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DYNAMIC BEHAVIORS FOR A MODEL OF PREDATOR-PREY IN THE CHEMOSTAT WITH MICROBIAL IMPULSIVE INPUTS

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Abstract. In this paper, on the basis of the theories and methods of ecology and differential equations, a model of predator-prey in the chemostat with microbial impulsive inputs is established. By using the theories of impulsive equations, small amplitude perturbation skills and comparison techniques, we get the conditions which guarantee the globally asymptotical stability of the prey and predator eradication periodic solution. At the same time, we also prove that the system is permanent if some parameters satisfy certain conditions. Finally, some numerical simulations are carried out to illustrate the influences of impulsive inputs on the dynamic behaviors of system.

Keywords: Chemostat; Predator-prey; Impulsive input; Globally asymptotical stability; Permanence; Competitive exclusion.

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1. Introduction

The chemostat, a laboratory apparatus used for the continuous culture of microorganisms, has been playing an important role in microbiology and population ecology. H.smith and P.waltman

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[1] proposed the chemostat version of predator-prey equations, which showed that if the competition is moved up one level then coexistence may occur, i.e., if the competition occurs among predators of an organism growing on the nutrient. The model is as follows:

$$\begin{cases} S'(t) = (S^0 - S)D - \dfrac{m_1 S x}{r_1(a_1+S)}, \\ x'(t) = x(\dfrac{m_1 S}{a_1+S} - D - \dfrac{m_2 y}{r_2(a_2+x)} - \dfrac{m_3 z}{r_3(a_3+x)}), \\ y'(t) = y(\dfrac{m_2 x}{a_2+x} - D), \\ z'(t) = z(\dfrac{m_3 x}{a_3+x} - D), \\ S(0) = S_0 \geq 0, x(0) = x_0 \geq 0, y(0) = y_0 \geq 0, z(0) = z_0 \geq 0 \end{cases} \tag{1.1}$$

where $S(t)$ and $x(t)$ are the concentrations of nutrient and organism at time t; $y(t)$ and $z(t)$ are the concentrations of two predators at time t, S^0 is the input concentration of nutrient, D is the washout rate of the chemostat, $m_i (i = 1,2,3)$ represent the maximal growth rates of the organism $x(t)$ and two predators $y(t)$, $z(t)$, $r_i (i = 1,2,3)$ represent the yield constants of the organism $x(t)$ and two predators $y(t), z(t)$, $a_i (i = 1,2,3)$ are the half saturation constants. Here $x(t)$ is growing on the nutrient $S(t)$; $y(t)$ and $z(t)$ feed on $x(t)$.

In the discussion, it is helpful to consider the scaling for the system (1.1) and letting $\Sigma = 1 - S - x - y - z$, the system (1.1) may be rewritten. Furthermore, $\lim_{t\to\infty} \Sigma(t) = 0$ and hence the omega limit set of any trajectory lies in the set $\Sigma = 0$. Trajectories in the omega limit set are solutions of the following system:

$$\begin{cases} x'(t) = x(\dfrac{m_1(1-x-y-z)}{1+a_1-x-y-z} - 1 - \dfrac{m_2 y}{a_2+x} - \dfrac{m_3 z}{a_3+x}), \\ y'(t) = y(\dfrac{m_2 x}{a_2+x} - 1), \\ z'(t) = z(\dfrac{m_3 x}{a_3+x} - 1), \\ x(0) = x_0 \geq 0, y(0) = y_0 \geq 0, z(0) = z_0 \geq 0. \end{cases} \tag{1.2}$$

Many evolutionary processes are characterized by the fact at certain moments of time their experience subject to instantaneous perturbations whose duration is very short and negligible in comparison with the duration of the process considered [2, 3]. It is natural to assume that these perturbations are “momentary” changes or impulse. In the recent years, the research of impulsive differential equations about biological control can be thought of as a new growing interesting area (e.g., see [4, 5, 6, 7, 8, 9]). S.L. Sun et al. [10, 11, 12] studied some chemostat models with nutrient impulsive inputs.

G. Robledo et al. [13] proposed a model of competition of n species in a chemostat with constant input of some species. They proved that if the inputs satisfy a constraint, the coexistence between the species is obtained in the form of a globally asymptotically stable positive equilibrium, while a globally asymptotically stable equilibrium without the dominant species is achieved if the constraint is not satisfied.

Motivated by the idea of adding impulsive inputs to predator, we give predator $z(t)$ of system (1.2) an impulsive inputs. We will consider the following system with periodic constant impulsive inputting predator.

$$\begin{cases} \left.\begin{aligned} & x'(t) = x\left(\frac{m_1(1-x-y-z)}{1+a_1-x-y-z} - 1 - \frac{m_2 y}{a_2+x} - \frac{m_3 z}{a_3+x}\right), \\ & y'(t) = y\left(\frac{m_2 x}{a_2+x} - 1\right), \\ & z'(t) = z\left(\frac{m_3 x}{a_3+x} - 1\right), \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} & \Delta x(t) = 0, \\ & \Delta y(t) = 0, \\ & \Delta z(t) = p, \end{aligned}\right\} t = nT, \\ x(0^+) = x_0 \geq 0, y(0^+) = y_0 \geq 0, z(0^+) = z_0 \geq 0, \end{cases} \tag{1.3}$$

where $x(t), y(t), z(t)$ are the densities of one prey and two predators at time t, respectively. $\Delta x(t) = x(t^+) - x(t), \Delta y(t) = y(t^+) - y(t), \Delta z(t) = z(t^+) - z(t)$. $p > 0$ is the release amount of predator z at $t = nT$, other parameters have the same biological meaning as in system (1.1). In order to get some conditions which guarantees the system (1.3) is permanent, we will only release amount of predator z at $t = nT$ in that we assume that the predator y is a dominant competitor in the system (1.3).

The paper is organized as follows. Section 2 give some notions and definitions. Section 3 give the conditions which guarantees the globally asymptotical stability of the prey x and predator y eradication periodic solution and the system is permanent via the method of the comparison involving multiple Lyapunov functions. Section 4, by using the numerical simulation, we investigate the influences on the dynamic behaviors of the system for the impulsive inputs.

2. Notations and definitions

Let $R_+ = [0,\infty)$, $R^3_+ = \{X \in R^3 \mid X \geq 0\}$. Denote as $f = (f_1, f_2, f_3)$ the map defined by the right hand of the first three equations of system (1.3). Let $V : R_+ \times R^3_+ \to R_+$, then V is said to belong to class V_0 if:

(1) V is continuous in $((n-1)T, nT] \times R^3_+$, and for each $X \in R^3_+, n \in N$, $\lim_{(t,y)\to(nT^+,X)} V(t,y) = V(nT^+,X)$ exist;

(2) V is locally Lipschitzian in X.

Definition 2.1. *Let $V \in V_0$, then for $(t,X) \in ((n-1)T, nT] \times R^3_+$,the upper right derivative of $V(t,X)$ with respect to the impulsive differential system (1.3) is defined as*

$$D^+V(t,X) = \limsup_{h\to 0^+} \frac{1}{h}[V(t+h, X+hf(t,X)) - V(t,X)].$$

The solution of system (1.3) is a piecewise continuous function $X : R_+ \to R^3_+$, $X(t)$ is left continuous on $((n-1)T, nT], n \in N$, $X(nT^+) = \lim_{t\to nT^+} X(t)$ exist. The smoothness properties of f guarantee the global existence and uniqueness of solution of system (1.3); for the detail see [14, 15].

Definition 2.2. *system (1.3) is said to be permanent if there exists a compact $\Omega \subset \mathrm{int}R^3_+$ such that every solution $(x(t), y(t), z(t))$ of system (1.3) will eventually enter and remain in the region Ω.*

The following lemmas are obvious.

Lemma 2.3. *Let $X(t)$ be a solution of system (1.3) with $X(0^+) \geq 0$, then $X(t) \geq 0$ for all $t \geq 0$. And further $X(t) > 0, t > 0$ if $X(0^+) > 0$.*

Lemma 2.4. *There exists a constant M such that $x(t) \leq M, y(t) \leq M, z(t) \leq M$ for each solution $(x(t), y(t), z(t))$ of system (1.3) for t large enough.*

We will use an important comparison theorem on impulsive differential equation.

Lemma 2.5. [12] *Suppose $V \in V_0$. Assume that*

$$\begin{cases} D^+V(t,X) \leq g(t, V(t,X)), t \neq nT, \\ V(t, X(t^+)) \leq \psi_n(V(t, X(t))), t = nT, \end{cases} \tag{2.1}$$

where $g \in C[R_+ \times R_+, R_+]$, $\psi_n : R_+ \to R_+$ is nondecreasing. Let $r(t)$ be maximal solution of the scalar impulsive differential equation

$$\begin{cases} u'(t) = g(t,u(t)), t \neq nT, \\ u(t^+) = \psi_n(u(t)), t = nT, \\ u(0^+) = u_0 \end{cases} \tag{2.2}$$

existing on $[0,\infty)$. Then $V(0^+, X_0) \leq u_0$, implies that $V(t,X(t)) \leq r(t), t \geq 0$,where $X(t)$ is any solution of system (1.3).

If the prey x and predator y are absent, that is, $x(t) = 0$ and $y(t) = 0$, then system (1.3) reduces to

$$\begin{cases} z'(t) = -z(t), t \neq nT, \\ z(t^+) = z(t) + p, t = nT, \\ z(0^+) = z_0. \end{cases} \tag{2.3}$$

Clearly, system (2.3) has a positive periodic solution $z^*(t) = \frac{p\exp(-(t-(n-1)T))}{1-\exp(-T)}, t \in ((n-1)T, nT], n \in N$. The solution of system (2.3) is $z(t) = (z_0 - \frac{p}{1-\exp(-T)})\exp(-t) + z^*(t), t \in ((n-1)T, nT]$, where $z^*(0^+) = \frac{p}{1-\exp(-T)}$. Hence, the following result is hold.

Lemma 2.6. *For a positive periodic solution $z^*(t)$ of system (2.3) and every solution $z(t)$ of system (2.3) with $z_0 \geq 0$, have $|z(t) - z^*(t)| \to 0, t \to \infty$. Moreover, $z(t) \geq z^*(t)$ when $z(0^+) \geq z^*(0^+)$; $z(t) < z^*(t)$ when $z(0^+) < z^*(0^+)$.*

Therefore, we obtain the complete expression of the prey x and predator y eradication periodic solution $(0,0,z^*(t))$ of system (1.3). Now, we study the stability of the prey x and predator y eradication periodic solution.

3. Extinction and permanence

Theorem 3.1. *Let $(x(t), y(t), z(t))$ be any solution of system (1.3), then $(0,0,z^*(t))$ is globally asymptotically stable if*

$$-\frac{a_1 m_1}{1+a_1}\left(T + \ln\frac{(1+a_1)(1-\exp(-T)) - p\exp(-T)}{(1+a_1)(1-\exp(-T)) - p}\right) + (m_1 - 1)T - \frac{m_3 p}{a_3} < 0.$$

Proof. The local stability of periodic solution $(0,0,z^*(t))$ may be determined by considering the behavior of small amplitude perturbations of the solution. Define

$$x(t)=u(t), y(t)=v(t), z(t)=w(t)+z^*(t). \tag{3.1}$$

Substituting (3.1) into (1.3), the linearization of system becomes

$$\begin{cases} \left.\begin{array}{l} u'(t)=\left(\dfrac{m_1(1-z^*(t))}{1+a_1-z^*(t)}-1-\dfrac{m_3 z^*(t)}{a_3}\right)u(t), \\ v'(t)=-v(t), \\ w'(t)=\dfrac{m_3 z^*(t)}{a_3}u(t)-w(t)\ , \end{array}\right\} t\neq nT, \\ \left.\begin{array}{l} \Delta u(t)=0, \\ \Delta v(t)=0, \\ \Delta w(t)=0, \end{array}\right\} t=nT. \end{cases} \tag{3.2}$$

Therefore, we have

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix}, 0\leq t<T,$$

where $\Phi(t)$ satisfies

$$\Phi'(t)=\begin{pmatrix} \frac{m_1(1-z^*(t))}{1+a_1-z^*(t)}-1-\frac{m_3 z^*(t)}{a_3} & 0 & 0 \\ 0 & -1 & 0 \\ \frac{m_3 z^*(t)}{a_3} & 0 & -1 \end{pmatrix}\Phi(t)$$

and $\Phi(0)=I$ is the identity matrix. in addition,

$$\begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}.$$

It follows from the Floquet theory that the stability of the periodic solution $(0,0,z^*(t))$ is determined by the eigenvalues of the monodromy matrix

$$M=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\Phi(T)=\Phi(T).$$

If the absolute values of all multipliers are less than 1, then the periodic solution $(0,0,z^*(t))$ is locally asymptotical stable. All eigenvalues of M are as follows:

$$\lambda_1 = \exp(\int_0^T (\frac{m_1(1-z^*(t))}{1+a_1-z^*(t)} - 1 - \frac{m_3 z^*(t)}{a_3})dt),$$

$$\lambda_2 = \exp(-T) < 1,$$

$$\lambda_3 = \exp(-T) < 1.$$

According to Floquet theory, $(0,0,z^*(t))$ is locally asymptatically stable if $|\lambda_1| < 1$, that is to say

$$-\frac{a_1 m_1}{1+a_1}(T + \ln\frac{(1+a_1)(1-\exp(-T)) - p\exp(-T)}{(1+a_1)(1-\exp(-T)) - p}) + (m_1 - 1)T - \frac{m_3 p}{a_3} < 0.$$

Next, we will prove the global attractivity of the periodic solution $(0,0,z^*(t))$. Since $|\lambda_1| < 1$, we can select a $\varepsilon > 0$ such that

$$\bar{\sigma} = \int_0^T [\frac{m_1(1-(z^*(t)-\varepsilon))}{1+a_1-(z^*(t)-\varepsilon)} - 1 - \frac{m_3(z^*(t)-\varepsilon)}{a_3}]dt < 0.$$

Note that $z'(t) \geq -z(t)$, from Lemma 2.6 and comparison theorem, we have

$$z(t) > z^*(t) - \varepsilon \tag{3.3}$$

for all t large enough. For simplicity, we may assume that (3.3) hold for all $t \geq 0$. From(1.3) and (3.3) we can obtain

$$x'(t) \leq (\frac{m_1(1-(z^*(t)-\varepsilon))}{1+a_1-(z^*(t)-\varepsilon)} - 1 - \frac{m_3(z^*(t)-\varepsilon)}{a_3})x(t). \tag{3.4}$$

Which leads to

$$\begin{aligned} x(t) &\leq x(0^+)\exp(\textstyle\int_0^{nT}[\frac{m_1(1-(z^*(t)-\varepsilon))}{1+a_1-(z^*(t)-\varepsilon)} - 1 - \frac{m_3(z^*(t)-\varepsilon)}{a_3}]dt) \\ &= x(0^+)\exp(n\bar{\sigma}). \end{aligned} \tag{3.5}$$

Hence, $x(t) \to 0$ as $n \to \infty$.

Analogously, we can obtain

$$y'(t) \leq -y, \tag{3.6}$$

and

$$y(t) \leq y(0^+)\exp(-nT). \tag{3.7}$$

Thus, $y(t) \to 0$ as $n \to \infty$.

Next, we prove that $z(t) \to z^*(t)$ as $t \to \infty$. For $0 < \varepsilon$ sufficiently small, there must exists a $T' > 0$ such that $0 < x(t) < \varepsilon$, $0 < y(t) < \varepsilon, t > T'$. Without loss of generality, we may assume that $0 < x(t) < \varepsilon$, $0 < y(t) < \varepsilon$ for all $t \geq 0$, then from system (1.3) we obtain

$$-z(t) \leq z'(t) \leq (-1 + r\varepsilon)z(t), (r = \frac{m_3}{a_3}).$$

From Lemma 2.6 and 2.5 we have $v_1(t) \leq z(t) \leq v_2(t)$ and $v_1(t) \to z^*(t), v_2(t) \to z^*(t)$ as $t \to \infty$, where $v_1(t)$ and $v_2(t)$ are solutions of

$$\begin{cases} {v_1}'(t) = -v_1(t), t \neq nT, \\ v_1(t^+) = v_1(t) + p, t = nT, \\ v_1(0^+) = z(0^+), \end{cases}$$

and

$$\begin{cases} {v_2}'(t) = (-1 + r\varepsilon)v_2(t), t \neq nT, \\ v_2(t^+) = v_2(t) + p, t = nT, \\ v_2(0^+) = z(0^+), \end{cases}$$

respectively.

Because of $v_2^*(t) = \frac{p\exp((-1+r\varepsilon)(t-(n-1)T))}{1-\exp((-1+r\varepsilon)T)}$ for $t \in ((n-1)T, T]$. So for any $\varepsilon_1 > 0$ there exists a $T_1 > 0$ such that $v_2^*(t) - \varepsilon_1 < v_2(t) < v_2^*(t) + \varepsilon_1, t > T_1$. Let $\varepsilon \to 0$. We have $z^*(t) - \varepsilon_1 < z(t) < z^*(t) + \varepsilon_1$ for t large enough. Which implies $z(t) \to z^*(t)$ as $t \to \infty$. This completes the proof. □

Now, we investigate the permanence of the system (1.3).

Theorem 3.2. *The system (1.3) is permanent if the following conditions hold:*

1) $-\frac{a_1 m_1}{1+a_1}(T + \ln\frac{(1+a_1)(1-\exp(-T))-p\exp(-T)}{(1+a_1)(1-\exp(-T))-p}) + (m_1 - 1)T - \frac{m_3 p}{a_3} > 0$;

2) $[\frac{m_1}{1+a_1} - 1 - (\frac{m_1}{1+a_1} + \frac{m_2}{a_2})M]T - (\frac{m_1}{1+a_1} + \frac{m_3}{a_3})p > 0.$

Proof. suppose that $X(t) = (x(t), y(t), z(t))$ is any solution of the system (1.3) with $X(0) > 0$. From Lemma 2.4 we assume that $x(t) \leq M, y(t) \leq M, z(t) \leq M$ with $t \geq 0$. From (3.3) we have $z(t) > z^*(t) - \varepsilon$ for all t large enough, and $z(t) \geq \frac{p\exp(-T)}{1-\exp(-T)} - \varepsilon = \xi_1$ for t large enough. Thus we only need to find a ξ_2 such that $x(t) > \xi_2, y(t) > \xi_2$ for t large enough.

We will prove the existence of ξ_2 in the following two steps. First, Since the condition 2), we can select $0<\xi_4<\frac{a_3}{m_3}, \varepsilon_1>0$ be small enough such that

$$\eta_1=\exp\{[\frac{m_1(1-\xi_4)}{1+a_1}-1-(\frac{m_1}{1+a_1}+\frac{m_2}{a_2})M-(\frac{m_1}{1+a_1}+\frac{m_3}{a_3})\varepsilon_1]T-(\frac{m_1}{1+a_1}+\frac{m_3}{a_3})\frac{a_3p}{a_3-m_3\xi_4}\}>1.$$

It is easy to prove that $x(t)<\xi_4$ cannot hold for all $t>0$. Otherwise,

$$\begin{cases} z'(t)\le-(1-\frac{m_3x(t)}{a_3})z(t)\le-(1-\frac{m_3\xi_4}{a_3})z(t), t\ne nT,\\ z(t^+)=z(t)+p, t=nT,\\ z(0^+)=z_0. \end{cases} \tag{3.8}$$

Then we have $z(t)\le v_3(t)$ and $v_3(t)\to v_3^*(t), (t\to\infty)$, where v_3 is the solution of

$$\begin{cases} v_3'(t)=-(1-\frac{m_3\xi_4}{a_3})v_3(t), t\ne nT,\\ v_3(t^+)=v_3(t)+p, t=nT,\\ v_3(0^+)=z_0, \end{cases} \tag{3.9}$$

$$v_3^*(t)=\frac{p\exp(-(1-\frac{m_3\xi_4}{a_3})(t-(n-1)T))}{1-\exp(-(1-\frac{m_3\xi_4}{a_3})T)}, (n-1)T<t<nT.$$

Therefore, there exists a $T_1>0$ such that

$$z(t)\le v_3(t)<v_3^*(t)+\varepsilon_1$$

and

$$x'(t)\ge[\frac{m_1(1-\xi_4)}{1+a_1}-1-(\frac{m_1}{1+a_1}+\frac{m_2}{a_2})M-(\frac{m_1}{1+a_1}+\frac{m_3}{a_3})(v_3^*(t)+\varepsilon_1)]x(t) \tag{3.10}$$

for $t>T$.

Let $N_1\in N$ and $(N_1-1)T\ge T_1$. Integrating (3.10) on$((n-1)T, nT], n>N_1$, we can get

$$\begin{aligned} &x(nT)\ge\\ &x((n-1)T^+)\exp\{\int_{(n-1)T}^{nT}(\tfrac{m_1(1-\xi_4)}{1+a_1}-1-(\tfrac{m_1}{1+a_1}+\tfrac{m_2}{a_2})M-(\tfrac{m_1}{1+a_1}+\tfrac{m_3}{a_3})(v_3^*(t)+\varepsilon_1))dt\}\\ &=x((n-1)T)\eta_1. \end{aligned} \tag{3.11}$$

Then $x((N_1+n)T)\ge x(N_1T)\eta_1^n\to\infty$ as $n\to\infty$. Which is a contradiction to the boundedness of $x(t)$. Hence there exists a $t_1>0$ such that $x(t_1)\ge\xi_4$.

Second, if $x(t)\geq\xi_4$, $y(t)\geq\xi_4$, for all $t\geq t_3$, then our aim is obtained. Hence we only need to consider those solution which leave the region $\Theta=\{X(t)\in R^3_+ : x(t)\leq\xi_4\}$ and reenter it again. Let $t^*=\inf_{t\geq t_3}\{x(t)<\xi_4\}$. Then $x(t)\geq\xi_4$, for $t\in[t_3,t^*]$, and $x(t^*)=\xi_4$, suppose that $t^*\in((n_1-1)T,n_1T),n_1\in N$. There are two possible cases for $t\in(t^*,n_1T)$

Case1: $x(t)\leq\xi_4$ for all $t\in(t^*,n_1T)$. Select $n_2,n_3\in N$, such that

$$(n_2-1)T>\frac{\ln(\frac{\varepsilon_1}{M+p})}{-1+\frac{m_3\xi_4}{a_3}},$$

$$\exp(n_2\eta_2T)\eta_1^{n_3}>\exp((n_2+1)\eta_2T)\eta_1^{n_3}>1,$$

where $\eta_2=(\frac{m_1(1-\xi_4)}{1+a_1}-1-(\frac{2m_1}{1+a_1}+\frac{m_2}{a_2})M-\frac{m_3}{a_3}M)<0$ for $\xi_4>0$ small enough and $M>0$ large enough. Let$\bar{T}=n_2T+n_3T$. We claim that there must be a $t_2\in(t^*,t^*+\bar{T})$ such that $x(t_2)>\xi_4$. Otherwise, consider (3.9) with $v_3(t^{*+})=z(t^{*+})$, and we have

$$v_3(t)=(v_3(n_1T^+)-\frac{p}{1-\exp((-1+\frac{m_3\xi_4}{a_3})T)})\exp((-1+\frac{m_3\xi_4}{a_3})(t-n_1T))+v_3^*(t)$$

for $(n-1)T<t\leq nT$ and $n_1+1\leq n\leq n_1+n_2+n_3$. Then

$$|v_3(t)-v_3^*(t)|<(M+p)\exp((-1+\frac{m_3\xi_4}{a_3})(t-n_1T))<\varepsilon_1$$

and $z(t)\leq v_3(t)\leq v_3^*(t)+\varepsilon_1$, $n_1T\leq t\leq t^*+\bar{T}$, which implies that (3.10) hold for $t^*+n_2T\leq t\leq t^*+\bar{T}$. As in the first step, we have $x(t^*+\bar{T})\geq x(t^*+n_2T)\eta_1^{n_3}$.

The first equation of the system (1.3) gives

$$x'(t)\geq(\frac{m_1(1-\xi_4)}{1+a_1}-1-(\frac{2m_1}{1+a_1}+\frac{m_2}{a_2})M-\frac{m_3}{a_3}M)x(t). \tag{3.12}$$

Integrating (3.12) on $[t^*,t^*+n_2T]$, We have

$$x(t^*+n_2T)\geq\xi_4\exp(n_2\eta_2T).$$

Thus $x(t^*+\bar{T})\geq\xi_4\exp(n_2\eta_2T)\eta_1^{n_3}>\xi_4$, and this is contradiction. Let $\bar{t}=\inf_{t\geq t^*}\{x(t)>\xi_4\}$, then $x(t)\leq\xi_4$ for $t\in(t^*,\bar{t})$ and $x(\bar{t})=\xi_4$. For $t\in(t^*,\bar{t})$, we have

$$x(t)\geq x(t^*)\exp(\eta_2(t-t^*))\geq\xi_4\exp((n_2+n_3+1)\eta_2T).$$

Let $\xi_2=\xi_4\exp((n_2+n_3+1)\eta_2T)$, so $x(t)\geq\xi_2$ for $t\in(t^*,\bar{t})$. For $t>\bar{t}$, the same arguments can be continued since $x(\bar{t})\geq\xi_4$.

Case2. There exists a $t \in (t^*, n_1 T)$ such that $x(t) > \xi_4$. Let $\tilde{t} = \inf_{t \geq t^*}\{x(t) > \xi_4\}$, then $x(t) \leq \xi_4$ for $t \in (t^*, \tilde{t})$ and $x(\tilde{t}) = \xi_4$. For $t \in (t^*, \tilde{t})$, (3.12) holds. Integrating (3.12) on $(t^*, \tilde{t})$ yields

$$x(t) \geq x(t^*)\exp(\eta_2(t-t^*)) \geq \xi_4 \exp(\eta_2 T) > \xi_2.$$

Since $x(t) \geq \xi_4$ for $t > \tilde{t}$, the same arguments can be continued. Hence $x(t) \geq \xi_2$ for all $t > \tilde{t}$. The proved method for $y(t)$ is similar with proved method for $x(t)$. Set $\Omega = \{(x,y,z) : x \geq \xi_2, y \geq \xi_2, z \geq \xi_1, x+y+z \leq 3M\}$. Obviously, we can see that every solution of system (1.3) will eventually enter and remain in the region Ω. Therefore, system (1.3) is permanent. The proof is completed. □

4. Numerical analysis.

Now we will study the dynamic of a chemostat version of predator-prey models with predator pulsed input through the numerical simulation. We choose the parameters: $a_1 = 0.08, a_2 = 0.38, a_3 = 0.9, m_1 = 1.6, m_2 = 1.4, m_3 = 0.5, T = 20$ to simulation the system (1.3): we can see the prey $x(t)$ and the intermediate predator $y(t)$ can coexist, but the top predator $z(t)$ will decrease to zero if $p = 0$(there is no impulsive input), see figure 1, This is in line with competitive exclusion [13]. In order to prevent the predator $z(t)$ extinction, we should give the predator $z(t)$ the impulsive input. The prey x and the predator y eradication periodic $(0,0,z^*(t))$ of system (1.3) is globally asymptotically stable provided that $p > 1.08$, see figure 2, where we may observe how the top predator $z(t)$ oscillates in a stable cycle, which is consistent with theorem 3.1. The prey $x(t)$ and two predators $y(t), z(t)$ can coexist if the release amount of p on the predator z is less than 1.08, see figure 3. This verified the correctness of the theorem 3.2.

5. Conclusions and remarks

In this paper, the dynamic behaviors of a chemostat version of predator-prey models with predator pulsed input is studied by theoretical analysis and numerical simulation. We obtain the sufficient conditions which ensure that the periodic solution $(0,0,z^*(t))$ of the system (1.3) is globally asymptotically stable and the system (1.3) is permanent. These results are very meaningful.

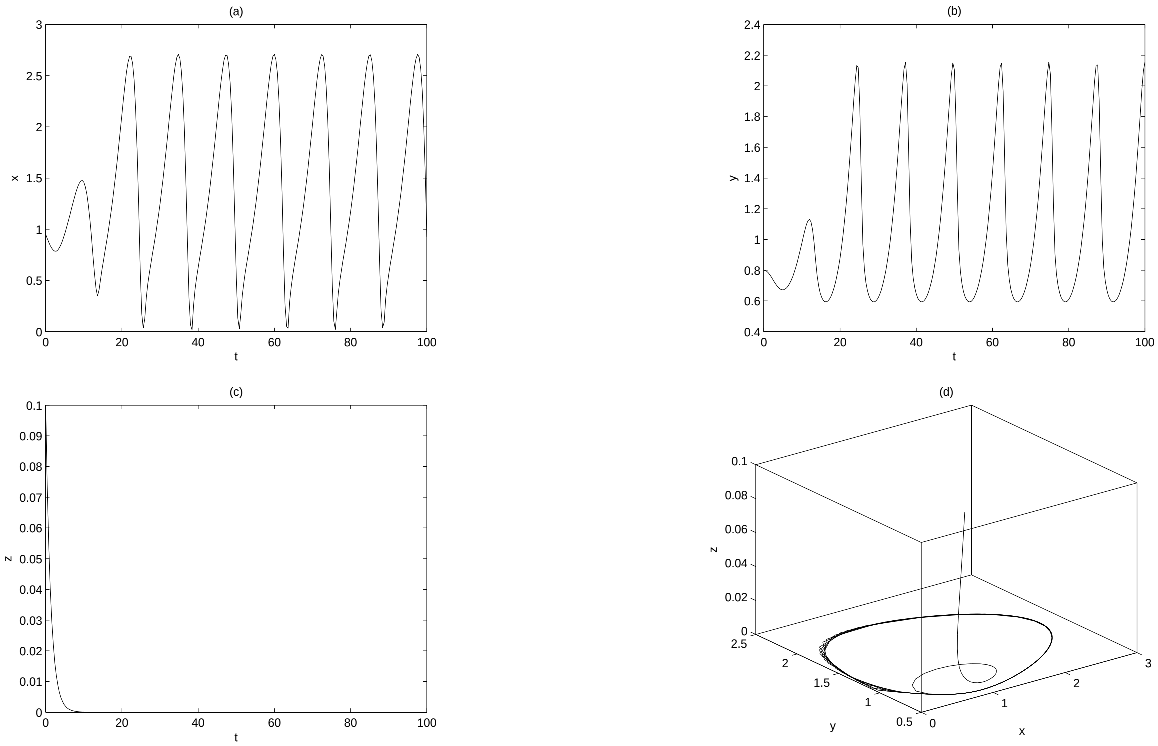

FIGURE. 1. The dynamics of system(1.3) with $x(0) = 0.95, y(0) = 0.8, z(0) = 0.1$ and $a_1 = 0.08, a_2 = 0.38, a_3 = 0.9, m_1 = 1.6, m_2 = 1.4, m_3 = 0.5, T = 20$. When $p = 0$, the prey x and the predator y can coexist, the predator z becomes extinction.

In particular, we consider the microbial pulsed input , which is different from the continuous input substrate and pulsed input substrate in before papers. Compared with [13](they considered a chemostat model with n species competing for a single limiting substrate), pulsed input microorganism is more practical and economic than continuous input microorganism in real work. This paper show that if we don't perform pulsed inputs on the microorganism predator z, then microorganism prey x and microorganism predator y can coexist, but microorganism predator z becomes extinction. This is according with the competitive exclusion. In order to prevent the predator $z(t)$ extinction, we should give the predator $z(t)$ the impulsive release. When $p < 1.08$, the prey $x(t)$ and two predators $y(t), z(t)$ can coexist; when $P > 1.08$, the prey x and the predator y go extinction, but the predator z presents a periodic oscillation.

Conflict of Interests

The authors declare that there is no conflict of interests.

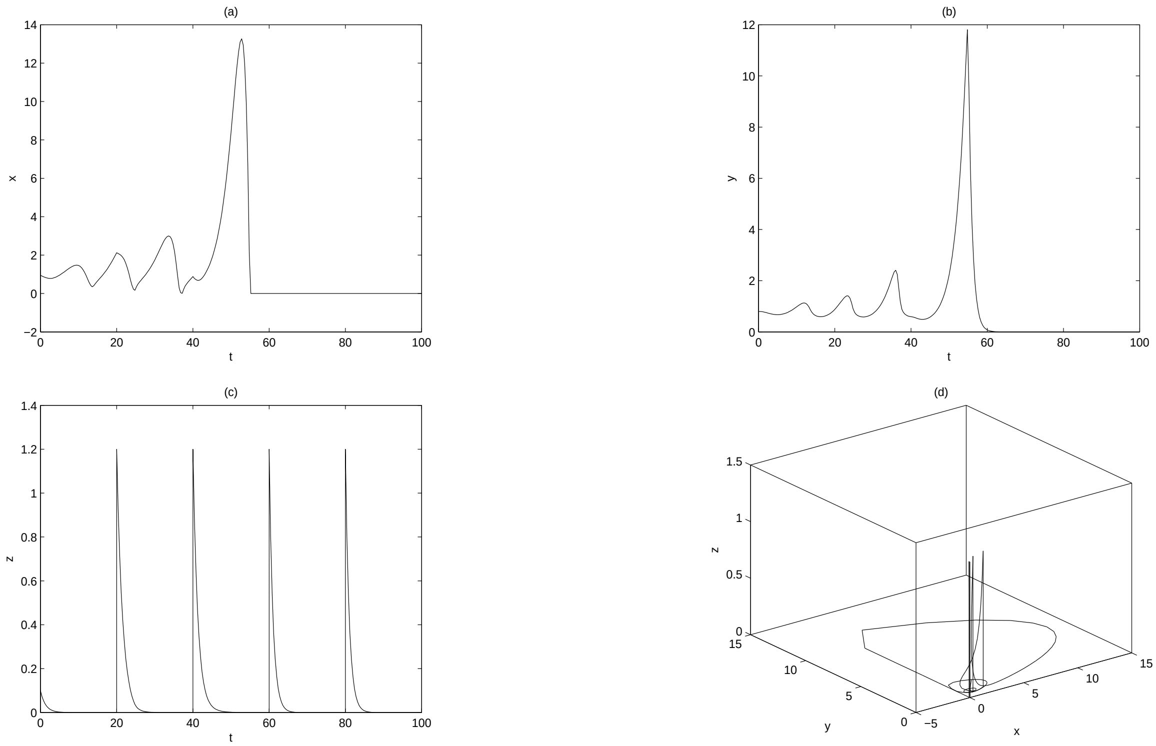

FIGURE. 2. The dynamics of system(1.3) with $x(0) = 0.95, y(0) = 0.8, z(0) = 0.1$ and $a_1 = 0.08, a_2 = 0.38, a_3 = 0.9, m_1 = 1.6, m_2 = 1.4, m_3 = 0.5, T = 20$. When $p = 1.2$, $(0,0,z^*(t))$ is globally asymptotically stable:(a) time series of the prey population $x(t)$; (b) time series of the predator population $y(t)$; (c) time series of the predator population $z(t)$; (d) phase portrait of system (1.3).

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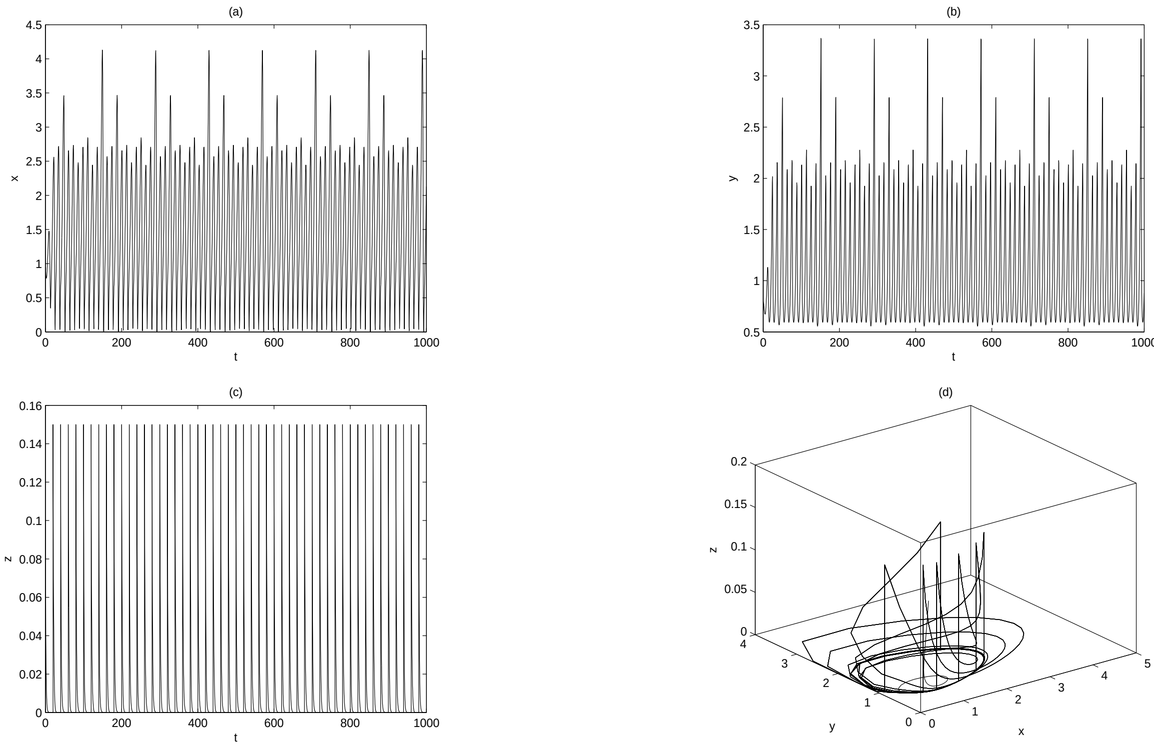

FIGURE. 3. The dynamics of system(1.3) with $x(0) = 0.95, y(0) = 0.8, z(0) = 0.1$ and $a_1 = 0.08, a_2 = 0.38, a_3 = 0.9, m_1 = 1.6, m_2 = 1.4, m_3 = 0.5, T = 20$. When $p = 0.15$, system (1.3) is permanence in chaos.

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