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FIXED POINT COMPUTING BASED ON FIREWORKS ALGORITHMS

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Abstract. Fixed point theory is of importance in calculus equation, operational research, program analysis and mathematical economics. The calculation of fixed point is the core of fixed point theory, and it is a difficult problem. In this paper, the fixed point problem is transformed into an optimization problem, and the fireworks algorithm is used to solve the optimization problem. Experiments show that the algorithm is feasible, but the convergence speed and accuracy of the algorithm decrease when solving complex problems and the algorithm is easy to fall into local optimal solution. For this reason, adjusting the explosion amplitude of fireworks algorithm, it shows that the adjusted algorithm can greatly improve the convergence speed and accuracy, and effectively solve the local convergence problem.

Keywords: fireworks algorithm; fixed point; local convergence.

2010 AMS Subject Classification: 47H05,47H10.

1. INTRODUCTION

The fixed point theory was founded in 1909 by the Dutch mathematician Brouwer. He formally proposed and proved Brouwer's fixed point theorem in 1912 using knowledge related to algebraic topology. The famous Polish mathematician Banach proposed and proved Banach's fixed point theorem in 1922, which greatly promoted the development of the fixed point theorem, and at the same time gave rise to the iterative idea of using iterative methods to solve the

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fixed point. The fixed point method can combine tools and equations in geometric, topological and functional analysis, which plays an important role in studying the existence, uniqueness and specific calculation methods of equation solutions.

The firework algorithm [1] (FWA) is a new type of swarm intelligence optimization algorithm proposed by Professor Tan Ying and others in 2010. It is mainly composed of several main operators such as explosion operator, mutation operator, mapping operator and selection operator. A large number of scholars have conducted in-depth research on these important operators, and enhanced the parallelism of the firework algorithm, the convergence of the non-origin solution, and the adaptability [2-4] and so on. It is widely used, such as solving Nash equilibrium of non-cooperative games [5] and so on.

When the traditional iterative algorithm solves the fixed point problem, the solution result has a great relationship with the choice of the initial value. In addition, it also has the disadvantages of slow convergence, low accuracy, and instability. When the firework algorithm solves the problem, it combines the meta-heuristic algorithm with random factors, and works together in multiple dimensions, thereby improving the convergence of the entire algorithm.

Inspired by the above research, based on the firework algorithm, this paper discusses the solution of several fixed points that satisfy the conditions of Brouwer's fixed point theorem and Banach's fixed point theorem. According to the analysis of the advantages and disadvantages of the firework algorithm, the explosion amplitude of the firework algorithm is adjusted to improve the accuracy and convergence speed of the algorithm when solving the fixed point problem.

2. Fixed Point Theory

This section gives some basic concepts and conclusions involved in the article.

Definition 2.1 Let X be a non-empty set and $A : X \to X$ be a mapping. If there exist $x^* \in X$ satisfies$A(x^*) = x^*$, then x^* is called the fixed point of the mapping A.

Definition 2.2 (Contractive map) Let X be a metric space and $A : X \to X$ be a mapping. If there is a constant $\alpha \in (0,1)$, for any $x, y \in X$, there is $d(Ax, Ay) \leq \alpha d(x, y)$.

Banach's fixed point theorem Suppose X is a complete metric space and $A : X \to X$ is a contractive map, then A has a unique fixed point in X, that is, there is a unique x^* such that $A(x^*) = x^*$.

Brouwer fixed point theorem[7] Suppose $C \subset R^n$ is a non-empty bounded closed convex set, $f : C \to C$ is continuous, then the fixed point of the mapping f exists, that is, there is $x^* \in C$ such that $f(x^*) = x^*$.

Convert the problem of solving the fixed point to the problem of solving the solution of the equation, let X be a non-empty set and $g : X \to X$ be a mapping,If the map has a fixed point, let x^* be the solution of the equation $G(x) = g(x) - x = 0$, then x^* is the fixed point of the map g.

Let $F(x) = \|G(x)\|_\infty$, where $x = (x_1, x_2, \cdots, x_n)$ and $\|G(x)\|_\infty$denote the infinite norm (the largest of the absolute values of all the elements of the vector function $G(x)$). Therefore, the above problem can be described as the following optimization problem:

$$\min\{F(x)\} \tag{2.1}$$

Convert the problem of solving a fixed point into a problem of solving the solution of an equation.

For the optimization problem (2.1), the point in the feasible region X is the feasible solution. If $\hat{x} \in X$, and satisfies $F(\hat{x}) \le F(x)$ for any $x \in X$, then $F(\hat{x})$ is the global optimal solution, and f takes the global optimal value. It is easy to know that when $F(\hat{x}) = 0$, then $x^* = \hat{x}$, so at this time $\hat{x}$ is the fixed point of the mapping g.

3. Introduction to Fireworks Algorithm

Use the firework algorithm to solve general optimization problems:

$$\min f(x) \in R, x \in [x_{\min}, x_{\max}] \tag{3.1}$$

Among them, $x = (x_1, x_2, \cdots, x_d)$ represents the possible value of x, $f(x)$ represents the fitness function, and $[x_{\min}, x_{\max}]$ represents the value range of x.

Define the number of sparks generated by firework $i(i = 1, 2, \cdots, n)$ as:

$$s_i = m \cdot \frac{y_{\max} - f(x_i) + \xi}{\sum_{i=1}^{n} (y_{\max} - f(x_i)) + \xi} \tag{3.2}$$

The explosion radius is:

$$A_i = A_0 \cdot \frac{y_{\min} - f(x_i) + \xi}{\sum\limits_{i=1}^{n} (y_{\min} - f(x_i)) + \xi} \tag{3.3}$$

Where m and A_0 represent the total number of conventional sparks and the sum of the initial firework radius respectively, $y_{\min} = \min(f(x_i))$and $y_{\max} = \max(f(x_i))$. ξ represents the minimum amount of the machine to avoid division by zero.

In order to avoid too few or too many sparks. The number of sparks generated can be adjusted according to the following formula (a, b are constants):

$$\hat{s}_i = \begin{cases} round(a \cdot m), s_i < am \\ round(b \cdot m), s_i > bm, a < b < 1 \\ round(s_i), others \end{cases} \tag{3.4}$$

Randomly generate z dimension, d represents the dimension of firework x_i:

$$z = round(d \cdot rand(0,1))) \tag{3.5}$$

To update any z dimension of firework x_i, the kth dimension coordinate update formula is as follows:

$$x_i^k = x_i^k + A_i \cdot rand(-1,1) \tag{3.6}$$

The out-of-bounds detection is performed on the generated sparks. If the kth dimension of the sparks generated by firework x_i is out of bounds, it will be mapped into the space according to the following formula.

$$x_i^k = x_{\min}^k + |x_i^k| \% (x_{\max}^k - x_{\min}^k) \tag{3.7}$$

In order to ensure the diversity of the firework population, avoid the firework algorithm from falling into local convergence prematurely, and can search for the global optimal solution. Design another spark generation method (Gaussian mutation spark). To mutate $m1$ fireworks, let's set the selected fireworks to x_j, and then perform the following operations on the z dimensions x_j^k of the fireworks x_j:

$$x_i^k = x_j^k \cdot Gaussian(1,1) \tag{3.8}$$

In the firework algorithm, the selection method is based on the distance-based roulette method [8]. In this algorithm, the current best firework, that is, the firework with the smallest fitness value, is always selected, and the other $n-1$ fireworks are selected. The distance-based roulette method is generated.

The distance between firework (spark)x_i and other firework sparks is defined as follows:

$$R(x_i) = \sum_{j\in K} d(x_i, x_j) = \sum_{j\in K} \|x_i - x_j\| \tag{3.9}$$

Where K contains all fireworks and sparks, and the probability of x_i being selected is:

$$k(x_i) = \frac{R(x_i)}{\sum\limits_{j\in K} R(x_i)} \tag{3.10}$$

4. Flow Chart for Solving Fixed Point by Firework Algorithm

According to the analysis in Chapter 2, select $F(x) = \|G(x)\|_\infty$ as the fitness function of FWA. The following are the steps and flowcharts of the firework algorithm to solve the fixed point problem:

step1 Initialize the parameters, randomly select n initial points in the feasible region space as the initial positions of the fireworks.

step2 Calculate the fitness function value of each firework, and judge the value of $F(x)$, if $F(x) < e_0$, turn to **step8**, otherwise turn to **step3**.

step3 Explode n fireworks, calculate the number of sparks $s_i$generated by each firework according to formula (3.2), and adjust the number of sparks generated by each firework according to formula (3.3). In addition, calculate the spark generation radius of the corresponding firework according to formula (3.4) for each firework.

step4 Calculate the value of changing several dimension directions according to formula (3.5), calculate the updated spark coordinates according to formula (3.6), and perform cross-border detection on the spark. If it crosses the boundary, map the spark to a new position according to formula (3.7),generate regular sparks.

step5 Calculate the value of changing several dimension directions according to formula (3.5), calculate the updated spark coordinates according to formula (3.8), and perform cross-border detection on the spark. If it crosses the boundary, map the spark to a new position according to formula (3.7),generate Gaussian sparks.

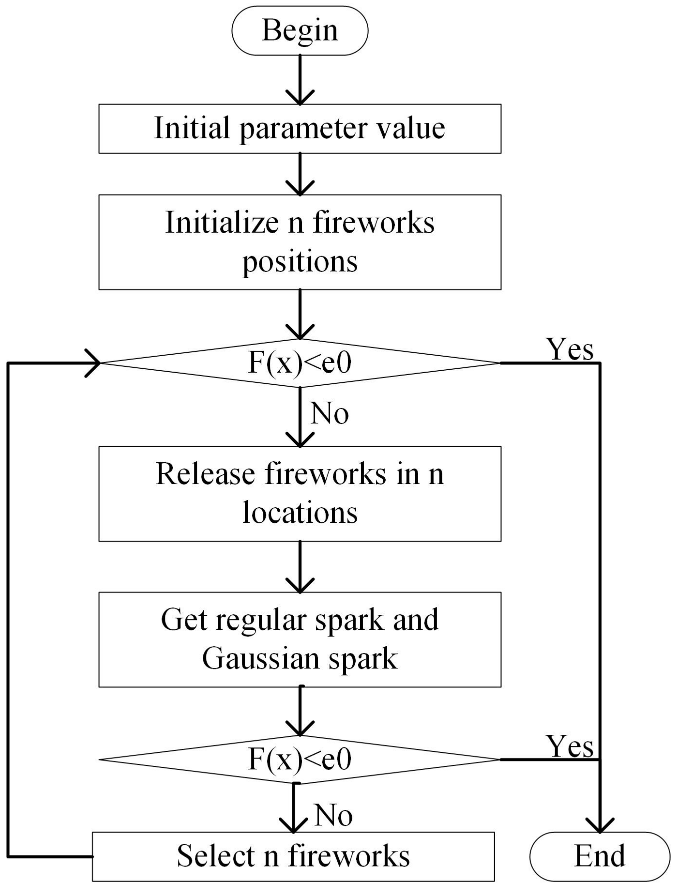

FIGURE 1. Flowchart of Fireworks Algorithm for Solving Fixed Points

step6 Calculate the fitness value of all regular sparks and Gaussian mutation sparks. If the accuracy e_0 is met and the maximum number of iterations N is not reached, then turn to **step7**, otherwise, turn to **step8**.

step7 First select the current optimal firework to enter the next iteration, and then select the remaining $n-1$ fireworks according to the distance-based roulette method, and save them in the next-generation fireworks population. Turn to **step3**.

step8 Stop.

5. Simulation Examples and Improvement Analysis

Example 1 Find the fixed point of $f(x_1,x_2,x_3) = (x_1 \cdot sin((x_2+x_3)\cdot\pi), \frac{1}{9}\cdot x_2 \cdot e^{x_1+x_3}, (x_1^2 - x_2^2)\cdot x_3)$, where, $x_1 \in [-1,1], x_2 \in [-1,1], x_3 \in [-1,1]$.

Let $S = [-1,1] \times [-1,1] \times [-1,1]$, it is easy to know that $f : S \to S$ and f is continuous. From the Brouwer fixed point theorem, we know that the mapping f has a fixed point in S, and $x^* = (0,0,0), x^* = (1,0,\frac{1}{2}), x^* = (-1,0,\frac{1}{2})$are calculated.

The fitness function is:

$F(x) = \|x_1 \cdot sin((x_2+x_3)\cdot\pi) - x_1, \frac{1}{9}\cdot x_2 \cdot e^{x_1+x_3} - x_2, (x_1^2 - x_2^2)\cdot x_3 - x_3\|_\infty.$

Repeat the experiment 10 times, T represents the number of iterations, and get the following experimental data:

TABLE 1. The fixed point value of Example 1 solved by the firework algorithm

T	Precision	The value of the fixed point x
321	$9.44679866 \times 10^{-5}$	$(0.99990608, -0.00000655, 0.50298852)$
585	$8.93468552 \times 10^{-5}$	$(-0.99991098, 0.00003954, 0.50189716)$
31	$5.58805267 \times 10^{-5}$	$1.0 \times 10^{-4} \times (-0.55875215, -0.06626965, -0.23632344)$
21	$4.77084447 \times 10^{-5}$	$1.0 \times 10^{-4} \times (-0.47703737, 0.02488224, -0.33901175)$
635	$8.53480479 \times 10^{-5}$	$(-0.99993775, -0.00006592, 0.50422482)$
428	$2.80903013 \times 10^{-5}$	$(-0.99999621, 0.00002464, 0.50236121)$
35	$5.66815349 \times 10^{-5}$	$1.0 \times 10^{-4} \times (0.01073003, -0.03412811, 0.56681534)$
252	$9.94549453 \times 10^{-5}$	$(0.99989963, 0.00003766, 0.49547931)$
42	$5.12299626 \times 10^{-5}$	$1.0 \times 10^{-4} \times (-0.14759885, -0.09613304, 0.51229962)$
421	$7.79565739 \times 10^{-5}$	$(0.99995151, -0.00015558, 0.50199048)$

It can be seen from Table 1 that the firework algorithm can be used to solve the fixed point problem, but the convergence speed of the firework algorithm is reduced for solving the non-origin problem of the optimal solution, and it is easy to fall into local convergence.

Analyzing why the firework algorithm falls into local convergence, formula (3.4) shows that when $y_{\min} - f(x_i) = 0$, the value of A_i tends to 0, which makes the firework generate a large number of sparks that are the same as x_i, causing the firework algorithm to fall into local convergence. Therefore, adjust the explosion range of the firework algorithm, add $f(x_i)$ to the formula (3.4), and dynamically adjust the explosion radius of the firework algorithm.

Example 2 Consider the system of linear equations$x_i - \sum_{j=1}^{n} a_{ij}x_i = b_i(i = 1,2,\cdots,n))$, where$\alpha = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}| < 1$, try to prove that the system of equations has a unique solution.

Proof: According to the principle of Banach compressed mapping, this equation system has a unique solution. The fitness function of this problem is constructed and the augmented matrix is randomly generated as:

$$F(x) = \|(\sum_{j=1}^{n} a_{1,j}x_j + b_1 - x_1, \sum_{j=1}^{n} a_{2,j}x_j + b_2 - x_2, \cdots, \sum_{j=1}^{n} a_{n,j}x_j + b_n - x_n)\|_\infty$$

$$A = \begin{bmatrix} 0.1115 & 0.1565 & 0.0866 & 0.3405 & 0.2763 \\ 0.1424 & 0.2713 & 0.1511 & 0.0219 & 0.0187 \\ 0.0741 & 0.0111 & 0.2840 & 0.0309 & 0.0007 \\ 0.0152 & 0.0465 & 0.0154 & 0.1028 & 0.0824 \\ 0.2182 & 0.0646 & 0.0543 & 0.0004 & 0.0158 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.6325 \\ 2.3205 \\ 4.2993 \\ 5.8333 \\ 3.9410 \end{bmatrix}$$

It is easy to verify that$\alpha = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}| = 0.5914 < 1$, solving the above equations can get:$x^* = (7.54188380, 6.55454951, 7.2250194, 7.69089553, 6.50828879)$.

Initialize each parameter value, T is the number of iterations, and get the following experimental data in Table 2.

In Table 2, the first five sets of data are before improvement, and the last five sets of data are after improvement. It is found by comparison that the convergence speed of the improved firework algorithm has increased dozens of times.

The following figure 2 is the convergence process diagram of 5000 iterations of the original firework algorithm solution example 2 (X represents the number of iterations, Y represents the accuracy). It can be seen from the figure that the accuracy of 10^{-3} is achieved when iterations 3289 times.

TABLE 2. Data comparison before and after improvement

T	Precision	The value of the fixed point x
2488	$5.89332166 \times 10^{-4}$	$(7.54189326, 6.55494236, 7.22422821, 7.69148245, 6.50825003)$
4251	$9.80698750 \times 10^{-4}$	$(7.54062050, 6.55267667, 7.22362798, 7.69175424, 6.50760544)$
3110	$9.97347451 \times 10^{-4}$	$(7.53981572, 6.55271630, 7.22332401, 7.68954305, 6.50709385)$
2918	$9.94848434 \times 10^{-4}$	$(7.54114809, 6.55375738, 7.22601257, 7.69180561, 6.50711806)$
3037	$8.81145097 \times 10^{-4}$	$(7.54266797, 6.55591924, 7.22637467, 7.69141287, 6.50866550)$
47	$9.70464563 \times 10^{-4}$	$(7.54330715, 6.55435010, 7.22652340, 7.69097892, 6.50957396)$
79	$6.16997743 \times 10^{-4}$	$(7.54178173, 6.55539360, 7.22503820, 7.69134574, 6.50828122)$
70	$9.93426502 \times 10^{-4}$	$(7.53999999, 6.55381315, 7.22403000, 7.69036125, 6.50721191)$
65	$9.83942765 \times 10^{-4}$	$(7.53982929, 6.55323583, 7.22389780, 7.68954409, 6.50684478)$
39	$8.04711536 \times 10^{-4}$	$(7.54206779, 6.55564787, 7.22619112, 7.69117956, 6.50824583)$

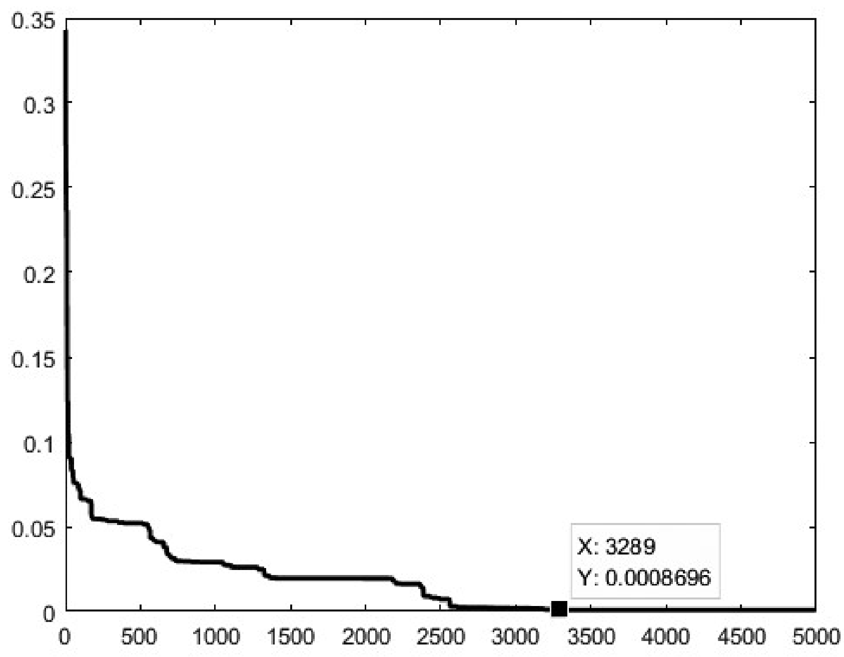

FIGURE 2. Original Firework Algorithm Solution Example 2 Iterative Convergence Process Diagram

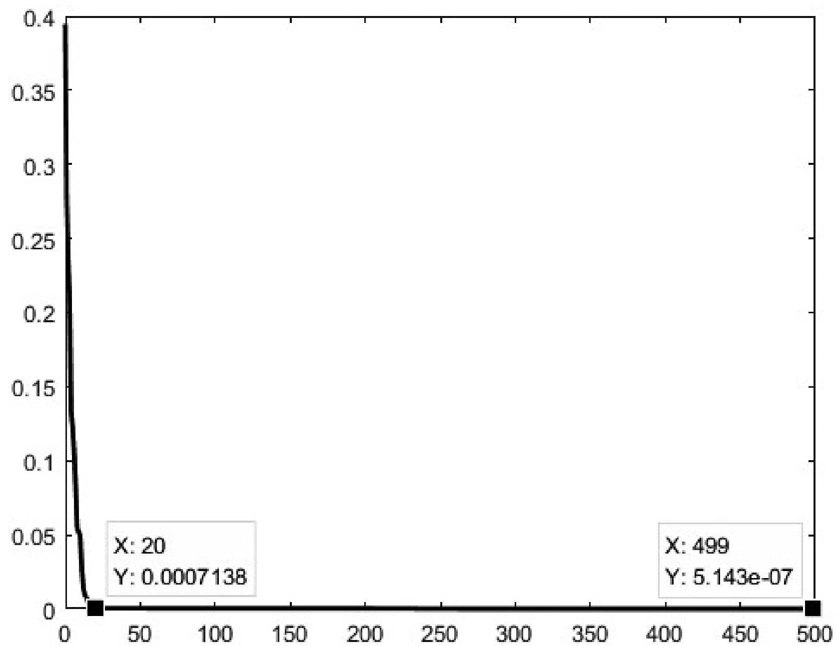

FIGURE 3. Improved Firework Algorithm Solution Example 2 Iterative Convergence Process Diagram

The following figure 3 is the convergence process diagram of the improved firework algorithm solution example 2 iterations for 500 times. It can be seen from the figure that the accuracy of 10^{-3} is reached at 20 iterations, and the accuracy of 10^{-6} is reached at 499 iterations. It can be seen that the improved firework algorithm solves the problem of local convergence to a certain extent.

6. CONCLUSION

This paper uses experiments to prove the feasibility of the firework algorithm to solve the fixed point problem. The firework algorithm shows great advantages in solving the problem of the fixed point as the origin. However, when solving the problem that the fixed point is not the origin, the iterative effect is reduced. The firework algorithm may not reach the ideal accuracy. Adjusting the explosion range of the firework can improve the accuracy and convergence speed of the firework algorithm.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] Y. Tan, Y. Zhu, Fireworks Algorithm for Optimization, in: Y. Tan, Y. Shi, K.C. Tan (Eds.), Advances in Swarm Intelligence, Springer Berlin Heidelberg, Berlin, Heidelberg, 2010: pp. 355–364.

[2] K. Ding, S. Zheng, Y. Tan, A GPU-based parallel fireworks algorithm for optimization, in: Proceeding of the Fifteenth Annual Conference on Genetic and Evolutionary Computation Conference - GECCO 13, ACM Press, Amsterdam, The Netherlands, 2013: p. 9.

[3] S. Zheng, A. Janecek, Y. Tan, Enhanced Fireworks Algorithm, in: 2013 IEEE Congress on Evolutionary Computation, IEEE, Cancun, Mexico, 2013: pp. 2069–2077.

[4] J. Li, S. Zheng, Y. Tan, Adaptive Fireworks Algorithm, in: 2014 IEEE Congress on Evolutionary Computation (CEC), IEEE, Beijing, China, 2014: pp. 3214–3221.

[5] Y.L. Yang, S.W. Xiang, S.Y. Xia, Solving Nash equilibrium problem of non-cooperative game based on firework algorithm, Comput. Appl. Softw. 35(5) (2018), 215-218.

[6] J. Sun, W.Y. Wang, T.W. Hao, Functional analysis, Higher Education Press, (2010), 14–55.

[7] J. Yu, Game theory and nonlinear analysis, Science Press, (2008), 51–56.

[8] L.N. Xu, Neural network control, Publishing House of Electronics Industry, (2009), 133–134.