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COMMON FIXED POINT THEOREMS FOR A PAIR OF MAPPINGS IN COMPLEX VALUED b-METRIC SPACES

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Abstract. In this paper, we generalize the results of Verma and Pathak [9], by improving the conditions of the contraction to establish the existence and the uniqueness of common fixed points for a pair of self mappings on complex valued b-metric spaces. Some examples are given to illustrate the main results.

Keywords: common fixed point; contraction mapping; complex valued b-metric space; uniqueness.

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1. Introduction

The fixed point theorem introduced by Banach in [4], was the source of metric fixed point theory in the field of Nonlinear Analysis. In 1989, Bakhtin [3], introduced the concept of b-metric space as a generalization of metric spaces. The concept of complex valued b-metric spaces was introduced in 2013 by Rao et al [6], which was more general than the well known complex valued metric spaces that was introduced in 2011 by Azam et al [2]. The main purpose of this paper is to present common fixed point results of a pair of self mappings satisfying some conditions on complex valued b-metric spaces.

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2. Preliminaries

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } \operatorname{Re}(z_1) \le \operatorname{Re}(z_1),\ \operatorname{Im}(z_1) \le \operatorname{Im}(z_2).$$

Thus $z_1 \precsim z_2$ if one of the following holds:

$$(1)\ \operatorname{Re}(z_1) = \operatorname{Re}(z_2) \quad \textit{and} \quad \operatorname{Im}(z_1) = \operatorname{Im}(z_2),$$

$$(2)\ \operatorname{Re}(z_1) < \operatorname{Re}(z_2) \quad \textit{and} \quad \operatorname{Im}(z_1) = \operatorname{Im}(z_2),$$

$$(3)\ \operatorname{Re}(z_1) = \operatorname{Re}(z_2) \quad \textit{and} \quad \operatorname{Im}(z_1) < \operatorname{Im}(z_2),$$

$$(4)\ \operatorname{Re}(z_1) < \operatorname{Re}(z_2) \quad \textit{and} \quad \operatorname{Im}(z_1) < \operatorname{Im}(z_2).$$

We will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (2),(3) and (4) is satisfied, also we will write $z_1 \prec z_2$ if only (4) is satisfied.

Remark 2.1. We can easily check that the following statements are hold:
(i) If $a, b \in \mathbb{R}$ and $a \le b$, then $az \precsim bz$ for all $z \in \mathbb{C}$,
(ii) If $0 \precsim z_1 \precnsim z_2$, then $|z_1| < |z_2|$,
(iii) If $z_1 \precsim z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

Definition 2.1. [1] Let X be a nonempty set and let $s \ge 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is called a b-metric if for all $x, y, z \in X$ the following conditions are satisfied:
(i) $d(x, y) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$;
(iii) $d(x, y) \le s[d(x, z) + d(z, y)]$.
The pair (X, d) is called a b-metric space. The number $s \ge 1$ is called the coefficient of (X, d).

Example 2.1. [7] Let (X, d) be a metric space and $\rho(x, y) = (d(x, y))^p$, where $p > 1$ is a real number. Then (X, ρ) is a b-metric space with $s = 2^{p-1}$.

Definition 2.2. [2] Let X be a nonempty set. A function $d : X \times X \to \mathbb{C}$ is called a complex valued metric on X if for all $x, y, z \in X$ the following conditions are satisfied:
(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$;

(iii) $d(x,y) \precsim d(x,z) + d(z,y)$.

The pair (X,d) is called a complex valued metric space.

Example 2.2. [5] Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by

$$d(x,y) = i|x-y|, \text{ for all } x,y \in X.$$

Then (X,d) is a complex valued metric space.

Example 2.3. [8] Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by

$$d(x,y) = e^{ik}|x-y|, \ where \ k \in \mathbb{R} \text{ and for all } x,y \in X.$$

Then (X,d) is a complex valued metric space.

Definition 2.3. [6] Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{C}$ is called a complex valued b-metric on X if for all $x,y,z \in X$ the following conditions are satisfied:

(i) $0 \precsim d(x,y)$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$;

(iii) $d(x,y) \precsim s[d(x,z) + d(z,y)]$.

The pair (X,d) is called a complex valued b-metric space.

Example 2.4. [6] Let $X = [0,1]$. Define the mapping $d : X \times X \to \mathbb{C}$ by

$$d(x,y) = |x-y|^2 + i|x-y|^2, \text{ for all } x,y \in X.$$

Then (X,d) is a complex valued b-metric space with $s = 2$.

Definition 2.4. [6] Let (X,d) be a complex valued b-metric space.

(i) A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x,r) := \{y \in X : d(x,y) \prec r\} \subseteq A$.

(ii) A point $x \in X$ is called a limit point of a set A whenever for every $0 \prec r \in \mathbb{C}$, $B(x,r) \cap (A - X) \neq \emptyset$.

(iii) A subset $A \subseteq X$ is called open whenever each element of A is an interior point of A.

(iv) A subset $A \subseteq X$ is called closed whenever each element of A belongs to A.

(v) A sub-basis for a Hausdorff topology τ on X is a family $F = \{B(x,r) : x \in X \text{ and } 0 \prec r\}$.

Definition 2.5. [6] Let (X,d) be a complex valued b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

(i) If for every $c \in \mathbb{C}$, with $0 \prec r$ there is $N \in \mathbb{N}$ such that for all $n > \mathbb{N}, d(x_n,x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. We denote this by $\lim_{n\to\infty} x_n = x$ or $\{x_n\} \to x \ as \ n \to \infty$.

(ii) If for every $c \in \mathbb{C}$, with $0 \prec r$ there is $N \in \mathbb{N}$ such that for all $n > \mathbb{N}, d(x_n,x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then $\{x_n\}$ is said to be Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent, then (X,d) is said to be a complete complex valued b-metric space.

Lemma 2.1. [6] Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n,x)| \to 0 \ as \ n \to \infty$.

Lemma 2.2. [6] Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n,x_{n+m})| \to 0 \ as \ n \to \infty$, where $m \in \mathbb{N}$.

3. Main results

Theorem 3.1. *Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let λ be nonnegative real number such that $0 \leq \lambda < \frac{1}{s^2+s}$. Suppose that $S,T : X \to X$ are a pair of self mappings satisfying:*

$$d(Sx,Ty) \precsim \lambda \max\{d(x,y), d(x,Sx), d(y,Ty), d(x,Ty), d(y,Sx)\}, \tag{3.1}$$

for all $x,y \in X$. Then S,T have a unique common fixed point in X.

Proof. For any arbitrary point $x_0 \in X$, define sequence $\{x_n\}$ in X such that

$$x_{2n+1} = Sx_{2n}, x_{2n+2} = Tx_{2n+1}, \quad \text{for } \ n = 0,1,2,3,....$$

Let $x = x_{2n}$ and $y = x_{2n+1}$ in (3.1). It follows that

$$\begin{aligned} d(x_{2n+1},x_{2n+2}) &= d(Sx_{2n},Tx_{2n+1}) \\ &\precsim \lambda \max\{d(x_{2n},x_{2n+1}),d(x_{2n},Sx_{2n}),d(x_{2n+1},Tx_{2n+1}), \\ &\quad d(x_{2n},Tx_{2n+1}),d(x_{2n+1},Sx_{2n})\} \\ &= \lambda \max\{d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}), \\ &\quad d(x_{2n},x_{2n+2}),d(x_{2n+1},x_{2n+1})\} \\ &= \lambda \max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),d(x_{2n},x_{2n+2}),0\} \\ &\precsim \lambda \max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}), \\ &\quad s[d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})]\}. \end{aligned} \tag{3.2}$$

If $x_{2n+1} = x_{2n}$, for some n then by using (3.2) we get

$$d(x_{2n+1},x_{2n+2}) \precsim s\lambda d(x_{2n+1},x_{2n+2}),$$

which implies that

$$|d(x_{2n+1},x_{2n+2})| \leq s\lambda |d(x_{2n+1},x_{2n+2})|. \tag{3.3}$$

By using the fact that $0 \leq \lambda < \frac{1}{s^2+s}$, it is easy to see that $0 < s\lambda < \frac{1}{2}$. The inequality (3.3) is true only if $|d(x_{2n+1},x_{2n+2})| = 0$, which implies that $d(x_{2n+1},x_{2n+2}) = 0$. Hence, $x_{2n+1} = x_{2n+2}$. Continuing this process one easily can show that $x_{2n} = x_{2n+1} = x_{2n+2} = x_{2n+3} = \ldots.$ Hence, $\{x_n\}$ is a Cauchy sequence. Assume that $x_{2n+1} \neq x_{2n}$, for all n in (3.2) and if

$$\max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),s[d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})]\} = d(x_{2n+1},x_{2n+2})$$

then

$$d(x_{2n+1},x_{2n+2}) \precsim \lambda d(x_{2n+1},x_{2n+2}),$$

which implies that

$$|d(x_{2n+1},x_{2n+2})| \leq \lambda |d(x_{2n+1},x_{2n+2})|. \tag{3.4}$$

Since $0 < \lambda < \frac{1}{2}$. The inequality (3.4) is true only if $|d(x_{2n+1},x_{2n+2})| = 0$, which implies that $d(x_{2n+1},x_{2n+2}) = 0$. Hence, $x_{2n+1} = x_{2n+2}$. Which is a contradiction with $x_{2n+1} \neq x_{2n}$, for all n.

Therefore, (3.2) becomes

$$d(x_{2n+1},x_{2n+2}) \precsim \lambda \max\{d(x_{2n},x_{2n+1}), s[d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})]\}.$$

Thus,

$$d(x_{2n+1},x_{2n+2}) \precsim \max\{\lambda, \frac{s\lambda}{1-s\lambda}\} d(x_{2n},x_{2n+1}).$$

Let $\beta = \max\{\lambda, \frac{s\lambda}{1-s\lambda}\}$, then

$$d(x_{2n+1},x_{2n+2}) \precsim \beta d(x_{2n},x_{2n+1}). \tag{3.5}$$

So, we have two cases

Case 1: If $\beta = \lambda$ then

$$s\beta = \frac{s}{s^2+s} = \frac{1}{1+s} \le \frac{1}{2} < 1.$$

Case 2: If $\beta = \frac{s\lambda}{1-s\lambda}$ then

$$s\beta = s\frac{s\lambda}{1-s\lambda} < s\frac{\frac{s}{s^2+s}}{1-\frac{s}{s^2+s}} = 1.$$

From the two cases, we conclude that $s\beta < 1$. Taking the modulus of (3.5), we get

$$|d(x_{2n+1},x_{2n+2})| \le \beta |d(x_{2n},x_{2n+1})|. \tag{3.6}$$

Similarly, we obtain

$$|d(x_{2n+2},x_{2n+3})| \le \beta |d(x_{2n+1},x_{2n+2})|.$$

Therefore, for all $n \ge 0$ and consequently, we have

$$|d(x_{2n+1},x_{2n+2})| \le \beta |d(x_{2n},x_{2n+1})| \le \beta^2 |d(x_{2n-1},x_{2n})| \le ... \le \beta^{2n+1} |d(x_0,x_1)|. \tag{3.7}$$

Thus for any $m > n$, $m,n \in \mathbb{N}$, we get

$$
\begin{aligned}
|d(x_n,x_m)| &\leq s|d(x_n,x_{n+1})| + s|d(x_{n+1},x_m)| \\
&\leq s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^2|d(x_{n+2},x_m)| \\
&\leq s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^3|d(x_{n+2},x_m)| + s^3|d(x_{n+2},x_m)| \\
&\leq s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^3|d(x_{n+2},x_m)| \\
&\quad + \ldots + s^{m-n-2}|d(x_{m-3},x_{m-2})| + s^{m-n-1}|d(x_{m-2},x_{m-1})| + s^{m-n-1}|d(x_{m-1},x_m)| \\
&\leq s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^3|d(x_{n+2},x_m)| \\
&\quad + \ldots + s^{m-n-2}|d(x_{m-3},x_{m-2})| + s^{m-n-1}|d(x_{m-2},x_{m-1})| + s^{m-n}|d(x_{m-1},x_m)|.
\end{aligned}
$$

Using (3.7), we get

$$
\begin{aligned}
|d(x_n,x_m)| &\leq s\beta^n|d(x_0,x_1)| + s^2\beta^{n+1}|d(x_0,x_1)| + s^3\beta^{n+2}|d(x_0,x_1)| \\
&\quad + \ldots + s^{m-n-2}\beta^{m-3}|d(x_0,x_1)| + s^{m-n-1}\beta^{m-2}|d(x_0,x_1)| + s^{m-n}\beta^{m-1}|d(x_0,x_1)|. \\
&= s\beta^n|d(x_0,x_1)| \sum_{i=0}^{m-n-1} (s\beta)^i \\
&\leq s\beta^n|d(x_0,x_1)| \sum_{i=0}^{\infty} (s\beta)^i = \frac{s\beta^n}{1-s\beta}|d(x_0,x_1)|.
\end{aligned}
$$

Since $s\beta, \beta < 1$, we deduce

$$
|d(x_n,x_m)| \leq \frac{s\beta^n}{1-s\beta}|d(x_0,x_1)| \to 0 \quad as \quad m,n \to \infty.
$$

Thus, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there exists some $u \in X$ such that $x_n \to u$ as $n \to \infty$. Assume not, then there exist $z \in X$ such that

$$
|d(u,Su)| = |z| > 0. \tag{3.8}
$$

Using the triangular inequality and (3.1), we get

$$\begin{aligned} z &= d(u,Su) \\ &\precsim sd(u,x_{2n+2}) + sd(x_{2n+2},Su) \\ &= sd(u,x_{2n+2}) + sd(Tx_{2n+1},Su) \\ &\precsim sd(u,x_{2n+2}) + s\lambda \max\{d(u,x_{2n+1}),d(u,Su),d(x_{2n+1},Tx_{2n+1}), \\ &\quad d(u,Tx_{2n+1}),d(x_{2n+1},Su)\} \\ &= sd(u,x_{2n+2}) + s\lambda \max\{d(u,x_{2n+1}),d(u,Su),d(x_{2n+1},x_{2n+2}), \\ &\quad d(u,x_{2n+2}),d(x_{2n+1},Su)\}. \end{aligned} \tag{3.9}$$

Taking the modulus of (3.9) and using $|a+b| \leq |a|+|b|$ for all $a,b \in \mathbb{C}$ we get

$$\begin{aligned} |z| &= |d(u,Su)| \\ &\leq s|d(u,x_{2n+2})| + s\lambda \max\{|d(u,x_{2n+1})|,|d(u,Su)|,|d(x_{2n+1},x_{2n+2})|, \\ &\quad |d(u,x_{2n+2})|,|d(x_{2n+1},Su)|\}. \end{aligned} \tag{3.10}$$

Letting n $\to \infty$ for (3.10), we have

$$|z| = |d(u,Su)| \leq s\lambda \max\{0,|z|,0,0,|z|\},$$

we obtain that $|z| = |d(u,Su)| \leq s\lambda|z| < |z|$, which is a contradiction with (3.8). So $|z| = 0$. Hence $Su = u$. Similarly, we obtain that $Tu = u$. Now, we show that S and T have unique common fixed point of S and T. To show this, we assume that u^* is another common fixed point of S and T. Then

$$\begin{aligned} d(u,u^*) &= d(Su,Tu^*) \\ &\precsim \lambda \max\{d(u,u^*),d(u,Su),d(u^*,Tu^*),d(u,Tu^*),d(u^*,Su)\} \\ &= \lambda \max\{d(u,u^*),0,0,d(u,u^*),d(u^*,u)\} \\ &= \lambda d(u,u^*). \end{aligned}$$

This implies that $|d(u,u^*)| < \lambda|d(u,u^*)|$, which leads us to a contradiction. Hence, $|d(u,u^*)| = 0$, and that is $u = u^*$ which proves the uniqueness of common fixed point in X as required. This completes the proof.

Corollary 3.1. *Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let λ be nonnegative real number such that $0 \leq \lambda < \frac{1}{s^2+s}$. Suppose that $S,T : X \to X$ are a pair of self mappings satisfying:*

$$d(Sx,Ty) \precsim \lambda \max\{d(x,y), d(x,Sx), d(y,Ty)\}, \tag{3.11}$$

for all $x,y \in X$. Then S,T have a unique common fixed point in X.

Proof. For any arbitrary point $x_0 \in X$, define sequence $\{x_n\}$ in X such that

$$x_{2n+1} = Sx_{2n}, x_{2n+2} = Tx_{2n+1}, \quad \text{for } n = 0,1,2,3,....$$

Now, we show that the sequence $\{x_n\}$ is Cauchy. Letting $x = x_{2n}$ and $y = x_{2n+1}$ in (3.11), we have

$$\begin{aligned} d(x_{2n+1},x_{2n+2}) &= d(Sx_{2n},Tx_{2n+1}) \\ &\precsim \lambda \max\{d(x_{2n},x_{2n+1}), d(x_{2n},Sx_{2n}), d(x_{2n+1},Tx_{2n+1})\} \\ &= \lambda \max\{d(x_{2n},x_{2n+1}), d(x_{2n},x_{2n+1}), d(x_{2n+1},x_{2n+2})\} \\ &= \lambda \max\{d(x_{2n},x_{2n+1}), d(x_{2n+1},x_{2n+2})\} \\ &\precsim \lambda d(x_{2n},x_{2n+1}). \end{aligned}$$

We can prove this corollary by following the same procedure in Theorem 3.1. This completes the proof.

Corollary 3.2. *Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let λ be nonnegative real number such that $0 \leq \lambda < \frac{1}{s^2+s}$. Suppose that $T : X \to X$ is a self mapping satisfying:*

$$d(Tx,Ty) \precsim \lambda d(x,y),$$

for all $x,y \in X$. Then T has a unique fixed point in X.

Proof. Putting $S = T$ in Corollary 3.1, we can include the desired conclusion easily.

Remark 3.1. Corollary 3.2. is the generalization of the Banach contraction principle in [4], on the complex valued b-metric spaces.

Remark 3.2. By taking $s = 1$ in Theorem 3.1. we can get Verma and Pathak Theorem in [9]

Example 3.1. Let $X = [0,1]$. Define a function $d : X \times X \to \mathbb{C}$ such that

$$d(z_1,z_2) = |z_1 - z_2|^2 + i|z_1 - z_2|^2.$$

To verify that (X,d) is a complete complex valued b-metric space with $s = 2$, it's enough to verify the triangular inequality condition. Let z_1, z_2 and $z_3 \in X$, then

$$\begin{aligned}
d(z_1,z_2) &= |z_1 - z_2|^2 + i|z_1 - z_2|^2 \\
&= |z_1 - z_3 + z_3 - z_2|^2 + i|z_1 - z_3 + z_3 - z_2|^2 \\
&\precsim |z_1 - z_3|^2 + |z_3 - z_2|^2 + 2|z_1 - z_3||z_3 - z_2| \\
&\quad + i[|z_1 - z_3|^2 + |z_3 - z_2|^2 + 2|z_1 - z_3||z_3 - z_2|] \\
&\precsim |z_1 - z_3|^2 + |z_3 - z_2|^2 + |z_1 - z_3|^2 + |z_3 - z_2|^2 \\
&\quad + i[|z_1 - z_3|^2 + |z_3 - z_2|^2 + |z_1 - z_3|^2 + |z_3 - z_2|^2] \\
&= 2\{|z_1 - z_3|^2 + |z_3 - z_2|^2 + i[|z_1 - z_3|^2 + |z_3 - z_2|^2]\} \\
&= 2[d(z_1,z_3) + d(z_3,z_2)].
\end{aligned}$$

Therefore, $s = 2$. Now, define a self mapping $T : X \to X$ such that:

$$Tx = \frac{x}{3}.$$

Note that,

$$\begin{aligned}
d(Tz_1,Tz_2) &= d(\frac{z_1}{3},\frac{z_2}{3}) \\
&= |\frac{z_1}{3} - \frac{z_2}{3}|^2 + i|\frac{z_1}{3} - \frac{z_2}{3}|^2 \\
&= \frac{1}{9}(|z_1 - z_2|^2 + i|z_1 - z_2|^2) \\
&= \frac{1}{9}d(z_1,z_2) \\
&\precsim \lambda d(z_1,z_2) \\
&\prec \frac{1}{6}d(z_1,z_2),
\end{aligned}$$

for all $z_1, z_2 \in X$ and $0 \le \lambda < \frac{1}{s^2+s}$. So all conditions in Corollary 3.2 are satisfied to get a unique fixed point 0 of T.

Conflict of Interests

The author declares that there is no conflict of interests.

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