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VECTOR BASIS {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-CORDIAL LABELING OF CERTAIN TREES AND FLOWER GRAPHS

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Abstract. Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $<x,y>$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $<\phi(u),\phi(v)>$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. In this paper, we investigate the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behavior of some new graphs like the olive tree, lobster graph, $I_{m,n}$ graph, shrub graph, rose flower graph, clematis flower graph, cherry blossom flower graph, armed crown graph, rocket graph and sandat graph.

Keywords: olive tree; lobster graph; shrub graph; clematis flower graph; rocket graph; sandat graph.

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1. INTRODUCTION

In this paper, a graph $G = (V,E)$ is finite, simple, connected and undirected. The idea of graph labeling technique was introduced by Rosa in 1967 [16]. Arithmetic number labeling for banana tree, olive tree, shrub, jelly fish, tadpole graphs were discussed by Uma Maheswari

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and Purnalakshmi [18]. The existence of (s,d) magic labeling in some trees such as a coconut tree, regular bamboo tree, symmetrical tree, olive tree and spider tree have been determined by Sumathi and Mala [14]. A novel encryption algorithm using skolem graceful labeling was studied in [17]. Square Sum labeling has been investigated in [19] for lobster and fan graph. Mean cordial labeling of tadpole and olive tree has been investigated by Deshmukh and Shaikh [3]. Rathod and Kanani [12] proved that the middle graph, total graph and splitting graph of the path are 4-cordial. Kavitha and Sumathi [7] have proved that the caterpillar and lobster graphs are quotient-4 cordial. Prime cordial labeling behavior of the crown, armed crown, H-graph and butterfly graph were examined in [13]. Amuthavalli and Shanmuga Sundaram [1] have studied the super fibonacci graceful anti-magic labeling for rose flower graph, clematis flower graph and cherry blossom flower graph. Cycle and path related near mean cordial graphs were explored in [7]. Susilawati and Salman [15] have determined the rainbow connection number of a rocket graph. Cordial labeling was first introduced by Cahit [2]. We follow the terminologies and relevant notations of graph theory and algebra in [5, 6]. For a survey on graph labeling, we refer the book of Gallian [4]. We have introduced the new labeling technique called vector basis S-cordial labeling in [9] and also investigated the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behavior of several graphs like path, cycle, comb, star, complete graph, fan graph, friendship graph, lilly graph, bistar graph, generalized friendship graph, tadpole graph, gear graph and thorn related graphs in [9, 11, 12]. In this paper, we examines the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behavior of some new graphs like the olive tree, lobster graph, $I_{m,n}$ graph, shrub graph, rose flower graph, clematis flower graph, cherry blossom flower graph, armed crown graph, rocket graph and sandat graph.

In this paper, we consider the inner product space R^n and the standard inner product $<x,y>= x_1y_1+x_2y_2+\cdots+x_ny_n$ where $x=(x_1,x_2,\ldots,x_n), y=(y_1,y_2,\ldots,y_n)$, $x_i,y_i \in R$.

2. PRELIMINARIES

We use some basic definitions which are needed for the upcoming section.

Definition 2.1. [16] *The olive tree O_n is a rooted tree consisting of n branches where the i^{th} branch is a path of length i.*

Definition 2.2. [17] *The lobster graph $L_n(q,r)$, $n \geq 2$ is a graph with n vertices on backbone path, each of which is connected to q hand vertices and each hand vertex is connected to r finger vertices.*

Definition 2.3. [18] *The shrub graph $S_{n_1,n_2,\ldots,n_k}$ is a graph got by connecting a vertex v_0 to the central vertex of each of k number of stars.*

Definition 2.4. [4] *Let $P_n : u_1u_2\ldots u_n$ and $P_n' : v_1v_2\ldots v_n$ be the two paths and consider two stars with vertex set $\{x,x_i,y,y_i\}$ and $\{xx_i,yy_i\}$. The graph $I_{m,n}$, $n \geq 2$ is obtained by the two paths P_n, P_n' and two stars by identifying the vertices x,y,u_n,v_1. Note that u is the identifying vertex.*

Definition 2.5. [1] *The rose flower graph R_n is obtained by joining n copies of the cycle C_6 with a common vertex. That is R_n is the one point union of n copies of C_6.*

Definition 2.6. [1] *The clematis flower graph $C_{m,n}$ is obtained by joining m copies of C_4+e and n copies of K_2 with a common vertex.*

Definition 2.7. [1] *The cherry blossom flower graph $CB_{n,n-1}$ is obtained by joining m copies of C_3 and n copies of K_2 with a common vertex.*

Definition 2.8. [17] *The armed crown $AC_{n,m}$ is a cycle with a path of length $m-1$ attached at each vertex of the cycle.*

Definition 2.9. [15] *The rocket $R_{m,n}$ is a graph with the vertex set $V(R_{m,n}) = \{x_i, y_i, z_0, x_{m,j}, z_j, y_{m,j} \mid 1 \leq i \leq m \, and \, 1 \leq j \leq n\}$ and the edge set $E(R_{m,n}) = \{x_iy_i, x_mz_0, y_mz_0 \mid 1 \leq i \leq m\} \cup \{x_ix_{i+1}, x_iy_{i+1}, y_iy_{i+1} \mid 1 \leq i \leq m-1\} \cup \{x_mx_{m,j}, y_my_{m,j}, z_0z_j \mid 1 \leq j \leq n\}$.*

Definition 2.10. [1] *A sandat graph ST_n is a graph with the vertex set $V(ST_n) = \{u, u_i, u_{i,j} \mid 1 \leq i \leq n \, and \, 1 \leq j \leq 2\}$ and the edge set $E(ST_{m,n}) = \{uu_i, uu_{i,j} \mid 1 \leq i \leq n \, and \, 1 \leq j \leq 2\}$.*

3. VECTOR BASIS S-CORDIAL LABELING

Definition 3.1. *Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $< \phi(u), \phi(v) >$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. A Simple example of a vector basis S-cordial labeling of graph is given in the following figure (1) where $S = \{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$ is a basis of R^4.*

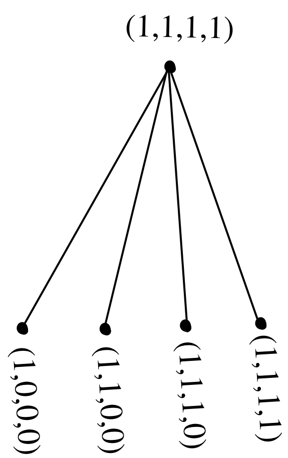

Figure 1: Vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial graph.

4. MAIN RESULTS

In this section, we investigate the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behavior of some new graphs like the olive tree, lobster graph, $I_{m,n}$ graph, shrub graph, rose flower graph, clematis flower graph, cherry blossom flower graph, armed crown graph, rocket graph and sandat graph.

Theorem 4.1. *The olive tree O_n is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial graph for all n.*

Proof. Let $V(O_n) = \{v, v_{ij} \mid 1 \leq i \leq n \text{ and } j = 1\} \cup \{v_{ij} \mid 1 \leq i \leq n-1 \text{ and } j = 2\} \cdots \cup \{v_{ij} \mid i = 1,2 \text{ and } j = n-1\} \cup \{v_{ij} \mid i = 1 \text{ and } j = n\}$ and $E(O_n) = \{vv_{1i} \mid 1 \leq i \leq n\} \cup \{v_{i1}v_{(i+1)1} \mid$

$1 \leq i \leq n-1\} \cup \{v_{i2}v_{(i+1)2} \mid 1 \leq i \leq n-2\} \cdots \cup \{v_{1(n-1)}v_{2(n-1)}\}$. Note that $p = |V(O_n)| = \frac{n(n+1)}{2} + 1$ and $q = |E(O_n)| = \frac{n(n+1)}{2}$. Assign the vectors to the olive tree O_n in the following order $v, v_{11}, v_{21}, v_{31}, \ldots, v_{(k-1)1}, \ldots, v_{n1}, v_{12}, v_{22}, \ldots, v_{(n-1)2}, \ldots, v_{1(n-1)}, v_{2(n-1)}, v_{1n}$.

Case (i): $p \equiv 0 \pmod 4$

Then $p = 4t$. Assign the vector $(1,1,1,1)$ to the first t vertices. We assign the vector $(1,1,1,0)$ to the next t vertices. Next, assign the vector $(1,1,0,0)$ to the next t vertices. Moreover, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (ii): $p \equiv 1 \pmod 4$

Note that $p = 4t+1$. Then, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We now assign the vector $(1,1,1,0)$ to the next t vertices. Also, assign the vector $(1,1,0,0)$ to the next t vertices. Finally, assign the vector $(1,0,0,0)$ to the last t vertices.

Case (iii): $p \equiv 2 \pmod 4$

Then $p = 4t+2$. Now, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Assign the vector $(1,1,0,0)$ to the next t vertices. Further, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (iv): $p \equiv 3 \pmod 4$

Then $p = 4t+3$. We assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. Assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Assign the vector $(1,1,0,0)$ to the next $t+1$ vertices. So assign the vector $(1,0,0,0)$ to the next t vertices.

Therefore the above labeling technique is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of the olive tree O_n. □

Theorem 4.2. *The lobster graph $L_n(2,r)$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph for all n, r.*

Proof. Let $V(L_n(2,r)) = \{u_i, v_i, w_i, v_{ij}, w_{ij} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq r\}$ and $E(L_n(2,r)) = \{u_iu_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_iv_i, u_iw_i, v_iv_{ij}, w_iw_{ij} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq r\}$. Note that $p = |V(L_n(2,r))| = n(2r+3)$ and $q = |E(L_n(2,r))| = n(2r+3)-1$. Assign the vectors in the following order $u_1, v_1, w_1, v_{11}, v_{12}, \ldots, v_{1r}, w_{11}, w_{12}, \ldots, w_{1r}, u_2, v_2, w_2, v_{21}, v_{22}, \ldots, v_{2r}, w_{21}, w_{22}, \ldots,$ $w_{2r}, \ldots, u_n, v_n, w_n, v_{n1}, v_{n2}, \ldots, v_{nr}, w_{n1}, w_{n2}, \ldots, w_{nr}$.

Case (i): $L_1(2,r), L_3(2,r), L_5(2,r), \ldots$

Subcase (i): $p \equiv 1 \pmod 4$

Then $p = 4t$. Assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We assign the vector $(1,1,1,0)$ to the next t vertices. Next, assign the vector $(1,1,0,0)$ to the next t vertices. Moreover, assign the vector $(1,0,0,0)$ to the next t vertices.

Subcase (ii): $p \equiv 3 \pmod 4$

Note that $p = 4t+1$. Then, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We now assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Also, assign the vector $(1,1,0,0)$ to the next $t+1$ vertices. Finally, assign the vector $(1,0,0,0)$ to the last t vertices.

Case (ii): $L_4(2,r), L_8(2,r), L_{12}(2,r), \ldots$

Then $p \equiv 0 \pmod 4$. Note that $p = 4t$. Now, assign the vector $(1,1,1,1)$ to the first t vertices. We assign the vector $(1,1,1,0)$ to the next t vertices. Assign the vector $(1,1,0,0)$ to the next t vertices. Further, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (iii): $L_2(2,r), L_6(2,r), L_{10}(2,r), \ldots$

We see that $p \equiv 2 \pmod 4$. Then $p = 4t+2$. We assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. Assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Assign the vector $(1,1,0,0)$ to the next t vertices. So assign the vector $(1,0,0,0)$ to the next t vertices.

Thus the above labeling technique is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of the lobster graph $L_n(2,r)$. □

Theorem 4.3. *The shrub graph $S_{n_1,n_2,\ldots,n_k}$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph for all n,r.*

Proof. Let $V(S_{n_1,n_2,\ldots,n_k}) = \{u, u_i, u_{ij} \mid 1 \leq i \leq k \,\text{and}\, 1 \leq j \leq i\}$ and $E(S_{n_1,n_2,\ldots,n_k}) = \{uu_i \mid 1 \leq i \leq k\} \cup \{u_i u_{ij} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq i\}$. Note that $p = |V(S_{n_1,n_2,\ldots,n_k})| = \frac{k(k+1)}{2} + k + 1$ and $q = |E(S_{n_1,n_2,\ldots,n_k})| = \frac{k(k+1)}{2} + k$. Assign the vectors in the following order $u, u_1, u_2, \ldots, u_k, u_{11}, u_{21}, u_{22}, u_{31}, u_{32}, u_{33}, u_{41}, u_{42}, u_{43}, u_{44}, \ldots, u_{k1}, u_{k2}, \ldots, u_{kk}$.

Case (i): $p \equiv 0 \pmod 4$

Then $p = 4t$. Assign the vector $(1,1,1,1)$ to the first t vertices. We assign the vector $(1,1,1,0)$ to the next t vertices. Next, assign the vector $(1,1,0,0)$ to the next t vertices. Moreover, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (ii): $p \equiv 1 \pmod 4$

Note that $p = 4t+1$. Then, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We now assign the vector $(1,1,1,0)$ to the next t vertices. Also, assign the vector $(1,1,0,0)$ to the next t vertices. Finally, assign the vector $(1,0,0,0)$ to the last t vertices.

Case (iii): $p \equiv 2 \pmod 4$.

Note that $p = 4t+2$. Now, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Assign the vector $(1,1,0,0)$ to the next t vertices. Further, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (iv): $p \equiv 3 \pmod 4$.

Then $p = 4t+3$. We assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. Assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Assign the vector $(1,1,0,0)$ to the next $t+1$ vertices. So assign the vector $(1,0,0,0)$ to the next t vertices.

Thus the above labeling technique is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of the shrub graph $S_{n_1,n_2,\ldots,n_k}$. □

Theorem 4.4. *The graph $I_{m,n}$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph for all n,m.*

Proof. Let $V(I_{m,n}) = \{u, u_i, v_i, x_j, y_j \mid 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\}$ and $E(I_{m,n}) = \{u_iu_{i+1}, uu_1, vv_1,$
$v_iv_{i+1}, uw_j, ux_j \mid 1 \leq i \leq n-2 \text{ and } 1 \leq j \leq m\}$. Note that $p = |V(I_{m,n})| = 2(m+n)-1$ and $q = |E(I_{m,n})| = 2(m+n)-2$. Assign the vectors in the following order $u, u_1, u_2, \ldots, u_{n-1}, v_1, v_2, \ldots,$ $v_{n-1}, x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_m$.

Case (i): $p \equiv 1 \pmod 4$

Then $p = 4t+1$. Assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We assign the vector $(1,1,1,0)$ to the next t vertices. Next, assign the vector $(1,1,0,0)$ to the next t vertices. Moreover, assign the vector $(1,0,0,0)$ to the next t vertices.

Case (ii): $p \equiv 3 \pmod 4$

Note that $p = 4t+3$. Then, assign the vector $(1,1,1,1)$ to the first $t+1$ vertices. We now assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Also, assign the vector $(1,1,0,0)$ to the next $t+1$ vertices. Finally, assign the vector $(1,0,0,0)$ to the last t vertices.

Clearly the above labeling method is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of the graph $I_{m,n}$. □

Theorem 4.5. *The rose flower graph R_n is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial if and only if $n \equiv 0,1 \pmod 4$.*

Proof. Let $V(R_n) = \{u, u_i, v_i, w_i \mid 1 \leq i \leq 2n\}$ and $E(R_n) = \{uu_i, u_iv_i \mid 1 \leq i \leq 2n\} \cup \{w_iv_{2i}, w_iv_{2i-1} \mid 1 \leq i \leq n\}$. Note that $p = |V(R_n)| = 5n+1$ and $q = |E(R_n)| = 6n$. Assign the vectors in the following order $u, u_1, v_1, w_1, v_2, u_2, u_3, v_3, w_2, v_4, u_4, u_5, v_5, w_3, \ldots, u_{2n-1}, v_{2n-1}, w_n, v_{2n}, u_{2n}$.

Case (i): $n \equiv 0 \pmod 4$

Then $n = 4t$. Then $p = 5(4t)+1 = 20t+1$. Assign the vector $(1,1,1,1)$ to the first vertex u. We assign the vector $(1,1,1,1)$ to the next $5t$ vertices. Next, assign the vector $(1,1,1,0)$ to the next $5t$ vertices. Moreover, assign the vector $(1,1,0,0)$ to the next $5t$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $5t$ vertices.

Case (ii): $n \equiv 1 \pmod 4$

Note that $n = 4t+1$. Then, $p = 5(4t+1)+1 = 20t+6$. Assign the vector $(1,1,1,1)$ to the first vertex u. We now assign the vector $(1,1,1,1)$ to the next $5t+1$ vertices. Also, assign the vector $(1,1,1,0)$ to the next $5t+2$ vertices. Also, assign the vector $(1,1,0,0)$ to the next $5t+1$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $5t+1$ vertices.

Case (iii): $n \equiv 2 \pmod 4$

Note that $n = 4t+2$. Then, $p = 5(4t+2)+1 = 20t+11$ and $q = 6(4t+2)+1 = 24t+12$. To get $6t+3$ edges for the label 4, we have to label $(1,1,1,1)$ for atleast $5t+4$ vertices, a contradiction.

Case (iv): $n \equiv 3 \pmod 4$

Note that $n = 4t+3$. Then, $p = 5(4t+3)+1 = 20t+16$ and $q = 6(4t+3)+1 = 24t+18$. To get $6t+4$ edges for the label 4, we should label $(1,1,1,1)$ for atleast $5t+5$ vertices, This is a contradiction. □

Theorem 4.6. *The clematis flower graph $C_{n,n}$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0), (1,0,0,0)}-cordial graph for all n.*

Proof. Let $V(C_{n,n}) = \{u, u_i, u_{i,j} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq 3\}$ and $E(C_{n,n}) = \{uu_i, uu_{i,j}, u_{i,1}u_{i,2}, u_{i,2}u_{i,3} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq 3\} \cup \{w_iv_{2i}, w_iv_{2i-1} \mid 1 \leq i \leq n\}$. Note that $p = |V(C_{n,n})| = 4n+1$ and $q = |E(C_{n,n})| = 6n$.

Case (i): $n \equiv 0 \pmod 4$

Note that $n = 4t$. Then, $p = 4(4t)+1 = 16t+1$. Assign the vector $(1,1,1,1)$ to the first vertex u. Also assign the vector $(1,1,1,1)$ to the vertices of t copies of C_4+e i.e., $u, u_{1,1}, u_{1,2}, u_{1,3}, u_{2,1}, u_{2,2}, u_{2,3}, \ldots, u_{t,1}, u_{t,2}, u_{t,3}$ and t copies of vertices of K_2 i.e., $u_1, u_2, \ldots, u_t$. Assign the vector $(1,1,1,0)$ to the vertices of next t copies of C_4+e i.e., $u_{t+1,1}, u_{t+1,2}, u_{t+1,3}, u_{t+2,1}, u_{t+2,2}, u_{t+2,3}, \ldots, u_{2t,1}, u_{2t,2}, u_{2t,3}$ and t copies of K_2 i.e., $u_{t+1}, u_{t+2}, \ldots, u_{2t}$. Thereafter assign the vector $(1,1,0,0)$ to the vertices of next t copies of C_4+e i.e., $u_{2t+1,1}, u_{2t+1,2}, u_{2t+1,3}, u_{2t+2,1}, u_{2t+2,2}, u_{2t+2,3}, \ldots, u_{3t,1}, u_{3t,2}, u_{3t,3}$ and t copies of K_2 i.e., $u_{2t+1}, u_{2t+2}, \ldots, u_{3t}$. Finally assign the vector $(1,0,0,0)$ to the vertices of next t copies of C_4+e i.e., $u_{3t+1,1}, u_{3t+1,2}, u_{3t+1,3}, u_{3t+2,1}, u_{3t+2,2}, u_{3t+2,3}, \ldots, u_{4t,1}, u_{4t,2}, u_{4t,3}$ and t copies of K_2 i.e., $u_{3t+1}, u_{3t+2}, \ldots, u_{4t}$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4t+1$. Then, $p = 4(4t+1)+1 = 16t+5$. **Step I:** Assign the vector to the vertices as in case (i). **Step II:** Then assign the vector $(1,1,1,1)$ to the $t+1^{th}$ copy of K_2 i.e., u_{t+1}. Assign the vector $(1,1,1,0)$ to the vertex $u_{t+1,1}$. Next, assign the vector $(1,1,0,0)$ to the vertex $u_{t+1,2}$. Further assign the vector $(1,0,0,0)$ to the vertex $u_{t+1,3}$.

Case (iii): $n \equiv 2 \pmod 4$

Then $n = 4t+2$. So, $p = 4(4t+2)+1 = 16t+9$. **Step I:** Assign the vector to the vertices as in case (i). **Step II:** Then assign the vector $(1,1,1,1)$ to the vertices $u_{t+1,1}$ and $u_{t+1,2}$. Assign the vector $(1,1,1,0)$ to the vertices $u_{t+1,3}$ and u_{t+1}. Next, assign the vector $(1,1,0,0)$ to the vertices $u_{t+2,1}$ and $u_{t+2,2}$. More over assign the vector $(1,0,0,0)$ to the vertices $u_{t+2,3}$ and u_{t+2}.

Case (iv): $n \equiv 3 \pmod 4$

Then $n = 4t+3$. So, $p = 4(4t+3)+1 = 16t+13$. **Step I:** Assign the vector to the vertices as in case (i). **Step II:** Then assign the vector $(1,1,1,1)$ to the vertices of $t+1^{th}$ copy of C_4+e i.e., $u_{t+1,1}, u_{t+1,2}, u_{t+1,3}$. Assign the vector $(1,1,1,0)$ to the vertices of $t+2^{th}$ copy of C_4+e i.e., $u_{t+2,3}, u_{t+2,2}u_{t+2,3}$. Further, assign the vector $(1,1,0,0)$ to the vertices $u_{t+3,1}, u_{t+2,2}, u_{t+2,3}$. More over assign the vector $(1,0,0,0)$ to the vertex $u_{t+3,3}$ and the vertices u_{t+2} and u_{t+3}. □

Theorem 4.7. *The cherry blossom flower graph $CB_{n,n-1}$ is a vector basis {(1,1,1,1),(1,1,1,0), (1,1,0,0),(1,0,0,0)}-cordial graph for all n.*

Proof. Let $V(CB_{n,n-1}) = \{u, u_i, v_i, w_j \mid 1 \le i \le n \text{ and } 1 \le j \le n-1\}$ and $E(CB_{n,n-1}) = \{uu_i, uv_i, u_iv_i, uw_j \mid 1 \le i \le n \text{ and } 1 \le j \le n-1\}$. We have $p = |V(CB_{n,n-1})| = 3n$ and $q = |E(CB_{n,n-1})| = 4n-1$.

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4t$. Assign the vector $(1,1,1,1)$ to the first vertex u. Also assign the vector $(1,1,1,1)$ to the vertices $u_1, v_1, u_2, v_2, \ldots, u_t, v_t$ and the vertices $w_1, w_2, \ldots, w_{t-1}$. Next assign the vector $(1,1,1,0)$ to the vertices $u_{t+1}, v_{t+1}, u_{t+2}, v_{t+2}, \ldots, u_{2t}, v_{2t}$ and the vertices $w_t, w_{t+1}, \ldots, w_{2t-1}$. Further assign the vector $(1,1,0,0)$ to the vertices $u_{2t+1}, v_{2t+1}, u_{2t+2}, v_{2t+2}, \ldots, u_{3t}, v_{3t}$ and the vertices $w_{2t}, w_{2t+1}, \ldots, w_{3t-1}$. Finally assign the vector $(1,0,0,0)$ to the vertices $u_{3t+1}, v_{3t+1}, u_{3t+2}, v_{3t+2}, \ldots, u_{4t}, v_{4t}$ and the vertices $w_{3t}, w_{3t+1}, \ldots, w_{4t-1}$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4t+1$. Assign the vector $(1,1,1,1)$ to the first vertex u. Then assign the vector $(1,1,1,1)$ to the vertices $u_1, v_1, u_2, v_2, \ldots, u_t, v_t$ and the vertices $w_1, w_2, \ldots, w_t$. Next assign the vector $(1,1,1,0)$ to the vertices $u_{t+1}, v_{t+1}, u_{t+2}, v_{t+2}, \ldots, u_{2t}, v_{2t}$ and the vertices $w_{t+1}, w_{t+2}, \ldots, w_{2t}$. Further assign the vector $(1,1,0,0)$ to the vertices $u_{2t+1}, v_{2t+1}, u_{2t+2}, v_{2t+2}, \ldots, u_{3t}, v_{3t}$ and the vertices $w_{2t+1}, w_{2t+2}, \ldots, w_{3t}$. Assign the vector $(1,0,0,0)$ to the vertices $u_{3t+1}, v_{3t+1}, u_{3t+2}, v_{3t+2}, \ldots, u_{4t}, v_{4t}$ and the vertices $w_{3t+1}, w_{3t+2}, \ldots, w_{4t}$. More over assign the vector $(1,1,1,0)$ to the vertex $u_n = u_{4t+1}$ and assign the vector $(1,1,0,0)$ the vertex $v_n = v_{4t+1}$.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4t+2$. Then assign the vector $(1,1,1,1)$ to the vertices $u, u_1, v_1, u_2, v_2, \ldots, u_t, v_t$ and the vertices $w_1, w_2, \ldots, w_{t+1}$. Thereafter assign the vector $(1,1,1,0)$ to the vertices $u_{t+1}, v_{t+1}, u_{t+2}, v_{t+2}, \ldots, u_{2t}, v_{2t}$ and the vertices $w_{t+2}, w_{t+3}, \ldots, w_{2t+1}, w_{2t+2}$. Also assign the vector $(1,1,0,0)$ to the vertices $u_{2t+1}, v_{2t+1}, u_{2t+2}, v_{2t+2}, \ldots, u_{3t}, v_{3t}$ and the vertices $w_{2t+1}, w_{2t+2}, \ldots, w_{3t+1}$. Assign the vector $(1,0,0,0)$ to the vertices $u_{3t+1}, v_{3t+1}, u_{3t+2}, v_{3t+2}, \ldots, u_{4t}, v_{4t}$ and the vertices $w_{3t+2}, w_{3t+3}, \ldots, w_{4t+1}$. More over assign the vector $(1,1,1,1)$ to the vertex $v_n = v_{4t+2}$ and assign the vector $(1,1,1,0)$ the vertex $u_n = u_{4t+2}$. Finally assign the vector $(1,1,0,0)$ to the vertex $v_{n-1} = v_{4t+1}$ and assign the vector $(1,0,0,0)$ the vertex $u_{n-1} = u_{4t+1}$.

Case (iv): $n \equiv 3 \pmod 4$

Let $n = 4t+3$. So assign the vector $(1,1,1,1)$ to the vertices $u, u_1, v_1, u_2, v_2, \ldots, u_t, v_t$ and the vertices $w_1, w_2, \ldots, w_t$. Then assign the vector $(1,1,1,0)$ to the vertices $u_{t+1}, v_{t+1}, u_{t+2}, v_{t+2}, \ldots,$
u_{2t}, v_{2t} and the vertices $w_{t+1}, w_{t+2}, \ldots, w_{2t}$. Further assign the vector $(1,1,0,0)$ to the vertices $u_{2t+1}, v_{2t+1}, u_{2t+2}, v_{2t+2}, \ldots, u_{3t}, v_{3t}$ and the vertices $w_{2t+1}, w_{2t+2}, \ldots, w_{3t}$. Assign the vector $(1,0,0,0)$ to the vertices $u_{3t+1}, v_{3t+1}, u_{3t+2}, v_{3t+2}, \ldots, u_{4t}, v_{4t}$ and the vertices $w_{3t+1}, w_{3t+2}, \ldots,$ w_{4t+2}. More over assign the vector $(1,1,1,1)$ to the vertex u_n, v_n and assign the vector $(1,1,1,0)$ the vertex u_{n-1}, v_{n-1}. Finally assign the vector $(1,1,0,0)$ to the vertex u_{n-2}, v_{n-2}. and assign the vector $(1,0,0,0)$ the vertex $u_{n-1} = u_{4t+1}$. □

Example 4.8. *A vector basis* $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$*-cordial labeling of the cherry blossom flower graph* $CB_{6,5}$ *is given in figure (2).*

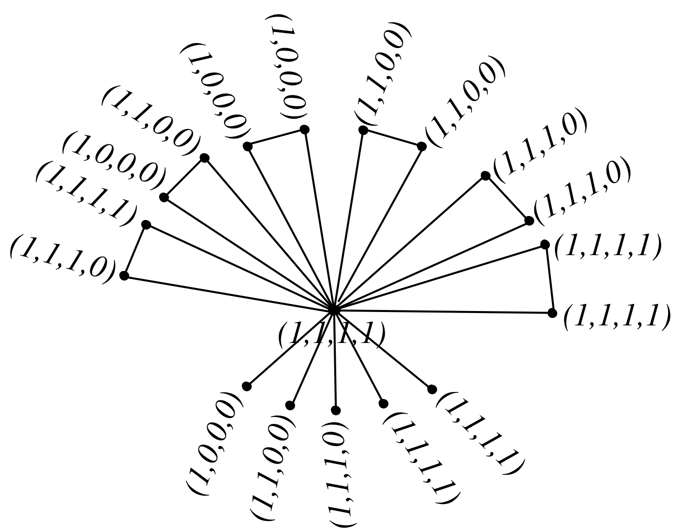

Figure 2: Vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of the cherry blossom flower graph $CB_{6,5}$.

Theorem 4.9. *The armed crown* $AC_{n,m}$ *is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1, 0,0,0)}-cordial graph for all* n,m.

Proof. Let $V(AC_{n,m}) = \{u_{i,j} \mid 1 \le i \le n \,\text{and}\, 1 \le j \le m\}$ and $E(AC_{n,m}) = \{u_{i,1}u_{i+1,1}, u_{n,1}u_{1,1} \mid 1 \le i \le n-1\} \cup \{u_{i,j}u_{i,j+1} \mid 1 \le i \le n \,\text{and}\, 1 \le j \le m-1\}$. We have $p =$

$|V(AC_{n,m})| = mn$ and $q = |E(AC_{n,m})| = mn$. Assign the vectors in the following order $u_{1,1}, u_{2,1}, u_{3,1}, \ldots, u_{n,1}, u_{1,2}, u_{2,2}, u_{3,2}$,

$\ldots, u_{n,2}, u_{1,m}, u_{2,m}, u_{3,m}, \ldots, u_{n,m}$.

Case (i): $p \equiv 0 \pmod 4$

Let $p = 4t$. Then assign the vector $(1,1,1,1)$ to the first t vertices and assign the vector $(1,1,1,0)$ to the next t vertices. Further, assign the vector $(1,1,0,0)$ to the next t vertices and assign the vector $(1,1,0,0)$ to the next t vertices.

Case (ii): $n \equiv 1 \pmod 4$

Let $p = 4t+1$. First assign the vector $(1,1,1,1)$ to the $t+1$ vertices and assign the vector $(1,1,1,0)$ to the next t vertices. Moreover, assign the vector $(1,1,0,0)$ to the next t vertices and assign the vector $(1,1,0,0)$ to the next t vertices.

Case (iii): $n \equiv 2 \pmod 4$

Let $p = 4t+1$. Assign the vector $(1,1,1,1)$ to the first $t+1$ vertices and assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Finally, assign the vector $(1,1,0,0)$ to the next t vertices and assign the vector $(1,1,0,0)$ to the next t vertices.

Case (iv): $n \equiv 3 \pmod 4$

Let $p = 4t+1$. First assign the vector $(1,1,1,1)$ to the $t+1$ vertices and assign the vector $(1,1,1,0)$ to the next $t+1$ vertices. Thereafter, assign the vector $(1,1,0,0)$ to the next $t+1$ vertices and assign the vector $(1,1,0,0)$ to the next t vertices.

Therefore the above labeling technique is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of the armed crown $AC_{n,m}$. □

Theorem 4.10. *The rocket graph $R_{m,3}$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph for all m.*

Proof. Let $V(R_{m,3}) = \{x_i, y_i, z_0, x_{1,j}, y_{1,j}, z_{1,j} \mid 1 \leq i \leq m \text{ and } 1 \leq j \leq 3\}$ and $E(R_{m,3}) = \{x_i y_i, x_1 z_1,$

$y_1 z_1, x_1 x_{1,j}, y_1 y_{1,j}, z_1 z_{1,j} \mid 1 \leq i \leq m \text{ and } 1 \leq j \leq 3\} \cup \{x_i x_{i+1}, x_i y_{i+1}, y_i y_{i+1} \mid 1 \leq i \leq m-1\}$. We have $p = |V(R_{m,3})| = 2m+10$ and $q = |E(R_{m,3})| = 4m+8$.

Case (i): $p \equiv 0 \pmod 4$

Let $p = 4t$. **Step I:** Then assign the vector $(1,1,1,0)$ to the vertices $x_{1,1}, x_{1,2}, x_{1,3}$ and assign the vector $(1,1,0,0)$ to the vertices $y_{1,1}, y_{1,2}, y_{1,3}$. Also assign the vector $(1,0,0,0)$ to the vertices $z_1, z_{1,1}, z_{1,2}, z_{1,3}$. **Step II:** Assign the vector to the vertices in the following $x_1, y_1, x_2, y_2, x_3, y_3, \ldots,$ x_m, y_m. Assign the vector $(1,1,1,1)$ to the first t vertices and assign the vector $(1,1,1,0)$ to the next $t-3$ vertices. Further, assign the vector $(1,1,0,0)$ to the next $t-3$ vertices and assign the vector $(1,0,0,0)$ to the next $t-4$ vertices.

Case (ii): $p \equiv 2 \pmod 4$

Let $p = 4t+2$. **Step I:** Assign the vector $(1,1,1,0)$ to the vertices $x_{1,1}, x_{1,2}, x_{1,3}$ and assign the vector $(1,1,0,0)$ to the vertices $y_{1,1}, y_{1,2}, y_{1,3}$. Also assign the vector $(1,0,0,0)$ to the vertices $z_1, z_{1,1}, z_{1,2}, z_{1,3}$. **Step II:** Then assign the vector to the vertices in the following $x_1, y_1, x_2, y_2, x_3, y_3, \ldots, x_m, y_m$. Assign the vector $(1,1,1,1)$ to the first t vertices and assign the vector $(1,1,1,0)$ to the next $t-2$ vertices. Moreover, assign the vector $(1,1,0,0)$ to the next $t-2$ vertices and assign the vector $(1,0,0,0)$ to the next $t-4$ vertices.

Clearly the above labeling pattern is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of the rocket graph $R_{m,3}$. □

Theorem 4.11. *A sandat graph ST_n is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial graph if and only if $n \equiv 0,3 \pmod 4$.*

Proof. Let $V(ST_n) = \{u, u_i, u_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq 2\}$ and $E(ST_{m,n}) = \{uu_i, uu_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq 2\}$. Note that $p = |V(ST_n)| = 3n+1$ and $q = |E(ST_n)| = 5n$.

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4t$ and $p = 12t+1$. Then assign the vector $(1,1,1,1)$ to the vertex u. Assign the vector $(1,1,1,1)$ to the vertices of first t leaves i.e., first $3t$ vertices $u_{1,1}, u_1, u_{1,2}, u_{2,1}, u_2, u_{2,2}, \ldots, u_{t,1}, u_t,$ $u_{t,2}$ and assign the vector $(1,1,1,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{t+1,1}, u_{t+1},$ $u_{t+1,2}, u_{t+2,1}, u_{t+2}, u_{t+2,2}, \ldots, u_{2t,1}, u_{2t}, u_{2t,2}$. Thereafter, assign the vector $(1,1,0,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{2t+1,1}, u_{2t+1}, u_{2t+1,2}, u_{2t+2,1}, u_{2t+2}, u_{2t+2,2}, \ldots, u_{3t,1}, u_{3t},$ $u_{3t,2}$ and assign the vector $(1,0,0,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{3t+1,1},$

$u_{3t+1}, u_{3t+1,2}, u_{3t+2,1}, u_{3t+2}, u_{3t+2,2}, \ldots, u_{4t,1}, u_{4t}, u_{4t,2}$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4t+1$. Then $p = 12t+4$ and $q = 20t+5$. To get the edge label 4, for atleast $5t+1$ vertices, we have to assign the vector for atleast $3t+2$ vertices, This is a contradiction.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4t+2$. Then $p = 12t+7$ and $q = 20t+10$. To get the edge label 4, for atleast $5t+2$ vertices, we have to assign the vector for atleast $3t+3$ vertices, a contradiction.

Case (iv): $n \equiv 3 \pmod 4$

Let $n = 4t+3$ and $p = 12t+10$. **Step I:** Then assign the vector $(1,1,1,1)$ to the vertex u. Assign the vector $(1,1,1,1)$ to the vertices of first t leaves i.e., first $3t$ vertices $u_{1,1}, u_1, u_{1,2}, u_{2,1}, u_2, u_{2,2}, \ldots, u_{t,1}, u_t, u_{t,2}$ and assign the vector $(1,1,1,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{t+1,1}, u_{t+1}, u_{t+1,2}, u_{t+2,1}, u_{t+2}, u_{t+2,2}, \ldots, u_{2t,1}, u_{2t}, u_{2t,2}$. Thereafter, assign the vector $(1,1,0,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{2t+1,1}, u_{2t+1}, u_{2t+1,2}, u_{2t+2,1}, u_{2t+2}$,
$u_{2t+2,2}, \ldots, u_{3t,1}, u_{3t}, u_{3t,2}$ and assign the vector $(1,0,0,0)$ to the vertices of next t leaves i.e., next $3t$ vertices $u_{3t+1,1}, u_{3t+1}, u_{3t+1,2}, u_{3t+2,1}, u_{3t+2}, u_{3t+2,2}, \ldots, u_{4t,1}, u_{4t}, u_{4t,2}$. **Step II:** Then assign the vector $(1,1,1,1)$ to the vertices $u_{4t+1,1}, u_{4t+1}$ and assign the vector $(1,1,1,0)$ to the vertices $u_{4t+1,2}, u_{4t+2,1}, u_{4t+2,2}$. Also assign the vector $(1,1,0,0)$ to the vertices $u_{4t+2}, u_{4t+3,1}$ and assign the vector $(1,0,0,0)$ to the vertices $u_{4t+3}, u_{4t+3}, u_{4t+3,2}$.

Clearly the above labeling method is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of the sandat graph ST_n if $n \equiv 0,3 \pmod 4$. □

Example 4.12. *A vector basis* $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$*-cordial labeling of the sandat graph* ST_4 *is given in figure (3).*

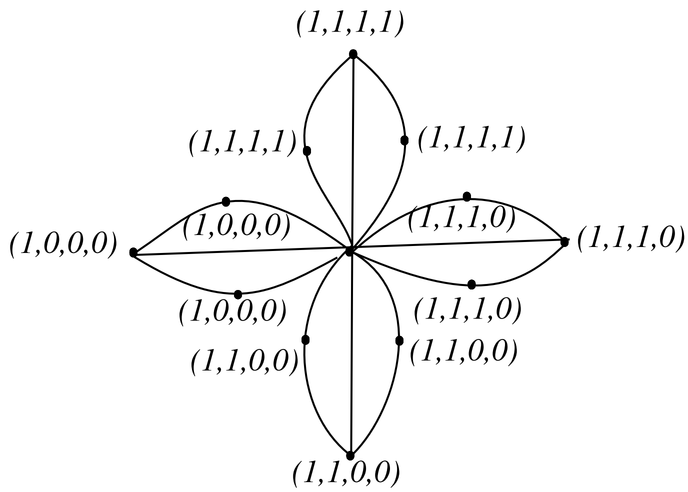

Figure 2: Vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$*-cordial labeling of the sandat graph* ST_4.

5. Conclusion

The vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behavior of some new graphs like the olive tree, Lobster graph, $I_{m,n}$ graph, shrub graph, rose flower graph, clematis flower graph, cherry blossom flower graph, armed crown, rocket graph and sandat graph has been investigated in this paper. Our future work will involve the investigation of vector basis S-cordial labelling for more graph families in trees and flower graphs.

Conflict of Interests

The authors declare that there is no conflict of interests.

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