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COMMON FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE SELF MAPPINGS IN MENGER SPACES

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Abstract: The aim of this paper is to establish common fixed point theorems for two pairs of maps satisfying a new contractive condition of integral type using the concept of occasionally weakly compatible single and multi-valued maps in probabilistic metric spaces. Our results neither require completeness of space nor the continuity of the maps involved there in .Our results extend, generalize and improve the results of existing in literature . Related examples have also been quoted.

Keywords: Compatible maps, fixed point results, menger spaces

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1 Introduction :

K. Menger [15] introduced the notion of probabilistic metric space, which is a generalization of the metric space. The study of this space was expanded rapidly with the pioneering works of Schweizer and Sklar [7,8] and many of their coworkers . Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physiological thresholds and physical quantities. It is also of fundamental importance in probabilistic functional analysis, nonlinear analysis and applications (see [11], [16], [17], [21], [24]). In 1972, Sehgal and Bharucha-Reid [26] initiated the study of contraction maps in probabilistic metric spaces (shortly, PM-spaces) which is an important step in the development of fixed point theorems.

Many authors formulated the definitions of compatible [21], weakly compatible maps [9] and occasionally weakly compatible maps [4, 10-11] in probabilistic settings and proved a number of

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fixed point theorems in this direction. Recently, Al-Thagafi and Shahzad [1] weakened the notion of weakly compatible maps by introducing occasionally weakly compatible maps. It is worth to mention that every pair of commuting self-maps is weakly commuting, each pair of weakly commuting self-maps is compatible, each pair of compatible self-maps is weak compatible and each pair of weak compatible self-maps is occasionally weak compatible but the reverse is not always true.. Several interesting and elegant results have been obtained by various authors on different settings (see [2-4,9-13,16-26]).

The study of fixed points for multi-valued contraction mappings using the Hausdorff metric was initiated by Nadler [20] and Markin [19] . Later an interesting and rich fixed point theory for such maps was developed which has found applications in control theory, convex optimization, differential inclusion and economics . A constructive proof of a fixed point theorem makes the theorem more valuable in view of the fact that it yields an algorithm for computing a fixed point. Indeed , many fixed point theorems have constructive proof , of which we mention the geometric fixed point results due to Banach and Nadler , for single valued and multivalued mappings . These results are of particular importance and play a fundamental role in nonlinear analysis and has numerous applications in the area such as varitional and linear inequalities , optimization and approximation theory.

The aim of this paper is to prove common fixed point theorems for single- valued and set-valued owc maps in menger spaces. Our results do not require conditions on completeness (or closedness) of the underlying space (or sub- spaces), containment of ranges and continuity of the involved maps.

2. Definitions and Preliminaries :

To set up our results in the next section we recall some definitions and facts .

Definition 2.1. A mapping $F : R \to [0,1)$ is said to be a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in R\} = 0$ and $\sup\{F(t) : t \in R\} = 1$.

We shall denote by Γ the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{Bmatrix} 0, t \le 0 \\ 1, t > 0 \end{Bmatrix}$$

If X is a non-empty set, $F : X \times X \to \Gamma$ is called a probabilistic distance on X and the value of F at $(x, y) \in X \times X$ is represented by $F(x, y)$ or $F_{x,y}$.

Definition 2.2. A binary operation * : [0,1] × [0,1]→ [0,1] is a continuous t-norm if

([0,1], *) is a topological abelian monoid with unit 1 s.t. a * b ≤ c * d whenever a ≤ c and

b≤ d , ∀ a, b, c, d ∈ [0,1]. Some examples are below:

(i) *(a, b) = ab,

(ii) *(a, b) = min.{a , b}.

Definition 2.3. The ordered pair (X, F) is called a probabilistic metric (PM) space if X is an arbitrary set and $F : X \times X \to \Gamma$ is a probabilistic distance on X satisfying the following conditions:

(PM-1) F(x, y, 0) = 0,

(PM-2) F(x, y, t) = 1 ,iff x=y and t > 0,

(PM-3) F(x, y, t) = F(y, x, t),

(PM-4) If $F(x, y, t_1) = 1$ and $F(y, z, t_2) = 1$ then $F(x, z, t_1 + t_2) = 1$

Definition 2.4 The ordered triplet $(X, F, *)$ is called a menger space if (X, F) is a PM space and * is a T-norm and the following inequality holds

$F(x, y, t_1 + t_2) \ge F(x, z, t_1) * F(z, y, t_2)$, for all $x, y, z \in X$ and $t, s > 0$.

Definition 2.5 Let $(X, F, *)$ be a menger space.

(i) A sequence $\{x_n\}$ is said to be convergent to a point x ∈ X if $\lim_{n\to\infty} F(x_n, x, t) = 1$ for all t > 0 .

(ii) A sequence $\{x_n\}$ is said to be Cauchy sequence if

$\lim_{m,n\to\infty} F\left(x_m, x_n, t\right) = 1$ for all $t > 0$.

(iii) A subset $A \subseteq X$ is said to be closed if each convergent sequence $\{x_n\}$ with $x_n \in A$ and $x_n \to x$, we have $x \in A$.

(iv) A subset $A \subseteq X$ is said to be compact if each sequence in A has a convergent subsequence .

Through out the paper X will represent the menger space (X,F ,∗) and $\kappa\left(X\right)$, the set of compact subsets of X. For A, B $\in \kappa\left(X\right)$ and for every t > 0 , denote

$$F_{\nabla}\left(A,B,t\right) = \min\left\{\min_{a\in A} F\left(a,B,t\right), \min_{b\in B} F\left(A,b,t\right)\right\}$$
$$F^{\Delta}\left(A,y,t\right) = \max\left\{F\left(x,y,t\right); x,y \in A\right\}$$

Remark : Obviously , $F_{\nabla}\left(A,B,t\right) \le F^{\Delta}\left(a,B,t\right)$ whenever $a \in A$ and $F_{\nabla}\left(A,B,t\right) \le 1 \Leftrightarrow A = B.$ Also $F^{\Delta}\left(A,y,t\right) = 1$ if $y \in A$.

Definition 2.6 A pair of self mappings *(f, g)* on menger space $(X, F, *)$ are said to be compatible if

$\lim_{n\to\infty} F(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = z$ for some z in X.

Thus the mappings f and g will be non-compatible if there exists at least one sequence $\{x_n\}$ such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = z$ for some z in X but either $\lim_{n\to\infty} F(fgx_n, gfx_n, t) \ne 1$ or the limit does not exist.

Definition 2.7 Let $(X, F, *)$ be a menger space. f and g be self maps on X. A point x in X is called a coincidence point of f and g iff $fx = gx$. In this case, $w = fx = gx$ is called a point of coincidence of f and g.

Definition 2.8. Apair of self mappings (f, g) on a menger space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu$ for some u in X , then $fgu = gfu$.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.9 Two self mappings f and g on a menger space $(X, F, *)$ are said to be occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of f and g at which f and g commute.

Lemma 1 Let $(X, F, *)$ be a menger space and f and g be owc maps on X having a unique point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f and g.

Proof: Since f and g are owc, there exists a point x in X such that $fx = gx = w$ and $fgx = gfx$. Thus, $ffx = fgx = gfx$, which says that ffx is also a point of coincidence of f and g. Since the point of coincidence $w = fx$ is unique by hypothesis, $gfx = ffx = fx$, and $w = fx$ is a common fixed point of f and g.

Moreover, if z is any common fixed point of f and g, then $z = fz = gz = w$ by the uniqueness of the point of coincidence.

Define $\Omega = \left\{ w : \left(R^{+}\right)^{5} \to R \,/\, \text{w is continuous and } w\left(x,1,x,1,x\right) = x \right\}$.

There are examples of $w \in \Omega$:

(1) $w\left(x_1, x_2, x_3, x_4, x_5\right) = \min\left\{x_1, x_2, x_3, x_4, x_5\right\};$

(2) $w\left(x_1, x_2, x_3, x_4, x_5\right) = \min\left\{x_1 \cdot x_2, x_3 \cdot x_4, x_5\right\};$

3 Main Results :

Theorem 3.1 Let $\left(X, F, *\right)$ be a menger space with A,B : X→X and $S, T : X \to \kappa\left(X\right)$ be the maps satisfying the following

(3.1) The pairs (A,S) and (B,T) are owc..

(3.2)

$$\gamma\left(\int_0^{F^\Delta(Ty,By,t)}\psi(t)dt\right)^p + w\left\{\begin{array}{l}\left(\int_0^{F^\Delta(Ax,By,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Ty,By,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Ty,Ax,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Sx,Ax,t)}\psi(t)dt\right)^p,\\ \left(\int_0^{F_\nabla(Sx,Ty,t)}\psi(t)dt\right)^p\end{array}\right\}$$

$$\geq \alpha\left(\int_0^{F^\Delta(Sx,Ax,t)}\psi(t)dt\right)^p + \beta\left(\int_0^{F_\nabla(Sx,Ty,t)}\psi(t)dt\right)^p,$$

for all $x, y \in X$, where $0 < \alpha, \beta < 1$ and $0 \leq \gamma < 1$ such that $\alpha + \beta - \gamma = 1$ and $\psi : R^+ \to R$ is a Lebseque –integrable mapping which is summable , nonnegative and such that $\int_0^{\varepsilon}\psi(t)dt > 0$ for each $\varepsilon > 0$ .Then A , B, S and T have a unique common fixed point in X .

Proof : Since the pairs (A,S) and (B.T) are owc, there exist u , v ∊ X such that $Au = Su, ASu \subset SAu$ and $Bv = Tv, BTv \subset TBv$. Suppose $Au \neq Bv$. From (3.2) , we have

$$\gamma\left(\int_0^{F^\Delta(Tv,Bv,t)}\psi(t)dt\right)^p + w\left\{\begin{array}{l}\left(\int_0^{F^\Delta(Au,Bv,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Tv,Bv,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Tv,Au,t)}\psi(t)dt\right)^p,\left(\int_0^{F^\Delta(Su,Au,t)}\psi(t)dt\right)^p,\\ \left(\int_0^{F_\nabla(Su,Tv,t)}\psi(t)dt\right)^p\end{array}\right\}$$

$$\geq \alpha\left(\int_0^{F^\Delta(Su,Au,t)}\psi(t)dt\right)^p + \beta\left(\int_0^{F_\nabla(Su,Tv,t)}\psi(t)dt\right)^p$$

i.e $\gamma + \left(\int_0^{F^\Delta(Au,Bv,t)}\psi(t)dt\right)^p \geq \alpha + \beta\left(\int_0^{F^\Delta(Au,Bv,t)}\psi(t)dt\right)^p$

$$(1-\beta)\left(\int_0^{F^\Delta(Au,Bv,t)}\psi(t)dt\right)^p \geq \alpha - \gamma \Rightarrow \left(\int_0^{F^\Delta(Au,Bv,t)}\psi(t)dt\right)^p = \frac{\alpha-\gamma}{1-\beta} = 1,$$

this gives, $Au = Bv$. Therefore, $Au = Bv = Su = Tv$. If there is another point z such that $Az = Sz$, then again by using inequality (3.2), it follows that $Az = Sz = Bv = Tv$ that is $Az = Au$. Hence $w = Au = Su$ is unique point of coincidence of A and S. By Lemma 1, w is the unique common fixed point of A and S. Similarly, there is unique point z in X such that $z = Bz = Tz$. Now, we claim that $w = z$. For this, put $x = w$ *and* $y = z$ in (3.2), we have

$$\gamma\left(\int_0^{F^{\Delta}(Tz,Bz,t)}\psi(t)dt\right)^p + w\left\{\begin{array}{l}\left(\int_0^{F^{\Delta}(Aw,Bz,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Tz,Bz,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Tz,Aw,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Sw,Aw,t)}\psi(t)dt\right)^p, \\ \left(\int_0^{F_{\nabla}(Sw,Tz,t)}\psi(t)dt\right)^p\end{array}\right\}$$

$$\geq \alpha\left(\int_0^{F^{\Delta}(Sw,Aw,t)}\psi(t)dt\right)^p + \beta\left(\int_0^{F_{\nabla}(Sw,Tz,t)}\psi(t)dt\right)^p,$$

$$\gamma + w\left\{\begin{array}{l}\left(\int_0^{F^{\Delta}(w,z,t)}\psi(t)dt\right)^p,1,\left(\int_0^{F^{\Delta}(z,w,t)}\psi(t)dt\right)^p,1, \\ \left(\int_0^{F_{\nabla}(w,z,t)}\psi(t)dt\right)^p\end{array}\right\} \geq \alpha + \beta\left(\int_0^{F_{\nabla}(w,z,t)}\psi(t)dt\right)^p$$

$$(1-\beta)\left(\int_0^{F^{\Delta}(w,z,t)}\psi(t)dt\right)^p \geq \alpha - \gamma \Rightarrow \left(\int_0^{F^{\Delta}(w,z,t)}\psi(t)dt\right)^p = \frac{\alpha-\gamma}{1-\beta} = 1$$

Thus, we have $w = z$. Hence, w is unique common fixed point of A, S, B and T in X.

Example : Let $X = [0,4]$ with $a * b = a \cdot b$ and $F(x,y,t) = \left\{\begin{array}{l}\dfrac{t}{t+|x-y|}, t > 0 \\ 0, t \leq 0\end{array}\right\}$. Then $(X,F,*)$ is a menger space .Define the single valued maps $A,B : X \to X$ and set valued maps $S,T : X \times X \to \kappa(X)$ as

$A(x)=\begin{Bmatrix} x, 0\le x\le 2 \\ 3, 2<x\le 4\end{Bmatrix}$, $B(x)=\begin{Bmatrix} 2, 0\le x\le 2 \\ \frac{x}{4}, 2<x\le 4\end{Bmatrix}$, $S(x)=\begin{Bmatrix} \{2\}, 0\le x\le 2 \\ \{0\}, 2<x\le 4\end{Bmatrix}$ and

$T(x)=\begin{Bmatrix} \{2\}, 0\le x\le 2 \\ \{4\}, 2<x\le 4\end{Bmatrix}$

Then we see that $A(2)=2\in S(2)$ and $AS(2)=\{2\}=SA(2)$

$B(2)=2\in T(2)$ and $BT(2)=\{2\}=TB(2)$. So the pairs (A,S) and (B,T) are owc. Also ' 2 ' is unique common fixed point of the maps A , B, S and T. On the other hand it is clear to see that the maps A,B,S and T are discontinuous at 2.

Also , $A(X)=\{3\}\cup[0,2]\not\subset\{0,2\}$ and $B(X)=\{2\}\cup\left[\frac{1}{2},1\right]\not\subset\{2,4\}$. So the example illustrates the generality of our result.

By setting $w(x_1,x_2,x_3,x_4,x_5)=\min\{x_1,x_2,x_3,x_4,x_5\}$ in theorem 3.1 , we have

Corollary 3.1 Let $(X,F,*)$ be a menger space with A,B : X→X and $S,T:X\to\kappa(X)$

be the maps satisfying the condition (3.1) and

$$(3.3)\gamma\left(\int_0^{F^{\Delta}(Ty,By,t)}\psi(t)dt\right)^p+\min\begin{Bmatrix}\left(\int_0^{F^{\Delta}(Ax,By,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Ty,By,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Ty,Ax,t)}\psi(t)dt\right)^p,\left(\int_0^{F^{\Delta}(Sx,Ax,t)}\psi(t)dt\right)^p, \\ \left(\int_0^{F_{\nabla}(Sx,Ty,t)}\psi(t)dt\right)^p\end{Bmatrix}$$

$$\geq\alpha\left(\int_0^{F^{\Delta}(Sx,Ax,t)}\psi(t)dt\right)^p+\beta\left(\int_0^{F_{\nabla}(Sx,Ty,t)}\psi(t)dt\right)^p$$

for all $x,y\in X$, where $0<\alpha,\beta<1$ and $0\le\gamma<1$ such that $\alpha+\beta-\gamma=1, p\ge 1$ and $\psi:R^+\to R$ is a Lebseque –integrable mapping which is summable , nonnegative and such that $\int_0^{\varepsilon}\psi(t)dt>0$ for each $\varepsilon>0$. Then A , B, S and T have a unique common fixed point in X .

By setting $w(x_1,x_2,x_3,x_4,x_5)=\min\left\{x_1,x_2,x_1^{\frac{1}{2}}x_3^{\frac{1}{2}},x_4,x_1^{\frac{1}{2}}x_5^{\frac{1}{2}}\right\}$ in theorem 3.1 and p=1, we get the following corollary

Corollary 3.2 Let $(X,F,*)$ be a menger space with A,B : X→X and $S,T:X\to\kappa(X)$ be the maps satisfying the condition (3.1) and

(3.4)

$$\gamma\left(\int_0^{F^{\Delta}(Ty,By,t)}\psi(t)dt\right)+\min\left\{\begin{array}{l}\int_0^{F^{\Delta}(Ax,By,t)}\psi(t)dt,\int_0^{F^{\Delta}(Ty,By,t)}\psi(t)dt,\left(\int_0^{F^{\Delta}(Ax,By,t)}\psi(t)dt\right)^{\frac{1}{2}}\left(\int_0^{F^{\Delta}(Ty,Ax,t)}\psi(t)dt\right)^{\frac{1}{2}},\\ \int_0^{F^{\Delta}(Sx,Ax,t)}\psi(t)dt,\left(\int_0^{F^{\Delta}(Ax,By,t)}\psi(t)dt\right)^{\frac{1}{2}}\left(\int_0^{F_{\nabla}(Sx,Ty,t)}\psi(t)dt\right)^{\frac{1}{2}}\end{array}\right\}$$

$$\geq\alpha\left(\int_0^{F^{\Delta}(Sx,Ax,t)}\psi(t)dt\right)+\beta\left(\int_0^{F_{\nabla}(Sx,Ty,t)}\psi(t)dt\right),$$

for all $x,y\in X$, where $0<\alpha,\beta<1$ and $0\leq\gamma<1$ such that $\alpha+\beta-\gamma=1, p\geq 1$and $\psi:R^{+}\to R$ is a Lebseque –integrable mapping which is summable , nonnegative and such that $\int_0^{\varepsilon}\psi(t)dt>0$ for each $\varepsilon>0$. Then A , B, S and T have a unique common fixed point in X .

Theorem 3.2 Let $(X,F,*)$ be a menger space with A , B, S , T : X→X and be the maps satisfying the condition (3.1) and

(3.5) $$\gamma\int_0^{F(Ty,By,t)}\psi(t)dt+w\left\{\begin{array}{l}\int_0^{F(Ax,By,t)}\psi(t)dt,\int_0^{F(Ty,By,t)}\psi(t)dt,\int_0^{F(Ty,Ax,t)}\psi(t)dt,\int_0^{F(Sx,Ax,t)}\psi(t)dt,\\ \int_0^{F(Sx,Ty,t)}\psi(t)dt\end{array}\right\}$$

$$\geq\alpha\int_0^{F(Sx,Ax,t)}\psi(t)dt+\beta\int_0^{F(Sx,Ty,t)}\psi(t)dt$$

for all $x, y \in X$, where $0 < \alpha, \beta < 1$ and $0 \le \gamma < 1$ such that $\alpha + \beta - \gamma = 1$ and $\psi : R^{+} \to R$ is a Lebseque –integrable mapping which is summable , nonnegative and such that $\int_0^{\varepsilon} \psi(t) dt > 0$ for each $\varepsilon > 0$. Then A , B, S and T have a unique common fixed point in X .

Proof : Proof follows easily from theorem 3.1 if we set $M^{\Delta} = M_{\nabla} = M, p = 1$ and replacing $S, T : X \to \kappa(X)$ by S , T : X →X .

If $\psi(t)$=1 in theorem 3.2 , then we get the following corollary

Corollary 3.3 Let $(X, F, *)$ be a menger space with A , B, S and T : X→X and be the maps satisfying the condition (3.1) and

(3.6) $\gamma F(Ty, By, t) + w\begin{Bmatrix} F(Ax, By, t), F(Ty, By, t), F(Ty, Ax, t), F(Sx, Ax, t), \\ F(Sx, Ty, t) \end{Bmatrix}$

$\ge \alpha M(Sx, Ax, t) + \beta M(Sx, Ty, t)$

for all $x, y \in X$, where $0 < \alpha, \beta < 1$ and $0 \le \gamma < 1$ such that $\alpha + \beta - \gamma = 1$. Then A , B, S and T have a unique common fixed point in X

Corollary 3.4 Let $(X, F, *)$ be a menger space with A , B, S and T : X→X and be the maps satisfying the condition (3.1) and

(3.7) $\gamma F(Ty, By, kt) + \min\begin{Bmatrix} F(Ax, By, kt), F(Ty, By, kt), F(Ty, Ax, kt), F(Sx, Ax, kt), \\ F(Sx, Ty, kt) \end{Bmatrix}$

$\ge \alpha F(Sx, Ax, kt) + \beta F(Sx, Ty, kt)$

for all $x, y \in X$, where $0 < \alpha, \beta < 1$ and $0 \le \gamma < 1$ such that $\alpha + \beta - \gamma = 1$ and $k \in (0,1)$. Then A , B, S and T have a unique common fixed point in X

If we set A = B and S =T in the theorem 3.2 , we get

Theorem 3.3 Let $(X,F,*)$ be a menger space with A , S : X→X and be the maps satisfying the condition (3.1) and

$$(3.7)\quad \gamma\int_0^{F(Sy,Ay,t)}\psi(t)dt + w\left\{\begin{array}{l}\int_0^{F(Ax,Ay,t)}\psi(t)dt, \int_0^{F(Sy,Ay,t)}\psi(t)dt, \int_0^{F(Sy,Ax,t)}\psi(t)dt, \int_0^{F(Sx,Ax,t)}\psi(t)dt, \\ \int_0^{F(Sx,Sy,t)}\psi(t)dt\end{array}\right\}$$

$$\geq \alpha\int_0^{F(Sx,Ax,t)}\psi(t)dt + \beta\int_0^{F(Sx,Sy,t)}\psi(t)dt$$

for all $x,y\in X$, where $0<\alpha,\beta<1$ and $0\leq\gamma<1$ such that $\alpha+\beta-\gamma=1$ and $\psi:R^{+}\to R$ is a Lebseque –integrable mapping which is summable , nonnegative and such that $\int_0^{\varepsilon}\psi(t)dt>0$ for each $\varepsilon>0$. Then A and S have a unique common fixed point in X .

If we set A = B and S =T in the theorem 3.3 and $\psi(t)=1$, we get

Corollary 3.5 Let $(X,F,*)$ be a menger space with A,S : X→X and be the maps satisfying the condition (3.1) and

$$(3.8)\quad \gamma F(Sy,Ay,t) + w\left\{\begin{array}{l}F(Ax,Ay,t), F(Sy,Ay,t), F(Sy,Ax,t), F(Sx,Ax,t), \\ F(Sx,Sy,t)\end{array}\right\}$$

$$\geq \alpha F(Sx,Ax,t) + \beta F(Sx,Sy,t)$$

for all $x,y\in X$, where $0<\alpha,\beta<1$ and $0\leq\gamma<1$ such that $\alpha+\beta-\gamma=1$. Then A and S have a unique common fixed point in X .

If we take $\alpha=\gamma=0, \beta=1$ in corollary 3.3 , we get

Corollary 3.6 Let $(X,F,*)$ be a menger space with A , B, S and T : X→X and be the maps satisfying the condition (3.1) and

(3.9) $$\phi\left[\min\left\{\begin{array}{l}F(Ax,By,t),F(Ty,By,t),F(Ty,Ax,t),F(Sx,Ax,t),\\F(Sx,Ty,t)\end{array}\right\}\right]\geq F(Sx,Ty,t)$$

for all $x, y \in X$, where $\phi: R^{+} \to R$ is a continuous non-decreasing map such that $\phi(t)=t$ for $t=0,1$ and $\phi(t)<t$ Then A , B, S and T have a unique common fixed point in X

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