Theorem 2.2, we have the following.

Theorem 2.6: Star graphs are not 3 – parity mean cordial graphs.

Proof: Let $K_{1,n}$ be a star graph and let u be the vertex of degree n.

Case (i) : $f(u) = 1$

Subcase (i) : n = 3t

Each of the three sets of t vertices receives 1, 2, 3 as labels. Thus we get 2t even edge labels and t odd edge labels which violates the condition $|\eta_e - \eta_o| \leq 1$.

Subcase (ii): n = 3t+1

Now we have to label 3t+1 vertices with 1, 2, 3 in such a way that they satisfy

$|V_f(i) - V_f(\text{j})| \leq 1$ $(i, j \in \{1,2,3\})$. There are three ways to do it.

Type I: t vertices receive the label 1, t vertices receive 2 and t+1 receive 3. Then 2t+1 even edge labels and t odd edge labels are obtained.

Type II: t vertices receive 1 as a label, t+1 vertices receive 2 and t vertices receive 3.t odd edge labels and 2t+1 even edge labels are obtained.

Type III: t+1 vertices receive the label 1, t vertices receive 2 and t remaining vertices receive 3. t+1 odd edge labels and 2t even edge labels are obtained.

All the three types violate the condition $|\eta_e - \eta_o| \leq 1$.

Subcase (iii): n = 3t+2

(t, t+1, t+1), (t+1, t+1, t) and (t+1, t, t+1) are the possibilities to divide 3t+2 vertices in such a way that it respects the condition $\left|v_f(i) - v_f(j)\right| \leq 1$ $(i, j \in \{1,2,3\})$. (t,2t+2), (t+1, 2t+1), (t+1, 2t+1) are the odd edge and even edge label sets corresponding to the mentioned divisions respectively. Clearly they violate $|\eta_e - \eta_o| \leq 1$.

Taking $f(u) = 2$ and $f(u) = 3$ as cases (ii) and (iii) respectively and proceeding in the same lines of arguments we see that the number of odd and even edge labels violate $|\eta_e - \eta_o| \leq 1$.

Hence Star graphs are not 3-parity mean cordial graphs.

Theorem 2.7: Ladder graphs L_n are 3-parity mean cordial graphs.

Proof:

Let L_n $(n \geq 9)$ be a ladder graph. For $n < 9$, it can be easily settled. Hence the labeling procedure below is for greater values of n. Let $u_1, u_2, \dots, u_n$ be the vertices on one side of the ladder and let $v_1, v_2, \dots, v_n$ be the vertices on the other side of the ladder. We have 2n vertices and 3n-2 edges.

Case (i): $n \equiv 0(mod\ 3)$

Subcase (i): $n \equiv 0(mod\ 12)$

Label $u_1, u_2, \dots, u_{\frac{n}{3}}$ and $v_1, v_2, \dots, v_{\frac{n}{3}}$ with 2.

Label $u_{\frac{n}{3}+1}, u_{\frac{n}{3}+2}, \dots, u_{\frac{n}{3}+\frac{n}{4}}$, $v_{\frac{n}{3}+1}, v_{\frac{n}{3}+2}, \dots, v_{\frac{n}{3}+\frac{n}{4}+1}$, v_n by 1.

Label $u_n, u_{n-1}, \dots, u_{n-\left(\frac{n}{4}+1\right)}$, $v_{n-1}, v_{n-2}, \dots, v_{n-\frac{n}{4}}$ by 3.

Now consider the remaining vertices in the u and v rows. There is going to be alternative labeling with 3s and 1s.

Label $u_{\frac{n}{3}+\frac{n}{4}+1}, u_{\frac{n}{3}+\frac{n}{4}+3}, \dots, u_{n-\left(\frac{n}{4}+3\right)}$, $v_{\frac{n}{3}+\frac{n}{4}+2}, v_{\frac{n}{3}+\frac{n}{4}+4}, \dots, v_{n-\frac{n}{4}-2}$ by 3 and label the remaining vertices by 1.

Thus we get $\frac{3n-2}{2}$ even edge labels and $\frac{3n-2}{2}$ odd edge labels. Also each of the labels 1, 2, 3 is given to $\frac{n}{3}$ vertices.

Thus the two conditions $\left|v_f(i) - v_f(\mathrm{j})\right| \leq 1$ $(i, j \in \{1,2,3 \dots k\})$ and $|\eta_e - \eta_o| \leq 1$ for 3 – parity mean cordial labeling are satisfied.

Subcase (ii): $n \not\equiv 0(mod\ 12)$

n can be expressed as one among the following: 15+12t, 18+12t, 21+12t $(k \geq 0)$.

Label $u_1, u_2, \dots, u_{\frac{n}{3}}$ and $v_1, v_2, \dots, v_{\frac{n}{3}}$ with 2.

Finding the appropriate value of t depending on n, label the last 3+t vertices of the u row and the last $\frac{2n}{3} - (3 + t)$ vertices of the v row by 3. All the remaining vertices are labeled with 1.

If n is of the form 18+12t, then $\frac{3n-2}{2}$ is an even natural number and hence we get $\frac{3n-2}{2}$ even edge labels and $\frac{3n-2}{2}$ odd edge labels. If n is of the form 15+12t or 21+12t, then consider $\left\lfloor \frac{3n-2}{2} \right\rfloor$ and $\left\lceil \frac{3n-2}{2} \right\rceil$. One of them is an even number and the other is odd. With respect to the above mentioned labeling, we get odd number of even edge labels and even number of odd edge labels which satisfy $|\eta_e - \eta_o| \leq 1$.

Case (ii): $n \equiv 1(mod\ 3)$

Label $u_1,\ v_1$ with 3, 2 respectively. Now consider the subgraph $L_n \backslash \{u_1,\ v_1\}$ as the ladder L_m with $m \equiv 0(mod\ 3)$. Labeling procedure as in Case(i) is followed. The resulting graph is a 3 – parity mean cordial labeling.

Case(iii): $n \equiv 2(mod\ 3)$

Label u_1, v_1, u_2, v_2 with 1, 1, 3, 2 respectively. Consider the subgraph $L_n \backslash \{u_1, v_1, u_2, v_2\}$ as the ladder L_m with $m \equiv 0(mod\ 3)$. Labeling procedure as in Case(i) is followed. This will be a 3 – parity mean cordial labeling.

Hence Ladder graphs are 3 - parity mean cordial graphs.

CONCLUSION

In this paper, we have introduced and studied 'k – parity mean cordial labeling' for some standard graphs like paths, cycles, star, complete graphs, wheel and ladder. Further investigation can be done for standard graphs and derived graphs. This could pave way for more general results for this labeling.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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