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EFFORT DYNAMICS OF TILAPIA – NILE PERCH FISHERY MODEL IN POLLUTED ENVIRONMENT OF TANZANIAN WATERS OF LAKE VICTORIA

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Abstract. In this paper we analyse the effort dynamics model for Tilapia (Oreochromis niloticus) – Nile perch (Lates niloticus) fishery in polluted environment of Tanzanian waters of Lake Victoria. The model is analysed to get the maximum sustainable yield (MSY) points and the corresponding conditions for their existence have been established. The equilibrium points of the model are found with the conditions for their existence being established. The stability analysis of the interior equilibrium point has been investigated by using Routh – Hurwitz criteria method. Later, numerical simulations and their corresponding graphs are presented. It was revealed that the water pollution has significant effects to the maximum sustainable yields of both Tilapia and Nile perch produces. This effect also manifests the rapid changes of efforts invested in harvested the two species at the interior equilibrium point.

Keywords: Effort dynamics model; fishery, Tilapia; Nile perch; harvesting and water pollution.

2010 AMS Subject Classification: 47H17, 47H05, 47H09.

1. Introduction

Lake Victoria is the largest fresh water lake in Africa, and the second largest lake in the world.

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It covers an area of approximately $68\ 000km^2$ shared by Kenya (6%), Uganda (43%) and Tanzania (51%). Lake Victoria has a mean depth of $40m$, maximum depth of $84m$, shoreline of approximately $3\ 450km$ and a catchment area of $193\ 000km^2$ which extends into Rwanda and Burundi (Matsuishi *et al.* 2006).

The fishery resources of Lake Victoria are in record of being the source of protein and employment opportunities, especially for the lakeside communities (Twong'o and Sikoyo, 2004). Approximately 200 000 metric tons of fish are harvested annually from Tanzanian waters of Lake Victoria in which Nile perch fishery contributes more than 60% of the total produce (Bulayi, 2001).

Studies suggest that the fishing efforts in the Lake Victoria can take a dynamic behavior as opposed to static. The number of fishermen and fishing boats has been increasing from day to day, between the year 1990 and 2000 – the ten years interval – the fishing effort raised to more than double, rising from about $5\ 000$ to $15\ 500$ in Tanzania (Matsuishi *et al.* 2006).

Regardless the economic importance of Tilapia and Nile perch fishery in Tanzania and other riparian states, water quality has been declining due to both point and non-point sources from domestic, industrial and agricultural activities (Rwetabula *et al.* 2005). The decline of water quality in the lake threatens both the Tilapia and Nile perch stocks and other species available in the lake's ecosystem. It is availed that the environmental factors are responsible, primarily for the decline of Nile perch stocks rather than fishing (Kolding *et al.* 2013).

The population dynamics of Tilapia and Nile Perch therefore suffers from a number of factors. This study is an extension of bioeconomic model developed in Mayengo *et al. (2014*) which studied the dynamical population of Tilapia and Nile perch species in a prey – predator interaction, which are harvested continuously with the constant fishing efforts. In this paper we consider the same model taking into account the dynamic nature of harvesting efforts.

2. The model and its properties

In this section, we consider the Tilapia – Nile perch model in prey – predator interaction. The ecological setup considers the following. The Tilapia and Nile perch are continuously harvested; the Nile perch is fed on the Tilapia as its favorite food in the lake. However, in the absence of Tilapia it can feed on other species available in the lake. So, the growth of both Tilapia and Nile perch is assumed to be logistic. Again, both species respond to the effects of water pollution. Let us assume $x(t)$ and $y(t)$ are respectively the size of Tilapia and Nile perch population at time t. Thus, the model becomes

(1) $$\frac{dx}{dt} = r_1\left(1 - \frac{x}{k_1}\right)x - h_1(t) - \frac{\alpha yx}{A+x} - d_1 x$$

(2) $$\frac{dy}{dt} = r_2\left(1 - \frac{y}{k_2}\right)y - h_2(t) + \frac{\beta yx}{A+x} - d_2 y$$

where r_1 and r_2 are the intrinsic growth rate of Tilapia and Nile perch respectively; k_1 and k_2 are the environmental carrying capacity of Tilapia and Nile perch respectively; $h_1(t)$ and $h_2(t)$ are the harvesting of Tilapia and Nile perch respectively at time t; α is the maximal relative increase of predation, β is the conversion factor, A is the saturation constant, d_1 and d_2 are the death rate of Tilapia and Nile perch respectively due to water pollution.

Due to the preference of Nile perch to feed on the Tilapia other than other species, we assume that the intrinsic growth rate of Nile perch is relatively smaller than that of Tilapia (say $r_2 < r_1$). It was also availed that the breeding of Tilapia specie takes place at exclusively near – shore, confined to shallow waters of up to about $20\ m$ deep. Contrary to Tilapia, the breeding of Nile perch is in record of taking place throughout the lake including the shallow waters (Balirwa *et al.* 2004). This conclusion were reached basing on the fact that in the catch assessment survey conducted the Nile perch juveniles were caught in both near shore waters and in the off shore waters. However, it is also possible that the Nile perch juveniles might have had moved from deep waters of the lake to the near shore waters for feeding purposes. The near shore waters region is at high risk of water pollution, thus suggesting that Tilapia receives relatively greater effects from water pollution than that of Nile perch (say $d_2 < d_1$).

We consider the functional form harvesting using the ***catch-per-unit-effort (CPUE)*** phenomenon to describe an assumption that CPUE is proportional to the stock level. Thus we have

(3) $$h_1(t) = q_1 E_1 x(t)$$

(4) $$h_2(t) = q_2 E_2 y(t)$$

where q_1 and q_2 are the catchability co-efficient of Tilapia and Nile perch respectively, E_1 and E_2 are the harvesting effort used to harvest Tilapia and Nile perch population respectively.

According to Kar and Chakraborty (2010), we assume that the fishery effort itself is a dynamic variable that satisfy

(5) $$\frac{dE_1}{dt} = \mu_1(p_1 q_1 x - c_1)E_1$$

(6) $$\frac{dE_2}{dt} = \mu_2(p_2 q_2 y - c_2)E_2$$

where c_1 and c_2 are the constant fishing cost per unit effort for Tilapia (x) and Nile perch (y) respectively, p_1 and p_2 are the constant price per unit biomass of Tilapia and Nile perch respectively, μ_1 and μ_2 are stiffness parameters for Tilapia and Nile perch respectively.

Thus model equations (1) and (2) can be extended to;-

(7) $$\frac{dx}{dt} = \left(a_1 - \frac{r_1}{k_1}x\right)x - q_1 E_1 x - \frac{\alpha yx}{A+x}$$

(8) $$\frac{dy}{dt} = \left(a_2 - \frac{r_2}{k_2}y\right)y - q_2 E_2 y + \frac{\beta yx}{A+x}$$

(9) $$\frac{dE_1}{dt} = \mu_1(p_1 q_1 x - c_1)E_1$$

(10) $$\frac{dE_2}{dt} = \mu_2(p_2 q_2 y - c_2)E_2$$

where $a_i = r_i - d_i$, for $i = 1, 2$ with initial conditions $x(0) \geq 0, y(0) \geq 0, E_1(0) \geq 0$ and

$E_2(0) \geq 0$. All parameters are considered to be non-negative.

Model equation (7) and (8) can be written as

(11) $$\frac{dx}{dt} = f(x) - q_1 E_1 x - \frac{\alpha yx}{A+x}$$

(12) $$\frac{dy}{dt} = g(y) - q_1 E_1 y + \frac{\beta yx}{A+x}$$

Such that $f'(x) > 0$ for $x < x_{msy}$ and $f'(x)|_{x=x_{msy}} = 0$. Similarly, we have $g'(y) > 0$ for $y < y_{msy}$ and $g'(y)|_{y=y_{msy}} = 0$ where $msy = maximum\ sustained\ yield.$

Thus we have:

(13) $$f(x) = a_1 x - \frac{r_1}{k_1} x^2$$

(14) $$g(y) = a_2 y - \frac{r_2}{k_2} y^2$$

Taking the derivatives of (13) and (14) with respect to x and y respectively we get

$$f'(x) = a_1 - \frac{2r_1}{k_1} x > 0, \quad \forall x < x_{msy}$$

$$g'(y) = a_2 - \frac{2r_2}{k_2} y > 0, \quad \forall y < y_{msy}$$

Therefore, at $x = x_{msy}$ we get

(15) $$x_{msy} = \frac{a_1 k_1}{2r_1}$$

In a similar manner, we have

(16) $$y_{msy} = \frac{a_2 k_2}{2r_2} \quad \text{when } y = y_{msy}$$

3. Equilibrium points

In this section, we establish conditions for existence of equilibrium points model equations (7)

– (10) by zero. We find that the system has at least six (6) equilibrium points found by setting $\frac{dx}{dt}=\frac{dy}{dt}=\frac{dE_1}{dt}=\frac{dE_2}{dt}=0$ and solving the system of simultaneously equations we get;

$P_0(0,0,0,0)$, $P_1\left(\frac{a_1k_1}{r_1},0,0,0\right)$, $P_2\left(0,\frac{a_2k_2}{r_2},0,0\right)$, $P_3\left(\omega_1,0,\frac{a_1k_1-r_1\omega_1}{k_1q_1},0\right)$,

$$P_4\left(0,\omega_2,0,\frac{a_2k_2-r_2\omega_2}{k_2q_2}\right) \text{ and } P^*\begin{pmatrix}\omega_1\\ \omega_2\\ \frac{(A+\omega_1)(a_1k_1-\omega_1r_1)-\alpha\omega_2k_1}{k_1q_1(A+\omega_1)}\\ \frac{(A+\omega_1)(a_2k_2-\omega_2r_2)+\beta\omega_1k_2}{k_2q_2(A+\omega_1)}\end{pmatrix}$$

where $\omega_i=\frac{c_i}{p_iq_i}>0$ for $i=1,2.$

The existence of $P_0(0,0,0,0)$ is trivial. The equilibrium point P_1 exist if and only if $a_1>0,$ that is if

(17) $$r_1-d_1>0$$

Similarly, P_2 exists if and only if and only if $a_2>0$ implying that

(18) $$r_2-d_2>0$$

Again, in order to get positive equilibrium point P_3 we have

(19) $$(r_1-d_1)>\frac{r_1\omega_1}{k_1}$$

Similarly P_4 exist if the following condition is satisfied

(20) $$(r_2-d_2)>\frac{r_2\omega_2}{k_2}$$

It follows that, satisfaction of inequality condition $(r_i-d_i)>\frac{r_i\omega_i}{k_i}$ for $i=1,2.$ guarantee the existence of predator extinction equilibria when $i=1$ and prey extinction equilibria at $i=2.$

The existence of the interior equilibrium point $P(x^*, y^*, E_1^*, E_2^*)$ is subject to the satisfaction of the following conditions.

Conditions (17) and (18) are necessary but not sufficient conditions for existence of interior equilibrium point. Therefore, the other necessary conditions for its existence are:-

(20) $$\alpha < \frac{1}{\omega_2 k_1}(A + \omega_1)(a_1 k_1 - \omega_1 r_1)$$

(21) $$\beta > \frac{1}{\omega_1 k_2}(A + \omega_1)(\omega_2 r_2 - a_2 k_2)$$

4. Stability analysis

To analyze the stability of the interior equilibrium point P^* above, we consider the Jacobian matrix J evaluated at P^*. The Jacobian matrix J of the model is given by:

$$\begin{pmatrix} \dfrac{(A+x)^2(a_1 - q_1 E_1 - 2x) - \alpha A y}{k_1(A+x)^2} & -\dfrac{\alpha x}{A+x} & -q_1 x & 0 \\ \dfrac{A\beta x}{(A+x)^2} & \dfrac{(A+x)(a_2 - q_2 E_2 - 2y) + \beta k_2 x}{k_2(A+x)} & 0 & -q_2 y \\ \mu_1 p_1 q_1 E_1 & 0 & \mu_1(p_1 q_1 x - c_1) & 0 \\ 0 & \mu_2 p_2 q_2 E_2 & 0 & \mu_2(p_2 q_2 y - c_2) \end{pmatrix}$$

Evaluating the matrix at P^* have matrix J^*

$$\begin{pmatrix} \dfrac{(r_1 - 2)(A - \omega_1)^2 \omega_1}{k_1(A+\omega_1)^2} & -\dfrac{\alpha\omega_1}{A+\omega_1} & -q_1\omega_1 & 0 \\ \dfrac{A\beta\omega_1}{(A+\omega_1)^2} & \dfrac{\omega_2(r_2 - 2)}{k_2} & 0 & -q_2\omega_2 \\ \dfrac{\mu_1 p_1\big((A+\omega_1)(a_1 k_1 - \omega_1 r_1) - \alpha\omega_2 k_1\big)}{k_1(A+\omega_1)} & 0 & 0 & 0 \\ 0 & \dfrac{\mu_2 p_2\big((A+\omega_1)(a_2 k_2 - \omega_2 r_2) + \beta\omega_1 k_2\big)}{k_2(A+\omega_1)} & 0 & 0 \end{pmatrix}$$

By using Routh – Hurwitz criteria the corresponding characteristic polynomial equation of the matrix J^* is $f(\lambda) = \lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4$ where:

$$b_1 = \frac{b_6 - b_5}{k_1 k_2 (A+\omega_1)^2},\ b_2 = \frac{b_7 - b_8}{k_1 k_2 (A+\omega_1)^3},\ b_3 = \frac{b_{10} - b_9}{k_1 k_2 (A+\omega_1)^2},\ b_4 = \frac{b_{11} - b_{12}}{k_1 k_2 (A+\omega_1)^2}$$

$$b_5 = A k_1 r_2 \omega_2 (A + 2\omega_1) + k_2 r_1 \omega_1 (A^2 + \omega_1^2) + \omega_1^2 (k_1 r_2 \omega_2 + 4 A k_2)$$

$$b_6 = 2Ak_2\omega_1(r_1\omega_1 + A) + 2Ak_1\omega_2(A + 2\omega_1) + 2\omega_1^2(k_1\omega_2 + k_2\omega_1)$$

$$b_7 = \left((r_1r_2 + 4)\omega_2 + \mu_1p_1q_1k_2a_1k_1\right)\omega_1^4$$

$$+ \left(\left((2r_1 + 2r_2)\omega_2 + 3\mu_1p_1q_1k_2a_1k_1\right)A + \mu_2p_2q_2\omega_2k_1a_2k_2\right)\omega_1^3$$

$$+ 3\left(\left(\left(\frac{2}{3}r_1 + \frac{2}{3}r_2\right)\omega_2 + \mu_1p_1q_1k_2a_1k_1\right)A\right.$$

$$\left. + \left(a_2\omega_2q_2p_2\mu_2 + \frac{1}{3}\alpha\beta\right)k_1k_2\right)A\omega_1^2$$

$$+ \left(\left((r_1r_2 + 4)\omega_2 + \mu_1p_1q_1k_2a_1k_1\right)A + 3\mu_2p_2q_2\omega_2k_1a_2k_2\right)A^2\omega_1$$

$$+ \mu_2p_2q_2\omega_2k_1A^3a_2k_2$$

$$b_8 = \left(\mu_1p_1q_1k_2r_1\omega_1^4 + \left((2r_1 + 2r_2)\omega_2 + 2\mu_1p_1q_1k_2Ar_1\right)\omega_1^3\right.$$

$$+ \left(\mu_2p_2q_2\omega_2^2k_1r_2\right.$$

$$+ \left((r_2 - 2)(r_1 - 2)A + k_1k_2(\mu_1p_1q_1\alpha + \mu_2p_2q_2\beta)\right)\omega_2$$

$$\left. + \mu_1p_1q_1k_2A^2r_1\right)\omega_1^2$$

$$+ 2A\left(\mu_2p_2q_2k_1r_2\omega_2 + (r_1 + r_2)A\right.$$

$$\left.\left. + \frac{1}{2}k_1k_2(\mu_1p_1q_1\alpha + \mu_2p_2q_2\beta)\right)\omega_2\omega_1 + \mu_2p_2q_2\omega_2^2k_1A^2r_2\right)(A + \omega_1)$$

$$b_9 = \left(2\mu_1p_1q_1r_1\omega_1^4\right.$$

$$+ \left(6\mu_1p_1q_1Ar_1 + \mu_1p_1q_1a_1k_1r_2\right.$$

$$\left. + p_2q_2\mu_2(a_2k_2r_1 + 2k_2\beta + 2r_2\omega_2)\right)\omega_1^3$$

$$+ \left(6\mu_1p_1q_1A^2r_1\right.$$

$$+ \left(3\mu_1 p_1 q_1 a_1 k_1 r_2 + 2q_2\left(\left(\left(k_2\beta + \frac{1}{2}\omega_2 r_2\right) r_1 + a_2 k_2\right)\mu_2 p_2\right) A\right.$$

$$\left. + 2\mu_1 p_1 q_1 k_1 \omega_2 \alpha\right) \omega_1^2$$

$$+ 2A\left(\mu_1 p_1 q_1 A^2 r_1\right.$$

$$+ \left(\frac{3}{2}\mu_1 p_1 q_1 k_1 a_1 r_2 + \left(\frac{1}{2} r_1 r_2 \omega_2 + k_2(a_2 + \beta)\right) q_2 \mu_2 p_2\right) A$$

$$\left. + 2\mu_1 p_1 q_1 k_1 \omega_2 \alpha\right) \omega_1$$

$$+ A^2 \left(\left(\mu_1 p_1 q_1 a_1 k_1 r_2 + p_2 q_2 \mu_2 (a_2 k_2 r_1 + 2 r_2 \omega_2)\right) A\right.$$

$$\left. + 2\mu_1 p_1 q_1 k_1 \omega_2 \alpha)\right) \omega_1 \omega_2$$

$$b_{10} = 2\left(\frac{1}{2}\mu_1 p_1 q_1 r_1 \omega_1^4 r_2\right.$$

$$+ \left(\frac{3}{2}\mu_1 p_1 q_1 A r_1 r_2 + \mu_1 p_1 q_1 a_1 k_1\right.$$

$$\left. + q_2\left(\frac{1}{2} r_1 r_2 \omega_2 + k_2\left(\frac{1}{2} r_1 \beta + a_2\right)\right)\mu_2 p_2\right) \omega_1^3$$

$$+ \left(\frac{3}{2}\mu_1 p_1 q_1 A^2 r_1 r_2\right.$$

$$+ \left(3\mu_1 p_1 q_1 a_1 k_1 + \frac{1}{2}\left(2\omega_2 r_2 + k_2(a_2 r_1 + 4\beta)\right) q_2 \mu_2 p_2\right) A$$

$$\left. + \frac{1}{2}\mu_1 p_1 q_1 k_1 r_2 \omega_2 \alpha\right) \omega_1^2$$

$$+ \frac{1}{2} A\left(\mu_1 p_1 q_1 A^2 r_1 r_2\right.$$

$$+ \left(6\mu_1 p_1 q_1 a_1 k_1 + q_2\left(2\omega_2 r_2 + k_2 r_1 (a_2 + \beta)\right)\mu_2 p_2\right) A$$

$$\left. + 2\mu_1 p_1 q_1 k_1 r_2 \omega_2 \alpha\right) \omega_1$$

$$+ A^2\left(\left(\mu_1 p_1 q_1 a_1 k_1 + q_2 \mu_2 p_2 \left(a_2 k_2 + \frac{1}{2} r_1 r_2 \omega_2\right)\right) A + \frac{1}{2}\mu_1 p_1 q_1 k_1 r_2 \omega_2 \alpha\right)\right)\omega_1 \omega_2$$

$$b_{11} = p_2 q_1 \mu_1 \mu_2 \omega_1 \Big(r_1 (k_2 \beta + r_2 \omega_2)\omega_1^3 + \big(2\omega_2 A r_1 r_2 + k_2(A r_1 \beta + a_1 a_2 k_1)\big)\omega_1^2 + (\omega_2^2 k_1 r_2 \alpha + (A^2 r_1 r_2 + k_1 k_2 \alpha\beta)\omega_2 + 2A a_1 a_2 k_1 k_2)\omega_1 + A k_1 (A a_1 a_2 k_2 + r_2 \omega_2^2 \alpha)\Big) p_1 q_2 \omega_2$$

$$b_{12} = q_1 \mu_2 \mu_1 p_2 \omega_2 q_2 \omega_1 (A + \omega_1)\big(\omega_1^2 a_2 k_2 r_1 + (a_1(k_2\beta + r_2 \omega_2)k_1 + A a_2 k_2 r_1)\omega_1 + k_1 \omega_2 (A a_1 r_2 + a_2 k_2 \alpha)\big) p_1$$

Such that $b_i > 0$ for $i = 5, 6, \dots, 12$.

We know that the necessary and sufficient condition for local stability is when $b_1, b_3, b_4 > 0$ and $b_1 b_2 b_3 > b_3^2 + b_1^2 b_4$. We therefore have the following stability conditions

(22) $$b_6 - b_5 > 0$$

(23) $$b_{10} - b_9 > 0$$

(24) $$b_{11} - b_{12} > 0$$

(25) $$(b_6 - b_5)(b_7 - b_8)(b_{10} - b_9) > k_1 k_2 (A + \omega_1)^3 (b_{10} - b_9) + (b_6 - b_5)^2 (b_{11} - b_{12})(A + \omega_1)$$

5. Illustrative examples

Numerical examples and their graphical illustrations are as shown below. We use Runge – Kutta method of order four MAT Lab algorithms to validate the qualitative analysis results. We compare the variation of both the maximum sustained yield values and the equilibrium point values between cases where the pollution parameters are included against the pollution free parameter cases.

Example 1. In this example, let us consider the following set of estimated parameter values

$r_1 = 0.9, k_1 = 6\ 000, q_1 = 0.0000005, \alpha = 0.000005, A = 2\ 000, d_1 = 0.3, p_1 = 7\ 500,$

$c_1 = 1.2, \mu_1 = 0.1, r_2 = 0.8, k_2 = 5\ 000, q_2 = 0.0000012, \beta = 0.000003, d_2 = 0.2,$

$p_2 = 7\ 000, c_2 = 1.8,$ and $\mu_2 = 0.12$ with $x(0) = 5\ 000,$ $y(0) = 5\ 000$ $E_1(0) = 5\ 000$

and $E_2(0) = 5\ \ 000$

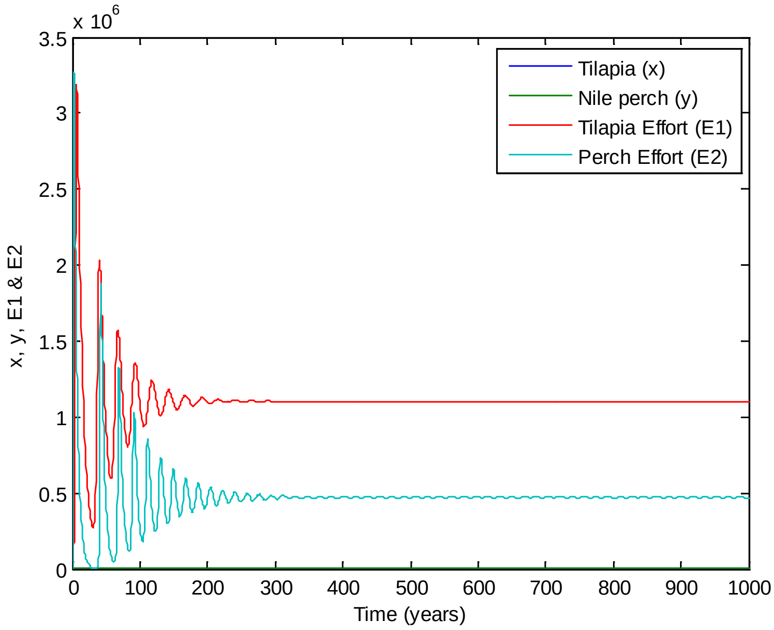

Figure 1 Variation of $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{E_1}$ *and* $\boldsymbol{E_2}$ *with the increasing time when* $\mathbf{d_1 = 0.3}$ *and* $\mathbf{d_2 = 0.2}$

Using this set of estimated parameter values we see that in pollution free environment (i.e. using the same set of parameters but setting $d_1 = d_2 = 0$) the maximum sustainable yield for Tilapia rises by 50%, rising from 2000 to 3000, while Nile perch increases by more than 30% from 1875 to 2500. For the case of interior equilibrium point, the population of both Tilapia and Nile perch at the interior equilibrium point is independent of the state of pollution in the lake while fishing effort for Tilapia goes up by at least 50%, raising from 1 104 000 to 1 704 000, and that of Nile perch raises by 35%, raising from 470 460 to 636 760 (See Fig. 1 and 2).

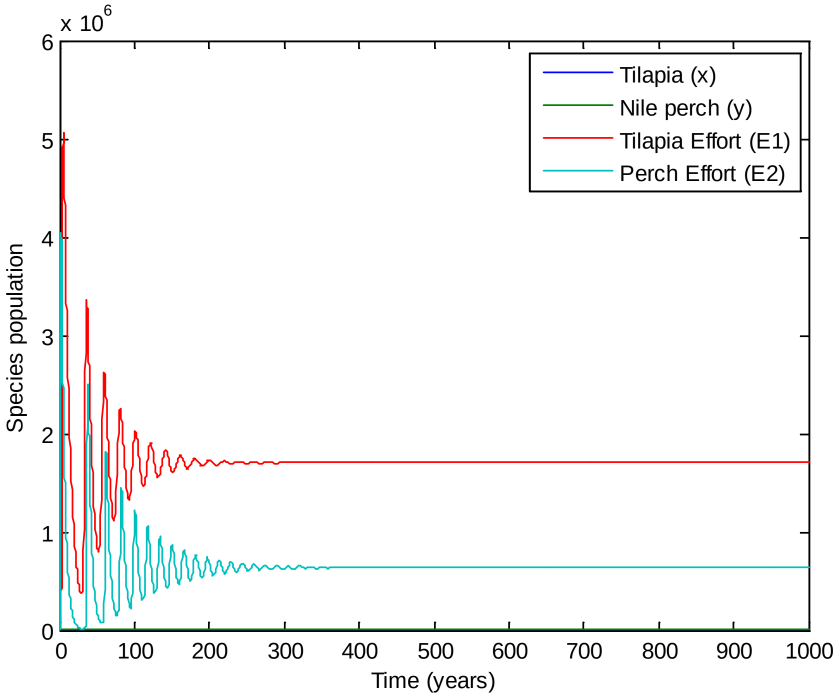

Figure 2 Variation of $\boldsymbol{x, y, E_1}$ *and* $\boldsymbol{E_2}$ *with the increasing time*

when $\boldsymbol{d_1 = d_2 = 0}$

Example 2. In this example, let us consider the following set of estimated parameter values

$r_1 = 0.8, k_1 = 12\ 000, q_1 = 0.000001, \alpha = 0.000005, A = 2\ 000, d_1 = 0.3, p_1 = 7\ 500,$

$c_1 = 1.5, \mu_1 = 0.1, r_2 = 0.65, k_2 = 10\ 000, q_2 = 0.0000012, \beta = 0.000003, d_2 = 0.2,$

$p_2 = 7\ 000, c_2 = 2.0,$ and $\mu_2 = 0.1$ with $x(0) = 8\ 000,$ $y(0) = 8\ 000$ $E_1(0) = 8\ 000$ and $E_2(0) = 8\ 000$. For this set of parameters we have $x_{msy} = 3\ 750, y_{msy} = 3\ 461.5,$ $E_1^* = 481\ 250$ and $E_2^* = 359\ 620$.

When the same set of estimated parameter values are used with $d_1 = d_2 = 0$ we get $x_{msy} = 6\ 000, y_{msy} = 5\ 000,$ $E_1^* = 778\ 680$ and $E_2^* = 525\ 64$. This suggests that, in a pollution free environment the maximum sustained yield for Tilapia population raises by 60% of its original population in polluted environment, Nile perch also raises its maximum sustained yield population by more than 40%. The same is true for the interior equilibrium point values.

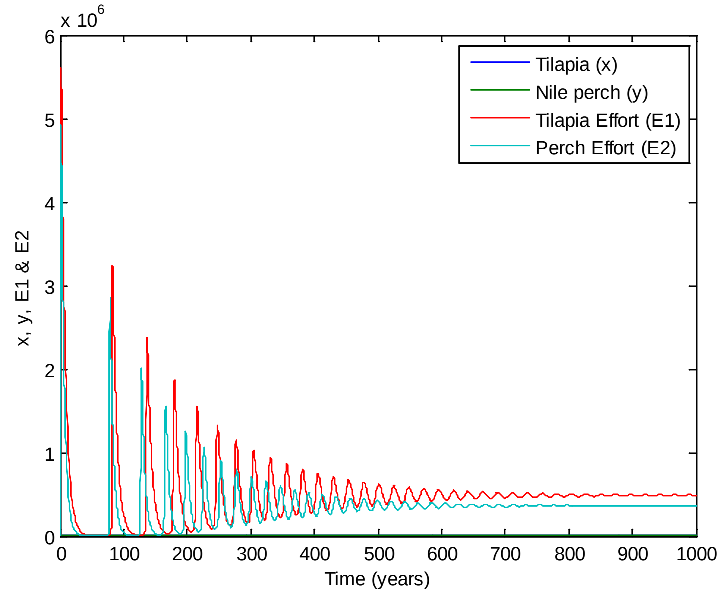

Figure 3 Variation of x, y, E_1 *and* E_2 *with the increasing time when* $d_1 = 0.3$ *and* $d_2 = 0.2$

There is a significant increase in the fishing efforts for both Tilapia and Nile perch. This is because of the fact that E_1^* and E_2^* are the composite functions of d_1 and d_2 respectively. (see Fig. 3 and 4)

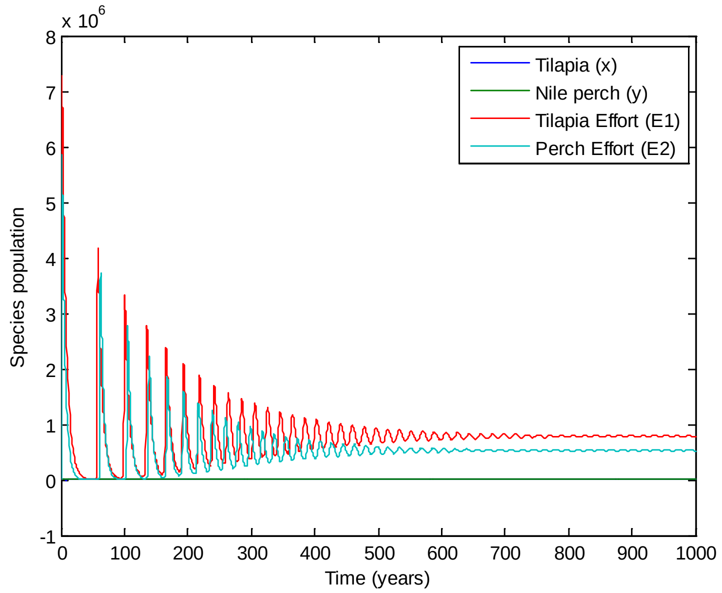

Figure 4 Variation of x, y, E_1 *and* E_2 *with the increasing time when* $d_1 = d_2 = 0$

6. Discussions and conclusion

In this paper we analyzed the effort dynamics of Tilapia – Nile perch fishery model in polluted environment of Tanzanian waters of Lake Victoria. The model has been analyzed to get the maximum sustainable yield (MSY) points and the corresponding conditions for their existence have been established. The equilibrium points of the model were found with the conditions for their existence being established. The stability analysis of the interior equilibrium point has been investigated by using Routh – Hurwitz criteria method. Later, numerical simulations and their corresponding graphs are presented. It was revealed that the water pollution has significant effects to the maximum sustainable yields of both Tilapia and Nile perch produces. This effect also manifests the rapid changes of efforts invested in harvested the two species at the interior equilibrium point.

Conflict of Interests

The author declares that there is no conflict of interests.

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