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FIXED POINT RESULTS IN CONTROLLED FUZZY METRIC SPACES WITH AN APPLICATION TO THE CONVERSION OF SOLAR ENERGY INTO ELECTRIC POWER

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Abstract. Aim of this paper is to provide sufficient conditions for a sequence to be Cauchy in the setting of controlled fuzzy metric space. Further, we generalize the concept of Banach's contraction principle by utilizing certain new contraction conditions and prove fixed point results of quadratic type. Additionally we provide a number of examples to validate our main results. Conclusively, we provide an important application to the conversion of solar energy to electric power.

Keywords: fuzzy-metric space; contraction principles; Green's Function.

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1. Introduction

The existence of a unique fixed point for self mapping under suitable contraction conditions over complete metric spaces is guaranteed by Banach's fixed point theory. New extensions and generalizations of fixed point results are significant because they expand our understanding of mathematical systems, empower the solution of specific problems, extend usual theorems, and show to the development of new theories and applications. They are significant aspect of

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mathematical study and have far-reaching implication in hetergeneity fields. Some quadratic type fixed point results have been established in different metric spaces.(see[22]-[24]).

The fuzzy logic was established by Zadeh [27]. Unlike the theory of conventional logic, some numbers are not contained within the set and fuzzy logic connection of the numbers in the set defines an element within the interval [0,1]. Uncertainly, the necessary section of real hardness, has helped Zadeh to learn theories of fuzzy sets to bear the difficulty of indefinity. The theory is a seen as a fixed point in the fuzzy metric space for various processes one of them applying a fuzzy logic. Later on following Zadeh's outcomes, Heilpern [8] established the fuzzy mapping notion and a theorem on an fixed point for fuzzy contraction mapping in quadratic metric space, expressing a fuzzy general form of Banach's contraction theory. In the definition of fuzzy metric space provided by Kaleva and Seikkala [11], the ambiguity is introduced if the distance between the elements is not precise integer. After the first by Kramosil and Michalek [13] and further work by George and Veeramani[4], the notion of an fuzzy metric space was introduced. Branga and Olaru [2] proved various fixed point results for self-mappings by applying generalized contractive conditions in context of altered metric spaces. Czerwik [3] found the solution of the well known Banachs fixed point theorem in the context of b-metric spaces (b-MS). Mlaiki [16] defined controlled metric space as a generalization of b-metric space by applying a control function of other side of the b-triangular inequality. The relation between b-metric space and fuzzy metric space has been discussed by Hassan Zadeh and Sedghi[7]. Li et al. [14] used Kaleva -Seikkala's type fuzzy b-metric space and proved various fixed point results by using contraction mappings.

Ishtiaq et al. [10] established the theory of double- controlled instuitionistic fuzzy metric like spaces by "considering the case where the self -distance is not zero," if the metrics value is 0 afterward, it must be a self-distance and also an established fixed point theorem for contraction mappings for triangular norm (TN), Continuous triangular norm(CTN), and TN of H-type worked on complete fuzzy metric spaces by applying orthogonality and pentagonal complete fuzzy metric spaces and proved various fixed point results for contraction mappings. Rakic[17] proved a fuzzy version of Banach's fixed point theorem by using Ciric-quasi-contraction in the context of fuzzy b-metric space. Mehmood et al.[15] introduced the concept of extended fuzzy b-metric spaces and generalized the Banach contraction principle. Younis et al. [26] proved various fixed point results in the context of dislocated b-metric spaces and solved the turning circuit problem.

Now, we provide various definitions and results that are helpful to understand the main section.

Definition 1.1. [18] *A binary operation* $\Gamma : [0,1]\times[0,1] \to [0,1]$ *is a CTN if it verifies the below conditions:*

(C_1) Γ *is a commutative and associative,*

(C_2) Γ *is a continuous,*

(C_3) $\Gamma(\chi,1) = \chi \ \ \forall \chi \in [0,1]$

(C_4) $\Gamma(\chi,\rho) \leq \Gamma(c,d)$ for $\chi,\rho,c,d \in [0,1]$ *such that* $\chi \leq c$ *and* $\rho \leq d$*. Examples of CTN are* $T_p(a,b) = a.b,\ T_{\min}(a,b) = \min\{a,b\}$ *and* $T_L(a,b) = \max\{a+b-1,0\}$.

Definition 1.2. [5] *Suppose that* Γ *is a TN and Suppose that* $\Gamma_\tau : [0,1] \to [0,1], \tau \in \mathbb{N}$, *express the process given below:* $\Gamma_1(b) = \Gamma(b,b), \Gamma_{\tau+1} = \Gamma(\Gamma_\tau(b),b), \tau \in \mathbb{N}, b \in [0,1]$. *Then, TN* Γ *is H -type if the family* $\{\Gamma_\tau(b)\}_{\tau\in\mathbb{N}}$ *is equi-continuous at* $b = 1$*. A TN of H-type is* $\Gamma_{\min}$ *and each t-norms can be generalized in a different way to an n-ary process associative taking* $(b_1,.....b_n) \in [0]^n$ *for the values*

$$\Gamma^1_{i=1} b_i = b_1,\ \Gamma^1_{i=1} b_i = \Gamma(\Gamma b_i,....b_\tau) = \Gamma(b_1,......b_\tau).$$

Definition 1.3. [4] *A three tuple* (Ξ,M,Γ) *is known as an FMS if* Ξ *is a random set* Γ *is a CTN,* M *is an FS on* $\Xi^2 \times (0,\infty)$ *and satisfies the following conditions for all* $(b,\omega,z \in \Xi)$ *and* $t,s > 0$*:*

(fm_1) $M(b,\omega,t) > 0$

(fm_2) $M(b,\omega,t) = 1 iff b = \omega,$

(fm_3) $M(b,\omega,t) = M(\omega,b,t)$

(fm_4) $\Gamma(M(b,\omega,t),M(\omega,z,s)) \leq M(b,z,t+s)$

(fm_5) $M(b,\omega,.) : (0,\infty) \to [0,1]$ *is continuous.*

Definition 1.4. [19] *A three tuple* (Ξ,M,Γ) *is called an FbMS if* Ξ *is a random set* Γ *is a CTN,* M *is an FS on* $\Xi^2 \times (0,\infty)$ *and satisfies the following conditions for all* $(b,\omega,z \in \Xi)$ *and* $t,s > 0$ *and* $\rho \geq 1$ *as a real number:*

(b_1) $M(b,\omega,t) > 0,$

(b_2) $M(b,\omega,t) = 1$ *iff* $b = \omega$,

(b_3) $M(b,\omega,t) = M(\omega,b,t)$,

(b_4) $\Gamma(M(b,\omega,t), M(\omega,z,s)) \leq M(b,z,\rho(t+s))$,

(b_5)=$M(b,\omega,.) : (0,\infty) \to [0,1]$ *is continuous.*

Lemma 1.5. [19] *Let $M(b,\omega,.)$ be an fuzzy b-metric space. Then, $M(b,\omega,t)$ is b-non-decreasing with respect to t for all $b,\omega \in \Xi$.*

Definition 1.6. [15] *Let Ξ be a non-empty set, $\alpha : \Xi \times \Xi \to [1,\infty)$, Γ is a CTN and M_α is an FS on $\Xi^2 \times [0,\infty)$ and satisfies the following conditions for all $b,\omega,z \in \Xi$, s and $t > 0$:*

$(EM_\alpha 1)$ $M_\alpha(b,\omega,0) > 0$,

$(EM_\alpha 2)$ $M_\alpha(b,\omega,t) = 1$ *iff* $b = \omega$,

$(EM_\alpha 3)$ $M_\alpha(b,\omega,t) = M_\alpha(\omega,b,t)$,

$(EM_\alpha 4)$ $(M_\alpha(b,z,\alpha(b,z)(t+s)) \geq \Gamma(M_\alpha(b,\omega,t), M_\alpha(\omega,z,s))$,

$(EM_\alpha 5)$ $M(b,\omega,.) : (0,\infty) \to [0,1]$ *is left continuous.*

Then, the triple (Ξ, M_α, Γ) is said to be an extended fuzzy b-metric space and M_α is said to be controlled fuzzy metric on Ξ.

Definition 1.7. [21] *Let Ξ be a non-empty set, $\alpha : \Xi \times \Xi \to [1,\infty)$, Γ is a CTN and M_α is an FS on $\Xi^2 \times [0,\infty)$ and satisfies the following conditions for all $b,\omega,Z \in \Xi$, s and $t > 0$:*

$(EM_\alpha 1)$ $M_\alpha(b,\omega,0) > 0$,

$(EM_\alpha 2)$ $M_\alpha(b,\omega,t) = 1$ *iff* $b = \omega$,

$(EM_\alpha 3)$ $M_\alpha(b,\omega,t) = M_\alpha(\omega,b,t)$,

$(EM_\alpha 4)$ $(M_\alpha(b,z,(t+s)) \geq \Gamma(M_\alpha(b,\omega,\frac{t}{\alpha(b,\omega)}), M_\alpha(\omega,z,\frac{s}{\alpha(\omega,z)}))$,

$(EM_\alpha 5)$ $M(b,\omega,.) : (0,\infty) \to [0,1]$ *is left continuous.*

Then, the triple (Ξ, M_α, Γ) is said to be a CFMS and M_α is said to be controlled FM on Ξ. Sezen [21] *proved the following Banach contraction principle in the context of control fuzzy metric space.*

Definition 1.8. [21] *Suppose $M(b,\omega,t)$ is a CFMS. For $t > 0$, the open ball $B(b,l,t)$ with center $b \in \Xi$ and radius $0 < l < 1$ is express as a sequence $\{b_\tau\}$*

(a) G-Convergent to b if $M(b_\tau,b,t) \to 0$ as $\tau \to \infty$ or for every $t > 0$. We write $\lim_{\tau\to\infty} b_\tau = b$.

(b) is said to be Cauchy sequence (CS) if for all $0 < \varepsilon < 1$ *and* $t > 0$ *then there exist satisfying* $\tau_0 \in \mathbb{N}$ *such that*

$$M(b_\tau, b_m, t) > 1 - \varepsilon, \ \forall\ \tau, m \geq \tau_0.$$

(c) The CFMS (Ξ, M, Γ) *is a G-complete if every CS is convergent in* Ξ.

2. Main Result

In this part, we explain various new results in the surrounding of complete fuzzy metric space.

Lemma 2.1. *Assume that* (Ξ, M, Γ) *is a complete controlled fuzzy metric space with* $\alpha : \Xi \times \Xi \to [1, \infty)$ *assume that* $\lim_{t\to\infty} M_\alpha^2(b, \omega, t) = 1$ *for all* $b, \omega \in \Xi$. *If* $f : \Xi \to \Xi$ *satisfies the following for some* $\aleph \in (0,1)$, *such that* $M_\alpha^2(f_b, f_\omega, t) \geq M_\alpha^2(b, \omega, \frac{t}{\aleph})$, *for all* $b, \omega \in \Xi$, $t > 0$. *Also, suppose that for arbitrary* $b_0 \in Xi$ *and* $n, q \in \mathbb{N}$, *we have* $\alpha(b_n, b_{n+q}) \leq \frac{1}{\aleph}$ *where* $b_n = f^n(b_0)$. *Then,* f *has a unique fixed point in* Ξ.

Lemma 2.2. *Assume* $\{b_\tau\}$ *is a sequence in a controlled fuzzy metric space* (Ξ, M_α, Γ). *Let* $\aleph \in (0,1)$ *exist such that*

$$M^2(b_\tau, b_{\tau+1}, t) \geq M^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph}), \tau \in \mathbb{N}, t > 0, \tag{1}$$

and $b_0, b_1 \in \Xi$ *and* $v \in (0,1)$ *exist such that*

$$\lim_{\tau\to\infty} M^2(b, \omega, t) = 1,\ t > 0. \tag{2}$$

Then, $\{b_\tau\}$ *is a CS.*

Proof. Suppose $\chi \in (\aleph, 1)$ and the $\sum_{i=1}^{\infty} \chi^i$ is convergent, $\tau_0 \in \mathbb{N}$ exist such that $\sum_{i=1}^{\infty} \chi^i < 1$ for every $\tau > \tau_0$. Let $\tau > m > \tau_0$. Since M_α is b-non-decreasing, by $(FM_\alpha 4)$ for every $t > 0$, we get

$$M_\alpha^2(b_\tau, b_{\tau+m}, t) \geq M_\alpha^2\left(b_\tau, b_{\tau+m}, \frac{\sum_{i=\tau}^{\tau+m-1} \aleph^i}{\alpha(b_\tau, b_{\tau+m})}\right)$$

$$\geq \Gamma\left(\begin{array}{c} M_\alpha^2\left(b_\tau, b_{\tau+1}, \frac{tx^\tau}{\alpha(b_\tau, b_{\tau+1})\alpha(b_\tau, b_{\tau+m})}\right), \\ M_\alpha^2\left(b_{\tau+1}, b_{\tau+m}, \frac{t\sum_{i=\tau+1}^{\tau+m-1} \aleph^i}{\alpha(b_{\tau+1}, b_{\tau+m})\alpha(b_\tau, b_{\tau+m})}\right) \end{array}\right)$$

$$\geq \Gamma \begin{pmatrix} M_{\alpha}^{2}\left(b_{\tau}, b_{\tau+1}, \frac{tx^{\tau}}{\alpha(b_{\tau},b_{\tau+1})\alpha(b_{\tau},b_{\tau+m})}\right), \\ M_{\alpha}^{2}\left(b_{\tau+1}, b_{\tau+2}, \frac{tx^{\tau-1}}{\alpha(b_{\tau+1},b_{\tau+2})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}\right), \\ M_{\alpha}^{2}\left(b_{\tau+2}, b_{\tau+m}, \frac{t\sum_{i=\tau+2}^{\tau+m-1}\aleph^{i}}{\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}\right) \end{pmatrix}$$

$$M_{\alpha}^{2}(b_{\tau}, b_{\tau+m}, t) \geq \Gamma \begin{pmatrix} M_{\alpha}^{2} b_{\tau}, b_{\tau+1}, \frac{t\chi^{\tau}}{\alpha(b_{\tau},b_{\tau+1})\alpha(b_{\tau},b_{\tau+m})}, \\ M_{\alpha}^{2}\left(b_{\tau+1}, b_{\tau+2}, \frac{t\chi^{\tau+1}}{\alpha(b_{\tau+1},b_{\tau+2})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}\right), \\ M_{\alpha}^{2}(b_{\tau+2}, b_{\tau+3}, \\ \frac{t\chi^{\tau+2}}{\alpha(b_{\tau+2},b_{\tau+3})\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}), \\ M_{\alpha}^{2}(b_{\tau+3}, b_{\tau+4}, \\ \frac{t\chi^{\tau+3}}{\alpha(b_{\tau+3},b_{\tau+4})\alpha(b_{\tau+2},b_{\tau+3})\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}), \\ M_{\alpha}^{2}(b_{\tau+m-2}, b_{\tau+m-1}, \\ \frac{t\chi^{\tau+3}}{\alpha(b_{\tau+3},b_{\tau+4})\alpha(b_{\tau+2},b_{\tau+3})\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})}) \\ \vdots \\ M_{\alpha}^{2}(b_{\tau+m-2}, b_{\tau+m-1}, \\ \frac{t\chi^{\tau+m-2}}{\alpha(b_{\tau+m-2},b_{\tau+m-1})\prod_{i=\tau}^{\tau+m-2}\alpha(b_{\tau},b_{\tau+m})}) \\ M_{\alpha}^{2}(b_{\tau+m-1}, b_{\tau+m}, \\ \frac{t\chi^{\tau+m-1}}{\prod_{i=\tau}^{\tau+m-1}\alpha(b_{\tau},b_{\tau+m})}) \end{pmatrix}.$$

From inequality (2), we conclude that

$$M_{\alpha}^{2}(b_{\tau}, b_{\tau+1}, t) \geq M_{\alpha}^{2}(b_{0}, b_{1}, \frac{t}{\aleph^{\tau}}),\ \tau \in \mathbb{N},\ t > 0,$$

and since $\tau > m$ and $\alpha : \Xi \times \Xi \to [1, \infty)$, we get

$$M_{\alpha}^{2}(b_{\tau}, b_{\tau+m}, t) \geq \Gamma \begin{pmatrix} M_{\alpha}^{2}\left(b_{0}, b_{1}, \frac{t\chi^{\tau}}{\alpha(b_{\tau},b_{\tau+1})\alpha(b_{\tau},b_{\tau+m})\aleph^{\tau}}\right), \\ M_{\alpha}^{2}\left(b_{0}, b_{1}, \frac{t\chi^{\tau+1}}{\alpha(b_{\tau+1},b_{\tau+2})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_{\tau},b_{\tau+m})\aleph^{\tau+1}}\right), \\ \vdots \\ M_{\alpha}^{2}\left(b_{0}, b_{1}, \frac{t\chi^{\tau+m-2}}{\alpha(b_{\tau+m-2},b_{\tau+m-1})\prod_{i=\tau}^{\tau+m-2}\alpha(b_{i},b_{\tau+m})\aleph^{\tau+m-1}}\right), \\ M_{\alpha}^{2}\left(b_{0}, b_{1}, \frac{t\chi^{\tau+m-1}}{\prod_{i=\tau}^{\tau+m-1}\alpha(b_{i},b_{\tau+m})\aleph^{\tau+m-1}}\right) \end{pmatrix}$$

as $\tau \to +\infty$ by applying (2), we get

$$M_{\alpha}^{2}(b_{\tau}, b_{\tau+m}, t) \geq \Gamma(1, 1, 1, ..., 1) = 1.$$

Hence, $\{b_\tau\}$ is a Cauchy sequence. □

Lemma 2.3. *Assume $\{b_\tau\}$ is a sequence in controlled fuzzy metric space (Ξ, M_α, Γ) and Γ is H-type. If $\aleph \in (0,1)$ exist such that*

$$M_\alpha^2(b_\tau, b_{\tau+1}, t) \geq M_\alpha^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph}),\ \tau \in \mathbb{N},\ t > 0, \tag{3}$$

Then, $\{b_\tau\}$ is a CS.

Lemma 2.4. *If for $b, \omega \in \Xi$ and some $\aleph \in (0,1)$,*

$$M^2(b, \omega, t) \geq M_\alpha^2(b, \omega, \frac{t}{\aleph}), t > 0. \tag{4}$$

Then, $b = \omega$.

Proof. Inequality (4) implies that

$$M_\alpha^2(b, \omega, t) \geq M_\alpha^2(b, \omega, \frac{t}{\aleph^\tau}),\ \tau \in \mathbb{N},\ t > 0.$$

Now

$$M_\alpha^2(b, \omega, t) \geq \lim_{\tau \to \infty} M_\alpha^2(b, \omega, \frac{t}{\aleph^\tau}) = 1\ t > 0,$$

and by $(FM_\alpha 2)$, it is easy to see that $b = \omega$. □

Theorem 2.5. *Assume that (Ξ, M, Γ) is a complete controlled fuzzy metric space and assume that $f : \Xi \to \Xi$. Let then exists $\aleph \in (0,1)$ such that*

$$M_\alpha(f_b, f_\omega, t) \geq \max \left\{ \begin{array}{c} M_\alpha^2(b, \omega, \frac{t}{\aleph}), M_\alpha^2(b, f_\omega, \frac{t}{\aleph}), \\ M_\alpha^2(f_b, \omega, \frac{t}{\aleph}), \\ \frac{M_\alpha(b, f_\omega, \frac{t}{\aleph}).M_\alpha(f_b, \omega, \frac{t}{\aleph})}{2}, \\ \frac{M_\alpha^2(b, f_\omega, \frac{t}{\aleph}) + M_\alpha^2(f_b, \omega, \frac{t}{\aleph})}{1 + M_\alpha^\alpha(b, \omega, \frac{t}{\aleph})} \end{array} \right\}^{\frac{1}{2}},\ b, \omega \in \Xi,\ t > 0, \tag{5}$$

and $b, \omega \in \Xi$ such that

$$\lim_{t \to \infty} M_\alpha^2(b, \omega, t) = 1,\ t > 0. \tag{6}$$

Then f has unique fixed point in Ξ.

Proof. Assume that $b_0 \in \Xi$ and $b_{\tau+1} = fb_\tau,\ \tau \in \mathbb{N}$. Consider $b = b_\tau$ and $\omega = b_{\tau-1}$ in (5), then we get

$$M_\alpha^2(b_\tau, b_{\tau+1}, t) = M_\alpha^2(fb_{\tau-1}, fb_\tau, t),$$

or

$$M_\alpha^2(b_\tau, b_{\tau+m}, t) \geq \max \left\{ \begin{array}{c} M_\alpha^2\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right), \Gamma\left(M_\alpha^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph}), M_\alpha^2\left(b_\tau, b_{\tau+1}, \frac{t}{\aleph}\right)\right), \\ M_\alpha^2\left(b_\tau, b_\tau, \frac{t}{\aleph}\right), \\ \frac{M_\alpha\left(b_{\tau-1}, b_{\tau+1}\frac{t}{\aleph}\right).M_\alpha\left(b_{\tau-1}, b_{\tau+1}\frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha^2(b_{\tau-1}, b_{\tau+1}, \frac{t}{\aleph}) + M_\alpha^2(b_\tau, b_\tau, \frac{t}{\aleph})}{1 + M_\alpha^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph})} \end{array} \right\}$$

$$\geq \max\left\{M_\alpha^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph}), M_\alpha^2(b_\tau, b_{\tau+1}, \frac{t}{\aleph})\right\}.$$

If

$$M_\alpha^2(b_\tau, b_{\tau+1}, t) \geq M_\alpha^2(b_\tau, b_{\tau+1}, \frac{t}{\aleph}),\ \tau \in \mathbb{N},\ t > 0,$$

then by Lemma 2.4 for $b_\tau = b_{\tau+1}, \tau \in \mathbb{N}$, we have $M_\alpha^2(b_\tau, b_{\tau+1}, \frac{t}{\aleph}) \geq M_\alpha^2(b_{\tau-1}, b_\tau, \frac{t}{\aleph}),\ \tau \in \mathbb{N},\ t > 0$, and by Lemma 2.2 it follows that $\{b_\tau\}$ is Cauchy sequence. Since (Ξ, M, Γ) is complete $b \in \Xi$ exist such that $\lim_{\tau\to\infty} b_\tau = b$ and

$$\lim_{\tau\to\infty} M_\alpha^2(b, b_\tau, t) = 1,\ t > 0. \tag{7}$$

By applying (5) and $(FM_\alpha 4)$ it is clear that b is fixed point for f. Assume $\chi_1 \in (\aleph, 1)$ *and* $\chi_2 = 1 - \chi_1$ by (5), we get

$$M_\alpha^2(fb, b, t) \geq \Gamma\left(M_\alpha^2\left(fb, fb_{\tau-1}, \frac{t\chi_1}{2\alpha(fb, b_\tau)}\right), M_\alpha^2\left(b_\tau, b, \frac{t\chi_2}{2\alpha(b_\tau, b)}\right)\right),$$

$$\geq \Gamma\left(\max\left\{\begin{array}{c} M_\alpha^2\left(b, b_{\tau-1}, \frac{t\chi_1}{2\alpha(fb, b_\tau)\aleph}\right), M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{2\alpha(fb, b_\tau)\aleph}\right), \\ \Gamma\left(M_\alpha^2\left(fb, b, \frac{t\chi_1}{(2)^2\alpha(fb, b)\alpha(fb, b_\tau)\aleph}\right), M_\alpha^2\left(fb, b, \frac{t\chi_1}{(2)^2\alpha(fb, b)\alpha(fb, b_\tau)\aleph}\right)\right) \\ M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{2\alpha(fb, b_\tau)\aleph)}\right) + \Gamma\left(\begin{array}{c} M_\alpha^2\left(b, b_{\tau-1}, \frac{t\chi_1}{(2)^2\alpha(b, b_{\tau-1})\alpha(fb, b_\tau)\aleph}\right) \\ M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{(2)^2\alpha(b, b_{\tau-1})\alpha(fb, b_\tau)\aleph}\right)2 \end{array}\right) \\ M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{2\alpha(fb, b_\tau)\aleph)}\right).\Gamma\left(\begin{array}{c} M_\alpha^2\left(b, b_{\tau-1}, \frac{t\chi_1}{(2)^2\alpha(b, b_{\tau-1})\alpha(fb, b_\tau)\aleph}\right) \\ M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{(2)^2\alpha(b, b_{\tau-1})\alpha(fb, b_\tau)\aleph}\right)2 \end{array}\right) \end{array}\right\}\right),$$

$\forall\, t > 0$. By (7) and as $\tau \to \infty$, we get

$$M_\alpha^2(fb,b,t) \geq \Gamma\left(\max\left\{\begin{array}{c} 1,1, \\ \Gamma\left(M_\alpha^2\left(fb,b,\frac{t\chi_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right),1\right), \\ \frac{1+\Gamma\left(\begin{array}{c} M_\alpha^2\left(fb,b,\frac{t\chi_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\chi}\right), \\ 1 \end{array}\right)}{2} \\ \frac{1+\Gamma\left(\begin{array}{c} M_\alpha^2\left(fb,b,\frac{t\chi_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\chi}\right), \\ 1 \end{array}\right)}{1+1} \\ 1 \end{array}\right\}\right),$$

$$= 1.$$

Assume that b and ω are two different fixed point for f. Then, by applying (5), we get

$$M_\alpha^2(b,\omega,t) = M_\alpha^2(fb,f\omega,t)$$

$$\geq \max\left\{\begin{array}{c} M_\alpha^2\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha^2\left(b,f\omega,\frac{t}{\aleph}\right), M_\alpha^2\left(fb,\omega,\frac{t}{\aleph}\right), \\ \frac{M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right).M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha^2\left(b,f\omega,\frac{t}{\aleph}\right)+M_\alpha^2\left(fb,\omega,\frac{t}{\aleph}\right)}{1+M_\alpha^2(b,\omega,\frac{t}{\aleph})} \end{array}\right\}$$

$$= \max\left\{\begin{array}{c} M_\alpha^2(b,\omega,\frac{t}{\aleph}), M_\alpha^2(b,\omega,\frac{t}{\aleph}), M_\alpha^2(b,\omega,\frac{t}{\aleph}), \\ \frac{M_\alpha(b,\omega,\frac{t}{\aleph}).M_\alpha(b,\omega,\frac{t}{\aleph})}{2}, \\ \frac{M_\alpha^2(b,\omega,\frac{t}{\aleph})+M_\alpha^2(b,\omega,\frac{t}{\aleph})}{1+M_\alpha^2(b,\omega,\frac{t}{\aleph})} \end{array}\right\}$$

$$= M_\alpha^2(b,\omega,\frac{t}{\aleph}),\ t > 0,$$

and by Lemma 2.4, it is clear that $b = \omega$. □

Remark 2.6. *If we take*

$$\max\left\{\begin{array}{c} M_\alpha^2(b,\omega,\frac{t}{\aleph}), M_\alpha^2(b,f\omega,\frac{t}{\aleph}), M_\alpha^2(fb,\omega,\frac{t}{\aleph}), \\ \frac{M_\alpha(b,f\omega,\frac{t}{\aleph}).M_\alpha(fb,\omega,\frac{t}{\aleph})}{2}, \\ \frac{M_\alpha^2(b,f\omega,\frac{t}{\aleph})+M_\alpha^2(fb,\omega,\frac{t}{\aleph})}{1+M_\alpha^2(b,\omega,\frac{t}{\aleph})} \end{array}\right\} = M_\alpha^2(b,\omega,\frac{t}{\aleph}),$$

in the above theorem then we get a fuzzy version of the Banach contraction principle of quadratic type.

Theorem 2.7. *Assuming that* (Ξ, M_α, Γ) *is complete controlled fuzzy metric space assuming that* $f : \Xi \to \Xi$ *then* $\aleph \in (0,1)$ *exists*

$$M_\alpha^2(fb, f\omega, t) \geq \min \left\{ \begin{matrix} M_\alpha^2(b, \omega, \frac{t}{\aleph}), M_\alpha^2(fb, b, \frac{t}{\aleph}), \\ M_\alpha^2(f\omega, \omega, \frac{t}{\aleph}), \\ M_\alpha(b, f_\omega, \frac{t}{\aleph}).M_\alpha(fb, \omega, \frac{t}{\aleph}) \end{matrix} \right\}, \tag{8}$$

for all $b, \omega \in \Xi$, $t > 0$ *such that*

$$\lim_{t\to\infty} M_\alpha^2(b, \omega, t) = 1, \ \forall\, t > 0. \tag{9}$$

Then, f *has a unique fixed point in* Ξ.

Proof. Assume that $b_0 \in \Xi$, $b_{\tau+1} = fb_\tau$ and $\tau \in \mathbb{N}$ from (9) with $b = b_\tau$ *and* $\omega = b_{\tau-1}$, for every $\tau \in \mathbb{N}$ and $t > 0$, we get

$$\begin{aligned} M_\alpha^2(b_{\tau+1}, b_\tau, t) &\geq \min\{M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph}), M_\alpha^2(b_{\tau+1}, b_\tau, \frac{t}{\aleph}), M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph})\} \\ &\geq \min\left\{M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph}), M_\alpha^2(b_{\tau+1}, b_\tau, \frac{t}{\aleph})\right\}. \end{aligned}$$

If

$$M_\alpha^2(b_{\tau-1}, b_\tau, t) \geq M_\alpha^2(b_{\tau+1}, b_\tau, \frac{t}{\aleph}), \ \tau \in \mathbb{N}, \ t > 0.$$

Then, Lemma 2.2 implies that $b_\tau = b_{\tau+1}, \ \tau \in \mathbb{N}$, such that

$$M_\alpha^2(b_{\tau+1}, b_\tau, t) \geq M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph}), \ \tau \in \mathbb{N}\ t > 0.$$

Moreover, by Lemma 2.2 $\{b_\tau\}$ is a CS. Hence, $b \in \Xi$ exists such that

$$\lim_{\tau\to\infty} M_\alpha^2(b, b_\tau, t) = 1, \ t > 0. \tag{10}$$

Now, we demonstrate that b is an fixed point for f. Letting $\chi_1 \in (\aleph, 1)$ and $\chi_2 = 1 - \chi_1$ by (8) we get

$$M_\alpha^2(fb,b,t) \geq \Gamma\left(\left(M_\alpha^2(fb, fb\tau, \frac{t\chi_1}{\alpha(fb, fb_\tau)}\right), M_\alpha^2\left(b_{\tau+1}, fb_\tau, \frac{t\chi_2}{\alpha(b_{\tau+1}, fb_\tau)}\right)\right)$$

$$\geq \Gamma\left(\begin{array}{c} \min\left\{\begin{array}{c} M_\alpha^2\left(b, fb_\tau, \frac{t\chi_1}{\alpha(b, fb_\tau)}\right), M_\alpha^2\left(b, fb, \frac{t\chi_1}{\alpha(b,fb)}\right), \\ M_\alpha^2\left(b_\tau, b_{\tau+1}, \frac{t\chi_2}{\alpha(b, fb_\tau)}\right) \end{array}\right\}, \\ M_\alpha^2\left(b_{\tau+1}, b, \frac{t\chi_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right) \end{array}\right)$$

taking $\tau \to \infty$ and applying (10), we conclude that

$$\begin{aligned} M_\alpha^2(fb,b,t) &\geq \Gamma(\min\left\{1, M_\alpha^2(b, fb, \frac{t\chi_1}{\alpha(b, fb)\aleph}), 1\right\}) \\ &= \Gamma(M_\alpha^2(b, fb, \frac{t\chi_1}{\alpha(b, fb)\aleph}), 1) \\ &= M_\alpha^2(fb,b,t) \\ &\geq M_\alpha^2(fb,b,\frac{t}{\nu}),\ t > 0, \end{aligned}$$

where $\nu = \frac{\alpha(b,fb)\aleph}{\chi_1} \in (0,1)$ and by Lemma 2.4, we have $fb = b$. Assume that b *and* ω are two different fixed point for f, that is, $fb = b$ and $f\omega = \omega$. By (8), we conclude that

$$\begin{aligned} M_\alpha^2(fb, f\omega, t) &\geq \min\left\{M_\alpha^2(b, \omega, \frac{t}{\aleph}), M_\alpha^2(b, \omega, \frac{t}{\aleph}), M_\alpha^2(\omega, f\omega, \frac{t}{\aleph})\right\} \\ &= \min\left\{M_\alpha^2(b, \omega, \frac{t}{\aleph}), 1, 1\right\} \\ &= M_\alpha^2(b, \omega, \frac{t}{\aleph}) \\ &= M_\alpha^2(fb, f\omega, \frac{t}{\aleph}) \end{aligned}$$

for $t > 0$ and by applying Lemma 2.4, we have $fb = f\omega$, which gives $b = \omega$. □

Remark 2.8. *If we take*

$$\min\left\{M_\alpha^2(b, \omega, \frac{t}{\aleph}), M_\alpha^2(fb, b, \frac{t}{\aleph}), M_\alpha^2(f\omega, \omega, \frac{t}{\aleph})\right\} = M_\alpha^2(b, \omega, \frac{t}{\aleph}),$$

in the above theorem then we obtain another fuzzy version of the Banach contraction principle of quadratic type.

Example 2.9. *Assume* $\Xi = (0,2)$, $M_\alpha(b,\omega,t) = e^{-\frac{(b-\omega)}{t}}$, *and* $\Gamma = \Gamma_p$. *Then,* (Ξ,M,Γ) *is a complete control fuzzy metric space with* $\alpha(b,\omega) = b+\omega+1$. *Let*

$$f(b) = \begin{cases} 2-b, & \text{if } b \in (0,1) \\ 1, & \text{if } b \in [1,2). \end{cases}$$

Part 1: *If* $b,\omega \in [1,2)$, *then* $M_\alpha(fb,f\omega,t) = 1$ *and* $M^2_\alpha(fb,f\omega,t) = 1,\ t>0$.

Part 2: *If* $b \in [1,2)$, *and* $\omega \in (0,1)$, *such that* $\aleph \in (\frac{1}{4},\frac{1}{2})$, *one can obtain*

$$M^2_\alpha(fb,f\omega,t) = e^{-[\frac{(1-\omega)}{t}]^2} \geq e^{-\frac{4\aleph(1-\omega)^2}{t^2}} = M^2_\alpha(f\omega,\omega,\frac{t}{\aleph}),\ t>0.$$

Part 3: *As in the preceding section, for* $\aleph \in (\frac{1}{4},\frac{1}{2})$, *we obtain*

$$M^2_\alpha(fb,f\omega,t) \geq M^2_\alpha(fb,b,\frac{t}{\aleph}), b \in (0,1)\ \omega \in [1,2), t>0.$$

Part 4: *If* $b,\omega \in (0,1)$, *then for* $\aleph \in (\frac{1}{4},\frac{1}{2})$ *we have*

$$M^2_\alpha(fb,f\omega,t) = e^{-[\frac{(1-\omega)^2}{t^2}]} \geq e^{-[\frac{4\aleph(1-\omega)^2}{t^2}]} = M^2_\alpha(f\omega,\omega,\frac{t}{\aleph}),\ b>\omega,\ t>0,$$

and

$$M^2_\alpha(fb,f\omega,t) \geq M^2_\alpha(f\omega,\omega,\frac{t}{\aleph}),\ b<\omega,\ t>0.$$

So, condition (9) *is fulfilled for all* $b,\omega \in (0,2),\ t>0$ *by Theorem* 2.2 *it follows that* $b=1$ *is a unique fixed point for* f.

Theorem 2.10. *Assuming that* $(\Xi,M,\Gamma),\ \Gamma \geq \Gamma p$ *is a complete controlled fuzzy metric space, assume that* $f:\Xi \to \Xi$ *for some* $\aleph \in (0,1)$, *let*

$$M^2_\alpha(fb,f\omega,t) \geq \min \left\{ \begin{array}{c} M^2_\alpha(b,\omega,\frac{t}{\aleph}), M^2_\alpha(fb,b,\frac{t}{\aleph}), \\ M^2_\alpha(f\omega,\omega,\frac{t}{\aleph}), \sqrt{M^2_\alpha(fb,\omega,\frac{2t}{\aleph})}, \\ M^2_\alpha(b,f\omega,\frac{t}{\aleph}) \end{array} \right\},\ b,\omega \in \Xi\ ,t>0, \tag{11}$$

and $b,\omega \in \Xi$ *exists such that*

$$\lim_{\tau\to\infty} M^2_\alpha(b,\omega,t) = 1,\ t>0. \tag{12}$$

Then, f *has a unique fixed point in* Ξ.

Proof. Let $b_0 \in \Xi$ *and* $b_{\tau+1} = fb_\tau, \tau \in \mathbb{N}$. Taking $b = b_\tau$ *and* $\omega = b_{\tau-1}$ in (12) by $(FM_\alpha 4)$ and $\Gamma \geq \Gamma p$, we get

$$M_\alpha^2(b_{\tau+1}, b_\tau, t) \geq \min \left\{ \begin{array}{c} M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph}), M_\alpha^2(fb, b, \frac{t}{\aleph}), \\ M_\alpha^2(b_{\tau+1}, b_\tau, \frac{t}{\aleph}), M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\aleph}), \\ \sqrt{M_\alpha(fb, \omega, \frac{2t}{\aleph}).M_\alpha(b_\tau, b_{\tau-1}} \\ \frac{t}{\alpha(b_\tau, b_{\tau-1})}\aleph) M_\alpha^2(b_\tau, b_\tau, \frac{t}{\aleph}), \end{array} \right\},$$

for all $b, \omega \in \Xi,\ t > 0$. Since $M_\alpha(b, \omega, t)$ is b-non decreasing in t and $\sqrt{\chi.\rho} \geq \min\{\chi, \rho\}$, we deduce

$$M_\alpha^2(b_{\tau+1}, b_\tau, t) \geq \min \left\{ M_\alpha^2(b_{\tau+1}, b_\tau, \frac{t}{\alpha(b_{\tau+1}, b_\tau)\aleph}), M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}) \right\},$$

for all $\tau \in \mathbb{N},\ t > 0$. By Lemmas 2.2 *and* 2.4 we have

$$M_\alpha^2(b_{\tau+1}, b_\tau, t) \geq M_\alpha^2(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}),\ \tau \in \mathbb{N}\ t > 0.$$

Hence $\{b_\tau\}$ is a CS. Since (Ξ, M, Γ) is complete $b \in \Xi$ exists such that

$$\lim_{\tau\to\infty} b_\tau = b \textit{ and } \lim_{\tau\to\infty} M_\alpha^2(b, b_\tau, t) = 1,\ t > 0. \tag{13}$$

Assuming $\chi_1 \in (\aleph, 1)$ *and* $\chi_2 = 1 - \chi_1$ by (13), we get

$$M_\alpha^2(fb, b, t) \geq \Gamma\left(M_\alpha^2\left(fb, fb_\tau, \frac{t\chi_1}{\alpha(fb, b_{\tau+1})}\right), M_\alpha^2\left(fb_\tau, b, \frac{t\chi_2}{\alpha(b_{\tau+1}, b)}\right)\right),$$

$$\geq \Gamma\left(\min\left\{ \begin{array}{c} M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{\alpha(b, b_\tau)\aleph}\right), M_\alpha^2\left(b, fb, \frac{t\chi_1}{\alpha(b, fb)\aleph}\right), M_\alpha^2\left(b_\tau, b_{\tau+1}, \frac{t\chi_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right), \\ \sqrt{M_\alpha\left(fb, b, \frac{t\chi_1}{\alpha(fb, b)\alpha(fb, b_\tau)\aleph}\right).M_\alpha\left(b, b_\tau, \frac{t\chi_1}{\alpha(fb, b_\tau)\alpha(b, b_\tau)\aleph}\right)} \\ M_\alpha^2\left(b, b_{\tau+1}, \frac{t\chi_1}{\alpha(b, b_{\tau+1})\aleph}\right), \\ M_\alpha^2\left(b_{\tau+1}, b, \frac{t\chi_2}{\alpha(b_{\tau+1}, b)\aleph}\right) \end{array} \right\}\right)$$

$$\geq \Gamma\left(\min\left\{ \begin{array}{c} M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{\alpha(b, b_\tau)\aleph}\right), M_\alpha^2\left(b, fb, \frac{t\chi_1}{\alpha(b, fb)\aleph}\right), M_\alpha^2\left(b_\tau, b_{\tau+1}, \frac{t\chi_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right), \\ \min\{M_\alpha^2\left(fb, b, \frac{t\chi_1}{\alpha(fb, b)\alpha(fb, b_\tau)\aleph}\right).M_\alpha^2\left(b, b_\tau, \frac{t\chi_1}{\alpha(fb, b_\tau)\alpha(b, b_\tau)\aleph}\right)\} \\ M_\alpha^2\left(b, b_{\tau+1}, \frac{t\chi_1}{\alpha(b, b_{\tau+1})\aleph}\right), \\ M_\alpha^2\left(b_{\tau+1}, b, \frac{t\chi_2}{\alpha(b_{\tau+1}, b)\aleph}\right) \end{array} \right\}\right),$$

for all $\tau \in \mathbb{N}$ and $t > 0$. Taking $\tau \to \infty$ and applying (13), we have

$$M_\alpha^2(fb,b,t) \geq \Gamma\left(\min\left\{\begin{array}{c} 1, M^2\left(b, fb, \frac{t\chi_1}{\alpha(b,fb)\aleph}\right), 1 \\ \min\{M^2\left(fb, b, \frac{t\chi_1}{\alpha(fb,b).\alpha(fb,b_\tau)\aleph}\right), 1\} \\ = M_\alpha^2\left(fb, b, \frac{t\chi_1}{\alpha(fb,b).\alpha(fb,b_\tau)\aleph}\right). \end{array}\right\}\right), t > 0,$$

and by Lemma 2.4 with $\nu = \frac{\alpha(fb,b).\alpha(fb,b_\tau)\aleph}{\chi_1} \in (0,1)$ such that $fb = b$.

Let b *and* ω are two different fixed points for f. By (12), we get

$$\begin{aligned} M_\alpha^2(fb,f\omega,t) &\geq \Gamma\left(\begin{array}{c} M_\alpha^2\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha^2\left(fb,b,\frac{t}{\aleph}\right), M_\alpha^2\left(f\omega,\omega,\frac{t}{\aleph}\right), \\ \sqrt{M_\alpha\left(fb,b,\frac{t}{\alpha(fb,b)\aleph}\right) M_\alpha\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right)}, M_\alpha^2\left(b,\omega,\frac{t}{\aleph}\right), \end{array}\right) \\ &\geq \Gamma\left(M_\alpha^2\left(b,\omega,\frac{t}{\aleph}\right), 1, 1 \min\{1, M_\alpha^2\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right)\}, M^2(b,\omega,\frac{t}{\aleph})\right) \\ &= M_\alpha^2\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right) \\ &= M_\alpha^2\left(fb,f\omega,\frac{t}{\alpha(fb,f\omega)\aleph}\right), \ t > 0, \end{aligned}$$

and thus by Lemma 2.4, we have $b = \omega$ □

3. An Application to the Transformation of Solar Energy to Electric Power

Sun-based boards are usually being distributed and shown generally to reduce people's assurance on petroleum derivatives which are less environmentally friendly. Closely 19 trillion kilowatts of power were converted internationally in 2007. In association, the amount of daylight that inserts the Earth's surface in a single hour is enough to illuminate the whole planet for a complete year. The question is: how do those glittering and heat beams of light obtain power? A numerical model of the electric flow in an RLC equal circuit, often known as a "tuning" circuit, can be displayed with a basic appreciate of how light is changed into power. In the fields of radio and communication engineering, this circuit has various uses. The version that is being displayed can be applied to calculate the production of electric power, supplied tools to improve building performance, and can be utilized as a decision-making implement when designing a hybrid renewable electricity system based on solar power. Every side of this system is mathematically expressed as a differential equation in [26] utilizing the following equation

$$\begin{cases} \frac{d^2\rho^{\cdot}}{d\omega^2} = \Omega(\omega, \rho^{\cdot}(\omega)) - \frac{\mathfrak{R}}{\mathfrak{L}}\frac{d\rho}{d\omega} \\ \rho^{\cdot}(0) = 0, \rho'(0) = m, \end{cases} \tag{14}$$

where $\omega : [0,1] \times \mathbb{R}^+ \to \mathbb{R}$ is a continuous function that is condition to the integral equation to which it is equivalent.

$$\rho(\omega) = \int_0^{\omega} N(\omega, l)\Omega(l, \rho^{\cdot}(l))dl, \omega \in [0,1], \tag{15}$$

where $N(\omega, b)$ is the Green's function it follows :

$$N(\omega, l) = \begin{cases} (\omega - l)e^{\Omega(M(b,\omega)(\omega-l))}, & 0 \leq l \leq \omega \leq 1, \\ 0, & 0 \leq \omega \leq 1. \end{cases} \tag{16}$$

where $\Omega(M(b,\omega)) > 0$ is constant , as intent by the values of $\mathfrak{R}$ and $\mathfrak{L}$. Let $\Xi = C([0,\omega,]\mathbb{R}^+)$ be the set of all real continuous positive functions that are expressed on the set $[0,\omega]$. Let Ξ be provide with the controlled fuzzy metric space given by the following

$$M(b, \omega, t) = \begin{cases} 0 & if\ t = 0 \\ \sup_{t\in[0,1]} \frac{\min\{b,\omega\}+t}{\sup\{b,\omega\}+t} & otherwise,\ for\ all\ b, \omega \in \Xi. \end{cases} \tag{17}$$

One can verify that (Ξ, M, Γ) is a complete controlled fuzzy metric space with controlled function $\alpha : \Xi \times \Xi \to [0,\infty)$, defined by $\alpha(b,\omega) = (b+\omega+1)$. It is clear that b^* is a solution of integral equation (17), and as result, a solution of differential equation (16) which control the system of converting solar energy into electric power if and only if b^* is an fixed point of f. It is put as guarantee of the existence of fixed point of f.

Theorem 3.1. *Infer the following problems accomplish:*

(I) $f : [0,\omega] \times [0,\omega] \to \mathbb{R}^+$ *is a continuous function,*

(II) there exists a continuous function $N : [0,\omega] \times [0,\omega] \to \mathbb{R}^+$ *such that*

$\sup_{\alpha\in[0,\omega]} \int_0^{\omega} N(\alpha, l) \geq 1,$

(III) $\max\{f(\alpha, l, b(l), f(\alpha, l, \omega(l))\} \geq N(\alpha, b) \max\{D(b(l), \omega(l))\}$, *and*

$\min\{f(\alpha,l,b(l),f(\alpha,l,\omega(l))\} \geq N(\alpha,b)\min\{D(b(l),\omega(l))\}$,

$\forall\ \alpha,l \in [0,1]\ b,\omega \in \mathbb{R}$ *and* $\aleph \in (0,1)$ *exists such that*

$$D(b(l),\omega(l)) = \max\left\{\begin{matrix} M^2_\alpha(b(l),\omega(l),\frac{t}{\aleph}), M^2_\alpha(b(l),f\omega(l),\frac{t}{\aleph}), M^2_\alpha(fb(l),\omega(l),\frac{t}{\aleph}) \\ \frac{M_\alpha(b(l),f\omega(l),\frac{t}{\aleph}).M_\alpha(fb(l),\omega(l),\frac{t}{\aleph})}{2} \\ \frac{M^2_\alpha(b(l),f\omega(l),\frac{t}{\aleph})+M^2_\alpha(fb(l),\omega(l),\frac{t}{\aleph})}{1+M^2_\alpha(b(l),\omega(l),\frac{t}{\aleph})} \end{matrix}\right\}.$$

The differential equation (14) *that represent the solar energy problem has a solution as a result and the integral equation* (15) *also has a solution.*

Proof. For $b,\omega \in \Xi$, by use of assumption (I) and (III), we have

$$\begin{aligned} M^2(fb,f\omega,t) &= \sup_{t\in[0,1]} \frac{\min\{\int_0^\omega N(\omega,l)\Omega(l,b(l))dl, \int_0^\omega N(\omega,l)\Omega(l,\omega(l))dl\}+t}{\max\{\int_0^\omega N(\omega,l)\Omega(l,b(l))dl, \int_0^\omega N(\omega,l)\Omega(l,\omega(l))dl\}+t'} \\ &= \sup_{t\in[0,1]} \frac{\int_0^\omega \min\{N(\omega,l)\Omega(l,b(l)),N(\omega,l)\Omega(l,\omega(l))\}dl+t}{\int_0^\omega \max\{N(\omega,l)\Omega(l,b(l)),N(\omega,l)\Omega(l,\omega(l))\}dl+t'} \\ &= \sup_{t\in[0,1]} \frac{\int_0^\omega N(\omega,l)\min\{\Omega(l,b(l)),N(\omega,l)\Omega(l,\omega(l))\}dl+t}{\int_0^\omega N(\omega,l)\max\{\Omega(l,b(l)),N(\omega,l)\Omega(l,\omega(l))\}dl+t'} \\ &\geq \sup_{t\in[0,1]} \frac{\int_0^\omega N(\omega,l)\min\{D(b(l),\omega(l))\}dl+t}{\int_0^\omega N(\omega,l)\max\{D(b(l),\omega(l))\}dl+t'} \\ &= M^2(D(b,\omega,t)). \end{aligned}$$

Thus, all conditions of Theorem 2.5 are satisfied. That is, operator f has an fixed point which is a solution to differential equation (14) regulating the change of solar energy to electric power. □

4. Conclusion

In the relevant of controlled fuzzy metric spaces, this stenography holds a number of fixed point theorems and sufficient condition for a sequence to be a Cauchy. As a outcome, we merged the well- known contraction requirements with controlled fuzzy metric spaces to simplify the proof of various fixed point theorems of quadratic type. Furthermore, we consider an application to convert solar energy to electric power.

Conflict of Interests

The authors declare that there is no conflict of interests.

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