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A FIXED POINT THEOREM FOR (F, Φ) -WEAK CONTRACTION ON COMPLETE METRIC SPACE

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Abstract. By introducing the notion of (F, ϕ) - weak contraction, we establish a fixed point result using this (F, ϕ) -weak contraction. Our result generalize and extend some well known results in the literature.

Keywords: metric space; fixed point; F -contraction.

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1. INTRODUCTION

Throughout this article, we denote by (X, d) a metric space (MS) with metric d , by \mathbb{N} the set of all natural numbers, by \mathbb{R} the set of all real numbers and by \mathbb{R}_+ the set of all positive real numbers.

2. PRELIMINARIES

In 1922, a famous result called Banach Contraction Principle (BCP) was proved by S. Banach [1] and can be stated as follows.

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Theorem 2.1. [1] *Let (X, d) be a complete MS and $T : X \rightarrow X$ be a mapping satisfying*

$$(2.1) \quad d(Tx, Ty) \leq kd(x, y)$$

$\forall x, y \in X$ and $k \in (0, 1)$. Then T has a unique fixed point (FP) in X .

Since then many generalizations were developed with modifications of the right of inequality (2.1) (see [2], [4], [6],[7]).

Replacing the constant k in (2.1) by a function, Boyd and Wong [2] established the following result.

Theorem 2.2. [2]. *Let (X, d) be a complete MS and $T : X \rightarrow X$ be a mapping. If there exists an upper semicontinuous from the right function $\phi : [0, \infty) \rightarrow [0, \infty)$ s.t. $\phi(t) \leq t$ for $t \in (0, \infty)$ and $\forall x, y \in X$*

$$(2.2) \quad d(Tx, Ty) \leq \phi d(x, y)$$

then the mapping T has a unique FP x^ and the iterative sequence $\{T^n x\}$ converges to x^* , $\forall x \in X$.*

Conditions on the function ϕ were modified by many authors (see[5], [6], [7][8]).

Another approach of modifying right of (2.1) is due to Ciric[4].

Theorem 2.3. [4]. *Let (X, d) be a complete MS. Let $T : X \rightarrow X$ be a mapping s.t. for some constant $\alpha \in (0, 1)$ and $\forall x, y \in X$,*

$$(2.3) \quad d(Tx, Ty) \leq \alpha \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$$

Then T has a unique FP.

A generalization similar to the combination of forms (2.2) and (2.3) was used by Pant [7].

Theorem 2.4. [7]. *Let T be a self-mapping of a complete MS (X, d) s.t $\forall x, y \in X$,*

$$(2.4) \quad d(Tx, Ty) \leq \phi (M(x, y))$$

where $M(x, y) = \max \left\{ d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\}$, $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ denotes a function s.t. $\phi(t) < t$ for each $t > 0$ and $\lim_{n \rightarrow \infty} \phi^n(t) = 0$. Then T has a unique FP say z . Moreover, T is continuous at z iff $\lim_{x_n \rightarrow z} M(x_n, z) = 0$.

Besides these, Wardowski [11] introduced a new notion called F -contraction and gave examples showing that his new notion is a generalization of Banach contraction.

Definition 2.5. [11] Let $F : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a mapping such that

(F1) F is strictly increasing i.e $\forall \alpha, \beta \in \mathbb{R}_+$ s.t. $\alpha < \beta$, then $F(\alpha) < F(\beta)$

(F2) for each sequence $\{\alpha_n\}$ of positive numbers, $\lim_{n \rightarrow \infty} \alpha_n = 0$ iff

$$\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$$

(F3) there exists $k \in (0, 1)$ s.t. $\lim_{\alpha \rightarrow 0^+} \alpha^k F(\alpha) = 0$.

A mapping $T : X \rightarrow X$ is called F -contraction if $\exists \tau > 0$ such that

$$(2.5) \quad \forall x, y \in X, d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(d(x, y))$$

Examples of F -contraction are given in [11].

Using F -contraction, Wardowski [11] proved the following theorem generalizing BCP.

Theorem 2.6. [11] Let (X, d) be a complete MS and let $T : X \rightarrow X$ be an F -contraction. Then T has a unique FP $x^* \in X$ and for every $x_0 \in X$ a sequence $\{T^n x_0\}_{n \in \mathbb{N}}$ is convergent to x^* .

In 2014, Wardowski and Dung [12] generalized F -contraction into F -weak contraction as follows.

Definition 2.7. [12] Let (X, d) be a MS. $T : X \rightarrow X$ is called F -weak contraction on (X, d) if $\exists \tau > 0$ s.t. $\forall x, y \in X$ with $d(Tx, Ty) > 0$, implies that

$$(2.6) \quad \tau + F(d(Tx, Ty)) \leq F \left(\max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\} \right)$$

Remark 2.8. [12]

Every F -contraction is an F -weak contraction but the converse is not necessarily true (see example [12]).

Using this notion of F - weak contraction, the authors proved the following theorem extending Wardowski [11] theorem (2.1)

Theorem 2.9. [12] *Let (X, d) be a complete MS and $T : X \rightarrow X$ be a F - weak contraction. If $T \circ F$ is continuous, then*

- (1) T has a unique FP $x^* \in X$.
- (2) $\forall x \in X, \{T^n x\}$ is convergent to x^* .

Recently, Lukacs and Kajanto[5], proved the following Lemma.

Lemma 2.10. [5] *If $F : (0, \infty) \rightarrow \mathbb{R}$ is an increasing function and $\{\alpha_n\}_{n \in \mathbb{N}} \subset (0, +\infty)$ is a decreasing sequence s.t.*

$$\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty \text{ then } \lim_{n \rightarrow \infty} \alpha_n = 0.$$

Also in [8], Peri and Kumam used new condition $(F3')$ instead of $(F3)$ of Wardowski [11] which is stated as

$(F3')$ F is continuous on $(0, \infty)$.

Now, we denote by \mathfrak{S} the family of all functions satisfying $(F1)$ and $(F3')$.

Example 2.11. *Let $F_1(x) = 2x, F_2(x) = x^n + x, n > 0$. Then $F_1, F_2, F_3 \in \mathfrak{S}$.*

It is noted that condition $(F3)$ and $(F3')$ are independent ([8], Remark 1.1).

$$\text{Let } M(x, y) = \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\}.$$

$$\text{Clearly, } \max \left\{ a, b, c, \frac{e+f}{2} \right\} \leq \max \{a, b, c, e, f\} \forall a, b, c, e, f \in \mathbb{R}.$$

In this paper, by considering $M(x, y) = \max \{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$, we define (F, ϕ) - weak contraction and prove a FP theorem for (F, ϕ) - weak contraction which generalised some well known result in the literature.

3. MAIN RESULTS

In the following we use the definition

Definition 3.1. *Let Φ be a family of functions $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfying:*

- ($\phi 1$) ϕ is non decreasing and continuous,
- ($\phi 2$) $\phi(2t) < t$ for $t \in \mathbb{R}_+$.

Remark 3.2. For $t \in (0, \infty)$, we consider that $\phi(2t) < t$ implies that $\phi(t) < t$ but not conversely.

Example 3.3. Let $\phi: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be defined by $\phi(t) = \frac{2}{3}t$. Then ϕ is non-decreasing, continuous and $\phi(t) < t \forall t \in \mathbb{R}_+$. But $\phi(2t) = \frac{2}{3}(2t) = \frac{4}{3}t > t$, so that $\phi(t) < t$ does not imply $\phi(2t) < t$. Now, we define (F, ϕ) -we contraction as follows.

Definition 3.4. Let (X, d) be a MS. A mapping $T: X \rightarrow X$ is called (F, ϕ) -weak contraction on (X, d) if $\exists \tau > 0$ with $F \in \mathfrak{F}$ and $\phi \in \Phi$ s.t $\forall x, y \in X$,

$$(3.1) \quad d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(M(x, y))$$

where

$$(3.2) \quad M(x, y) = \phi(\max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\})$$

Remark 3.5. Let T be (F, ϕ) -weak contraction. Then from (3.1), $\forall x, y \in X, Tx \neq Ty$

$$F(d(Tx, Ty)) < \tau + F(d(Tx, Ty)) \leq F(\phi(\max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}))$$

Since $\phi(2t) < t \implies \phi(t) < t$, for $t > 0$, therefore $\forall x, y \in X, Tx \neq Ty$

$$F(d(Tx, Ty)) < \tau + F(d(Tx, Ty)) \leq F(\max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\})$$

Then by $(F1)$, we have

$$d(Tx, Ty) < \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\} \forall x, y \in X, Tx \neq Ty$$

Example 3.6. Let $X = [0, 1]$ and d be the usual metric on X i.e

$$d(x, y) = |x - y|, \forall x, y \in X$$

Define $T: X \rightarrow X$ as

$$Tx = \begin{cases} \frac{x}{3}, & x \in [0, 1) \\ \frac{1}{3}, & x = 1 \end{cases}$$

$$\forall x, y \in [0, 1], x \neq y,$$

$$d(Tx, Ty) = \left| \frac{x}{3} - \frac{y}{3} \right| = \frac{1}{3} |x - y| = \frac{1}{3} d(x, y)$$

and

$$d(x, y) \leq \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$$

Therefore, taking $F(x) = 2x$ and $\phi(x) = \frac{4x}{9}$, we can find a real no. $\tau > 0$ (say $\tau = \frac{1}{10}$) s.t. (3.1) holds. Hence T is (F, ϕ) - weak contraction. Now we state our main result.

Theorem 3.7. *Let (X, d) be a complete MS and let $T : X \rightarrow X$ be a (F, ϕ) - weak contraction. Then, T has a unique FP $x^* \in X$.*

Proof: Let $x_0 \in X$ be arbitrary and fixed. Define $\{x_n\}_{n \in \mathbb{N}} \subset X$ s.t. $x_{n+1} = Tx_n, n = 0, 1, 2, \dots$

If $\exists n_0 \in \mathbb{N}$ s.t. $x_{n_0+1} = x_{n_0}$, then $Tx_{n_0} = x_{n_0}$ and the proof is completed.

Suppose $x_{n+1} \neq x_n \forall n \in \mathbb{N}$. Then by (7), $\forall n \in \mathbb{N}$,

$$(3.3) \quad F(d(x_{n+1}, x_n)) = F(d(Tx_n, Tx_{n-1})) \leq F(M(x_n, x_{n-1})) - \tau$$

where

$$\begin{aligned} M(x_n, x_{n-1}) &= \phi(\max \{d(x_n, x_{n-1}), d(x_n, Tx_n), d(x_{n-1}, Tx_{n-1}), d(x_n, Tx_{n-1}), d(x_{n-1}, Tx_n)\}) \\ &= \phi(\max \{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_n), d(x_n, x_n), d(x_{n-1}, x_{n+1})\}) \\ &= \phi(\max \{d(x_n, x_{n-1}), d(x_{n+1}, x_n), d(x_n, x_{n-1}), d(x_n, x_n), d(x_n, x_{n-1}) + d(x_{n+1}, x_n)\}) \\ &= \phi(d(x_n, x_{n-1}) + d(x_{n+1}, x_n)) \end{aligned}$$

If $d(x_{n+1}, x_n) > d(x_n, x_{n-1})$, then from (3.3), using $(\phi 2)$

$$F(d(x_{n+1}, x_n)) \leq F(\phi(2d(x_{n+1}, x_n))) - \tau \leq F(d(x_{n+1}, x_n)) - \tau < F(d(x_{n+1}, x_n))$$

which is a contradiction. Hence, $d(x_{n+1}, x_n) < d(x_n, x_{n-1}) \forall n \in \mathbb{N}$.

Thus, from (3.3), $F(d(x_{n+1}, x_n)) \leq F(d(x_n, x_{n-1})) - \tau \forall n \in \mathbb{N}$.

It follows that

$$(3.4) \quad F(d(x_{n+1}, x_n)) \leq F(d(x_1, x_0)) - n\tau$$

$\forall n \in \mathbb{N}$.

Letting $n \rightarrow \infty$ in (3.4), we have $\lim_{n \rightarrow \infty} F(d(x_{n+1}, x_n)) = -\infty$.

Since $\{d(x_{n+1}, x_n)\}_{n \in \mathbb{N}}$ is a decreasing sequence of positive real numbers s.t. $\lim_{n \rightarrow \infty} F(d(x_{n+1}, x_n)) = -\infty$. Then by Lemma 2.10,

$$\lim_{n \rightarrow \infty} d(x_{n+1}, x_n) = 0$$

$$(3.5) \quad i.e \lim_{n \rightarrow \infty} d(Tx_n, x_n) = 0$$

Suppose that $\{x_n\}_{n \in \mathbb{N}}$ is not a Cauchy sequence. Then $\forall \varepsilon > 0 \exists \{p(n)\}_{n \in \mathbb{N}}$ and $\{q(n)\}_{n \in \mathbb{N}}$ with $p(n) > q(n) > n$ s.t.

$$(3.6) \quad d(x_{p(n)}, x_{q(n)}) \geq \varepsilon$$

Suppose $p(n)$ is the smallest integer greater than $q(n)$ which satisfy (3.6) i.e.

$$d(x_{p(n)-1}, x_{q(n)}) < \varepsilon \quad \forall n \in \mathbb{N}.$$

Then, $\forall n \in \mathbb{N}$

$$\begin{aligned} \varepsilon \leq d(x_{p(n)}, x_{q(n)}) &\leq d(x_{p(n)}, x_{p(n)-1}) + d(x_{p(n)-1}, x_{q(n)}) \\ &\leq d(x_{p(n)}, x_{p(n)-1}) + \varepsilon \\ &= d(Tx_{p(n)-1}, x_{p(n)-1}) + \varepsilon. \end{aligned}$$

Letting $n \rightarrow \infty$ and using (3.5).

$$(3.7) \quad \varepsilon \leq \lim_{n \rightarrow \infty} d(x_{p(n)}, x_{q(n)}) < \varepsilon \quad i.e. \lim_{n \rightarrow \infty} d(x_{p(n)}, x_{q(n)}) = \varepsilon$$

Since $\lim_{n \rightarrow \infty} d(Tx_n, x_n) = 0 \exists k \in \mathbb{N}$ s.t.

$$(3.8) \quad d(x_{p(n)}, Tx_{p(n)}) \leq \frac{\varepsilon}{4}$$

and

$$(3.9) \quad d(x_{q(n)}, Tx_{q(n)}) \leq \frac{\varepsilon}{4}$$

$\forall n \geq k$. For this k , we claim that

$$d(Tx_{p(n)}, Tx_{q(n)}) = \varepsilon > 0$$

$\forall n \geq k$. Assume that $\exists m \geq k$ s.t. $d(Tx_{p(m)}, Tx_{q(m)}) = 0$, i.e

$$(3.10) \quad d(x_{p(m)+1}, x_{q(m)+1}) = 0$$

From (3.6) using (3.8), (3.9) and (3.10), we have

$$\varepsilon \leq d(x_{p(m)}, x_{q(m)}) \leq d(x_{p(m)}, x_{p(m)+1}) + d(x_{p(m)+1}, x_{q(m)+1}) + d(x_{q(m)+1}, x_{q(m)})$$

$$\begin{aligned}
&= d(x_{p(m)}, Tx_{p(m)}) + d(x_{p(m)+1}, x_{q(m)+1}) + d(x_{q(m)}, Tx_{q(m)}) \\
&< \frac{\varepsilon}{4} + 0 + \frac{\varepsilon}{4} \\
&= \frac{\varepsilon}{2}
\end{aligned}$$

a contradiction. Therefore

$$d(Tx_{p(n)}, Tx_{q(n)}) = \varepsilon > 0$$

$\forall n \geq k$. Then by (3.1), $\forall n \geq k$

$$(3.11) \quad F(d(Tx_{p(n)}, Tx_{q(n)})) \leq F(M(x_{p(n)}, x_{q(n)})) - \tau$$

where

$$\begin{aligned}
M(x_{p(n)}, x_{q(n)}) &= \phi \left(\max \left\{ d(x_{p(n)}, x_{q(n)}), d(x_{p(n)}, Tx_{p(n)}), d(x_{q(n)}, Tx_{q(n)}), \right. \right. \\
&\quad \left. \left. d(x_{p(n)}, Tx_{q(n)}), d(x_{q(n)}, Tx_{p(n)}) \right\} \right) \\
&\leq \phi \left(\max \left\{ d(x_{p(n)}, x_{q(n)}), d(x_{p(n)}, Tx_{p(n)}), d(x_{q(n)}, Tx_{q(n)}), \right. \right. \\
&\quad \left. \left. d(x_{p(n)}, x_{q(n)}) + d(x_{q(n)}, Tx_{q(n)}), d(x_{p(n)}, x_{q(n)}) + d(x_{p(n)}, Tx_{p(n)}) \right\} \right) \\
&= \phi \left(d(x_{p(n)}, x_{q(n)}) + \frac{\varepsilon}{4} \right)
\end{aligned}$$

Letting $n \rightarrow \infty$ in (3.11) and using $(F1), (F3'), (\phi1)$ and $(\phi2)$,

$$\begin{aligned}
F(\varepsilon) &\leq F\left(\phi\left(\varepsilon + \frac{\varepsilon}{4}\right)\right) - \tau \\
&\leq F(\phi(2\varepsilon)) - \tau \\
&\leq F(\varepsilon) - \tau \\
&< F(\varepsilon)
\end{aligned}$$

a contradiction. Thus $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy.

Since (X, d) is complete, then $\exists x^* \in X$ s.t. $\lim_{n \rightarrow \infty} x_n = x^*$.

Now we shall prove that x^* is a FP of T .

By $(F3')$, we will consider two cases.

Case I: If there exists subsequence $\{x_{n_i}\}_{i \in \mathbb{N}}$ of $\{x_n\}_{n \in \mathbb{N}}$ s.t. $x_{n_i} = Tx^* \forall i \in \mathbb{N}$, then $x^* =$

$$\lim_{i \rightarrow \infty} x_{n_i} = \lim_{i \rightarrow \infty} Tx^* = Tx^*$$

Case II : If \exists no such subsequence of $\{x_n\}_{n \in \mathbb{N}}$, then $\exists n_0 \in \mathbb{N}$ s.t. $x_{n+1} \neq Tx^* \forall n \geq n_0$ i.e $d(Tx_n, Tx^*) > 0 \forall n \geq n_0$. Then, by (3.1)

$$(3.12) \quad \tau + F(d(x_{n+1}, Tx^*)) = \tau + F(d(Tx_n, Tx^*)) \leq F(M(x_n, x^*))$$

where

$$(3.13) \quad \begin{aligned} M(x_n, x^*) &= \phi(\max\{d(x_n, x^*), d(x_n, Tx_n), d(x^*, Tx^*), d(x_n, Tx^*), d(x^*, Tx_n)\}) \\ &\leq \phi(\max\{d(x_n, x^*), d(x_n, x_{n+1}), d(x^*, Tx^*), \\ &\quad d(x_n, x^*) + d(x^*, Tx^*), d(x^*, x_{n+1})\}) \end{aligned}$$

We know that $\lim_{n \rightarrow \infty} d(x_n, x^*) = \lim_{n \rightarrow \infty} d(x_{n+1}, x^*) = 0$, then letting $n \rightarrow \infty$ in (3.13) and using $(\phi 1)$ and $(\phi 2)$, we have

$$(3.14) \quad \begin{aligned} \lim_{n \rightarrow \infty} Mx_n, x^*) &= \phi(d(x^*, Tx^*)) \\ &\leq \phi(2d(x^*, Tx^*)) < d(x^*, Tx^*) \end{aligned}$$

Letting $n \rightarrow \infty$ in (3.12) and using $(F1), (F3')$ and (3.14) we have

$$\tau + F(d(x^*, Tx^*)) \leq F(d(x^*, Tx^*))$$

a contradiction as $\tau > 0$. Hence

$$d(x^*, Tx^*) = 0$$

that is x^* is a FP of T .

Now we prove that the FP is unique.

Let x_1^*, x_2^* be two FP of T . Suppose, if possible, that $x_1^* \neq x_2^*$. Then $Tx_1^* \neq Tx_2^*$. Then from (3.1)

$$(3.15) \quad \begin{aligned} \tau + F(d(x_1^*, x_2^*)) &= \tau + F(d(Tx_1^*, Tx_2^*)) \\ &\leq F(M(x_1^*, x_2^*)) \end{aligned}$$

where

$$\begin{aligned} M(x_1^*, x_2^*) &= \phi(\max\{d(x_1^*, x_2^*), d(x_1^*, Tx_1^*), d(x_2^*, Tx_2^*), d(x_1^*, Tx_2^*), d(x_2^*, Tx_1^*)\}) \\ &\leq \phi \max\{d(x_1^*, x_2^*), d(x_1^*, Tx_1^*), d(x_2^*, Tx_2^*), d(x_1^*, x_2^*) + d(x_2^*, Tx_2^*), d(x_1^*, x_2^*) + d(x_1^*, Tx_1^*)\} \end{aligned}$$

$$\begin{aligned}
&= \phi(d(x_1^*, x_2^*)) \\
&\leq \phi(2d(x_1^*, x_2^*)) \\
&< d(x_1^*, x_2^*)
\end{aligned}$$

Therefore, from (3.15), we have

$$\tau + F(d(x_1^*, x_2^*)) \leq F(d(x_1^*, x_2^*))$$

which implies that $\tau \leq 0$, a contradiction. Hence $x_1^* = x_2^*$.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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