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MAPPING NEARLY CONTINUOUS FUNCTION BY NEWTON-LIKE OPERATORS IN BAIRE'S SPACE

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Abstract: The paper presents a mapping $F_n: X \to Y$, where Facts on a topological space X into a regular topological space Y that is nearly continuous. We use Newton-like operators in Baire's space in the presence of singularities of Jacobian matrices with contributory factor of the Tikhonov regularization parameter introduced on the discontinuous system in order to enforce the homeomorphic image to be continuous under the induced closed graph theorem.

Keywords: nonlinear system; Baire's space; Newton-like methods; strange attractor.
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1.0 INTRODUCTION

In the paper, we study numerical solution of nonlinear system of equation

$$F(x) = 0, \tag{1.1}$$

where F is assumed to be nearly continuous on $D(x) \subset R^m \to R^n, m > n$.

Such problems are often encountered in Category theorems, e.g., the Baire's category theorems. We give a few properties of Baire space as a prelude to the discussion. Firstly, a complete metric space shares property of a Baire space and, a topological space that is homeomorphic to an open subset of a complete pseudo-metric space is also a Baire space. Such a good example of completely metrizable topology of Baire space would be the uniform boundedness principle , see e.g., McCoy (1975).We say that a subset T of a topological space X is nearly open if T

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belongs to the interior of its closure. A subspace T of a topological space X is said to be rare when $X \setminus T$ everywhere dense.

Particularly, as a measure of our discussion is that a bounded linear surjection of a Banach space X into a Banach space Y is an open mapping. The inverse mapping theorem states that a bounded linear bijection of a Banach space X onto a Banach space Y has a bounded inverse, that is in addition, a topological isomorphism.

In what follows, we relate the Jacobian matrices of the nonlinear system of Equation (1.1) to regularity spaces for $A \in \frac{\partial F_i}{\partial x_j}, (i = 1,2,...,n,\ j = 1,2,...,n)$ and for x near enough x^*, one can find $x_0 \in B(x^*, \varepsilon)$ such that

$$x_{k+1} = x_k + s_k, \qquad (k = 0,1,2,...,.) \tag{1.2}$$

Wherefrom,

$\|A(x_n)\, s_k + F(x_k)\| \le \eta \|F(x_k)\|$,the desired approximate solution converges to x^* for good enough starting point x_0.

Considering the perturbed Jacobian matrix $\hat{A} = A + \Delta A$ defined, and supposing there is a constant M in existence such that $\left\|\hat{A}^{-1}\right\| \le M$, $x \in B(x^*, \varepsilon)$, it would hold that $\left\|A(x_k) - \hat{A}_k\right\| \le K \Delta A$ for the Lipschitz matrix K. By using ideas closely related to Fowler and Kelley (2005) would lead us to write that:

$$\left\|A(x_k)^{-1}\right\| \le \frac{1}{\left(M^{-1} - K\Delta A\right)} \le 2M , \tag{1.3}$$

and

$$\|s_k\| \le 2M(\eta_k + 1)\|F(x_k)\| . \tag{1.4}$$

The monotone coerciveness of the mapping F is explained as follows:

Definition 1.1 (Ortaga and Rheinboldt (2000). A mapping $F : D \subset R^n \to R^n$ is weakly coercive on the open set $D_0 \subset D$ if there exists a point $t \in D_0$ with the property that, for any $\kappa > 0$, there is an open, bounded set $D_r \subset D_0$ containing t for which $\bar{D}_r \subset D_0$ and

$(x-t)^T F(x) > \kappa\|x-t\|_2, \forall x \in D_0$ in $\bar{D}_r$. By further setting $D = D_0 = R^n$, and $t = 0$ forces F to be coercive.

The concept of Uniform monotonicity theorem Ortega and Rheinboldt (2000) states that: If $F : R^n \to R^n$ is continuous and uniformly monotone, then F is a homeomorphism of R^n onto itself.

Via inclusion of nested sequences for the generated Jacobian matrices $A \in \frac{\partial F_i}{\partial x_j}, (i = 1,2,...,n, j = 1,2,...,n)$ the full rank matrix A in the topological space, which A_k occupy should be completely regular, uniformable, Hausdorff and Polish (i.e., metrizable , complete and separable).

Being inspired in this direction, we invoke the hereditary properties of Hausdorff spaces A_k for the matrices in the sense of Bourles (2014), namely:

∘ An open subset of a polish (resp. Suslin, Baire) space is such;

∘ Every polish space is Baire, the collections of countable many open subsets is dense;

∘ Every closed subsets of Baire space is a continuous image of the Baire space and every Polish space is a continuous image of the Baire space;

∘ A Banach space is barrelled just as ultrabornological space is barrelled, however, ultrabornological space need not be Baire as well.

Consequently following, a topological space is F polish space if F is a complete separable metrizable topological space, Hola (1990). We basically; may be interested in the effective Polish space, the complete separable metric space that has a computable representation.

A computable topological space is a computable locally compact separable Hausdorff space. Thus, such a computable topological space has effective intersection property, effective disjointness property, effective inclusion property and effective covering property. It must be stated that membership of a recursive closed set need not be decidable.

This inspires the following definition for a useful purpose.

Definition 1.1 Hola (1990). Let E, F be topological spaces. A mapping f from E to f is called nearly continuous if for every $x \in E$ and one can find a neighbourhood U of $f(x)$ such that the set $\overline{f^{-1}(U)}$ is a neighbourhood of x.

It holds that a function $f:E\to F$ will be called Borel measurable of class one if the pre-images of open sets are $F_\sigma-$sets.

Motivated by the above terminologies, we state the following relevant theorem.

Theorem 1.1, Hola (1990). Let E be a topological space with a $\sigma-$locally finite almost base and such that every singleton is a G_σ-set. Let F be a Polish space with at least two elements. The following statements are equivalent:

(1) E is a Baire space;

(2) Every nearly continuous mapping $f:E\to F$ which is Borel measurable of class one is continuous;

(3) Every nearly continuous mapping $f:E\to F$ whose graph is a G_σ-set is continuous.

We introduce in our presentation the homeomorphism induced by the following assertion pertaining to Baire Class one as follows:

Definition 1.3. A mapping $F:X\to Y$ acting between topological spaces X and Y that is point-wise limit of a sequence $(F_n:n\in N)$ of a continuous mapping is said to be of Baire Class one given that U is any open subset of Y for which

$$F^{-1}(U)\subseteq\bigcap_{n\in N}\left(\bigcup_{k\geq n}F_n^{-1}(U)\right)\subset F^{-1}(U),$$

implying that:

$F^{-1}\left(\bar{U}\right)\cap\operatorname{int}\overline{F^{-1}(U)}$ is a residual subset of $\operatorname{int}\overline{F^{-1}}(U)$.

The orbit of F is a sequence $\{x_i\}$ such that $x_{i+1}\in F(x_i)$ for every i .Similarly, the reverse of an orbit of F is an orbit of F^{-1}.

In the realm of computation, an operator F is Lower –semi-continuous if $F^{-1}(U)$ is open whenever U is open.

Consequently, we give the Cantor's theorem on Baire space for a useful purpose.

Cantor's theorem asserts that if X be a complete metric space and $x_1,x_2,...,x_n$ be a decreasing sequence of non-empty closed subsets of X , with diam $F_n\to 0$, there exists a point $x\in X\ni\bigcap_{n=1}^{\infty}F_n=\{x\}$, in addition it holds that $\bigcap_{n=1}^{\infty}X\neq\phi$.

The major task which every numerical analyst faces in arithmetical computation is how to relate a discontinuous system with continuity which says that if f be any map from a metric space X into another metric space Y there exists set of points of discontinuity of f that has empty interior.

2.0 PRELIMINARIES: THE NEWTON-LIKE METHODS

It may be difficult and boring re-evaluating the Jacobian at every iteration step. As a result, we use Broyden operators as a good alternative in the work. Broyden method is very useful in Chemical Engineering practices where the solution of interactions between chemical reactors converging to equilibrium condition may be required, Buhler and Barth (2001) and Jaulin et al (2001). The Broyden-rank one update in the sense of Golub and Vanloan (1983), Uwamusi (2004) for the case of system of nonlinear equation of order n is given by the equation

$$(B_k)d_k = F(x_k),\ (k = 0,1,...) \tag{2.1}$$

Where,

B_k is updated in the form**:**

$$B_{k+1} = B_k + \frac{(y_k - B_k d_k)d_k}{d_k^T d_k} \tag{2.2}$$

$d_k = x_k - x_{k-1}$, $y_k = F(x_k) - F(x_{k-1})$

The solution to system 1.1 will be written as

$$x_{k+1} = x_k + d_k,\ (k = 0,1,...,) \tag{2.3}$$

We take B_0 as a starting approximation for the Jacobian matrix $F^{/}(x_0)$ .The algorithmic structure is presented below for purposes of clarity:

ALGORITHM

1) Initialize $x_0, \varepsilon -$ order of accuracy
2) $k = 0$; compute

   $F(x_0),\quad J(x_0),$

   $B_0 d_0 = -F(x_0)$**,**

   $x_1 = x_0 + d_0$**.**

   $k = k + 1$

3) Update Jacobian Matrix

$$B_{k+1} = B_k + \frac{(y_k - B_k d_k) d_k^T}{(d_k^T d_k)}$$

$$d_k = x_k - x_{k-1}, y_k = F(x_k) - F(x_{k-1})$$

4) $x_{k+1} = x_k + d_k$

5) Compute the ratio

$$p_k = \frac{\left(\|F(x_k)\|_2^2 - \|F(x_k + d_k)\|_2^2\right)}{\theta(d_k)}$$

6) $\theta = \|F(x_k)\|_2^2 - \|F(x_k) + B_k(x) d_k\|_2^2$

7) If $\left\|B_k - \bar{J}(x)\right\|_2 \leq \Delta;\ and\ p_k < 1$write solution obtained at the k-th step

endif

end

We state the superlinear convergence of Broyden's method.

Let $F: R^n \times R^n$ be continuously differentiable in the open convex set $D \subset R^n$, and assume that $J \in \frac{\partial F_i}{\partial x_j}$. Given that there exists an $x^* \in D$ such that $F(x^*) = 0$ and $\frac{\partial F_i(x^*)}{\partial x_j}$ exists. Let $\varphi: R^n \times \Im(R^n) \to P\{\Im(R^n)\}$ be defined in a neighbourhood $N = N_1 \times N_2$ of $(x^*, J(x^*))$ where N_1 is contained in D and N_2 contains only the non-singular matrices. If in addition there are constants α_1, α_2 reals such that for each (x, B) in N such that $\bar{x} = x - B^{-1}F(x)$, the function φ satisfies

$$\|B - J(x^*)\|_F \leq \left(1 + \alpha_1 \max\left\{\left\|\bar{x} - x^*\right\|\right\}\right)\|B - J(x^*)\|_F + \alpha_2 \max\left\{\left\|\bar{x} - x^*\right\|, \|x - x^*\|\right\}$$

for each $\bar{B} \in (x, B)$. Then for arbitrary $r \in (0,1)$, there are positive constants $\varepsilon(r)$ and $\delta(r)$ such that for $\|x_0 - x^*\| < \varepsilon(r)$ and $\|B_0 - J(x^*)\| < \delta(r)$ and $B_{k+1} \in \varphi(x_k, B_k), k \geq 0$, the sequence $x_{k+1} = x_k - B_k^{-1}F(x_k)$ is well defined and converges to x^*.

Furthermore,

$$\|x_{k+1} - x^*\| \leq r\|x_k - x^*\| \quad \text{for each } k \geq 0 \text{ and } \{\|B_k\|\}, \left\{\left\|B_k^{-1}\right\|\right\}$$

3.0 THE METHOD OF SVD AND STRANGE ATTRACTORS FOR THE BALL.

The SVD approach for the system is given by

$$x_A(\varphi_A) = \sum_{i=1}^{n} \varphi(A) \frac{u_i^T b}{\sigma_i} v_i = V diag\left(U^T b\right) \Sigma^{-1} \varphi_A \ . \tag{3.1}$$

We give the Low rank approximation for the data (matrix) , the Jacobian matrix. Such applications are useful in image processing, signal processing as well as data analysis, Paige and Strakos (2006). Define $E = U \sum V^T$ as an approximation to A, we write that

$$\left\| E - U \sum V^T \right\|, \tag{3.2}$$

and,

$$\| EA - I_n \|_2^2 + \alpha \|x\|_2^2 . \tag{3.3}$$

With inverse matrix $E = V\left(\sum^T \sum + \alpha^2 I\right)^{-1} \sum U^T$, we discuss the iterated Landwebber method for system of Equation (3.1). Starting from the iterative system below

$$x_k = x_{k-1} + \omega A^T \left(b - A x_{k-1}\right), \tag{3.4}$$

where $x_0 = 0$ with a fixed parameter ω. Then after the kth iteration, we should have that

$$\begin{aligned} x_k &= \left(I - \omega A^T A\right)^k x_0 + \sum_{i=0}^{k-1} \left(I - \omega A^T A\right)^i \left(A^T b\right) \\ &= \left(A^T A\right)^{-1} \left(1 - \left(I - \omega A^T A\right)^k\right) \left(A^T b\right) \end{aligned} \tag{3.5}$$

Using $A^T A = V\left(\sum^T \sum\right) V^T$, $I = VV^T$, it would follow that

$$x_k = \sum_{i=1}^{k} \left(1 - \left(I - \omega \sigma_i^2\right)^k\right) \left(\frac{u_i^T b}{\sigma_i}\right) v_i \tag{3.6}$$

Method (3.6) converges if and only if the term $\left|1 - \omega \sigma_i^2\right| < 1$ which gives that $0 < \omega \sigma_i^2 < 2$ for which holds the estimate, $\omega < \dfrac{2}{\sigma_i^2} \ \forall i$.

As $\omega \to 0$ for large enough k, $\displaystyle \lim_{k\to\infty} \frac{\left(\omega + \sigma_i^2\right)^k - \omega^k}{\left(\omega + \sigma_i^2\right)^k} = \lim_{k\to\infty}\left(1 - \frac{\omega^k}{\left(\omega + \sigma_i^2\right)^k}\right) = 1.$

The Point of attraction for the Ball, the strange attractors using one –step stationary iterations in the sense of Ortega and Rheinboldt (2000) is defined:

$$x_{k+1} = G\left(x_k\right), k = 0,1,2,..., \ . \tag{3.7}$$

The operator $G : D \subset R^n \to R^n$ is assumed to be any of Newton methods, thus the k-step Newton –SOR and SOR-Newton methods for the map $G : D \subset R^n \to R^n$ has a fixed point $x^* \in \text{int}(D)$, G is F-differentiable at x^* and $G^/(x^*) = 0$. The x^* is a point of attraction for the process (3.7) and $R_1(\Im, x^*) = Q_1(\Im, x^*) = 0$. In addition, if for some ball, $S = S(x^*, r) \subset D$, the estimate

$\|G(x) - G(x^*)\| \le \kappa \|x - x^*\|^p$, $\forall x \in S$ with $O_R(\Im, x^*) \ge O_Q(\Im, x^*) \ge p$ for some $p > 1$, and for some $\beta > 0$ then, $O_R(\Im, x^*) = O_Q(\Im, x^*) = p$.

Utilizing the above preambles, we state the following definition.

Definition 3.1, Ortega and Rheinboldt(2000). Let $F : D \subset R^n \to R^n$ be F-differentiable at a point $x^* \in \text{int}(D)$ for which $F(x^*) = 0$. Assuming further that $R \in L(R^n)$ is such that $\tau = \rho(I - RF^/(x^*)) < 1)$. Then x^* is a point of attraction for the iteration $\ell : x_{k+1} = x_k - RF(x_k); k = 0,1,...,.$ and that $R_1(\Im, x^*) = \tau$.

For a continuous mapping $B : D \subset R^m \to L(R^n)$ at a point $x_0 \in D$ for which $B(x_0)$ is invertible, and a $\delta > 0$ such that $B(x)$ is invertible, and $\| B(x)^{-1} \| \le \eta, \forall x \in D \bigcap \bar{S}(x_0, \delta)$, the $B(x)^{-1}$ is feasible.

For verification of continuous map $F : (X, \rho) \to (Y, \eta)$, we set $u = u(x_0)$ in the metrizable space topology (X, ρ) for which $f(u) \subset V$. V is a neighbourhood of $f(x_0)$, that is , $V = V(f(x_0))$ .By open mapping theorem, we can find $\varepsilon > 0$ such that $B_\eta(f(x_0), \varepsilon) \subseteq V$ and $\eta(f(x), f(x_0))) < \varepsilon$ whenever $\rho(x, x_0) < \delta$ .That $f(B_\rho(x_0, \delta)) \subseteq B_\eta(f(x_0), \varepsilon)$ remains valid. The map $f : (X, \rho) \to (Y, \eta)$ is isomorphic to the defining base topology it generates. The connectedness for the map is implied by the Hausdorff space.

The path connected set for the map $f : (X, \rho) \to (Y, \eta)$ is constructed by choosing two arbitrary points $x_1, x_2 \in X$ for which f is continuous on (X, ρ). We take d as an open set in (Y, η). Inverse function theorem asserts that the homeomorphism defined on (X, ρ) implies that

$f^{-1}(d) = \left(f^{-1}(d) \cap x_1\right) \cup \left(f^{-1}(d) \cap x_2\right)$. Therefore, continuity of f on x_1 and x_2 implies that $f^{-1}(d) \cap x_1$, $f^{-1}(d) \cap x_2$ are open sets on x_1, respectively on x_2.

The component of the metric shows that f is connected on (X, ρ), the closure of f is also connected. Set as $X = \bigcup f_x$ and $x \in (X, \rho)$. Choose positive integers $m, n \in N$ we can find $f_n \cap f_m \neq \phi$ implying accumulation point is reached when $|f_m(x) - f_n(x)| < \varepsilon$.

The question is under what conditions will Broyden's method fail woefully even when the Nonlinear system of equations is well defined? The poised question is a major obstacle in enclosure problems for the Numerical analysts.

The above posed question is answered in the context of Corollary of Cantor's theorem.

Theorem 3.1 (Cantor's theorem). Supposing that X be a complete metric space, and $F_1 \supseteq F_2 \supseteq \ldots \supseteq$ be a decreasing sequence of nonempty closed subsets of X, with diam. $F_n \to O$. Then, there exists a point $x \in X$ such that $\bigcap_{n=1}^{\infty} F_n = \{x\}$. In particular, we have that $\bigcap_{n=1}^{\infty} F_n \neq \phi$.

We state a corollary to theorem 3.1 as follows:

Corollary : Let F be any map from a metric space X into another metric space Y. Then the set of points of discontinuity of F is either meagre or else has non empty interior.

Because of Equation 3.7, we suppose that there exists a map G that is a bounded linear map from a Banach space X into a normed linear map Y .Assuming further that $G[X]$ is non meagre in Y, it would hold that G is surjective , that is that, $G[X] = Y$ and an open mapping by implication..

4.0 MAIN RESULTS

We take the numerical example from Uwamusi (2004) given by the function

$$F(x) = 3x_1 - \cos(x_2 x_3) - 0.5 = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

$$x^{(0)} = \left(x_1^{(0)},\ x_2^{(0)},\ x_3^{(0)}\right) = (0.1\ ,0.1\ ,-0.1)^T$$

The results have been computed as in Uwamusi(2004) we used modified Broyden Jacobi method (MBJM) and modified Broyden SOR method (MBSORM) and obtained .

Table 1. Computed results for problem 1 with Modified Broyden Jacobi and Modified Broyden SOR methods.

Iteration k	Modified Broyden Jacobi method (MBJM) x_k	Modified Broyden SOR method (MBSORM) x_k
0	$(0.1,\ 0.1, -0.1)^T$	$(0.1,\ 0.1,\ -0.1)^T$
1	$\begin{pmatrix} 0.500000288 \\ 0.022430579 \\ -0.521505144 \end{pmatrix}$	$\begin{pmatrix} 0.499998229 \\ 0.022430546 \\ -0.521505164 \end{pmatrix}$
2	$\begin{pmatrix} 0.499997351 \\ 0.025064386 \\ -0.52305226 \end{pmatrix}$	$\begin{pmatrix} 0.499999974 \\ 0.025093054 \\ -0.523029910 \end{pmatrix}$
3	$\begin{pmatrix} 0.499999357 \\ 0.026585484 \\ -0.0522954796 \end{pmatrix}$	$\begin{pmatrix} 0.499999984 \\ 0.026583788 \\ -0.522944545 \end{pmatrix}$
4	$\begin{pmatrix} 0.5000000162 \\ 0.026771892 \\ -0.522914768 \end{pmatrix}$	$\begin{pmatrix} 0.499999999 \\ 0.026733511 \\ -0.522934902 \end{pmatrix}$

Table 2: Results for the function Evaluates for Modified Broyden Jacobi and Modified Broyden SOR methods.

Iteration k	Modified Broyden Jacobi method $F(x^{(k)})$	Modified Broyden SOR method $F(x^{(k)})$
0	$\begin{pmatrix} -1.199999985 \\ -2.171745328 \\ 8.462025346 \end{pmatrix}$	$\begin{pmatrix} -1.199999985 \\ -2.171745328 \\ 8.462025346 \end{pmatrix}$
1	$\begin{pmatrix} 0.000008714 \\ 0.086772061 \\ 0.030719995 \end{pmatrix}$	$\begin{pmatrix} 0.000008714 \\ 0.086772061 \\ 0.030719995 \end{pmatrix}$
2	$\begin{pmatrix} -0.000002938 \\ 0.002633807 \\ -0.001547119 \end{pmatrix}$	$\begin{pmatrix} -0.000007925 \\ 0.034852258 \\ -0.001523614 \end{pmatrix}$
3	$\begin{pmatrix} -0.000001905 \\ 0.003851255 \\ -0.00032515 \end{pmatrix}$	$\begin{pmatrix} -0.000000024 \\ 0.003885795 \\ -0.000119333 \end{pmatrix}$
4	$\begin{pmatrix} 0.000000510 \\ -0.000035404 \\ 0.000381915 \end{pmatrix}$	$\begin{pmatrix} 0.000000026 \\ -0.000098818 \\ -0.000000345 \end{pmatrix}$

Using the numerical information contained in Tables 1 and 2, we formed over determined linear systems for Modified Broyden Jacobi method and Modified Broyden SOR methods using averaging technique for the solution vectors and functional evaluates. We denote these systems as problems 2 and 3 respectively.

Problem 2

$$\begin{pmatrix} 1.0000 & 0.10000 & 0.0100 & 0.0010 \\ 1.0000 & 0.0003 & 0.0000 & 0.0000 \\ 1.0000 & 0.0012 & 0.0000 & 0.0000 \\ 1.0000 & 0.1581 & 0.0250 & 0.0040 \\ 1.0000 & 0.0013 & 0.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix} = \begin{pmatrix} 1.69676001 \\ 0.039166923 \\ 0.00036125 \\ 0.001174733 \\ 0.0001156737 \end{pmatrix}$$

With solution

$$\hat{m} = 1.0e+03 * \begin{pmatrix} 0.0001 \\ -0.0436 \\ 1.1631 \\ -5.6246 \end{pmatrix}.$$

Problem 3

$$\begin{pmatrix} 1.0000 & 0.10000 & 0.0100 & 0.0010 \\ 1.0000 & 0.0003 & 0.0000 & 0.0000 \\ 1.0000 & 0.0007 & 0.0000 & 0.0000 \\ 1.0000 & 0.0012 & 0.0000 & 0.0000 \\ 1.0000 & 0.0001 & 0.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 1.696760011 \\ 0.039166923 \\ 0.0111069063 \\ 0.0012554793 \\ -0.000003305 \end{pmatrix}$$

With solution

$$\bar{v} = 1.0e+05 * \begin{pmatrix} 0.0000 \\ 0.0007 \\ -0.6057 \\ 6.0057 \end{pmatrix}$$

The implication of the numerical example on the described method indicated that the map F has a closed graph for which $x_n \in D, x_n \to x$ and $f(x) = y$. We used Tikhonov type regularization procedure in conjunction with Cholesky –SVD type Factorization on problems 2 and 3 to obtain the solution to the linear systems of equation.

5.0 CONCLUSION

The paper described the inexact newton method often referred to as the Broyden's method which acts on a topological space $F: X \to Y$ for nonlinear system of equations. We used the Baire's category theorem as a prelude to our presentation. The modified Broyden's method using Jacobi and Successive Overrelaxation methods were adopted as earlier used in Uwamusi (2004) as a continuation path. It was established that computable topological space is a computable locally compact separable Hausdorff space. Thus such a computable topological space has effective intersection property, effective disjointness property, effective inclusion property and effective covering property. It was also pointed out that membership of a recursive closed set need not be decidable.

In the context of convergence, the point of attraction for the ball often seen as the strange attractor was stated in the sense of Ortega and Rheinboldt (2000). This enables us to have a focus on the ball-radius epsilon a strong criterion of Banach fixed point theory.

We described the Singular Values decomposition possible for adoption in the least squares solution occurring in the solution process which enables the noise filtering in the data space. We demonstrated the described methods on problem 1 using Cholesky –SVD factorization and results obtained are quite revealing.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] H.Bourles, On the Closed Graph theorem and Open mapping theorem, arXiv:1411.5500v1 [math.FA]. (2014).

[2] K.Buhler, W.Barth, A New intersection algorithm for parametric surfaces based on linear interval estimation. Scientific Computing, Validated Numerics, Interval Methods, (in Kramer and Wolf von Gudenberg edtn), Kluwer Academic/Plenum Publishers, New York, (2001), 179-190.

[3] J.Chung,M. Chung, D.P. O' Leary, Optimal filters from calibration Data for image deconvolution with Data acquisition error, J. Math. Imag. Vision, 44(3)(2012), 366-374.

[4] K.R.Fowler ,C.T.Kelley, Pseudo-Transient continuation for Nonsmooth nonlinear equation, SIAM J. Numer. Anal. 43(4)(2005), 1385-1406.

[5] G.Golub,F.Van-Loan, Matrix Computations. The Johns Hopkins University Press, Baltimore Maryland, (1983).

[6] L.Hola, Functional characterization of Baire space. Proc. Amer. Math. Soc. 109(4) (1990), 1121-1124.

[7] L.Jaulin, I.Breams, M.Kieffer, E.Walter, Nonlinear state estimation using forward-backward propagation of intervals in an algorithm. Scientific Computing, Validated Numerics, Interval Methods, (in Kramer and Wolf von Gudenberg edtn), Kluwer Academic/Plenum Publishers, New York (2001), 191-201.

[8] R.A.McCoy, Baire spaces and Hyper surfaces, Pac J Math. 58 (1)(1975), 133-142.

[9] J.M.Ortega, W.C. Rheinboldt, Iterative solution of Nonlinear equations in several variables. Society for Industrial and Applied Mathematics, Philadelphia, USA, (2000).

[10] C.Paige, Z. Strakos, Core problems in Linear Algebraic Systems, SIAM J. Matrix Anal. Appl. 27(3)(2006), 861-875.

[11] S.E.Uwamusi, On AOR Methods for modified Broyden's Method for Nonlinear Systems, Nigerian J. Appl. Sci. 22(2004), 41-47.