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COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. In this paper, we introduce a new class of implicit functions and common property (E.A) in intuitionistic fuzzy metric spaces, and utilize both to prove common fixed point theorems in intuitionistic fuzzy metric spaces. Related results and an illustrative example are also provided.

Keywords: fuzzy metric space; intuitionistic fuzzy metric space; property (E.A); common property (E.A).

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1. Introduction-preliminaries

The theory of fuzzy sets was initiated by Zadeh [24]. In the last four decades, like all other aspects of Mathematics, various authors have introduced the concept of fuzzy metric in several ways; see, e.g., [4, 6, 14]. George and Veeramani [6] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [14] and defined a Hausdorff topology on such fuzzy metric space which are often used in current researches. Grabiec [7] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively.

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Recently, Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a noted generalization of fuzzy sets which has inspired intense research progress around this new notion of intuitionistic fuzzy set. With a view to mention some relevant work, one may refer to [8, 19, 20, 22]. Most recently, Park [19] using the idea of intuitionistic fuzzy sets, defined intuitionistic fuzzy metric space (employing the notions of continuous t-norm and continuous t-conorm) as a generalization of fuzzy metric spaces (due to George and Veeramani [6]) and also proved some basic results which include Baire's theorem (a necessary and sufficient condition for completeness of the space), separability of the space, second countability of the space and it's relation with separability, uniform limit theorem besides other core results. Presently, it remains an important problem in fuzzy topology to obtain an appropriate concept of intuitionistic fuzzy metric spaces. This problem has been investigated by Saadati and Park [20] wherein they defined precompact sets in intuitionistic fuzzy metric spaces and proved that any subset of an intuitionistic fuzzy metric space is compact if and only if it is precompact and complete. Also they defined topologically complete intuitionistic fuzzy metrizable spaces and proved that any G_δ set in a complete intuitionistic fuzzy metric space is a topologically complete intuitionistic fuzzy metrizable space and vice versa. The varied concepts of fuzzy topology have already found vital applications in quantum particle physics particularly in connections with both string and $\varepsilon^{(\infty)}$ theory which were studied and formulated by El Naschie [5].

Definition 1.1. [17] A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is continuous t-norm if

(I) $*$ is commutative and associative;
(II) $*$ is continuous;
(III) $a*1=a$ for all $a\in[0,1]$;
(IV) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 1.2. [17] A binary operation $\diamondsuit:[0,1]\times[0,1]\to[0,1]$ is a continuous t-conorm if

(I) $\diamondsuit$ is commutative and associative;
(II) $\diamondsuit$ is continuous;
(III) $a\diamondsuit 0=a$ for all $a\in[0,1]$;
(IV) $a\diamondsuit b\le c\diamondsuit d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

The concepts of t-norms and t-conorms are known as the axiomatic skeletons which are respectively utilized in characterizing fuzzy intersections and unions. Several examples substantiating the utility of these concepts were proposed by many authors; see, e.g., [13, 21].

In respect of intuitionistic fuzzy metric spaces, the following definition along with some fundamental properties are available in [19].

Definition 1.3. [19] A 5-tuple $(X,M,N,*,\diamondsuit)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times(0,\infty)$ satisfying the following conditions:

(I) $M(x,y,t)+N(x,y,t)\leq 1$ for all $x,y\in X$ and $t>0$;

(II) $M(x,y,t)>0$ for all $x,y\in X$;

(III) $M(x,y,t)=1$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

(IV) $M(x,y,t)=M(y,x,t)$ for all $x,y\in X$ and $t>0$;

(V) $M(x,y,t)*M(y,z,s)\leq M(x,z,t+s)$ for all $x,y,z\in X$ and $s,t>0$;

(VI) $M(x,y,.):(0,\infty)\to(0,1]$ is continuous, for all $x,y\in X$.

(VIII) $N(x,y,t)>0$ for all $x,y\in X$;

(IX) $N(x,y,t)=0$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

(X) $N(x,y,t)=N(y,x,t)$ for all $x,y\in X$ and $t>0$;

(XI) $N(x,y,t)\diamondsuit N(y,z,s)\geq N(x,z,t+s)$ for all $x,y,z\in X$ and $s,t>0$;

(XII) $N(x,y,.):(0,\infty)\to(0,1]$ is continuous, for all $x,y\in X$;

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ respectively denote the degree of nearness and degree of nonnearness between x and y with respect to t.

Notice that every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space (in short IFMS) of the form $(X,M,1-M,*,\diamondsuit)$ such that t-norm $*$ and t-conorm $\diamondsuit$ are interrelated ([16]) by the relation $x\diamondsuit y=1-((1-x)*(1-y))$ for all $x,y\in X$. In intuitionistic fuzzy metric space $X,M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non-increasing for all $x,y\in X$.

Definition 1.4. [19] A sequence $\{x_n\}$ in an IFMS $(X,M,N,*,\diamondsuit)$ is said to be convergent to some $x \in X$ if for all $t > 0$, there is some $n_0 \in N$ such that $\lim_{n\to\infty} M(x_n,x,t) = 1$ and $\lim_{n\to\infty} N(x_n,x,t) = 0$ for all $n \geq n_0$.

Definition 1.5. [22] A pair (f,S) of self mappings defined on an IFMS $(X,M,N,*,\diamondsuit)$ is said be compatible if for all $t > 0$,

$$\lim_{n\to\infty} M(fSx_n,Sfx_n,t) = 1 \text{ and } \lim_{n\to\infty} N(fSx_n,Sfx_n,t) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Definition 1.6. A pair (f,S) of self mappings defined on an IFMS $(X,M,N,*,\diamondsuit)$ is said be non-compatible if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$ but $\lim_{n\to\infty} M(fSx_n,Sfx_n,t) \neq 1$ or nonexistent, or $\lim_{n\to\infty} N(fSx_n,Sfx_n,t) \neq 0$ or nonexistent for at least one $t > 0$.

Motivated by Aamri and Moutawakil [1], we define the following:

Definition 1.7. A pair (f,S) of self mappings of an IFMS $(X,M,N,*,\diamondsuit)$ is said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that for all $t > 0$

$$\lim_{n\to\infty} M(fx_n,Sx_n,t) = 1 \text{ and } \lim_{n\to\infty} N(fx_n,Sx_n,t) = 0.$$

Clearly, a pair of compatible mappings as well as non-compatible mappings satisfy the property (E.A).

Motivated by Liu *et al.* [15], we also define the following.

Definition 1.8. Two pairs (f,S) and (g,T) of self mappings of an IFMS $(X,M,N,*,\diamondsuit)$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} M(fx_n,Sx_n,t) = \lim_{n\to\infty} M(gy_n,Ty_n,t) = 1 \text{ and}$$

$$\lim_{n\to\infty} N(fx_n,Sx_n,t) = \lim_{n\to\infty} N(gy_n,Ty_n,t) = 0.$$

Definition 1.9. [12] A pair (f,S) of self mappings of a non-empty set X is said be weakly compatible if $fx = Sx$ for some $x \in X$ implies $fSx = Sfx$.

Definition 1.10. [11] Two finite families of self mappings $\{A_i\}_{i=1}^m$ and $\{B_k\}_{k=1}^n$ of a set X are said to be pairwise commuting if:

(i) $A_iA_j = A_jA_i \;\; i,j \in \{1,2,\ldots,m\}$,

(ii) $B_kB_l = B_lB_k \;\; k,l \in \{1,2,\ldots,n\}$,

(iii) $A_iB_k = B_kA_i \;\; i \in \{1,2,\ldots,m\}$ and $k \in \{1,2,\ldots,n\}$.

The purpose of this paper is to introduce a new class of implicit functions and common property $(E.A)$ in IFMS and utilize the both notions to prove some common fixed point theorems in intuitionistic fuzzy metric spaces.

2. Implicit Relations

Motivated by Ali and Imdad [2], we define an implicit function (abstracting both nearness and nonnearness) as follows:

Let Ψ be the set of all upper semi-continuous functions $F(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ satisfying the following conditions:

$(F_1) : F(u,1,u,1,1,u) < 0$, for all $u > 0$,

$(F_2) : F(u,1,1,u,u,1) < 0$, for all $u > 0$,

$(F_3) : F(u,u,1,1,u,u) < 0$, for all $u > 0$,

whereas Φ be the family of lower semi-continuous functions $\phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ satisfying the following conditions:

$(\phi_1) : \phi(u,0,u,0,0,u) > 0$, for all $u > 0$,

$(\phi_2) : \phi(u,0,0,u,u,0) > 0$, for all $u > 0$,

$(\phi_3) : \phi(u,u,0,0,u,u) > 0$, for all $u > 0$.

The following examples satisfy (F_1), (F_2), (F_3), (ϕ_1), (ϕ_2) and (ϕ_3).

Example 2.1. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1 - \alpha \min\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } \alpha > 1$$

and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \beta\max\{t_2,t_3,t_4,t_5,t_6\},\ \text{where } \beta \in [0,1).$$

Example 2.2. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^3 - c_1\min\{t_2^2,t_3^2,t_4^2\} - c_2\min\{t_3t_6,t_4t_5\},$$

where $c_1,c_2,c_3 > 0,\ c_1+c_2 > 1,\ c_1 \geq 1$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - k\max\{t_2,t_3t_5,t_4t_6\},\ \text{where } k \in [0,1).$$

Example 2.3. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^3 - a\min\{t_1^2t_2,t_1t_3t_4,t_5^2t_6,t_5t_6^2\},$$

where $a > 1$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - k[\max\{t_2^2,t_3t_4,t_5t_6,t_3t_5,t_4t_6\}]^{\frac{1}{2}},\ \text{where } k \in [0,1).$$

Example 2.4. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^3 - a\frac{t_3^2t_4^2+t_5^2t_6^2}{t_2+t_3+t_4},$$

where $a \geq \frac{3}{2}$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \alpha[\beta\max\{t_2,t_3,t_4,t_5,t_6\} + (1-\beta)(\max\{t_2^2,t_3t_4,t_5t_6,t_3t_6,t_4t_5\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ and $\beta \geq 0$.

Example 2.5. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = (1+pt_2)t_1 - p\min\{t_3t_4,t_5t_6\} - \psi(\min\{t_2,t_3,t_4,t_5,t_6\}),$$

where $p \geq 0$ and $\psi : [0,1] \to [0,1]$ is continuous function such that $\psi(t) > t$ for all $t \in (0,1)$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1^2 - \alpha\max\{t_2^2,t_3^2,t_4^2\} - \beta\max\{t_3t_5,t_4t_6\} - \gamma t_5t_6,$$

where $\alpha,\beta,\gamma \geq 0$ and $\alpha+\gamma < 1$.

Example 2.6. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^2 - a\frac{t_2^2+t_3^2+t_4^2}{t_5+t_6},$$

where $a \geq 2$ and

$$\phi(t_1,t_2,\cdots,t_6) = (1+\alpha t_2)t_1 - \alpha\max\{t_3t_4,t_5t_6\} - \beta\max\{t_2,t_3,t_4,t_5,t_6\},$$

where $\alpha \geq 0$ and $\beta \in [0,1)$.

Example 2.7. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1 - \psi(\min\{t_2,t_3,t_4,t_5,t_6\}),$$

where $\psi : [0,1] \to [0,1]$ is continuous function such that $\psi(t) > t$ for all $t \in (0,1)$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \beta\max\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } \beta \in [0,1).$$

Example 2.8. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^2 - c_1\min\{t_2^2,t_3^2,t_4^2\} - c_2\min\{t_3t_6,t_4t_5\},$$

where $c_1,c_2,c_3 > 0,\ c_1+c_2 > 1,\ c_1 \geq 1$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - k[\max\{t_2^2,t_3t_4,t_5t_6,t_3t_5,t_4t_6\}]^{\frac{1}{2}}, \text{ where } k \in [0,1).$$

Example 2.9. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^3 - a\min\{t_1^2t_2,t_1t_3t_4,t_5^2t_6,t_5t_6^2\},$$

where $a > 1$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - k\max\{t_2,t_3t_5,t_4t_6\}, \text{ where } k \in [0,1).$$

Example 2.10. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1 - k\min\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } k > 1$$

and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \alpha[\beta\max\{t_2,t_3,t_4,t_5,t_6\} + (1-\beta)(\max\{t_2^2,t_3t_4,t_5t_6,t_3t_6,t_4t_5\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ and $\beta \geq 0$.

Example 2.11. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = (1+pt_2)t_1 - p\min\{t_3t_4,t_5t_6\} - \psi(\min\{t_2,t_3,t_4,t_5,t_6\}),$$

where $p \geq 0$ and $\psi : [0,1] \to [0,1]$ is continuous function such that $\psi(t) > t$ for all $t \in (0,1)$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \alpha[\beta\max\{t_2,t_3,t_4,t_5,t_6\} + (1-\beta)(\max\{t_2^2,t_3t_4,t_5t_6,t_3t_6,t_4t_5\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ and $\beta \geq 0$.

Example 2.12. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^2 - a\frac{t_2^2+t_3^2+t_4^2}{t_5+t_6},$$

where $a \geq 2$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \beta\max\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } \beta \in [0,1).$$

Example 2.13. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^3 - a\frac{t_3^2t_4^2}{t_2+t_5+t_6},$$

where $a \geq 3$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1^2 - \alpha\max\{t_2^2,t_3^2,t_4^2\} - \beta\max\{t_3t_5,t_4t_6\} - \gamma t_5t_6,$$

where $\alpha,\beta,\gamma \geq 0$ and $\alpha+\gamma < 1$.

Example 2.14. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^2 - a\min\{t_2^2,t_3^2,t_4^2\} - b\frac{t_5}{t_5+t_6},$$

where $a \geq 1$ and $b > 0$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \alpha[\beta\max\{t_2,t_3,t_4,t_5,t_6\} + (1-\beta)(\max\{t_2^2,t_3t_4,t_5t_6,t_3t_6,t_4t_5\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ and $\beta \geq 0$.

Example 2.15. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1^2 - a\min\{t_2^2,t_5^2,t_6^2\} - b\frac{t_3}{t_3+t_4},$$

where $a \geq 1$ and $b > 0$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - k\max\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } k \in [0,1).$$

Example 2.16. Define $F(t_1,t_2,\cdots,t_6), \phi(t_1,t_2,\cdots,t_6) : [0,1]^6 \to \Re$ as

$$F(t_1,t_2,\cdots,t_6) = t_1 - a_1t_2 - a_2t_3 - a_3t_4 - a_4t_5 - a_5t_6,$$

where $a_1,a_2,a_3,a_4,a_5 > 0,\ a_2+a_5 \geq 1,\ a_3+a_4 \geq 1$ and $a_1+a_4+a_5 \geq 1$ and

$$\phi(t_1,t_2,\cdots,t_6) = t_1 - \beta\max\{t_2,t_3,t_4,t_5,t_6\}, \text{ where } \beta \in [0,1).$$

3. Main Results

We begin with the following lemma.

Lemma 3.1. *Let f,g,S and T be four self mappings of an IFMS $(X,M,N,*,\diamondsuit)$ satisfying the following conditions:*

(I) *the pair (f,S) (or (g,T)) satisfies the property (E.A),*

(II) *$f(X) \subset T(X)$ (or $g(X) \subset S(X)$),*

(II1) *for all $x,y \in X$, $F \in \Psi, \phi \in \Phi$*

$$\left.\begin{array}{l} F(M(fx,gy,t),M(Sx,Ty,t),M(gy,Ty,t),M(fx,Sx,t), \\ M(fx,Ty,t),M(Sx,gy,t)) \geq 0 \\ \text{and} \\ \phi(N(fx,gy,t),N(Sx,Ty,t),N(gy,Ty,t),N(fx,Sx,t), \\ N(fx,Ty,t),N(Sx,gy,t)) \leq 0. \end{array}\right\} \quad (3.1),$$

then the pairs (f,S) and (g,T) share the common property (E.A).

Proof. Since the pair (f,S) enjoys the property (E.A), there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in X,$$

implying thereby $\lim_{n\to\infty} M(fx_n,Sx_n,t) = 1$ and $\lim_{n\to\infty} N(fx_n,Sx_n,t) = 0$. Since $f(X) \subset T(X)$, therefore for each $\{x_n\}$ there exists $\{y_n\}$ in X such that $fx_n = Ty_n$. Therefore, $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Ty_n = z$. Thus in all we have $fx_n \to z,\ Sx_n \to z$ and $Ty_n \to z$. Now we assert that $\lim_{n\to\infty} M(gy_n,Ty_n,t) = 1$ and $\lim_{n\to\infty} N(gy_n,Ty_n,t) = 0$. If not, then using inequality (3.1.1), we have

$$F(M(fx_n,gy_n,t),M(Sx_n,Ty_n,t),M(gy_n,Ty_n,t),M(fx_n,Sx_n,t),M(fx_n,Ty_n,t),$$

$$M(Sx_n,gy_n,t)) \geq 0$$

and

$$\phi(N(fx_n,gy_n,t),N(Sx_n,Ty_n,t),N(gy_n,Ty_n,t),N(fx_n,Sx_n,t),N(fx_n,Ty_n,t),$$

$$N(Sx_n,gy_n,t)) \leq 0,$$

which on making $n \to \infty$, reduces to

$$F(\lim_{n\to\infty} M(Ty_n,gy_n,t),1,\lim_{n\to\infty} M(gy_n,Ty_n,t),1,1,\lim_{n\to\infty} M(Ty_n,gy_n,t)) \geq 0$$

and

$$\phi(\lim_{n\to\infty} N(Ty_n,gy_n,t),0,\lim_{n\to\infty} N(gy_n,Ty_n,t),0,0,\lim_{n\to\infty} N(Ty_n,gy_n,t)) \leq 0.$$

This derives contradictions to F_1 and ϕ_1 respectively yielding thereby $\lim_{n\to\infty} M(gy_n,Ty_n,t) = 1$ and $\lim_{n\to\infty} N(gy_n,Ty_n,t) = 0$, i.e. $\lim_{n\to\infty} gy_n = z$ which shows that the pairs (f,S) and (g,T) share the common property (E.A).

Our next result is a common fixed point theorem via the common property (E.A).

Theorem 3.1. *Let f,g,S and T be four self mappings of an IFMS $(X,M,N,*,\diamondsuit)$ satisfying the condition (3.1). Suppose that*

(I) *the pairs (f,S) and (g,T) share the common property (E.A) and*

(II) *$S(X)$ and $T(X)$ are closed subsets of X.*

Then pair (f,S) as well as (g,T) have a coincidence point. Moreover, f,g,S and T have a unique common fixed point in X provided both the pairs (f,S) and (g,T) are weakly compatible.

Proof. Since the pair (f,S) and (g,T) share the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \in X.$$

Since $S(X)$ is a closed subset of X, therefore $\lim_{n\to\infty} Sx_n = z \in S(X)$. Also, there exists a point $u \in X$ such that $Su = z$. Now we assert that $M(fu,z,t) = 1$ and $N(fu,z,t) = 0$. If not, then using inequality (3.1), we have

$$F(M(fu,gy_n,t),M(Su,Ty_n,t),M(gy_n,Ty_n,t),M(fu,Su,t),M(fu,Ty_n,t),$$
$$M(Su,gy_n,t)) \geq 0$$

and

$$\phi(N(fu,gy_n,t),N(Su,Ty_n,t),N(gy_n,Ty_n,t),N(fu,Su,t),N(fu,Ty_n,t),$$
$$N(Su,gy_n,t)) \leq 0,$$

which on making $n \to \infty$, reduces to

$$F(M(fu,z,t),1,1,,M(fu,z,t),M(fu,z,t),1) \geq 0$$

and

$$\phi(N(fu,z,t),0,0,N(fu,z,t),N(fu,z,t),0) \leq 0,$$

which give contradictions to F_2 and ϕ_2 respectively, implying thereby $M(fu,z,t) = 1$ and $N(fu,z,t) = 0$, so that $fu = z = Su$. So that, u is a coincidence point of the pair (f,S).

Since $T(X)$ is a closed subset of X, therefore $\lim_{n\to\infty} Ty_n = z \in T(X)$. Also, there exists a point $w \in X$ such that $Tw = z$. Now we assert that $M(gw,z,t) = 1$ and $N(gw,z,t) = 0$. If not, then using inequality (3.1), we have

$$F(M(fx_n,gw,t),M(Sx_n,Tw,t),M(gw,Tw,t),M(fx_n,Sx_n,t),M(fx_n,Tw,t),$$
$$M(Sx_n,gw,t)) \geq 0$$

and

$$\phi(N(fx_n,gw,t),N(Sx_n,Tw,t),N(gw,Tw,t),N(fx_n,Sx_n,t),N(fx_n,Tw,t),$$
$$N(Sx_n,gw,t))\leq 0,$$

which on making $n\to\infty$, reduces to

$$F(M(z,gw,t),1,M(gw,z,t),1,1,M(z,gw,t))\geq 0$$

and

$$\phi(N(z,gw,t),0,N(gw,z,t),0,0,N(z,gw,t))\leq 0,$$

which give contradictions to F_1 and ϕ_1 respectively, implying thereby $M(gw,z,t)=1$ and $N(gw,z,t)=0$, so that $gw=z=Tw$. So that, w is a coincidence point of the pair (g,T).
Since $fu=Su$ and the pair (f,S) is weakly compatible, therefore $fz=fSu=Sfu=Sz$. Now we need to show that z is a common fixed point of the pair (f,S). To accomplish this, we assert that $M(fz,z,t)=1$ and $N(fz,z,t)=0$. If not, then using inequality (3.1), we have

$$F(M(fz,gw,t),M(Sz,Tw,t),M(gw,Tw,t),M(fz,Sz,t),M(fz,Tw,t),M(Sz,gw,t))\geq 0$$

and

$$\phi(N(fz,gw,t),N(Sz,Tw,t),N(gw,Tw,t),N(fz,Sz,t),N(fz,Tw,t),N(Sz,gw,t))\leq 0.$$

These imply

$$F(M(fz,z,t),M(fz,z,t),1,1,M(fz,z,t),M(fz,z,t))\geq 0$$

and

$$\phi(N(fz,z,t),N(fz,z,t),0,0,N(fz,z,t),N(fz,z,t))\leq 0.$$

These give contradictions to F_3 and ϕ_3 respectively, yielding thereby $M(fz,z,t)=1$ and $N(fz,z,t)=0$ so that $fz=z$ which shows that z is a common fixed point of the pair (f,S). Also $gw=Tw$ and the pair (g,T) is weakly compatible, therefore $gz=gTw=Tgw=Sz$. Next, we show that z is a common fixed point of the pair (g,T). To do this, we assert that $M(gz,z,t)=1$ and $N(gz,z,t)=0$. If not, then using inequality (3.1), we have

$$F(M(fu,gz,t),M(Su,Tz,t),M(gz,Tz,t),M(fu,Su,t),M(fu,Tz,t),M(Su,gz,t))\geq 0$$

and

$$\phi(N(fu,gz,t),N(Su,Tz,t),N(gz,Tz,t),N(fu,Su,t),N(fu,Tz,t),N(Su,gz,t)) \leq 0$$

or

$$F(M(z,gz,t),M(z,gz,t),1,1,M(z,gz,t),M(z,gz,t)) \geq 0$$

and

$$\phi(N(z,gz,t),N(z,gz,t),0,0,N(z,gz,t),N(z,gz,t)) \leq 0,$$

which give contradictions to F_3 and ϕ_3 respectively, implying thereby $M(gz,z,t) = 1$ and $N(gz,z,t) = 0$ so that $gz = z$ which shows that z is a common fixed point of the pair (g,T). Uniqueness of the common fixed point is an easy consequence of the inequality (3.1) in view of condition F_3 (or ϕ_3).

Remark 3.1. Theorem 3.1 extends relevant results of Imdad and Ali [2, 9, 10] to IFMS.

Theorem 3.2. *The conclusions of Theorem 3.1 remain true if the condition (II) of Theorem 3.1 is replaced by following.*

(II') $\overline{f(X)} \subset T(X)$ *and* $\overline{g(X)} \subset S(X)$.

As a corollary of Theorem 3.2, we can have the following result which is also a variant of Theorem 3.1.

Corollary 3.1. *The conclusions of Theorems 3.1 and 3.2 remain true if the conditions (II) and* (II') *are replaced by following.*

(II'') $f(X)$ *and* $g(X)$ *are closed subset of X provided* $f(X) \subset T(X)$ *and* $g(X) \subset S(X)$.

Theorem 3.3. *Let* f,g,S *and* T *be four self mappings of an IFMS* $(X,M,N,*,\Diamond)$ *satisfying the conditions (3.1.1). Suppose that*

(I) *the pair* (f,S) *(or* (g,T)*) satisfies the property (E.A),*

(II) $f(X) \subset T(X)$ *(or* $g(X) \subset S(X)$*) and*

(III) $S(X)$ *(or* $T(X)$*) is a closed subset of* X.

Then pair (f,S) *as well as* (g,T) *have a coincidence point. Moreover,* f,g,S *and* T *have a unique common fixed point in X provided that the pairs* (f,S) *and* (g,T) *are weakly compatible.*

Proof. In view of Lemma 3.1, the pairs (f,S) and (g,T) share the common property (E.A) i.e. there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \in X.$$

As $S(X)$ is a closed subset of X, on the lines of Theorem 3.1, one can show that the pair (f,S) has a point of coincidence, say u i.e. $fu = Su$. Since $f(X) \subset T(X)$ and $fu \in f(X)$, there exists $w \in X$ such that $fu = Tw$. Now we assert that $M(gw,z,t) = 1$ and $N(gw,z,t) = 0$. If not, then using inequality (3.1.1), we have

$$F(M(fx_n,gw,t),M(Sx_n,Tw,t),M(gw,Tw,t),M(fx_n,Sx_n,t),M(fx_n,Tw,t),$$
$$M(Sx_n,gw,t)) \geq 0$$

and

$$\phi(N(fx_n,gw,t),N(Sx_n,Tw,t),N(gw,Tw,t),N(fx_n,Sx_n,t),N(fx_n,Tw,t),$$
$$N(Sx_n,gw,t)) \leq 0,$$

which on making $n \to \infty$, reduces to

$$F(M(z,gw,t),1,M(gw,z,t),1,1,M(z,gw,t)) \geq 0$$

and

$$\phi(N(z,gw,t),0,N(gw,z,t),0,0,N(z,gw,t)) \leq 0,$$

which give contradictions to F_1 and ϕ_1 respectively, implying thereby $M(gw,z,t) = 1$ and $N(gw,z,t) = 0$, so that $gw = z = Tw$. Therefore, w is a coincidence point of the pair (g,T). The rest of the proof can be completed on the lines of Theorem 3.1.

By choosing f,g,S and T suitably, one can deduce result for a pair of mappings.

Corollary 3.2. Let f and S be two self mappings of an IFMS $(X,M,N,*,\diamondsuit)$ satisfying the following conditions:

(I) the pair (f,S) satisfies the property (E.A),

(II) $S(X)$ is a closed subset of X and

(III) for all $x, y \in X$, $F \in \Psi, \phi \in \Phi$

$$F(M(fx,fy,t),M(Sx,Sy,t),M(fy,Sy,t),M(fx,Sx,t),M(fx,Sy,t),M(Sx,fy,t)) \geq 0$$

and

$$\phi(N(fx,fy,t),N(Sx,Sy,t),N(fy,Sy,t),N(fx,Sx,t),N(fx,Sy,t),N(Sx,fy,t)) \leq 0.$$

Then pair (f,S) has a coincidence point. Moreover, f and S have a unique common fixed point in X provided that the pair (f,S) is weakly compatible.

Remark 3.2. Above corollary extends and generalizes certain relevant results involving pair of mappings from the existing literature; see, e.g., [9, 10].

Corollary 3.3. *The conclusions of Theorem 3.1 remain true if inequality (3.1.1) is replaced by one of the following contraction conditions. For all $x, y \in X$,*

(I)

$$M(fx,gy,t) \geq \alpha \min\{M(Sx,Ty,t),M(gy,Sy,t),M(fx,Sx,t),$$

$$M(fx,Ty,t),M(Sx,gy,t)\}, \text{ where } \alpha > 1$$

and

$$N(fx,gy,t) \leq \beta \max\{N(Sx,Ty,t),N(gy,Ty,t),N(fx,Sx,t),$$

$$N(fx,Ty,t),N(Sx,gy,t)\}, \text{ where } \beta \in [0,1).$$

(II)

$$M(fx,gy,t)^2 \geq c_1 \min\{M(Sx,Ty,t)^2,M(gy,Ty,t)^2,M(fx,Sx,t)^2\}$$

$$-c_2 \min\{M(fx,Sx,t)M(fx,Ty,t),M(gy,Ty,t)M(Sx,gy,t)\},$$

where $c_1, c_2 > 0, c_1 + c_2 \geq 1, c_1 \geq 1$ and

$$N(fx,gy,t) \leq k \max\{N(Sx,Ty,t),N(gy,Ty,t)N(fx,Ty,t),$$

$$N(fx,Sx,t)N(Sx,gy,t)\}, \text{ where } k \in [0,1).$$

(III)

$$M(fx,gy,t)^3 \geq a\ \min\{M(fx,gy,t)^2M(Sx,Ty,t), M(fx,gy,t)M(gy,Ty,t)M(fx,Sx,t),$$

$$M(fx,Ty,t)^2M(Sx,gy,t), M(fx,Ty,t)M(Sx,gy,t)^2\},\ \textit{where } a > 1$$

and

$$N(fx,gy,t) \leq k[\max\{N(Sx,Ty,t)^2, N(gy,Ty,t)N(fx,Sx,t), N(fx,Ty,t)N(Sx,gy,t),$$

$$N(gy,Ty,t)N(fx,Ty,t), N(fx,Sx,t)N(Sx,gy,t)\}]^{\frac{1}{2}},\ \textit{where } k \in [0,1).$$

(IV)

$$M(fx,gy,t)^3 \geq a\ \frac{M(gy,Ty,t)^2M(fx,Sx,t)^2 + M(fx,Ty,t)^2M(Sx,gy,t)^2}{M(Sx,Ty,t)+M(gy,Ty,t)+M(fx,Sx,t)},$$

where $a > \frac{3}{2}$ *and*

$$N(fx,gy,t) \leq \alpha[\beta \max\{N(Sx,Ty,t), N(gy,Ty,t), N(fx,Sx,t), N(fx,Ty,t), N(Sx,gy,t)\}$$

$$+(1-\beta)(\max\{N(Sx,Ty,t)^2, N(gy,Ty,t)N(fx,Sx,t), N(fx,Ty,t)N(Sx,gy,t),$$

$$N(gy,Ty,t)N(fx,Ty,t), N(fx,Sx,t)N(Sx,gy,t)\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ *and* $\beta \geq 0$.

(V)

$$(1+pM(Sx,Ty,t))M(fx,gy,t) \geq p\min\{M(fx,Sx,t)M(gy,Ty,t), M(Sx,gy,t)M(fx,Ty,t)\}$$

$$+\psi(\min\{M(Sx,Ty,t), M(gy,Ty,t), M(fx,Sx,t), M(fx,Ty,t), M(Sx,gy,t)\}),$$

where $p \geq 0$ *and* $\psi : [0,1] \to [0,1]$ *is continuous function such that* $\psi(t) > t$ *for all* $t \in (0,1)$ *and*

$$N(fx,gy,t)^2 \leq c_1 \min\{N(Sx,Ty,t)^2, N(gy,Ty,t)^2, N(fx,Sx,t)^2\}$$

$$-c_2 \min\{N(fx,Sx,t)N(fx,Ty,t), N(gy,Ty,t)N(Sx,gy,t)\}$$

$$+\gamma N(fx,Ty,t), N(Sx,gy,t)$$

where $\alpha, \beta, \gamma \geq 0$, *and* $\alpha + \gamma < 1$.

(VI)

$$M(fx,gy,t)^2 \geq a\, \frac{M(Sx,Ty,t)^2 + M(gy,Ty,t)^2 + M(fx,Sx,t)^2}{M(fx,Ty,t) + M(Sx,gy,t)},$$

where $a > 2$ *and*

$$(1+\alpha N(Sx,Ty,t))N(fx,gy,t) \geq \alpha \min\{N(fx,Sx,t)N(gy,Ty,t), N(Sx,gy,t)N(fx,Ty,t)\}$$

$$+\beta \min\{N(Sx,Ty,t), N(gy,Ty,t), N(fx,Sx,t), N(fx,Ty,t), N(Sx,gy,t)\},$$

where $\alpha \geq 0$ *and* $\beta \in [0,1)$.

(VII)

$$M(fx,gy,t) \geq \psi(\min\{M(Sx,Ty,t), M(gy,Ty,t), M(fx,Sx,t), M(fx,Ty,t), M(Sx,gy,t)\}),$$

where $\psi : [0,1] \to [0,1]$ *is continuous function such that* $\psi(t) > t$ *for all* $t \in (0,1)$ *and*

$$N(fx,gy,t) \leq \beta \max\{N(Sx,Ty,t), N(gy,Ty,t), N(fx,Sx,t),$$

$$N(fx,Ty,t), N(Sx,gy,t)\}, \text{ where } \beta \in [0,1).$$

(VIII)

$$M(fx,gy,t)^2 \geq c_1 \min\{M(Sx,Ty,t)^2, M(gy,Ty,t)^2, M(fx,Sx,t)^2\}$$

$$-c_2 \min\{M(fx,Sx,t)M(fx,Ty,t), M(gy,Ty,t)M(Sx,gy,t)\},$$

where $c_1, c_2 > 0, c_1 + c_2 \geq 1, c_1 \geq 1$ *and*

$$N(fx,gy,t) \leq k[\max\{N(Sx,Ty,t)^2, N(gy,Ty,t)N(fx,Sx,t), N(fx,Ty,t)N(Sx,gy,t),$$

$$N(gy,Ty,t)N(fx,Ty,t), N(fx,Sx,t)N(Sx,gy,t)\}]^{\frac{1}{2}}, \text{ where } k \in [0,1).$$

(IX)

$$M(fx,gy,t)^3 \geq a\ \min\{M(fx,gy,t)^2 M(Sx,Ty,t), M(fx,gy,t)M(gy,Ty,t)M(fx,Sx,t),$$

$$M(fx,Ty,t)^2 M(Sx,gy,t), M(fx,Ty,t)M(Sx,gy,t)^2\}, \text{ where } a > 1$$

and

$$N(fx,gy,t) \leq k \max\{N(Sx,Ty,t), N(gy,Ty,t)N(fx,Ty,t),$$

$$N(fx,Sx,t)N(Sx,gy,t)\}, \text{ where } k \in [0,1).$$

(X)

$$M(fx,gy,t) \geq \alpha \min\{M(Sx,Ty,t),M(gy,Ty,t),M(fx,Sx,t),$$

$$M(fx,Ty,t),M(Sx,gy,t)\},\ \textit{where}\ \alpha > 1$$

and

$$N(fx,gy,t) \leq \alpha[\beta \max\{N(Sx,Ty,t),N(gy,Ty,t),N(fx,Sx,t),N(fx,Ty,t),N(Sx,gy,t)\}$$

$$+(1-\beta)(\max\{N(Sx,Ty,t)^2,N(gy,Ty,t)N(fx,Sx,t),N(fx,Ty,t)N(Sx,gy,t),$$

$$N(gy,Ty,t)N(fx,Ty,t),N(fx,Sx,t)N(Sx,gy,t)\})^{\frac{1}{2}}],$$

where $\alpha \in [0,1)$ *and* $\beta \geq 0$.

Proof. The proof follows from Theorem 3.1 and Examples 2.1–2.10.

Remark 3.3. Corollaries corresponding to contraction conditions (I–X) are new results as these results never require conditions on the containment of ranges of involved mappings as employed by earlier authors. Some contraction conditions embodied in the above corollary are well known, and extend and generalize corresponding relevant results (e.g., [2, 9, 10, 18, 22, 23]). Similarly corollaries can also be outlined in respect of Examples 2.11–2.16.

As an application of Theorem 3.1, we can have the following result for four finite families of self mappings.

Theorem 3.4. *Let* $\{f_1,f_2,\cdots,f_m\}$,$\{g_1,g_2,\cdots,g_p\}$,$\{S_1,S_2,\cdots,S_n\}$ *and* $\{T_1,T_2,\cdots,T_q\}$ *be four finite families of self mappings of an IFMS* $(X,M,N,*,\diamondsuit)$ *with* $f = f_1f_2\cdots f_m$, $g = g_1g_2\cdots g_p$, $S = S_1S_2\cdots S_n$ *and* $T = T_1T_2\cdots T_q$ *satisfying inequality (3.1.1) and the pairs* (f,S) *and* (g,T) *share the common property (E.A). If* $S(X)$ *and* $T(X)$ *are closed subsets of* X,

then the pair (f,S) *and* (g,T) *have a coincidence point each.*

Moreover, f_i,S_k,g_r *and* T_t *have a unique common fixed point provided the pairs of families* $(\{f_i\},\{S_k\})$ *and* $(\{g_r\},\{T_t\})$ *commute pairwise, where* $i \in \{1,\ldots,m\}, k \in \{1,\ldots,n\}, r \in \{1,\ldots,p\}$ *and* $t \in \{1,\ldots,q\}$.

Proof. Proof follows on the lines of the corresponding result contained in Imdad et al. [11].

By setting $f_1 = f_2 = \cdots = f_m = G,\ g_1 = g_2 = \cdots = g_p = H,\ S_1 = S_2 = \cdots = S_n = I$ and $T_1 = T_2 = \cdots = T_q = J$ in Theorem 3.4, we deduce the following:

Corollary 3.4. *Let G,H,I and J be four self mappings of an IFMS $(X,M,N,*,\diamondsuit)$, pairs (G^m,I^n) and (H^p,J^q) share the common property (E.A) and satisfying the condition for all $x,y\in X$, $F\in\Psi,\phi\in\Phi$*

$$F(M(G^mx,H^py,t),M(I^nx,J^qy,t),M(H^py,J^qy,t),M(G^mx,I^nx,t),M(G^mx,J^qy,t),$$
$$M(I^nx,H^py,t))\geq 0$$

and

$$\phi(N(G^mx,H^py,t),N(I^nx,J^qy,t),N(H^py,J^qy,t),N(G^mx,I^nx,t),N(G^mx,J^qy,t),$$
$$N(I^nx,H^py,t))\leq 0,$$

where m,n,p and q are positive integers. If $I^n(X)$ and $J^q(X)$ are closed subsets of X, then G,H,I and J have a unique common fixed point provided $GI=IG$ and $HJ=JH$.

Finally, we conclude this paper with the following example which furnishes an instance wherein Corollary 3.4 is applicable but Theorem 3.1 is not.

Example 3.1. Let $(X,M,N,*,\diamondsuit)$ be an IFMS, wherein $X=[0,1]$, $a*b=ab$ and $a\diamondsuit b=\min\{1,a+b\}$ with

$$M(x,y,t)=\begin{cases}\frac{t}{t+|x-y|} & \text{if } t>0\\ 0 & \text{if } t=0\end{cases} \text{ and } N(x,y,t)=\begin{cases}\frac{|x-y|}{t+|x-y|} & \text{if } t>0\\ 1 & \text{if } t=0.\end{cases}$$

Define mappings f,g,S and T on X by

$$f(x)=\begin{cases}1 & \text{if } x\in[0,1]\cap Q\\ \frac{1}{2} & \text{if } x\notin[0,1]\cap Q\end{cases},\ g(x)=\begin{cases}1 & \text{if } x\in[0,1]\cap Q\\ \frac{1}{4} & \text{if } x\notin[0,1]\cap Q\end{cases},$$

$$S(x)=\begin{cases}1 & \text{if } x=1\\ 0 & \text{if } x\in[0,1)\end{cases} \text{ and } T(x)=\begin{cases}1 & \text{if } x=1\\ \frac{1}{3} & \text{if } x\in[0,1)\end{cases}.$$

Then $f^2(X)=\{1\}\subset\{0,1\}=T^2(X)$ and $g^2(X)=\{1\}\subset\{\frac{1}{3},1\}=S^2(X)$. Define $F(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-\psi[\min\{t_2,t_3,t_4,t_5,t_6\}]$ where $\psi(s)=\sqrt{s}$ for all $s\in(0,1)$, and $\phi(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-k\max\{t_2,t_3,t_4,t_5,t_6\}$for all $k\in[0,1)$ where $F\in\Psi,\ \phi\in\Phi$.

Now, for all $x,y\in X$ and $t>0$, after verifying all possible cases, we find that

$$\psi\Big[\min\big\{M(S^2x,T^2y,t),M(g^2y,T^2y,t),M(f^2x,S^2x,t),M(f^2x,T^2y,t),M(S^2x,g^2y,t)\big\}\Big]$$

$$\leq 1=M(1,1,t)=M(f^2x,g^2y,t)$$

and

$$k\Big[\max\big\{N(S^2x,T^2y,t),N(g^2y,T^2y,t),N(f^2x,S^2x,t),N(f^2x,T^2y,t),N(S^2x,g^2y,t)\big\}\Big]$$

$$\geq 0=N(1,1,t)=N(f^2x,g^2y,t),$$

which demonstrates the verification of the esteemed implicit function. The remaining requirements of Corollary 3.4 can be easily verified. Notice that 1 is the unique common fixed point of f,g,S and T.

However this implicit function does not hold for the maps f,g,S and T in respect of Theorem 3.1. Otherwise, with $x=0$ and $y=\frac{1}{\sqrt{2}}$, we get

$$\psi\Big[\min\big\{M(Sx,Ty,t),M(gy,Ty,t),M(fx,Sx,t),M(fx,Ty,t),M(Sx,gy,t)\big\}\Big]$$

$$=\psi\Big[\min\big\{M(0,\frac{1}{3},t),M(\frac{1}{4},\frac{1}{3},t),M(1,0,t),M(1,\frac{1}{3},t),M(0,\frac{1}{4},t)\big\}\Big]$$

$$=\psi\Big[\min\{\frac{t}{t+\frac{1}{3}},\frac{t}{t+\frac{1}{12}},\frac{t}{t+1},\frac{t}{t+\frac{2}{3}},\frac{t}{t+\frac{1}{4}}\}\Big]$$

$$=\psi\{\frac{t}{t+1}\}=\sqrt{}\{\frac{t}{t+1}\}\leq M(1,\frac{1}{4},t)=M(fx,gy,t)$$

$$\sqrt{}\{\frac{t}{t+1}\}\leq\frac{t}{t+\frac{3}{4}},$$

which is not true for all $t>0$ (e.g. $t=\frac{1}{2}$).

Thus Corollary 3.4 is a partial generalization of Theorem 3.1 and can be situationally useful.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] M. Aamri, D.E. Moutawakil Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270 (2002), 181-188.

[2] J. Ali, M. Imdad, An implicit function implies several contraction conditions, Sarajevo J. Math. 17 (2008), 269-285.

[3] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986), 87-96.

[4] Z. Deng, Fuzzy pseudometric spaces, J. Math. Anal. Appl. 86 (1982), 74-95.

[5] El Naschie, A review of E-infinity theory and the mass spectrum of high energy particle physics, Chaos Solitons & Fractals 19 (2004), 209-236.

[6] A. George, P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets Syst. 64 (1994), 395-399.

[7] M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets Syst. 27 (1988), 385-389.

[8] X. Huang, C. Zhu, X. Wen, Common fixed point theorems for families of compatible mappings in intuitionistic fuzzy metric spaces, Ann Univ Ferrara, DOI 10.1007/s11565-010-0105-1.

[9] M. Imdad, J. Ali, Some common fixed point theorems in fuzzy metric spaces, Math. Commun. 11 (2006), 153-163.

[10] M. Imdad, J. Ali, A general fixed point theorem in fuzzy metric spaces via an implicit function, J. Appl. Math. Informatics, 26 (2008), 591-603.

[11] M. Imdad, J. Ali, M. Tanveer, Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces, Chaos Solitons & Fractals 42 (2009), 3121-3129.

[12] G. Jungck, Common fixed points for noncontinuous nonself maps on nonmetric spaces, Far East J. Math. Sci. 4 (1996), 199-215.

[13] E.P. Klement, Operations on fuzzy sets–an axiomatic approach, Inform. Sci. 27 (1982), 221-232.

[14] O. Kramosil, J. Michalek Fuzzy metric and statistical metric spaces, Kybernetica, 11 (1975), 326-334.

[15] Y. Liu, J. Wu, Z. Li, Common fixed points of single-valued and multi-valued maps, Int.. J. Math. Math. Sci. 19 (2005), 3045-3055.

[16] R. Lowen, Fuzzy Set Theory, Dordrecht: Kluwer Academic Publishers, 1996.

[17] K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. 28 (1942), 535-537.

[18] V. Pant, Some fixed point theorems in fuzzy metric space, Tamkang J. Math. 40 (2009), 59-66.

[19] J.H. Park, Intuitionistic fuzzy metric spaces, Chaos Solitons & Fractals, 22 (2004), 1039-1046.

[20] R. Saadati, J.H. Park, On the intuitionistic topological spaces, Chaos Solitons & Fractals, 27 (2006), 331-344.

[21] B. Schweizer, A. Sklar, Statistical metric spaces, Pacific J. Math. 10 (1960), 313-334.

[22] D. Turkoglu, C. Alaca, Y.J. Cho, C. Yildiz, Common fixed point theorems in intuitionistic fuzzy metric spaces, J. Appl. Math. Comput. 22 (2006), 411-424.

[23] P. Vijayaraju, Z. Sajath, Some common fixed point theorems in fuzzy metric spaces, Int. J. Math. Anal. 15 (2009), 701-710.

[24] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965), 338-353.