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ITERATIVE METHOD FOR SPLIT GENERALIZED EQUILIBRIUM AND FIXED POINT PROBLEMS FOR MULTIVALUED DEMICONTRACTIVE-TYPE MAPPINGS

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Abstract. In [4], [Fixed Point Theory and Applications (2018) 2018:6], W. Phuengrattana and K. Lerkchaiyaphum obtained some results on solving split generalized equilibrium problem and fixed point of multi-valued nonexpansive mappings in real Hilbert spaces. We observed a gap in the proof of their main result, Theorem 3.1. Motivated by their result, we first correct the observed error and study in this article, approximation of solution of generalized split equilibrium problem and common fixed point problem for a finite family of multi-valued demicontractive-type mappings in real Hilbert spaces.

Keywords: split equilibrium problem; demicontractive-type mapping; multi-valued mapping; quasi-nonexpansive mapping; Hilbert spaces.

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1. Introduction

Let H be a real Hilbert space with inner product $\langle , \rangle$ and norm $||.||$. Let C be a nonempty closed convex subset of H Suppose that $F, \varphi : C \times C \to \mathbb{R}$ are bi-functions, that is, $F(u,u) = \varphi(u.u) =$

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$0, \forall u \in C$. The generalized equilibrium problem is to find $x \in C$ such that

$$F(x,y) + \varphi(x,y) \geq 0,\ \forall y \in C. \tag{1.1}$$

We denote the set of solution of problem (1.1) by $GEP(F,\varphi)$. The generalized equilibrium problem is a unifying problem for many important problems araising from fixed point theory, physics, economics, optimization and so on (see e.g., [6, 7, 8]).

In 2013, Kazmi and Rizvi [9] introduced and studied the following split generalized equilibrium problem. Let H_1, H_2 be two real Hilbert spaces and $C \subseteq H_1$ and $Q \subseteq H_2$, let $F_1, \varphi_1 : C \times C \to \mathbb{R}$ and $F_2, \varphi_2 : Q \times Q \to \mathbb{R}$ be nonlinear bifunctions, and let $A : H_1 \to H_2$ be a bounded linear operator. The split generalized equilibrium problem is to find $\xi \in C$ such that

$$F_1(\xi, y) + \varphi_1(\xi, y) \geq 0\ \forall y \in C \text{ and such that} \tag{1.2}$$

$$y^* = A\xi \in Q \text{ solves } F_2(y^*, y) + \varphi_2(y^*, y) \geq 0\ \forall y \in C \tag{1.3}$$

The solution set of the split generalized equilibrium problem is denoted by$SGEP(F_1, \varphi_1, F_2, \varphi_2)$. That is:

$$SGEP(F_1, \varphi_1, F_2, \varphi_2) := \{\xi \in C : \xi \in GEP(F_1, \varphi_1) \text{ and } A\xi \in GEP(F_2, \varphi_2)\}.$$

The authors gave an iterative algorithm to find a common element of the solution set of the split generalized equilibrium problem in real Hilbert spaces; (see e.g. [9, 10, 11]). For $\varphi_1 = 0$ and$\varphi_2 = 0$, the split generalized equilibrium problem reduces to the split equilibrium problem studied by Moudafi [13] and Suantai *et.al.* [20]. For $F_2 = 0$ and $\varphi_2 = 0$, the split generalized equilibrium problem reduces to the equilibrium problem which has been studied extensively by many authors (see for instance, [6, 11, 15]).

Iterative approximation of fixed points of nonlinear mappings has been studied widely in the literature, (see e.g., [3, 12] and the references therein). One iterative method that has been used successfully to approximate fixed points of nonexpansive mappings is the *shrinking projection method* which was introduced by Takahashi et al. [16].This method has been studied and developed by many researchers under different settings, (see, for example, [14, 19]).

Recently, W. Phuengrattana and K . Lerkchaiyaphum, [4] proposed an iterative algorithm based on the shrinking projection method for finding a common element of the set of solutions of split generalized equilibrium problems and the set of common fixed points of a countable family of nonexpansive multivalued mappings in real Hilbert spaces. They proved strong convergence theorems that extend and improve the corresponding results of Kazmi and Rizvi [9], Suantai et al. [20], and others.

In the proof of the main result, Theorem 3.1 of [4], we observed some gap in their argument.

It is known that the study of multivalued mappings is in general more demanding than that of single-valued mappings. Motivated by the work of Kazmi and Rizvi [9], Takahashi et al. [16], W. Phuengrattana and K . Lerkchaiyaphum, [4] and the ongoing research in this direction, it is our purpose in this manuscript to first correct the observed error in the proof of Theorem 3.1 of W. Phuengrattana and K . Lerkchaiyaphum, [4]. Next, we propose an iterative algorithm based on the shrinking projection method for approximating a solution of split generalized equilibrium problem and a common fixed point of a finite family of multi-valued demicontractive -type mappings (see definition below) in real Hilbert spaces. The class of multi-valued demicontractive-type mappings is known to be more general than the class of multivalued nonexpansive mappings. Our result complements the result of Kazmi and Rizvi [9], improves and generalizes the results of W. Phuengrattana and K . Lerkchaiyaphum, [4] and many of other important results.

2. Preliminaries

Let X be a normed space. and $C \subseteq X$. A map $T : C \to C$ is called nonexpansive if

$$||Tx - Ty|| \leq ||x - y|| \ \forall \ x, y \in C \tag{2.1}$$

T is said to be quasi-nonexpansive if $F(T) \neq \emptyset$ and

(2.2) $||Tx - Tp|| \leq ||x - p|| \ \forall \ x \in C, p \in F(T),$ where $F(T)$ denotes the fixed point set of T.

In real Hilbert space H, Hicks and Kubicek [21] introduced the class of demicontractive mappings as follows;

A map $T : C \subseteq H \to C$ is called demicontractive if $F(T) \neq \emptyset$ and there exists $k \in [0, 1)$ such that

$$||Tx - Tp||^2 \leq ||x - p||^2 + k||x - Tx||^2 \ \forall \ x \in C, \ p \in F(T) \tag{2.3}$$

It is known that the class of demicontractive mappings is more general than the class of quasi nonexpansive mappings. Some researchers have studied this class of maps and obtained different important results (see for instance [3]). For a nonempty subset C of X, let $CB(C)$ denote the family of nonempty, closed and bounded subsets subsets of C. We denote the identity map on X by I, the weak topology of X by $\sigma(X,X^*)$, and the norm (or strong) topology of X by $(X,||.||)$. The Hausdorff metric on $CB(C)$ induced by metric d on X is defined by

$$H(A,B) = \ max\{sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\} \text{ for all } A,B \in CB(C).$$

Let $T : D(T) \subset X \to CB(X)$ be a multi-valued mapping on X. A point $x \in D(T)$ is called a fixed point of T if $x \in Tx$. The fixed points set of T is denoted by $F(T)$. A point $x \in D(T)$ is called a strict fixed point of T if $Tx = \{x\}$. The set $F_s(T) = \{x \in D(T) : Tx = \{x\}\}$ is called the strict fixed point set of T.

A multi-valued mapping $T : D(T) \subset X \to CB(X)$ is called $L-$ Lipschitzian if there exists $L > 0$ such that

$$H(Tx,Ty) \leq L||x-y|| \ \forall x,y \in D(T). \tag{2.4}$$

When $L \in (0,1)$ in (2.4), we say that T is a contraction, and T is called nonexpansive if $L = 1$. T is called quasi-nonexpansive mapping if $F(T) \neq \emptyset$, with

$$H(Tx,Tp) \leq ||x-p|| \ \forall x, \in D(T), \text{ and } \forall p \in F(T). \tag{2.5}$$

Clearly every multivalued nonexpansive mapping with nonempty fixed point set is multivalued quasi-nonexpansive.

Several papers have been published that deal with the problem of approximating fixed points of single valued and multi-valued nonexpansive mappings (see, for example [1, 2] and the references therein).

Recently, Isiogugu and Osilike [5] introduced and studied the class of multi-valued demicontractive-type mappings. Precisely, they gave the following definition.

Definition 2.1. Let X be a real normed space. A mapping $T : D(T) \subseteq X \to 2^X$ is said to be demicontractive-type in the terminology of Hicks and Kubicek [21] if $F(T) \neq \emptyset$ and for all

$p \in F(T), x \in D(T)$ there exists $k \in [0,1)$ such that

$$H^2(Tx,Tp) \leq ||x-p||^2 + kd^2(x,Tx), \tag{2.6}$$

where $H^2(Tx,Tp) = [H(Tx,Tp)]^2$ and $d^2(x,p) = [d(x,p)]^2$. If $k = 1$ in (2.6), then T is called a hemicontractive-type mapping.

The following are some examples of multivalued demicontractive-type mappings.

Example 2.2. Every multivalued quasi-nonexpansive mapping is demicontractive-type.

Example 2.3. Let $X = R$ (the reals with usual metric). Define $T : \mathbb{R} \to 2^{\mathbb{R}}$ by

$$Tx = \begin{cases} [-\frac{5}{2}x, -3x], \text{ if } x \in (-\infty, 0], \\ [-3x, -\frac{5}{2}x], \text{ if } x \in (0, \infty). \end{cases} \tag{2.7}$$

Then $F(T) = \{0\}$. Now

$$\begin{aligned} H^2\big(Tx,T0\big) &= |-3x-0|^2 = 9|x-0|^2 = |x-0|^2 + 8|x-0|^2 \\ d^2(x,Tx) &= |x-(-\frac{5}{2}x)|^2 = |\frac{7}{2}x|^2 = \frac{49}{4}|x|^2 \Rightarrow |x|^2 = \frac{4}{49}d^2(x,Tx) \\ 8|x|^2 &= \frac{32}{49}d^2(x,Tx). \text{ Hence,} \\ H^2\big(Tx,T0\big) &= |x-0|^2 + 8|x-0|^2 = |x-0|^2 + \frac{32}{49}d^2(x,Tx) \\ &\leq |x-0|^2 + \frac{32}{49}d^2(x,Tx) \end{aligned}$$

Thus, T is demicontractive-type mapping with $k = \frac{32}{49}$. However, for $x = 1, p = 0$,

$$\begin{aligned} H^2\big(Tx,T0\big) &= 9|x-0|^2 = 3^2|x-0|^2 > 1 = |x-0|^2. \text{ i.e.} \\ H\big(Tx,T0\big) &> |x-0|. \end{aligned}$$

Therefore, T is not quasi-nonexpansive. So the class of multivalued demicontractive-type mappings contains the class of multi-valued quasi-nonexpansive mappings, and also the class of multivalued nonexpansive mappings with fixed points. For more examples and details about the class of demicontractive-type mappings, see [5].

Lemma 2.4. *([17]) Let C be a nonempty closed convex subset of a real Hilbert space H, and let $P_C : H \to C$ be the metric projection. Then*

$$||y - P_C x||^2 + ||x - P_C x||^2 \leq ||x - y||^2, \ \forall x \in H, y \in C. \tag{2.8}$$

Lemma 2.5. *Let C be a nonempty, closed and convex subset of a real Hilbert space H. Given $x, y, z \in H$ and also given $a \in \mathbb{R}$, the set $\{v \in C : \|y - v\|^2 \leq \|x - v\|^2 + \langle z, v\rangle + a\}$ is convex and closed.*

For solving the generalized equilibrium problem, we assume that the bifunctions $F_1 : C \times C \to \mathbb{R}$ and $\varphi_1 : C \times C \to C$ satisfy the following assumption.

Assumption 2.6 Let C be nonempty closed and convex subset of a real Hilbert space H_1 . Let $F_1 : C \times C \to \mathbb{R}$ and $\varphi_1 : C \times C \to C$ be two bifunctions satisfy the following conditions:

(A1) $F_1(x,x) = 0 \ for \ all \ x \in C$,

(A2) F_1 is monotone, that is $F_1(x,y) + F_1(y,x) \leq 0 \ for \ all \ x,y \in C$,

(A3) F_1 is upper hemicontinuous, that is, for each $x,y,z \in C$, $\lim_{t \downarrow 0} F_1(tz + (1-t)x, y) \leq F_1(x,y)$.

(A4) $F(x,.)$ is convex and lower semi-continuous for each $x \in C$,

(A5) $\varphi_1(x,x) \geq 0$ for all $x \in C$,

(A6) for each $y \in C, x \to \varphi_1(x,y)$ is upper semicontinuous,

(A7) for each $x \in C, y \to \varphi_1(x,y)$ is convex and lower semicontinuous,

and assume that for fixed $r > 0$ and $z \in C$, there exists a nonempty compact convex subset K of H_1 and $x \in C \cap K$ such that

$$F_1(y,x) + \varphi_1(y_x) + \frac{1}{r}\langle y - x, x - z\rangle < 0 \ \forall y \in C \setminus K. \tag{2.9}$$

Lemma 2.6. *([18]) Let C be nonempty closed and convex subset of a real Hilbert space H_1. Let $F_1 : C \times C \to \mathbb{R}$ and $\varphi_1 : C \times C \to \mathbb{R}$ be two bifunctions satisfing Assumption 2.6. Assume that φ_1 is monotone. For $r > 0$ 0 and $x \in H_1$, define a mapping $T_r^{(F_1,\varphi_1)} : H_1 \to C$ as follows:*
$T_r^{(F_1,\varphi_1)}(x) = \{z \in C : F_1(z,y) + \varphi_1(z,y) + \frac{1}{r}\langle y - z, z - x\rangle \geq 0, \ y \in C\}$
for all $x \in H_1$. Then the following conclusions hold:

(1) for each $x \in H_1, T_r^{(F_1,\varphi_1)}(x) \neq \emptyset$,

(2) $T_r^{(F_1,\varphi_1)}$ is single- valued,

(3) $T_r^{(F_1,\varphi_1)}$ *is firmly nonexpansive, i.e.* $||T_r^{(F_1,\varphi_1)}x - T_r^{(F_1,\varphi_1)}y|| \leq \langle T_r^{(F_1,\varphi_1)}x - T_r^{(F_1,\varphi_1)}y, x-y\rangle$

(4) $F(T_r^{(F_1,\varphi_1)}) = GMEP(F_1,\varphi_1)$*,*

(5) $GMEP(F_1,\varphi_1)$ *is compact and convex.*

Further, assume that $F_2 : Q\times Q \to \mathbb{R}$ and $\varphi_2 : Q\times Q \to \mathbb{R}$ satisfy Assumption 2.6, where Q is a nonempty closed and convex subset of a real Hilbert space H_2. For all $s > 0$ and $w \in H_2$, define the mapping $T_r^{(F_2,\varphi_2)} : H_2 \to C$ as follows:

$T_r^{(F_2,\varphi_2)}(v) = \{\xi \in Q : F_2(\xi,y) + \varphi_2(\xi,y) + \frac{1}{r}\langle y-\xi, \xi - v\rangle \geq 0,\ y \in C\}$

for all $x \in H_2$.

Then we have:

(1) for each $v \in H_2, T_r^{(F_1,\varphi_1)}(v) \neq \emptyset$,

(2) $T_r^{(F_2,\varphi_2)}$ is single- valued,

(3) $T_r^{(F_2,\varphi_2)}$ is firmly nonexpansive, i.e. $||T_r^{(F_2,\varphi_2)}x - T_r^{(F_2,\varphi_2)}y|| \leq \langle T_r^{(F_2,\varphi_2)}x - T_r^{(F_2,\varphi_2)}y, x-y\rangle$

(4) $F(T_r^{(F_2,\varphi_2)}) = GMEP(F_2,\varphi_2)$,

(5) $GMEP(F_2,\varphi_2)$ is compact and convex,

where $GEP(F_2,\varphi_2)$ is the solution set of the following generalized equilibrium problem:

Find $y^* \in Q :$ such that $F_2(y^*,y) + \varphi_2(y^*,y) \geq 0$ for all $y \in Q$. Moreover, $SGEP(F_1,\varphi_1,F_2,\varphi_2)$ is a closed and convex set.

Lemma 2.7. *(*[19]*) Let H be a real Hilbert space and $\{x_i, i = 1,2,...,m\} \subset H$. For $\alpha_i \in (0,1), i = 1,2,\cdots,m$, such that $\sum_{i=1}^m \alpha_i = 1$,the following identity holds:*

$$||\sum_{i=1}^m \alpha_i x_i||^2 = \sum_{i=1}^m \alpha_i ||x_i||^2 - \sum_{i,j=1, i\neq j}^m \alpha_i\alpha_j ||x_i - x_j||^2.$$

3. Main Results

Remark 3.1. (1). Lemma 2.2 of the paper of Phuengrattana *et. al.* [4] holds for finitely many vectors in a real Hilbert space and finitely many scalars too. This Lemma played key role in obtaining many conclusions in the proof of their main result. For instance, conclutions (3.8), (3.9) and (3.10) all follow from the Lemma 2.2. This reduces their Theorem to the case of finite family of the operators considered and not countably infinite family as claimed by the authors.

(2). In order to prove step 5 of their main result, the authors made the following estimates

$$
\begin{aligned}
d(q,S_iq) &\leq ||q-u_n||+||u_n-y_n^i||+d(y_n^i,S_iq)\\
&\leq ||q-u_n||+dist(u_n,S_iu_n)+\mathscr{H}(S_iu_n,S_iq)\\
&\leq 2||q-u_n||+dist(u_n,S_iu_n)\\
&\leq 2||q-z_n||+||z_n-x_n||+dist(u_n,S_iu_n).
\end{aligned}
\tag{3.1}
$$

By inequality (3.1), the authors claim that $||u_n-y_n^i||\leq dist(u_n,S_iu_n),\ y_n^i\in S_iu_n$. This certainly, is not correct. However, the conclusion they got using this wrong assumption can be obtained without the assumption as follows.

$$
\begin{aligned}
d(q,S_iq) &\leq ||q-u_n||+||u_n-y_n^i||+d(y_n^i,S_iq)\\
&\leq ||q-u_n||+||u_n-y_n^i||+\mathscr{H}(S_iu_n,S_iq)\\
&\leq 2||q-u_n||+||u_n-y_n^i||\\
&\leq 2\big(||q-z_n||+||z_n-x_n||\big)+||u_n-y_n^i||.
\end{aligned}
\tag{3.2}
$$

Since $lim_{n\to\infty}z_n=q$, and from (3.7) and (3.14) of their proof, $\lim_{n\to\infty}||z_n-x_n||=0$ and $\lim_{n\to\infty}||u_n-y_n^i||=0$, respectively, it follows that $d(q,S_iq)=0$ as required.

Now we prove our main result.

Theorem 3.2. *Let C be a nonempty closed convex subset of a real Hilbert space H_1, and let Q be a nonempty closed convex subset of a real Hilbert space H_2 . Let $A:H_1\to H_2$ be a bounded linear operator, and let $\{S_i\}_{i=1}^m$ be a finite family of demicontractive-type multivalued mappings of C into $CB(C)$, with $k\in(0,1)$. Let $F_1,\varphi_1:C\times C\to\mathbb{R}$, $F_2,\varphi_2:Q\times Q\to\mathbb{R}$ be bifunctions satisfying Assumption 2.6. Let φ_1,φ_2 be monotone, φ_1 be upper hemicontinuous, and F_2 and φ_2 be upper semicontinuous in the first argument. Assume that $F(S_i)$ is nonempty, closed and convex for each $i=1,2,\cdots,m$. For $\Omega=\cap_{i=1}^mF(S_i)$, suppose $\Gamma:=\Omega\cap SGEP(F_1,\varphi_1,F_2,\varphi_2)\neq\emptyset$ with $T_ip=\{p\}$ for each $p\in\Omega$. Let $x_1\in C$ with $C_1=C$, and let $\{x_n\}$ be a sequence generated* *by*

$$
(3.3)\qquad \begin{cases} u_n = T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_2,\varphi_2)})A\big)x_n, \\ z_n = \alpha_n^{(0)}u_n + \alpha_n^{(1)}y_n^{(1)} + \cdots + \alpha_n^{(m)}y_n^{(m)},\ y_n^{(i)} \in S_i u_n, \\ C_{n+1} = \{z \in C_n : \|z_n - z\| \leq \|x_n - z\|\}, \\ x_{n+1} = P_{C_{n+1}}x_1, n \in \mathbb{N}, \end{cases}
$$

where $k = \max\{k_i\ i = 1,2,\cdots,m\}$ $\{\alpha_n^{(i)}\} \subset (k,1)$ *satisfies* $\sum_{i=0}^m \alpha_n^{(i)} = 1, \{r_n\} \subseteq (0,\infty)$, *and* $\gamma \in (0, \frac{1}{L})$, *where L is the spectral radius of* A^*A, *and* A^* *is the adjoint of* A. *Assume that the following conditions hold:*

(1) The $\lim_{n\to\infty} \alpha_n^{(i)} \in (0,1)$ *exists for all* $i \geq 0$,

(2) $liminf_{n\to\infty} r_n > 0$. *Then the sequence* $\{x_n\}$ *generated by* (3.24) *converges strongly to* $P_\Gamma x_1$.

Proof. We divide the proof into steps.

Step 1. We show that $\{x_n\}$ is well-defined for every $n \in \mathbb{N}$.

Since we assume that Ω is a closed and convex subset of C. Then, by Lemma 2.6 we obtain that Γ is a closed and convex subset of C. Also, from Lemma 2.5, C_{n+1} is closed and convex for each $n \in \mathbb{N}$.

Let $x^* \in \Gamma$, then using the nonexpansiveness of $T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_1,\varphi)})A$, we get

$$
\begin{aligned}
||u_n - x^*|| &= ||T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_1,\varphi)})Ax_n - T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_1,\varphi)})Ax^*|| \\
&\leq ||x_n - x^*||
\end{aligned}
$$

Since $S_i x^* = \{x^*\}\ \ forall x^* \in \cap_{i=1}^m F(S_i)$ we have:

$$
\begin{aligned}
||z_n - x^*||^2 &\leq \alpha_n^{(0)}||x_n - x^*||^2 + \sum_{i=1}^m \alpha_n^{(i)}||y_n^{(i)} - x^*||^2 \\
&= \alpha_n^{(0)}\|u_n - x^*\|^2 + \sum_{i=1}^m \alpha_n^{(i)}\|y_n^i - x^*\|^2 - \sum_{i=1}^m \alpha_0\alpha_{n,i}\|u_n - y_n^i\|^2 - \sum_{i,j=1,i\neq j} \alpha_{n,i}\,\alpha_{n,j}\|y_n^i - y_n^j\|^2 \\
&\leq \alpha_n^{(0)}\|u_n - x^*\|^2 + \sum_{i=1}^m \alpha_n^{(i)}H^2(S_i u_n, S_i x^*) - \sum_{i=1}^m \alpha_0\alpha_{n,i}\|u_n - y_n^i\|^2 \\
&\leq \alpha_n^{(0)}\|u_n - x^*\|^2 + \sum_{i=1}^m \alpha_n^{(i)}[\|u_n - x^*\|^2 + k_i d^2(u_n, S_i u_n)] - \sum_{i=1}^m \alpha_0\alpha_{n,i}\|u_n - y_n^i\|^2
\end{aligned}
$$

$$\leq \alpha_n^{(0)}\|u_n - x^*\|^2 + \sum_{i=1}^{m} \alpha_n^{(i)}[\|u_n - x^*\|^2 + k_i\|u_n - y_n^i\|^2] - \sum_{i=1}^{m} \alpha_0 \alpha_{n,i}\|u_n - y_n^i\|^2$$

$$= \|x_n - x^*\|^2 + (k - \alpha_n^0)\sum_{i=1}^{m} \alpha_n^{(i)}\|u_n - y_n^i\|^2 \tag{3.4}$$

$$\leq \|x_n - x^*\|^2$$

This shows that $x^* \in C_{n+1}$ and hence $\Gamma \subseteq C_{n+1} \subseteq C_n$. Therefore,$P_{Cn+1}x_1$ is well-defined for every $x_1 \in C$. Hence, $\{x_n\}$ is well-defined.

Step 2: We show that $\underset{n\to\infty}{lim}\|x_n - p\|$ exists, for some $p \in C$.

Since $x_n = P_{C_n}x_1$ and $x_{n+1} \in C_{n+1} \subset C_n \; \forall \; n \geq 0$. We have

$$\|x_n - x_1\| \leq \|x_{n+1} - x_1\| \tag{3.5}$$

For $x^* \in \Gamma \subset C_{n+1} \subset C_n$, we have

$$\|x_n - x_1\| \leq \|x^* - x_1\| \tag{3.6}$$

From (3.5) and (3.6) , we have that $\{||x_n - x_1||\}$ is a non-decreasing and bounded sequence. Therefore, $\lim_{n\to\infty}\|x_n - x_1\|$ exists.

We show that the sequence $\{x_n\}$ converges strongly to $p \in C$. Since $x_m = P_{C_m}x_1 \in C_m \subset C_n$ for $m > n$ we obtain from Lemma 2.4, we have that

$$\|x_m - x_n\|^2 \leq \|x_m - x_1\|^2 - \|x_n - x_1\|^2 \tag{3.7}$$

Since $\lim_{n\to\infty}\|x_n - x_1\|$ exists, we have from(3.7) that

$$\lim_{n\to\infty}\|x_m - x_n\| = 0$$

Thus $\{x_n\}$ is a Cauchy sequence. By the completeness of H and the closedness of C there exists $p \in C$ such that $\{x_n\}$ converges to p.

Step 3: We show that $\lim_{n\to\infty}\|y_n^{(i)} - x_n\| = 0, i = 1,2,...,m$. Since $x_{n+1} \in C_{n+1}$, from (3.1), we have

$$\|x_n - z_n\| \leq \|x_n - x_{n+1}\| + \|x_{n+1} - z_n\|$$

$$\leq 2\|x_n - x_{n+1}\| \to 0, \; as \; n \to \infty.$$

From (3.4), we have

$$
\begin{aligned}
(\alpha_n^{(0)}-k)\sum_{i=1}^{m}\alpha_n^i\|u_n-y_n^{(i)}\|^2 &\le \|u_n-x^*\|^2-\|z_n-x^*\|^2 \\
&\le \|x_n-x^*\|^2-\|z_n-x^*\|^2
\end{aligned}
$$

and for each $i=1,2,...,m$, we have

$$
\begin{aligned}
(\alpha_n^{(0)}-k)\alpha_{n,i}\|u_n-y_n^{(i)}\|^2 &\le \|x_n-x^*\|^2-\|z_n-x^*\|^2 \\
&\le M\|x_n-z_n\|\to 0,\ as\ n\to\infty, \qquad (3.8)
\end{aligned}
$$

Where $M=\sup_{n\ge 0}\{\|x_n-x^*\|+\|z_n-x^*\|\}$.
Using conditions (i) and (ii) in (3.8) , we have

$$
\lim_{n\to\infty}\|u_n-y_n^{(i)}\|^2=0, i=1,2,...,m \qquad (3.9)
$$

Claim: $lim_{n\to\infty}||u_n-x_n||=0$. Let $x^*\in\Gamma$, then

$$
\begin{aligned}
||u_n-x^*||^2 &= ||T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_1,\varphi)})Ax_n\big)-T_{r_n}^{(F_1,\varphi_1)}x^*||^2 \\
&\le ||\big(I-\gamma A^*(I-T_{r_n}^{(F_1,\varphi)})Ax_n\big)-x^*||^2 \\
&\le ||x_n-x^*||^2+\gamma^2||A^*(I-T_{r_n}^{(F_2,\varphi_2)})Ax_n||^2+2\gamma\langle x^*-x_n,A^*(I-T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle \\
&\le ||x_n-x^*||^2+\gamma^2\langle Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n,AA^*(I\text{–}T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle \\
&+ 2\gamma\langle A(x^*-x_n),Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n\rangle \\
&\le ||x_n-x^*||^2+L\gamma^2\langle Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n,Ax_n\text{–}T_{r_n}^{(F_2,\varphi_2)}Ax_n\rangle \\
&+ 2\gamma\langle A(x^*-x_n)+Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n-Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n,Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n\rangle \\
&\le ||x_n-x^*||^2+L\gamma^2||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2 \\
&+ 2\gamma\big(\langle Ax^*-T_{r_n}^{(F_2,\varphi_2)}Ax_n,Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n\rangle-||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2\big) \\
&\le ||x_n-x^*||^2+L\gamma^2||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2 \\
&+ 2\gamma\big(\frac{1}{2}||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2-||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2\big) \\
&= ||x_n-x^*||^2+\gamma(L\gamma-1)||Ax_n-T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2.
\end{aligned}
$$

Observe that from (3.4),

$$
\begin{aligned}
||z_n - x^*||^2 &\leq \alpha_{n,0}\|u_n - x^*\|^2 + \sum_{i=1}^{m}\alpha_{n,i}[||u_n - x^*\|^2 + k_i||u_n - y_n^i||^2] - \sum_{i=1}^{m}\alpha_0\alpha_{n,i}\|u_n - y_n^i\|^2 \\
&\leq \alpha_{n,0}\|x_n - x^*\|^2 + \sum_{i=1}^{m}\alpha_{n,i}\|u_n - x^*\|^2 \qquad (3.10) \\
&\leq \alpha_{n,0}\|x_n - x^*\|^2 + \sum_{i=1}^{m}\alpha_{n,i}[||x_n - x^*||^2 + \gamma(L\gamma - 1)||Ax_n - T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2.] \\
&= \|x_n - x^*\|^2 + \gamma(L\gamma - 1)\sum_{i=1}^{m}\alpha_{n,i}||Ax_n - T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2 \\
&= \|x_n - x^*\|^2 - \gamma(1 - L\gamma)(1 - \alpha_{n,0})||Ax_n - T_{r_n}^{(F_2,\varphi_2)}Ax_n||^2. \qquad (3.11)
\end{aligned}
$$

Using condition (1) and the fact that $\gamma(1 - L\gamma) > 0$, we get from (3.10) that

$$
\lim_{n\to\infty} ||Ax_n - T_{r_n}^{(F_2,\varphi_2)}Ax_n|| = 0. \qquad (3.12)
$$

But $T_{r_n}^{(F_1,\varphi_1)}$ is firmly nonexpansive and $(I - \gamma A^*(I - T_{r_n}^{(F_2,\varphi_2)}))$ is nonexpansive, so

$$
\begin{aligned}
||u_n - x^*||^2 &= ||T_{r_n}^{(F_1,\varphi_1)}\big(I - \gamma A^*(I - T_{r_n}^{(F_1,\varphi)})Ax_n\big) - T_{r_n}^{(F_1,\varphi_1)}x^*||^2 \\
&\leq \langle T_{r_n}^{(F_1,\varphi_1)}\big(I - \gamma A^*(I - T_{r_n}^{(F_1,\varphi)})Ax_n\big) - T_{r_n}^{(F_1,\varphi_1)}x^*, (I - \gamma A^*(I - T_{r_n}^{(F_2,\varphi_2)}))Ax_n - x^*\rangle \\
&= \langle u_n - x^*, (I - \gamma A^*(I - T_{r_n}^{(F_2,\varphi_2)}))Ax_n - x^*\rangle \\
&\leq \frac{1}{2}\big(||u_n - x^*||^2 + ||(I - \gamma A^*(I - T_{r_n}^{(F_1,\varphi)})Ax_n - x^*||^2 - ||(u_n - x_n - \gamma A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n||^2\big) \\
&\leq \frac{1}{2}\big(||u_n - x^*||^2 + ||x_n - x^*||^2 - (||u_n - x_n||^2 + \gamma^2||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n||) \\
&\quad - 2\gamma\langle u_n - x_n, A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle\big). \qquad (3.13)
\end{aligned}
$$

Hence

$$
\begin{aligned}
||u_n - x^*||^2 &\leq ||x_n - x^*||^2 - ||u_n - x_n||^2 + 2\gamma\langle u_n - x_n, A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle \\
&\leq ||x_n - x^*||^2 - ||u_n - x_n||^2 + 2\gamma||u_n - x_n||||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n|| \qquad (3.14)
\end{aligned}
$$

Using (3.14) in (3.10), we obtain

$$
\begin{aligned}
||z_n - x^*||^2 &\leq \alpha_{n,0}\|u_n - x^*\|^2 + \sum_{i=1}^{m}\alpha_{n,i}[||u_n - x^*\|^2 + k_i||u_n - y_n^i||^2] - \sum_{i=1}^{m}\alpha_0\alpha_{n,i}\|u_n - y_n^i\|^2 \\
&\leq \alpha_{n,0}\|x_n - x^*\|^2 + \sum_{i=1}^{m}\alpha_{n,i}[||x_n - x^*||^2 - ||u_n - x_n||^2 + 2\gamma||u_n - x_n||||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n||] \\
&= \|x_n - x^*\|^2 - (1-\alpha_{n,0})||u_n - x_n||^2 \\
&+ 2\gamma(1-\alpha_{n,0})K||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n||, \text{ where } K = \sup\{||u_n - x_n||\}. \qquad (3.15)
\end{aligned}
$$

Hence

$$
\begin{aligned}
(1-\alpha_{n,0})||u_n - x_n||^2 &\leq \|x_n - x^*\|^2 - \|z_n - x^*\|^2 + 2\gamma(1-\alpha_{n,0})K||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n|| \\
&\leq ||x_n - z_n||\big(\|x_n - x^*\| + \|z_n - x^*\|\big) + 2\gamma(1-\alpha_{n,0})K||A^*(I - T_{r_n}^{(F_2,\varphi_2)})Ax_n||. \qquad (3.16)
\end{aligned}
$$

Thus,

$$
\lim_{n\to\infty}||u_n - x_n|| = 0. \qquad (3.17)
$$

From (3.17) and (3.9) , we get that

$$
\lim_{n\to\infty}||y_n^{(i)} - x_n|| = 0 \qquad (3.18)
$$

completing the proof of **step 3**.

Step 4. We prove that $p \in \cap_{i=1}^m F(T_i)$.

For each $i = 1, 2, \cdots, m$,

$$
\begin{aligned}
d(p, S_i p) &\leq ||p - u_n|| + ||u_n - y_n^i|| + d(y_n^i, S_i p) \\
&\leq ||p - u_n|| + ||u_n - y_n^i|| + H(S_i u_n, S_i p) \\
&\leq ||p - u_n|| + ||u_n - y_n^i|| + H^2(S_i u_n, S_i p) \\
&\leq ||p - u_n|| + ||u_n - y_n^i|| + ||u_n - p||^2 + kd^2(u_n, S_i u_n) \\
&\leq ||p - u_n|| + ||u_n - y_n^i|| + ||u_n - p||^2 + k||u_n - y_n^{(i)}||^2 \qquad (3.19)
\end{aligned}
$$

Observe that

$$(3.20)\qquad ||p-u_n|| \leq ||p-z_n||+||z_n-x_n||+||x_n-u_n|| \to 0 \text{ as } n\to\infty. \text{ Hence,}$$

$$(3.21)\qquad ||p-u_n||^2 \to 0, \text{ as } n\to\infty.$$

Furthermore, from (3.9), we have that $k||u_n-y_n^{(i)}||^2 \to 0$ as $n\to\infty$.

Consequently, $d(p,S_ip)=0$, and so, $p\in\cap_{i=1}^m F(T_i)$.

The rest of the proof follows the same argument as steps 6 and 7 of [4]. We give details here, for completion.

Step 6. We show that $p\in SGEP(F_1,\varphi_1,F_2,\varphi_2)$. First, we show that $p\in GEP(F_1,\varphi_1)$. Since $u_n=T_{r_n}^{(F_1,\varphi_1)}\big(I-\gamma A^*(I-T_{r_n}^{(F_2,\varphi_2)})A\big)x_n$, we have

$$F_1(u_n,y)+\varphi_1(u_n,y)+\frac{1}{r_n}\langle y-u_n,u_n-x_n-\gamma A^*(I-T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle\geq 0,\ \forall y\in C,$$

which implies that

$$F_1(u_n,y)+\varphi_1(u_n,y)+\frac{1}{r_n}\langle y-u_n,u_n-x_n\rangle-\frac{1}{r_n}\langle y-u_n,\gamma A^*(I-T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle\geq 0,\ \forall y\in C,$$

It follows from the monotonicity of F_1 and φ_1 that

$$\frac{1}{r_n}\langle y-u_n,u_n-x_n\rangle-\frac{1}{r_n}\langle y-u_n,\gamma A^*(I-T_{r_n}^{(F_2,\varphi_2)})Ax_n\rangle\geq F_1(y,u_n)+\varphi_1(y,u_n),\ \forall y\in C,$$

Using conclusion (3.17)and the fact that and $lim_{n\to\infty}x_n=p$ we get that $\lim_{n\to\infty}u_n=p$. It follows by Condition (2) , (3.12), (3.10), Assumption 2.7, (A4) and (A7), that $0\geq F_1(y,p)+\varphi_1(y,p)\ \forall y\in C$. Put $y_t=ty+(1-t)p\ \forall\ t\in(0,1]$ and$y\in C$. Consequently, we get $y_t\in C$, and hence $F_1(y_t,p)+\varphi_1(y_t,p)\leq 0$. So by Assumption 2.7, $(A1)-(A7)$, we have

$$\begin{aligned}
0 &\leq F_1(y_t,y_t)+\varphi_1(y_t,y_t)\\
&\leq t\big(F_1(y_t,y)+\varphi_1(y_t,y)\big)+(1-t)\big(F_1(y_t,p)+\varphi_1(y_t,p)\big)\\
&\leq t\big(F_1(y_t,y)+\varphi_1(y_t,y)\big)+(1-t)\big(F_1(p,y_t)+\varphi_1(p,y_t)\big)\\
&\leq F_1(y_t,y)+\varphi_1(y_t,y).
\end{aligned}$$

Hence we have $F_1(y_t,y)+\varphi_1(y_t,y)\geq 0,\ \forall y\in C$.

Letting $t\to 0$, by Assumption 2.7 (A3) and the upper hemicontinuity of φ_1 we have $F_1(p,y)+$

$\varphi_1(p,y) \geq 0,\ \forall\, y \in C$. This implies that $p \in GEP(F_1,\varphi_1)$.

Since A is a bounded linear operator, we have $Ax_n \to Ap$. Then, it follows from (3.12) that

$$T_{r_n}^{(F_2,\varphi_2)}Ax_n \to Ap. \tag{3.22}$$

By the definition of $T_{r_n}^{(F_2,\varphi_2)}Ax_n$, we have

$$F_2(T_{r_n}^{(F_2,\varphi_2)}Ax_n,y)+\varphi_2(T_{r_n}^{(F_2,\varphi_2)}Ax_n,y)+\frac{1}{r_n}\langle y-T_{r_n}^{(F_2,\varphi_2)}Ax_n, T_{r_n}^{(F_2,\varphi_2)}Ax_n - Ax_n\rangle \geq 0,\ \forall y \in Q.$$

Since F_2 and φ_2 are upper semicontinuous in the first argument, it follows by (3.22) that

$$F_2(Ap,y)+\varphi_2(Ap,y) \geq 0,\ \forall\, y \in Q. \tag{3.23}$$

This shows that $Ap \in GEP(F_2,\varphi_2)$. Therefore $p \in SGEP(F_1,\varphi_1,F_2,\varphi_2)$.

Step 7. Finally, we show that $p = P_\Gamma x_1$.

Since $x_n = P_{C_n}x_1$, and $\Gamma \subset C_n$, we get $\langle x_1 - x_n, x_n - x^*\rangle \geq 0\ \ \forall x^* \in \Gamma$. Hence, $\langle x_1 - p, p - x^*\rangle \geq 0\ \forall x^* \in \Gamma$. This shows that $p = P_\Gamma x_1$. So we conclude that $\{x_n\}$ converges strongly to $P_\Gamma x_1$. □

As we observed earlier, every multivalued nonexpansive mapping with nonempty fixed point set is a multivalued demicontractive-type mapping. Consequently, we obtain the main result of Phuengrattana *et. al.* [4] as a corollary of the main result of our work.

Corollary 3.3. *Let C be a nonempty closed convex subset of a real Hilbert space H_1, and let Q be a nonempty closed convex subset of a real Hilbert space H_2 . Let $A : H_1 \to H_2$ be a bounded linear operator, and let $\{S_i\}$ be a finite family of multivalued nonexpansive mappings of C into $CB(C)$. Let $F_1,\varphi_1 : C\times C \to \mathbb{R}$, $F_2,\varphi_2 : Q\times Q \to \mathbb{R}$ be bifunctions satisfying Assumption 2.6. Let φ_1,φ_2 be monotone, φ_1 be upper hemicontinuous, and F_2 and φ_2 be upper semicontinuous in the first argument. Assume that $F(S_i)$ is nonempty for each $i = 1,2,\cdots,m$. For $\Omega = \cap_{i=1}^m F(S_i)$, suppose $\Gamma := \Omega \cap SGEP(F_1,\varphi_1,F_2,\varphi_2) \neq \emptyset$ with $S_ip = \{p\}$ for each $p \in \Omega$. Let $x_1 \in C$ with*

$C_1 = C$, and let $\{x_n\}$ be a sequence generated by

$$(3.24)\qquad \begin{cases} u_n = T_{r_n}^{(F_1,\varphi_1)}\big(I - \gamma A^*(I - T_{r_n}^{(F_2,\varphi_2)})A\big)x_n, \\ z_n = \alpha_n^{(0)}u_n + \alpha_n^{(1)}y_n^{(1)} + \cdots + \alpha_n^{(m)}y_n^{(m)},\ y_n^{(i)} \in S_i u_n, \\ C_{n+1} = \{z \in C_n : \|z_n - z\| \le \|x_n - z\|\}, \\ x_{n+1} = P_{C_{n+1}}x_1, n \in \mathbb{N}, \end{cases}$$

where $\{\alpha_n^{(i)}\} \subset (k,1)$ satisfies $\sum_{i=0}^m \alpha_n^{(i)} = 1, \{r_n\} \subseteq (0,\infty)$, and $\gamma \in (0, \frac{1}{L})$, where L is the spectral radius of A^*A, and A^* is the adjoint of A. Assume that the following conditions hold:
(1) The $\lim_{n\to\infty} \alpha_n^{(i)} \in (0,1)$ exists for all $i \ge 0$,
(2) $liminf_{n\to\infty} r_n > 0$. Then the sequence $\{x_n\}$ generated by (3.24) converges strongly to $P_\Gamma x_1$.

Remark 3.4. (1) The main result of our work Theorem 3.1 extends the main result of Phuengrattana and Lerkchaiyaphum [4] and many others, from the class of multivalued nonexpansive mappings to the class multivalued demicontractive-type mappings.
(2) It is our view that the authors of [4] either do not misunderstood the Ceasaro mean as was used in [11] or misapplied it in their main result and claimed that their results hold for countably infinite family of the class of operators they studied.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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