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FIXED POINTS FOR HYBRID PAIR OF COMPATIBLE MAPPINGS IN PARTIAL METRIC SPACES

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Abstract. In this paper coincidence and fixed point theorems for a pair of single-valued and multi-valued compatible mappings on complete partial metric spaces are presented. The notion of compatible mappings for a pair of single-valued and multi-valued mappings proved to be very useful as the existing metric fixed point theory contains numerous fixed point results for pair(s) of mappings established under compatibility condition and its generalizations. Partial metric spaces are one of generalizations of the notion of a metric space that allows non-zero self distance. The existing metric fixed point theory approaches, are adapted to establish the results. The main result generalizes, in particular, a fixed point theorem due to Kaneko and Sessa for hybrid pair of compatible mappings. An illustrative example is also provided.

Keywords: partial metric spaces; compatible mappings; hybrid pair of mappings.

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1. Introduction

The study of fixed points for compatible mappings on complete metric spaces was initiated by Jungck [2]. Kaneko and Sessa [4] extended the notion of compatible mappings to include multi-valued mappings, and established some metric fixed point results for single-valued and multi-valued compatible mappings.

Partial metric spaces were introduced by Mathews [7] while studying denotational semantics in data flow networks. Partial metric spaces are generalization of the notion of metric spaces such that the distance of a point of the space from itself is not necessarily zero [8]. Matthews established, as a tool for his study, a fixed point theorem, a generalization of the Banach contraction principle, for contraction mappings on complete partial metric spaces. Matthews therefore initiated the study of fixed points in the framework of partial metric spaces. Matthews [8] also studied topological aspects for partial metric spaces. Recently, there has been several studies on possible generalizations of the existing metric fixed point results to partial metric spaces. This paper forms a part of the studies for metric fixed point results for a hybrid pair of compatible mappings.

The purpose of this paper is to generalize a metric fixed point theorem due to Kaneko and Sessa [4] to partial metric spaces.

2. Preliminaries

The following definitions and preliminary results will be required to establish the results.

Definition 2.1. Let X be a non-empty set. Let $T : X \to 2^X$, where 2^X denotes the collection of all non-empty subsets of X, be a multi-valued mapping and $f : X \to X$ be a single-valued mapping. Then:

(i) a point $t \in X$ is called a common fixed point of T and f if $t = ft \in Tt$.

(ii) a point $s \in X$ is called a coincidence point of f and T if $fs \in Ts$.

Definition 2.2. [8] Let X be non-empty set. A partial metric space is a pair (X, p), where p is a function $p : X \times X \to [0, \infty)$, called the partial metric, such that for all $x, y, z \in X$ the following hold:

(P1) $x = y \Leftrightarrow p(x,y) = p(x,x) = p(y,y)$;

(P2) $p(x,x) \leq p(x,y)$;

(P3) $p(x,y) = p(y,x)$;

(P4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

Clearly, by (P1) - (P3) , if $p(x,y) = 0$, then $x = y$. But, the converse is in general not true.

Amongst classical examples of partial metric spaces is a pair $([0,\infty), p)$ where $p(x,y) = \max\{x,y\}$ for all $x,y \in [0,\infty)$. More examples of partial metric spaces may be found in [1 , 3].

Each partial metric p on X generates a T_0 topology τ_p on X whose basis is the collection of all open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$ where
$B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$, and ε is a real number.

Let (X, p) be a partial metric space. Let A be any non-empty subset of the set X and x be an element of the set X. It is well known [11] that $x \in \bar{A}$, where $\bar{A}$ is the closure of A, if and only if $p(x,A) = p(x,x)$. Also, the set A is said to closed in (X, p) if and only if $A = \bar{A}$.

Definition 2.3. [8] Let (X, p) be a partial metric space. Then:

(i) A sequence $\{x_n\}$ in (X, p) is said to be convergent to $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) A sequence $\{x_n\}$ in (X, p) is a Cauchy sequence if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.

(iii) A partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to the topology τ_p to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

Lemma 2.1. [1] *Let (X, p) be a partial metric space. Then the mapping $p^s : X \times X \to [0,\infty)$ given by*

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

for all $x,y \in X$ defines a metric on X.

Lemma 2.2. [1] *Let (X, p) be a partial metric space. Then:*

(i) *A sequence $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s).*

(ii) *A partial metric space* (X,p) *is complete if and only if the metric space* (X,p^s) *is complete.*

Let (X,d) be a metric space and $CB(X)$ denotes the collection of all non-empty bounded closed subsets of X. For $A,B \in CB(X)$, define

$$H(A,B) = \max\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\}$$

where $d(x,A) = \inf\{d(x,a) : a \in A\}$ is the distance from a point $x \in X$ to the set $A \in CB(X)$. It is well known [6] that H is a metric, called the Pompeiu-Hausdorff metric, on $CB(X)$ induced by the metric d. The metric space $(CB(X),H)$ is complete whenever (X,d) is complete.

Definition 2.4. [2] Let (X,d) be a metric space. The mappings $f,g : X \to X$ are called compatible if and only if $d(fgx_n, gfx_n)$ approaches 0 whenever $\{x_n\}$ is a sequence in X such that $\{fx_n\}$ approaches t, $\{gx_n\}$ approaches t for some point $t \in X$.

Definition 2.5. [4] Let (X,d) be a metric space. The mappings $f : X \to X$ and $T : X \to CB(X)$ are compatible if and only if $fTx \in CB(X)$ for all x in X and $H(Tfx_n, fTx_n)$ approaches 0, whenever $\{x_n\}$ is a sequence in X such that $\{Tx_n\}$ approaches $N \in CB(X)$ and $\{fx_n\}$ approaches $t \in N$.

Kaneko and Sessa [4] extended the Proposition 2.2 due to Jungck [2] to include a hybrid pair of mappings, and obtained the following result:

Lemma 2.3. [4] *Let* (X,d) *be a metric space. Let* $f : X \to X$ *and* $T : X \to CB(X)$ *be compatible mappings. If* $fy \in Ty$ *for some* $y \in X$*, then* $fTy = Tfy$ *.*

Definition 2.6. [10] Let (X,p) be a partial metric space. Mappings $f,g : X \to X$ are:

(i) compatible if and only if $p(fgx_n, gfx_n)$ approaches $p(t,t)$ whenever $\{x_n\}$ is a sequence in X such that both $\{fx_n\}$ and $\{gx_n\}$ approach $t \in X$.

(ii) non-compatible if there exists at least one sequence $\{x_n\}$ in X such that both $\{fx_n\}$ and $\{gx_n\}$ approach $t \in X$, but $p(fgx_n, gfx_n)$ diverges.

Definition 2.7. [3] Let (X,p) be a partial metric space and $CB^p(X)$ denotes the collection of all non-empty bounded and closed subsets of X. For $A,B \in CB^p(X)$, define

$$H_p(A,B) = \max\{\delta_p(A,B), \delta_p(B,A)\}$$

where $p(x,A)=\inf\{p(x,a):a\in A\}$, $\delta_p(A,B)=\sup\{p(a,B):a\in A\}$ and $\delta_p(B,A)=\sup\{p(b,A):b\in B\}$. Then the mapping H_p is a partial metric, called the partial Hausdorff metric, on $CB^p(X)$ induced by the partial metric p.

Proposition 2.1. [3] Let (X,p) be a partial metric space. For $A,B,C\in CB^p(X)$, the following hold:

(h1) $H_p(A,A)\leq H_p(A,B)$;

(h2) $H_p(A,B)=H_p(B,A)$;

(h3) $H_p(A,B)\leq H_p(A,C)+H_p(C,B)-\inf_{c\in C}p(C,C)$.

Lemma 2.3. [3] *Let (X,p) be a partial metric space. Let $A,B\in CB^p(X)$ and $q>1$. Then for any $a\in A$, there exists $b\in B$ that depends on a such that*

$$p(a,b)\leq qH_p(A,B).$$

Definition 2.8. Let (X,p) be a partial metric space. The mappings $f:X\to X$ and $T:X\to CB^p(X)$ are:

(i) compatible if and only if $fTx\in CB^p(X)$ for all $x\in X$ and

$$H_p(Tfx_n,fTx_n)=H_p(fTx_n,fTx_n)$$

whenever $\{x_n\}$ is a sequence in X such that $\{Tx_n\}$ approaches $N\in CB^p(X)$ and $\{fx_n\}$ approaches $t\in N$.

(ii) non-compatible if there exists at least one sequence $\{x_n\}$ in X such that $\{Tx_n\}$ approaches $N\in CB^p(X)$ and $\{fx_n\}$ approaches $t\in M$, but

$$H_p(Tfx_n,fTx_n)\neq H_p(fTx_n,fTx_n).$$

Lemma 2.4. *Let (X,p) be a partial metric space. Let $f:X\to X$ and $T:X\to CB^p(X)$ be compatible mappings. If $fy\in Ty$ for some $y\in X$, then $fTy=Tfy$.*

Proof. Let $x_n=y$ for each n, then $fx_n=fy\to fy$ and $Tx_n\Rightarrow N=Ty$. Suppose that $fy\in Ty$ for some $y\in X$, then

$H_p(fTy,Tfy)=H_p(fTx_n,Tfx_n)=H_p(fTx_n,fTx_n)$ by compatibility of the mappings f and

T (see Definition 2.8) Thus by Definition 2.2 (P2) we have $fTy = Tfy$. This completes the proof.

Motivated by Nadler [6] and inspired by Kubiak [9], Kaneko and Sessa [4] established the following metric fixed point result for a hybrid pair of compatible mappings.

Theorem 2.1. [4] *Let (X,d) be a complete metric space, $f : X \to X$ and $T : X \to CB(X)$ be compatible continuous mappings such that $T(X) \subseteq f(X)$ and*
$H(Tx,Ty)$
$\leq h\max\left\{d(fx,fy),d(fx,Tx),d(fy,Ty),\frac{1}{2}[d(fx,Ty)+d(fy,Tx)]\right\}$ *for all x,y in X, where $0 \leq h < 1$. Then, there exists a point $t \in X$ such that $ft \in Tt$.*

In this paper Theorem 2.1 is generalized to partial metric spaces in order to obtain a fixed point theorem for hybrid pair of compatible mappings in partial metric spaces.

3. Main Results

The following theorem is a generalization of Theorem 2.1 to partial metric spaces.

Theorem 3.1. Let (X,p) be a complete partial metric space. Let $f : X \to X$ and $T : X \to CB^p(X)$ be compatible continuous mappings such that

(i) $T(X) \subseteq f(X)$ and

(ii) $H_p(Tx,Ty)$
$\leq h\max\left\{p(fx,fy),p(fx,Tx),p(fy,Ty),\frac{1}{2}[p(fx,Ty)+p(fy,Tx)]\right\}$ for all x,y in X, where $0 \leq h < 1$.

Then, the mappings f and T have a common coincidence point. i.e. there exists a point $t \in X$ such that $ft \in Tt$.

Proof. Let x_0 be an arbitrary point in X. Using Theorem 3.1 (i) we can find $x_1 \in X$ such that $fx_1 \in Tx_0$. By the definition of H_p (refer Definition 2.7) , and the Theorem 3.1 (ii) for $h = 0$, we have

$$\begin{aligned} p(fx_1,Tx_1) \leq & H_p(Tx_0,Tx_1) = 0 \\ \leq & H_p(Tx_1,Tx_1) \end{aligned}$$

This implies that fx_1 is contained in Tx_1.

Consider a case when $h > 0$.

For define $q = \dfrac{1}{\sqrt{h}}$. So $q > 1$. By Lemma 2.3 , there exists a point $z_1 \in Tx_1$ such that $p(z_1, fx_1) \le qH_p(Tx_1, Tx_0)$. By Theorem 3.1 (i), we can find $x_2 \in X$ such that $z_1 = fx_2 \in Tx_1$. In general, after selecting x_n, we can choose $x_{n+1} \in X$ and set

$z_n = fx_{n+1} \in Tx_n$ satisfying:

$p(z_n, fx_n) = p(fx_{n+1}, fx_n) \le qH_p(Tx_n, Tx_{n-1})$ for each $n \ge 1$. Now,

$$
\begin{aligned}
(1) \qquad p(fx_n, fx_{n+1}) \le & \, qH_p(Tx_{n-1}, Tx_n) \\
\le & \, qh \max\{p(fx_{n-1}, fx_n), p(fx_{n-1}, Tx_{n-1}), p(fx_n, Tx_n), \\
& \tfrac{1}{2}[p(fx_{n-1}, Tx_n) + p(fx_n, Tx_{n-1})]\} \\
\le & \sqrt{h} \max\{p(fx_{n-1}, fx_n), p(fx_{n-1}, fx_n), p(fx_n, fx_{n-1}), \\
& \tfrac{1}{2}[p(fx_{n-1}, fx_{n+1}) + p(fx_n, fx_n)]\} \\
\le & \sqrt{h} \max\{p(fx_n, fx_{n-1}), \tfrac{1}{2}[p(fx_{n-1}, fx_{n+1}) + p(fx_n, fx_n)]\} \\
\le & \sqrt{h} \max\{p(fx_n, fx_{n-1}), p(fx_n, fx_{n+1})\}
\end{aligned}
$$

$$
(2) \qquad p(fx_n, fx_{n+1}) \le \sqrt{h}\, p(fx_n, fx_{n-1}) \quad \text{for all} \quad n \ge 2.
$$

By mathematical induction, we get

$$
(3) \qquad p(fx_n, fx_{n+1}) \le (\sqrt{h})^{n-1} p(fx_2, fx_1) \quad \text{for all} \quad n \in \mathbb{N}.
$$

By (3) and Definition 2.2 (P4), for any $m \in \mathbb{N}$ we have

$$
\begin{aligned}
p(fx_n, fx_{n+m}) \le & \, p(fx_n, fx_{n+1}) + p(fx_{n+1}, fx_{n+2}) + \cdots + p(fx_{n+m-2}, fx_{n+m-1}) \\
& + p(fx_{n+m-1}, fx_{n+m}) \\
\le & \left[(\sqrt{h})^{n-1} + (\sqrt{h})^{n} + \cdots + (\sqrt{h})^{n+m-3} + (\sqrt{h})^{n+m-2}\right] p(x_2, x_1) \\
\le & \, \frac{(\sqrt{h})^{n-1}}{1-\sqrt{h}} p(x_2, x_1) \to 0 \quad \text{as} \quad n \to \infty \quad \text{since} \quad 0 < h < 1.
\end{aligned}
$$

By the Lemma 2.1 , we get for any $m \in \mathbb{N}$, $p^s(fx_n, fx_{n+m}) \le 2p(fx_n, fx_{n+m}) \to 0$ as $n \to \infty$. This yields $\{fx_n\}$ is a Cauchy sequence with respect to p^s and hence convergent by Lemma 2.2.

Thus, there exist some $t \in X$ such that

$$p(t,t) = \lim_{n\to\infty} p(fx_n, t) = \lim_{n,m\to\infty} p(fx_n, fx_m). \tag{4}$$

From (1) and (2) we have:

$$qH_p(Tx_n, Tx_{n-1}) \leq \sqrt{h}p(fx_{n-1}, fx_n)$$
$$H_p(Tx_n, Tx_{n-1}) \leq hp(fx_{n-1}, fx_n). \quad \text{for} \quad n \geq 2.$$

This implies that $\{Tx_n\}$ is a Cauchy sequence as $\{fx_n\}$ is a Cauchy sequence. Hence $\{Tx_n\}$ is convergent by completeness of $(CB^p(X), H_p)$.

Now, let $Tx_n \to N \in CB^p(X)$. Then we have the following:

$$\begin{aligned} p(t,N) &\leq p(t, fx_n) + p(fx_n, N) - p(fx_n, fx_n) \\ &\leq p(t, fx_n) + H_p(Tx_{n-1}, N) - H_p(Tx_{n-1}, Tx_{n-1}) \\ &\leq p(t,t) + 0 \quad \text{as} \quad n \to \infty \quad \text{by} \quad (4) \\ &= p(t,t) \end{aligned}$$

This implies that $t \in N$, since N is closed.

By compatibility of the mappings as in Definition 2.8 ($H_p(Tfx_n, fTx_n) = H_p(fTx_n, fTx_n)$), Proposition 2.1 (h3) and continuities of the mappings f, T we have :

$$\begin{aligned} p(ft, Tt) &\leq p(ft, ffx_{n+1}) + p(ffx_{n+1}, Tt)) - p(ffx_{n+1}, ffx_{n+1}) \\ &\leq p(ft, ffx_{n+1}) + H_p(fTx_n, Tt) - H_p(fTx_n, fTx_n) \\ &\leq p(ft, ffx_{n+1}) + H_p(fTx_n, Tfx_n) \\ &\quad + H_p(Tfx_n, Tt) - H_p(fTx_n, fTx_n) - \inf_{z \in Tfx_n} p(z,z) \\ &\leq p(ft, ft). \end{aligned}$$

So we have $p(ft, ft) \leq p(ft, Tt) \leq p(ft, ft)$.

This implies $p(ft, Tt) = p(ft, ft)$. Therefore $ft \in Tt$ since Tt is closed. This completes the proof.

Corollary 3.1. *Let (X,p) be a complete partial metric space. Let $T : X \to CB^p(X)$ and $f : X \to X$ be a continuous mapping satisfying $H_p(Tx,Ty) \le hp(fx, fy)$ for all $x,y \in X$, where $0 \le h < 1$ and $Tfx = fTx$. If the mappings f,T satisfy condition (i) of Theorem 3.1 , then the mappings f and T have a common coincidence point.*

Remark 3.1. Let (X,p) be a partial metric space. We denote by $PB^p(X)$ the collection of all non-empty and bounded subsets G of X such that for each $x \in X$, there exists a point $y \in G$ with $p(x,y) = p(x,G)$. If we define $T : X \to PB^p(X)$, then the iterative process z_n in the above proof can be simplified to the modified iteration scheme of Smithson [5], where Tx is compact and therefore contained in $PB^p(X)$. This can be done as follows: after selecting x_n, let $x_{n+1} \in X$ be such that $z_n = fx_{n+1} \in Tx_n$ and $p(fx_n, z_n) = p(fx_n, Tx_n)$. Clearly $PB^p(X) \subseteq CB^p(X)$ and therefore we have the following as corollary of Theorem 3.1.

Corollary 3.2. *Let (X,p) be a complete partial metric space. Let $f : X \to X$ and $T : X \to PB^p(X)$ be continuous mappings satisfying $fTx \in PB^p(X)$ for all $x \in X$. If the mappings f and T are such that $H_p(Tfx, fTx) \le p(fx,Tx)$ for all $x \in X$ and satisfy conditions (i) and (ii) of the Theorem 3.1 , then the mappings f and T have a common coincidence point.*

We now present an illustrative example for Theorem 3.1.

Example 3.1. Let $X = [1,\infty)$ and $p : X \times X \to [0,\infty)$ be a function given by $p(x,y) = \max\{x,y\}$, for all $x,y \in X$. Clearly (X,p) is a partial metric space. Define $fx = 2x^4 - 1$ and $Tx = [1,x^2]$ for each $x \ge 1$. Clearly, the mappings f and T are continuous and satisfy condition (i) of the Theorem 3.1. Since $\{fx_n\} \to 1$ and $\{Tx_n\} \to \{1\}$ if and only if $\{x_n\} \to 1$, $H_p(fTx_n, Tfx_n) = H_p(fTx_n, fTx_n) = 2{x_n}^8 - 1$ if and only if $\{x_n\} \to 1$ and $fTx_n = [1, 2{x_n}^8 - 1] \in CB^p(X)$ for all $x,y \in X$, then f and T are compatible mappings. Now, $H_p(Tx,Ty) = H_p([1,x^2],[1,y^2])$. Without loss of generality we assume $y \le x$. This implies that

$$\begin{aligned} H_p(Tx,Ty) &= x^2 \\ &\le \frac{2(x^2+1)(x^2-1)+1}{2} \\ &= \frac{1}{2}\max\{fx, fy\} \\ &= \frac{1}{2}p(fx, fy) \quad \text{for all} \quad x,y \in X. \end{aligned}$$

Thus, the mappings f and T satisfy condition (ii) of the Theorem 3.1 for $h = \frac{1}{2}$. Therefore by the Theorem 3.1 , 1 is a coincidence point for the mappings f and T. i.e. $t = 1$.

We now present a fixed point result by imposing appropriate restrictions to the mappings f and T as defined in Theorem 3.1.

Theorem 3.2. Let (X, p) be a complete partial metric space. Let $f : X \to X$ and $T : X \to CB^p(X)$ be compatible continuous mappings satisfying both conditions (i) and (ii) of the Theorem 3.1. Furthermore, if for each $x \in X$ either $fx \neq f^2x$ implies $fx \notin Tx$ or $fx \in Tx$ implies that $f^n x \to y$ for some $y \in X$. Then the mappings f and T have a common fixed point in X.

Proof. Since f and T satisfy both conditions of the Theorem 3.1 , then $ft \in Tt$ for some $t \in X$. We assume $fx \neq f^2x$ implies $fx \notin Tx$ for each $x \in X$. Now, by continuity of f and the Lemma 2.8 we have $f^2t \in fTt = Tft$. Thus we have $ft = f^2t \in Tft$. i.e. $f(t)$ is a common fixed point for f and T.

We assume $fx \in Tx$ implies that $f^n x \to y$ for some $y \in X$. By continuity of f we have $fy = y$. We now show that y is also a fixed point for T. By the Lemma 2.4 , $f^n t \in Tf^{n-1}t$ for each natural number n, and the continuity of T we have:

$$\begin{aligned}
p(y, Ty) &\leq p(y, f^n t) + p(f^n t, Ty) - p(f^n t, f^n t) \\
&\leq p(y, f^n t) + H_p(Tf^{n-1}, Ty) - H_p(Tf^{n-1}t, Tf^{n-1}t) \\
&\leq p(y, y) \\
&= p(y, y).
\end{aligned}$$

Therefore, $y \in Ty$ since Ty is closed. This completes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests.

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