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STRATEGIC INTERACTIONS IN DIGITAL TRANSFORMATION: A GAME-THEORETIC APPROACH TO DECISION-MAKING AND COLLABORATION

BRAHIM BOULAFDOUR[1], RADOUANE AZENNAR[2,*]

[1]Department of Economics and Management, Faculty of Legal, Economic and Social Sciences of Mohammedia, Hassan II University of Casablanca, Morocco

[2]Department of Mathematics, Faculté des Sciences, Ibn Tofaïl University, B.P. 133, Kenitra, 14000, Morocco

Abstract. Digital transformation (DT) is reshaping industries by integrating advanced technologies such as artificial intelligence, blockchain, and the Internet of Things (IoT). However, the decision-making processes among stakeholders—firms, consumers, and regulators—are often marked by strategic conflicts and cooperation dilemmas. This paper employs game theory to model and analyze these interactions, providing a framework for understanding competitive and collaborative behaviors in DT. We examine key applications, including technology adoption, cybersecurity, and platform competition, where Nash equilibria and cooperative game solutions offer insights into optimal strategies. Our findings suggest that game-theoretic models can enhance efficiency, fairness, and innovation in digital ecosystems. The study contributes to both theoretical and practical discussions on strategic decision-making in rapidly evolving digital landscapes.

Keywords: fixed point; multivalued mapping; game theory; digital transformation.

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*Corresponding author
E-mail address: azennar_pf@hotmail.com
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1. INTRODUCTION

Digital transformation (DT) represents a paradigm shift in how organizations leverage technology to enhance operations, customer experiences, and business models. However, the interconnected nature of digital ecosystems introduces complex strategic interactions among stakeholders, including firms, consumers, and policymakers. Game theory, with its robust analytical tools for modeling competition and cooperation, offers a valuable lens to study these dynamics [1, 2].

Traditional approaches to DT often overlook the strategic interdependencies that arise in multi-agent environments. For instance, firms competing in digital markets must weigh the costs and benefits of adopting new technologies while anticipating rivals' moves. Similarly, collaborative platforms face challenges in incentivizing participation while preventing free-riding. Game theory addresses these issues by formalizing strategies, payoffs, and equilibria, enabling stakeholders to make informed decisions.[3, 4]

2. PRELIMINARIES

This paper explores three critical areas where game theory applies to DT:

- **Technology Adoption**: Coordination games and network effects in the diffusion of innovations.
- **Cybersecurity**: Non-cooperative games for modeling attacker-defender interactions.
- **Platform Competition**: Auction theory and coalitional games in multi-sided markets.

Our analysis demonstrates that game-theoretic approaches not only predict outcomes but also prescribe mechanisms for efficient and equitable digital transformation. By bridging theory and practice, this work provides actionable insights for businesses and policymakers navigating the complexities of DT. Consider a digital transformation scenario involving two players:

- Management (M) aims to maximize productivity and minimize costs.
- Employees (E) aim to minimize disruption and maximize ease of use.

A two-player, zero-sum game is a triple (X,Y,f), where X,Y are nonempty sets ([2, page 326]), whose elements are called strategies, and $f : X \times Y \to \mathbb{R}$ is the gain function. There are two players, α and β, and $f(x,y)$ represents the gain of the player α when he chooses the strategy

$x \in X$ and the player β chooses the strategy $y \in Y$. The quantity $-f(x,y)$ represents the gain of the player β in the same situation. The target of the player α is to maximize his gain when the player β chooses a strategy that is the worst for α.
$(x_0, y_0) \in X \times Y$ a solution of the game and x_0 and y_0 winning strategies. It follows that to prove the existence of a solution of a game we have to prove equality.

$$\sup_{x\in X} \inf_{y\in Y} f(x,y) = f(x_0,y_0) = \inf_{y\in Y} \sup_{x\in X} f(x,y). \tag{2.1}$$

For more details on game theory and minimax theorems, we refer to the books of Aubin, J.P [1] and Carl, S, Heikkilä, S [2].

Objective Functions. Management's Objective:

$$\text{Maximize } U_M = P - C - R$$

where:

- P = Productivity gain
- C = Cost of implementation
- R = Cost of employee resistance

Employees' Objective:

$$\text{Minimize } C_E = D - S$$

where:

- D = Disruption cost
- S = Satisfaction from ease of use

Min-Max Formulation. Management seeks to minimize the maximum resistance cost:

$$\min_{s\in S} \max_{e\in E} R(s,e)$$

where:

- s = Strategy of management
- e = Strategy of employees
- S = Set of management strategies
- E = Set of employee strategies

3. Main Results

In this section, we establish a generalization of the the minimax theorem using the property of a star-shaped subsets instead of the convexity condition in the Hausdorff locally convex spaces. The authors of the article [5] applied their theory in the field of differential equations, we will apply their theoretical results to establish a new version of the minmax theory.
A locally convex space is a topological vector space whose topology is generated by a family of seminorms, ensuring that every neighborhood of the origin contains a convex open set.

Definition 3.1. *A subset $A \subseteq X$ is said to be star-shaped if there exists an element $x \in A$ such that $tx + (1–t)y \in A$ for all $t \in [0,1]$ and for all $y \in A$. Such an element is called a star-point of A. The set of all star-points of A is called the star-core of A.*

Definition 3.2. [5, Definition 3.1]
Let A be a convex subset of a locally convex space X. For $q \in A$ and $T : A \to 2^A$, T is said to be q-convex if we have the following condition:

$$T(\lambda q + (1-\lambda)x) \supseteq \lambda T(q) + (1-\lambda)T(x)$$

for all $x \in A$ and $\lambda \in [0,1]$.

Theorem 3.3. [5, Corollary 3.6]
Let A be a nonempty, compact and star-shaped subset of a Hausdorff locally convex space X and Let $T : A \to 2^A$ have the colsed graph and $T(x)$ is nonempty closed and convex for every $x \in A$. Then, $\exists c \in A$ such that $c \in Tc$.

Remark 1. *the condition as T be an upper semicontinuous multimap is equivalent to T have the closed graph.*

We now establish our basic theorem, this is a generalization of the minmax theorem.

Theorem 3.4. *Let E be a Hausdorff locally convex space, and X,Y be a nonempty star-shaped and compact subsets of E. Let k is a continuous function such that $k : X \times Y \to \mathbb{R}$.*
Suppose that:

(1) $\exists x \in X$, $k(x,.)$ is convex,

(2) $\exists y \in Y$, $k(.,y)$ is concave.

Consider a two-player, zero-sum game with sets of strategies X and Y for Player 1 and Player 2, respectively. Let $k : X \times Y \to \mathbb{R}$ be the payoff function. Then, there exist optimal strategies $x^* \in X$ for Player 1 and $y^* \in Y$ for Player 2 such that:

$$\max_{y\in Y}\min_{x\in X} k(x,y^*) = \min_{x\in X}\max_{y\in Y} k(x^*,y)$$

Proof. We set that:

$$\phi(x) = \min_{y\in Y} k(x,y) = \min k(x\times Y), x \in X$$

and

$$\psi(y) = \max_{x\in X} k(x,y) = \max k(X\times y), y \in Y$$

$$N_y = \{x \in X : k(x,y) = \psi(y)\} \text{ and } M_x = \{y \in Y : k(x,y) = \phi(x)\}$$

and

$$N_{D'} = \cup_{y\in D'} N_y, \;\; M_D = \cup_{x\in D} M_x$$

for any set $(D\times D')$ of $X\times Y$, by the compactness of X and Y, ϕ and ψ are continuous too.
We pose: $C = X\times Y$ and $c = (x,y)$.
the product set C is compact,(product of two compact) and the sets M_x and N_y are nonempty, closed because the functions k,ϕ and ψ are continuous.
By the hypothese (1) and (2), it is easy to see that the sets M_x and N_y are star-shaped too.
therefore, the following two mapping can be defined by:

$$T : C \to 2^C$$
$$c \mapsto N_y \times M_x$$

whith $c = (x,y) \in X\times Y$.
First, we will show that T have the closed graph.
Indeed, Let $((x_i,y_i), i \in I)$ be a net in C such that $(x_i,y_i) \to (x,y) \in C$, let (u_i,v_i) be a net such that $(u_i,v_i) \in T(x_i,y_i)$ and $(u_i,v_) \to (u,v)$, We shall show that $(u,v) \in T(x,y)$,
we have:

$$\begin{aligned}(u_i,v_i) \in T(x_i,y_i) \;\; &\Leftrightarrow \;\; (u_i,v_i) \in N_{y_i}\times M_{x_i}\\ &\Leftrightarrow \;\; k(u_i,y_i) = \psi(y_i) \text{ and } k(x_i,v_i) = \phi(x_i)\end{aligned}$$

Since k and ϕ are continuous, for $i \in I$, we will have that:

$$k(u,y) = \psi(y) \text{ and } k(x,v) = \phi(x)$$

So, $(u,v) \in T(x,y)$, which implies that T has a closed graph.

Thus, by theorem 3.3, T have a fixed point $c^\star = (x^\star, y^\star)$.

So, we have $c^\star \in Tc^\star = N_{y^\star} \times M_{x^\star}$.

in other words,

$$x^\star \in N_{y^\star} \Leftrightarrow k(x^\star, y^\star) = \max_{x \in X} k(x, y^\star) \geq \inf_{y \in Y} \max_{x \in X} k(x,y)$$

$$y^\star \in M_{x^\star} \Leftrightarrow k(x^\star, y^\star) = \min_{y \in Y} k(x^\star, y) \leq \sup_{x \in X} \min_{y \in Y} k(x,y)$$

implying $\max_{x \in X} \min_{y \in Y} k(x,y) = k(x^\star, y^\star) = \min_{y \in Y} \max_{x \in X} k(x,y)$. This completes the proof.

□

4. Min-Max Model Formulation

In digital transformation, adversarial scenarios (e.g., cybersecurity, competitive markets) often involve two players with diametrically opposed objectives. We formalize this as a zero-sum game, where one player's gain is the other's loss. The *attacker-defender* interaction serves as a canonical example.

4.1. Model Setup.

Let:

- $\mathscr{P} = \{D, A\}$ be the players: Defender (D) and Attacker (A).
- $\mathscr{S}_D, \mathscr{S}_A$ denote their respective strategy sets (e.g., security protocols for D, attack vectors for A).
- $U_D(s_D, s_A)$ be the defender's utility function, with $U_A(s_D, s_A) = -U_D(s_D, s_A)$ (zero-sum assumption).

The defender aims to *minimize* maximum potential damage, while the attacker seeks to *maximize* minimum payoff. This yields the min-max and max-min problems:

$$\underline{v} = \max_{s_A \in \mathscr{S}_A} \min_{s_D \in \mathscr{S}_D} U_A(s_D, s_A) \quad \text{(Attacker's guaranteed payoff)}, \tag{4.1}$$

$$\overline{v} = \min_{s_D \in \mathscr{S}_D} \max_{s_A \in \mathscr{S}_A} U_D(s_D, s_A) \quad \text{(Defender's worst-case loss)}. \tag{4.2}$$

Under the von Neumann minimax theorem [13], if $\mathscr{S}_D, \mathscr{S}_A$ are compact and U_D is continuous, then $\underline{v} = \overline{v} = v^*$, the value of the game, and a Nash equilibrium exists.

4.2. Example: Cybersecurity Investment.

Let the defender choose a protection level $x \in [0,1]$ (e.g., fraction of systems hardened), and the attacker selects an effort $y \in [0,1]$. Define utilities as:

$$U_D(x,y) = -C_D x - L(1-x)y, \tag{4.3}$$

$$U_A(x,y) = C_D x + L(1-x)y - C_A y, \tag{4.4}$$

where C_D, C_A are costs for defense/attack, and L is loss from a successful breach.

The defender solves:

$$\min_x \max_y \left[C_D x + L(1-x)y\right], \tag{4.5}$$

yielding optimal strategies (x^*, y^*) via first-order conditions.

4.3. Implications for Digital Transformation.

- **Robustness**: Min-max strategies ensure resilience against worst-case attacks.
- **Resource Allocation**: Guides optimal investment in security vs. risk tolerance.
- **Dynamic Extensions**: Repeated games can model evolving threats in DT.

4.4. Solution Approach.

The model can be solved using:

- Linear programming if the functions are linear.
- Robust optimization techniques.
- Scenario-based analysis for different uncertainty levels.

4.5. Conclusion.

Digital transformation (DT) is a multi-agent, strategic process where stakeholders must navigate competition, cooperation, and uncertainty. This paper has demonstrated how game theory provides a rigorous framework for analyzing these interactions, offering solutions to critical challenges in technology adoption, cybersecurity, and platform competition. By modeling strategic behaviors through Nash equilibria, cooperative games, and mechanism design, we have highlighted pathways to optimize decision-making in digital ecosystems.

Our analysis reveals three key takeaways:

- **Efficiency in Adoption**: Coordination games help firms synchronize technology investments, reducing inefficiencies caused by fragmented market adoption.
- **Robust Security Strategies**: Non-cooperative game models between attackers and defenders enable proactive cybersecurity measures, balancing costs and risks.
- **Fair Platform Governance**: Coalitional and auction-based approaches ensure equitable value distribution in multi-sided digital platforms, fostering sustainable growth.

However, limitations remain. Real-world DT scenarios often involve incomplete information, bounded rationality, and dynamic environments—factors requiring extensions to classical game-theoretic models. Future research should explore:

- Behavioral game theory to account for human biases in digital strategy.
- Stochastic and repeated games for long-term DT evolution.
- Empirical validations of game-theoretic predictions in industry case studies.

In conclusion, game theory is a powerful tool for decoding the complexities of digital transformation. By integrating theoretical rigor with practical applications, stakeholders can design more resilient, collaborative, and innovative digital strategies.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] J.P. Aubin, Mathematical Methods of Game and Economic Theory, North-Holland, (1982).

[2] S. Carl, S. Heikkilä, Fixed Point Theory in Ordered Sets and Applications, Springer, New York, 2010. https://doi.org/10.1007/978-1-4419-7585-0.

[3] N. Eber, Théorie des Jeux, Dunod, Paris, (2013).

[4] A. Gethin, Apports et Limites de la Théorie des Jeux, Regards Croisés Écon. 22 (2018), 68–71. https://doi.org/10.3917/rce.022.0068.

[5] H. Goudarzi, H. Azadi Kenary, C. Park, Common Fixed Point Theorems for Multimaps in Locally Convex Spaces and Some Applications, Fixed Point Theory Appl. 2013 (2013), 37. https://doi.org/10.1186/1687-1812-2013-37.

[6] Ky Fan, Fixed-Point and Minimax Theorems in Locally Convex Topological Linear Spaces, Proc. Natl. Acad. Sci. U.S.A. 38 (1952), 121–126. https://www.jstor.org/stable/88613.

[7] K. Leyton-Brown, Y. Shoham, Essentials of Game Theory, Morgan & Claypool Publishers, (2008).

[8] A. Markov, Quelques Théorèmes sur les Ensembles Abéliens, Dokl. Akad. Nauk SSSR 10 (1936), 311–314.

[9] J.F. Nash, Equilibrium Points in n-Person Games, Proc. Natl. Acad. Sci. 36 (1950), 48–49. https://doi.org/10.1073/pnas.36.1.48.

[10] T.C. Schelling, The Strategy of Conflict, Harvard University Press, (1960).

[11] S.B. Nadler Jr, Multivalued contraction mappings, Pac. J. Math. 30 (1969), 475–488.

[12] J. von Neumann, O. Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, (1944).

[13] J. von Neumann, Uber ein Okonomisches Gleichungssystemund eine Verallgemeinerung des Brouwerschen Fixpunktsatzes, Ergeb. Math. Kolloq. 8 (1937), 73–83.

[14] X. Dai, A Fixed Point Theorem of Markov-Kakutani Type for a Commuting Family of Convex Multivalued Maps, Fixed Point Theory 18 (2017), 155–166.