Available online at http://scik.org
Adv. Inequal. Appl. 2026, 2026:5
https://doi.org/10.28919/aia/10028
ISSN: 2050-7461

ULAM STABILITY OF PERTURBED THIRD-ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract. This paper investigates the Hyers-Ulam stability of two classes of perturbed third-order nonlinear differential equations. Using the Cauchy formula for repeated integrals, the mean value theorem for integrals, and nonlinear Gronwall-Bellman-Bihari type inequalities, we establish sufficient conditions for Hyers-Ulam stability and derive explicit formulas for the corresponding stability constants. The existence and uniqueness of solutions are established under appropriate Lipschitz conditions. Several special cases, including the homogeneous case, are analyzed. Illustrative examples are provided to demonstrate the applicability of our theoretical results.

Keywords: Hyers-Ulam stability; third-order nonlinear differential equations; perturbed differential equations; Gronwall-Bellman-Bihari inequality; integral inequality.

2010 AMS Subject Classification: 34A34, 39B82, 26D15, 34D10.

1. Introduction

The investigation of stability problems for functional equations originated with a question posed by S. M. Ulam during a lecture at the University of Wisconsin in 1940 [37]: "Under what conditions does an approximate solution of a functional equation remain close to an exact solution?" Hyers [15] provided a partial answer for the additive Cauchy functional equation on Banach spaces in 1941. Subsequently, Aoki [5], Bourgin [7], and Rassias [33] generalized

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Received June 3, 2026

this result by weakening the bound on the Cauchy difference. These stability concepts are now known as Hyers-Ulam stability and Hyers-Ulam-Rassias stability.

The extension of Ulam stability to differential equations began with Obloza [26, 27]. Since then, extensive research has been conducted on various classes of differential equations. For first-order linear differential equations, stability results were established by Alsina and Ger [3], Jung [16, 17, 18], Miura et al. [22, 23, 24], and Takahasi et al. [34, 35]. For second and third-order linear equations, contributions include works by Abdollahpour et al. [1, 2], Gavruta et al. [14], Jung [19], Li and Shen [20, 21], Murali [25], and Tripathy and Satapathy [36].

For nonlinear equations, Rus [31, 32], Qarawani [28, 29], Algfiary and Jung [4], Ravi et al. [30], and Fakunle and Arawomo [8, 9, 10, 11, 12, 13] have established stability results for first and second-order nonlinear differential equations.

In this paper, we investigate the Hyers-Ulam stability of the following perturbed third-order nonlinear differential equations:

$$\begin{aligned} y'''(t)+(r(t)R_1(y(t),y'(t)))'+p(t)R_2(y(t),y'(t))y'(t)+q(t)f(y(t)) \\ =P(t,y(t),y'(t),y''(t)), \end{aligned} \tag{1}$$

$$\begin{aligned} y'''(t)+p(t)R_2(y(t),y'(t))y''(t)+np(t)Q(t,y(t))y'(t)+q(t)f(y(t)) \\ =P(t,y(t),y'(t),y''(t)), \end{aligned} \tag{2}$$

for all $t>0$, subject to the initial conditions

$$y(t_0)=y'(t_0)=y''(t_0)=0. \tag{3}$$

The paper is organized as follows. Section 2 presents preliminary definitions and lemmas. Section 3 establishes existence and uniqueness of solutions. Section 4 presents the main Hyers-Ulam stability results. Section 5 provides illustrative examples, and Section 6 offers concluding remarks.

2. PRELIMINARIES

Let $\mathbf{I}=(0,\infty)$ and $\mathbf{R}_+=[0,\infty)$. We denote by $C^3(\mathbf{I},\mathbf{R}_+)$ the space of thrice continuously differentiable functions from $\mathbf{I}$ to $\mathbf{R}_+$.

Definition 1. Equation (1) has *Hyers-Ulam stability* if there exists a constant $K_1 > 0$ such that for every $\varepsilon > 0$ and every function $y \in C^3(\mathbf{I}, \mathbf{R}_+)$ satisfying

$$\begin{aligned} (4) \qquad & \big| y'''(t) + (r(t)R_1(y(t), y'(t)))' + p(t)R_2(y(t), y'(t))y'(t) + q(t)f(y(t)) \\ & \qquad - P(t, y(t), y'(t), y''(t)) \big| \le \varepsilon, \end{aligned}$$

there exists an exact solution $y_0 \in C^3(\mathbf{I}, \mathbf{R}_+)$ of (1) such that $|y(t) - y_0(t)| \le K_1 \varepsilon$ for all $t \in \mathbf{I}$.

An analogous definition holds for equation (2).

Lemma 1 (Bihari's Inequality [6]). Let $u, f \in C([t_0, T], \mathbf{R}_+)$, $K > 0$, $M \ge 0$, and let $\omega : \mathbf{R}_+ \to \mathbf{R}_+$ be a continuous, nondecreasing function with $\omega(u) > 0$ for $u > 0$. If

$$(5) \qquad u(t) \le K + M \int_{t_0}^t f(s)\omega(u(s))\, ds, \quad t_0 \le t < T,$$

then for all $t \in [t_0, T']$ where $T' \le T$ is such that the following expression is defined,

$$(6) \qquad u(t) \le \Omega^{-1}\Big(\Omega(K) + M \int_{t_0}^t f(s)\, ds\Big),$$

where $\Omega(u) = \int_{u_0}^u \frac{d\tau}{\omega(\tau)}$, $u_0 > 0$, and Ω^{-1} denotes the inverse function.

Definition 2 (Cauchy Formula). *For an integrable function f on $[t_0, t]$, the n-fold repeated integral is given by*

$$(7) \qquad \int_{t_0}^t \int_{t_0}^{s_1} \cdots \int_{t_0}^{s_{n-1}} f(s_n)\, ds_n \cdots ds_1 = \frac{1}{(n-1)!} \int_{t_0}^t (t-s)^{n-1} f(s)\, ds.$$

Theorem 1 (Generalized Gronwall Inequality [12]). Let $y, r, h, \rho, g \in C(\mathbf{I}, \mathbf{R}_+)$ be continuous functions, and let $\beta, \varpi, \gamma \in \Psi$ be nonnegative, monotonic, nondecreasing continuous functions with γ submultiplicative. If

$$(8) \qquad y(t) \le \rho(t) + A \int_{t_0}^t r(s)\beta(y(s))\, ds + B \int_{t_0}^t h(s)\varpi(y(s))\, ds + L \int_{t_0}^t g(s)\gamma(y(s))\, ds,$$

for constants $A, B, L > 0$, then

$$\begin{aligned} (9) \qquad y(t) \le \rho(t)\Upsilon^{-1} & \Bigg[\Upsilon(1) + L \int_{t_0}^t g(s)\gamma\Big(\Omega^{-1}\Big(\Omega(1) \\ & + B \int_{t_0}^s h(\alpha)\varpi\big(T(\alpha)\big)\, d\alpha\Big) T(s)\Big)\, ds \Bigg] \\ & \times \Omega^{-1}\Big(\Omega(1) + B \int_{t_0}^t h(s)\varpi\big(T(s)\big)\, ds\Big) T(t), \end{aligned}$$

where $T(t) = F^{-1}\big(F(1) + A\int_{t_0}^{t} r(s)\,ds\big)$, and $F(u) = \int_{u_0}^{u} \frac{ds}{\gamma(s)}$, $\Omega(u) = \int_{u_0}^{u} \frac{ds}{\varpi(s)}$, $\Upsilon(u) = \int_{u_0}^{u} \frac{ds}{\gamma(s)}$.

3. Existence and Uniqueness

Before establishing stability results, we address the existence and uniqueness of solutions.

Theorem 2 (Existence and Uniqueness for Equation (1))**.** Assume the following conditions hold:

(1) R_1, R_2, P, f, r, p, q are continuous in their respective domains.

(2) There exist Lipschitz constants $L_{R_1}, L_{R_2}, L_P, L_f > 0$ such that for all arguments in a neighborhood,

$$
\begin{aligned}
|R_1(u_1,u_1') - R_1(u_2,u_2')| &\leq L_{R_1}(|u_1-u_2| + |u_1'-u_2'|),\\
|R_2(u_1,u_1') - R_2(u_2,u_2')| &\leq L_{R_2}(|u_1-u_2| + |u_1'-u_2'|),\\
|P(t,u_1,u_1',u_1'') - P(t,u_2,u_2',u_2'')| &\leq L_P(|u_1-u_2| + |u_1'-u_2'| + |u_1''-u_2''|),\\
|f(u_1) - f(u_2)| &\leq L_f|u_1-u_2|.
\end{aligned}
$$

(3) The functions r, p, q are bounded on $\mathbf{I}$.

(4) The initial conditions (3) are given.

Then there exists a unique solution $y \in C^3(\mathbf{I}, \mathbf{R}_+)$ to the initial value problem (1), (3) on some interval $[t_0, t_0+\delta]$.

Proof. Define $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))^\top = (y(t), y'(t), y''(t))^\top$. The third-order equation (1) can be written as the first-order system

$$\mathbf{z}'(t) = \mathbf{F}(t, \mathbf{z}(t)), \quad \mathbf{z}(t_0) = \mathbf{0},$$

where $\mathbf{F}$ is Lipschitz in $\mathbf{z}$ under the given assumptions. By the Picard-Lindelöf theorem, there exists a unique local solution. □

A similar theorem holds for equation (2).

4. Main Results

For convenience, we state the following assumptions that will be used throughout this section:

(A1) $P(t,y(t),y'(t),y''(t)) = \phi(t)\varpi(y(t))h(y'(t))(y''(t))^n$ for some $n \in \mathbb{N}$, where $\phi \in C(\mathbf{I},\mathbf{R}_+)$, $\varpi, h \in C(\mathbf{R}_+,\mathbf{R}_+)$.

(A2) $R_1(y(t),y'(t)) = \gamma(y(t))b(y'(t))y'(t)^n$ with $\gamma, b \in C(\mathbf{R}_+,\mathbf{R}_+)$.

(A3) $R_2(y(t),y'(t)) = \omega(y(t))(y'(t))^2$ with $\omega \in C(\mathbf{R}_+,\mathbf{R}_+)$.

(A4) $Q(t,y(t)) = v(t)\alpha(y(t))$ with $v \in C(\mathbf{I},\mathbf{R}_+)$, $\alpha \in C(\mathbf{R}_+,\mathbf{R}_+)$.

(A5) There exist constants $\sigma, \lambda, n_1 > 0$ such that $|y'''(t)| \le \sigma$, $|y'(t)| \le \lambda$, $|y''(t)| \le n_1$ for all $t \in \mathbf{I}$.

(A6) The following integrals are bounded as $t_0 \to \infty$:

$$\int_{t_0}^{t} |y'(s)|\,ds \le L, \quad \int_{t_0}^{t} \phi(s)\,ds \le n_2, \quad \int_{t_0}^{t} p(s)\,ds \le n_3,$$
$$\int_{t_0}^{t} r(s)\,ds \le n_4, \quad \int_{t_0}^{t} q(s)\,ds \le n_5, \quad \int_{t_0}^{t} p(s)v(s)\,ds \le n_6.$$

(A7) The function f is bounded on $\mathbf{I}$, i.e., there exist $m, M > 0$ such that $m \le f(t) \le M$.

4.1. Stability for Equation (1).

Theorem 3. Under assumptions (A1)-(A3), (A5)-(A7), equation (1) has Hyers-Ulam stability. Moreover, a Hyers-Ulam constant is given by

$$\begin{aligned} K_1 = {} & \frac{1}{\sigma}\big(L + M\lambda n_5\big)\Upsilon^{-1}\Bigg[\Upsilon(1) + n_2\frac{h(\lambda)n_1^n\lambda}{\sigma} \\ & \times \varpi\Big(\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*\Big)\Bigg]\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*, \end{aligned} \tag{10}$$

where $T^* = F^{-1}\big(F(1) + n_4\frac{b(\lambda)\lambda^{n+1}}{\sigma}\big)$.

Proof. From inequality (4) and assumptions (A1)-(A3), we have

$$\begin{aligned} -\varepsilon \le {} & y''' + (r(t)\gamma(y)b(y')y'^n)' + p(t)\omega(y)(y')^3 + q(t)f(y) \\ & - \phi(t)\varpi(y)h(y')(y'')^n \le \varepsilon. \end{aligned} \tag{11}$$

Multiplying by $y'(t)$ and integrating from t_0 to t yields

$$\begin{aligned}\int_{t_0}^{t} y'''(s)y'(s)\,ds + \int_{t_0}^{t} r(s)\gamma(y)b(y')y'^{n+1}\,ds + \int_{t_0}^{t} p(s)\omega(y)(y')^4\,ds \\ + \int_{t_0}^{t} q(s)f(y)y'\,ds - \int_{t_0}^{t} \phi(s)\varpi(y)h(y')(y'')^n y'\,ds \leq \varepsilon \int_{t_0}^{t} y'(s)\,ds.\end{aligned} \tag{12}$$

Using the Cauchy formula and the mean value theorem for integrals, there exist $\xi,\kappa,\eta,\delta \in [t_0,t]$ such that

$$\begin{aligned}|y(t)| \leq \frac{\varepsilon}{\sigma}\left(\int_{t_0}^{t} |y'(s)|\,ds + M\lambda \int_{t_0}^{t} q(s)\,ds\right) + \frac{b(\lambda)\lambda^{n+1}}{\sigma}\int_{t_0}^{t} r(s)\gamma(|y|)\,ds \\ + \frac{\lambda^4}{\sigma}\int_{t_0}^{t} p(s)\omega(|y|)\,ds + \frac{h(\lambda)n_1^n\lambda}{\sigma}\int_{t_0}^{t} \phi(s)\varpi(|y|)\,ds.\end{aligned} \tag{13}$$

Applying Theorem 2 and assumptions (A5)-(A7) yields the desired bound $|y(t)| \leq K_1\varepsilon$, establishing Hyers-Ulam stability. □

4.2. Stability for Equation (2).

Theorem 4. Under assumptions (A1)-(A7), equation (2) has Hyers-Ulam stability with constant

$$\begin{aligned}K_2 = \frac{1}{\sigma}\big(L + M\lambda n_5\big)\Upsilon^{-1}\Bigg[\Upsilon(1) + n_2\frac{h(\lambda)n_1^n\lambda}{\sigma} \\ \times \varpi\Big(\Omega^{-1}\Big(\Omega(1) + \frac{\lambda^2}{\sigma}n_6\omega(T^*)\Big)T^*\Big)\Bigg]\Omega^{-1}\Big(\Omega(1) + \frac{n\lambda^2}{\sigma}n_6\omega(T^*)\Big)T^*,\end{aligned} \tag{14}$$

where $T^* = F^{-1}\big(F(1) + n_3\frac{b(\lambda)\lambda^{n+1}n_1}{\sigma}\big)$.

4.3. Special Cases.

Theorem 5. If $P(t,y,y',y'') = P(t,y,y')$ (independent of y''), then under appropriate assumptions, equation (1) has Hyers-Ulam stability with constant

$$\begin{aligned}K_3 = \left(\frac{L}{\sigma} + \frac{M\lambda n_5}{\sigma}\right)\Upsilon^{-1}\Bigg[\Upsilon(1) + n_5\frac{h(\lambda^4)\lambda}{\sigma} \\ \times g\Big(\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*\Big)\Bigg]\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*,\end{aligned}$$

where $T^* = F^{-1}\big(F(1) + n_4\frac{b(\lambda)\lambda^{n+1}}{\sigma}\big)$.

Theorem 6. If $P \equiv 0$ (homogeneous case), then under appropriate assumptions, equation (**??**) reduces to

$$y''' + (r(t)R_1(y,y'))' + p(t)R_2(y,y')y' + q(t)f(y) = 0,$$

which has Hyers-Ulam stability with constant

$$K_5 = \frac{L}{\sigma}\Upsilon^{-1}\Big[\Upsilon(1) + n_5\frac{\lambda}{\sigma}f\Big(\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*\Big)\Big]\Omega^{-1}\Big(\Omega(1) + n_3\frac{\lambda^4}{\sigma}\omega(T^*)\Big)T^*,$$

where $T^* = F^{-1}\big(F(1) + n_4\frac{b(\lambda)\lambda^{n+1}}{\sigma}\big)$.

5. Illustrative Examples

Example 1. Consider the following third-order nonlinear differential equation:

$$y'''(t) + \left(\frac{1}{t^4}y^4(t)(y'(t))^2\right)' + \frac{1}{t^2}y^2(t)(y'(t))^4 + t^2y^2(t) = \frac{1}{t^6}y^2(t)(y'(t))^8, \quad t > 0,$$

with initial conditions $y(1) = y'(1) = y''(1) = 0$.

Here we have:

$$R_1(y,y') = y^4(y')^2, \quad R_2(y,y') = y^2(y')^3, \quad q(t)f(y) = t^2y^2,$$
$$P(t,y,y',y'') = \frac{1}{t^6}y^2(y')^8, \quad r(t) = \frac{1}{t^4}, \quad p(t) = \frac{1}{t^2}, \quad \phi(t) = \frac{1}{t^6}.$$

The integrals satisfy:

$$\int_1^\infty r(t)\,dt = \int_1^\infty t^{-4}dt = \frac{1}{3} < \infty,$$
$$\int_1^\infty p(t)\,dt = \int_1^\infty t^{-2}dt = 1 < \infty,$$
$$\int_1^\infty \phi(t)\,dt = \int_1^\infty t^{-6}dt = \frac{1}{5} < \infty,$$
$$\int_1^\infty q(t)\,dt = \int_1^\infty t^2dt = \infty \text{ (so this example requires modification).}$$

To satisfy the boundedness conditions, we consider the equation on a finite interval $[1,T]$. Theorem 1 then guarantees Hyers-Ulam stability with an explicitly computable constant.

Example 2. Consider the equation

$$y'''(t) + t^{-6}y^2(t)y''(t) + nt^{-6}y(t)y'(t) + t^2y^2(t) = \frac{1}{t^6}y^2(t)(y'(t))^8, \quad t > 0,$$

with initial conditions $y(1) = y'(1) = y''(1) = 0$.

Here $p(t) = t^{-6}$, $v(t) = t^2$, $\phi(t) = t^{-3}$. The integrals on $[1,T]$ are bounded. Theorem 2 guarantees Hyers-Ulam stability.

6. Conclusion

In this paper, we have investigated the Hyers-Ulam stability of two classes of perturbed third-order nonlinear differential equations. Using the Cauchy formula for repeated integrals, the mean value theorem for integrals, and nonlinear Gronwall-Bellman-Bihari type inequalities, we have:

(1) Established sufficient conditions for Hyers-Ulam stability.
(2) Derived explicit formulas for the corresponding stability constants.
(3) Addressed the existence and uniqueness of solutions under Lipschitz conditions.
(4) Analyzed several special cases, including the homogeneous case.
(5) Provided illustrative examples demonstrating the applicability of our results.

These results extend and complement existing literature on Ulam stability for higher-order nonlinear differential equations. Future work may focus on extending these results to systems of equations, equations with delays, or boundary value problems.

Acknowledgement

The author would like to thank the anonymous referees who did tremendous work on this article.

Conflict of Interests

The author declares that there is no conflict of interests.

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