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CATCHABILITY COEFFICIENT INFLUENCE ON THE FISHERMEN'S NET ECONOMIC REVENUES

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Abstract. The present paper describes a prey-predator type fishery model with two predators in competition. The aim of the paper is to maximize the net economic revenue earn from the fishery through implementing the sustainable properties of the fishery to keep the ecological balance. The existence of the steady states and the stability of the interior equilibrium point is studied using Routh Hurwitz criterion. The problem of determining the fishing effort that maximizes the net economic revenue of each fisherman results in a Generalized Nash Equilibrium Problem. More precisely, we are interested in equilibrium of mathematical game given by the situation where all fishermen try to optimize their strategies according to the strategies of all other fishermen. The importance of marine reserve is analyzed through the obtained results of the numerical simulations of proposed model system. The results depict that reserves will be most effective when the coefficient of catchability decreases.

Keywords: bio-economic model; catchability coefficients; proportionate harvesting; Routh Hurwitz criterion; generalized Nash equilibrium problem.

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1. Introduction

In the present study, we analyse a prey predator model, with the existence of one prey- and two predators populations. The differential system is based on the Lotka-Volterra schema, considering a logistic growth for each population. Biological and bioeconomic equilibria of the system are obtained, and criteria for local stability and instability of the system are derived. This work is an attempt to study the effect of intrinsic growth rates changes in the net economic rents of several fishermen exploiting the three marine populations. To achieve this objective, we have to solve the problem of maximization of fishermen's net economic rents by using the generalized Nash equilibrium problem and linear complementarity problem. Finally, some numerical examples are discussed. Our study indicates that balance between harvesting and biodiversity should be maintained for better ecosystem management.

In the recent years, the technical and economic developments have led to commercial exploitation of more and more fish stocks, and stocks which sustained fisheries for a very long time have been severely depleted. This is probably the main reason for the increased interest biologists and others have taken in the use of multi-species bioeconomic models in applied research. We can refer for example to [1–4]. Chaudhurie [1] proposed a problem of combined harvesting of two competing fish species, each of which obeys the law of logistic growth; the author gave the mathematical formulation of the optimal harvest policy and its solution is derived in the equilibrium case by using Pontryagin's maximal principle also he explained the biological and economic interpretations of the results associated with the optimal equilibrium solution. An other important examples are [5, 6]; the article [5] discusses bioeconomic analysis and different management strategies in fisheries, the paper [6] aims to study the problem of combined harvesting of a system involving one predator and two prey species fishery in which the predator feeds more intensively on the more abundant species.

In this context, we can also refer to [7–10]. In [7], authors defined a bioeconomic equilibrium model for 'n' fishermen who catch three species; these species compete with each other for space or food, the natural growth of each species is modeled using a logistic law. In [8], authors supposed the price of fish population depends on quantity harvested and they defined a bioeconomic model that merges a model of competition and a model of prey-predator of three fish populations. More specifically, they assumed that on the one hand, the evolution of the first and second fish population is described by a density dependent model taking into account the competition between fish populations which compete with each other for space or food; on the other hand, the evolution of the second and third fish population is described by a Lotka-Volterra model. Also, the paper [9] presented a bioeconomic model for several fish populations taking into consideration the fact that the prices of fish populations vary according to the quantity harvested; the fish populations compete with each other for space or food and the natural growth of each one is modeled using a logistic law.

An other important examples in this context are [11–16]. The parer [11] introduces and describes in detail the bioeconomic optimization model BEMCOM (BioEconomic Model to evaluate the Consequences of Marine protected areas) that has been developed to assess the economic effects of introducing Marine Protected Areas (MPA) for fisheries.

In this manuscript, we propose to study a bioeconomic model of three fish populations. This model combines a model of competition and a model of prey-predator. As assumptions we suppose that the three fish populations grows according to a logistic equation, the first fish population is a prey of the second and the third one, also we suppose that the second and the third fish populations are predators of the first one and competes with each other for space or food.

The main difficulty in modeling the dynamics of fisheries is the estimation of the relationship between fishing effort and fishing mortality. Theoretically, following [17], fishing mortality is related to the effort and the catchability coefficient by the relation $F = qE$, with F represent fishing mortality, E represent fishing effort and q represent catchability. Hence, the interest of the catchability coefficient.

In the present manuscript, we have two objectives. Firstly, we search to determine the fishing effort that maximizes the net economic revenues of fishermen exploiting the fish populations constrained by conservation of the biodiversity. The second objective is to discuss the influence of the catchability coefficient on the net economic revenue of each fisherman.

This paper is organized as follows. In Section 2, we propose the bioeconomic model of three fish populations and we steady the positivity and boundedness of solutions. In Section 3, we study the existence of the steady states and the stability of the interior steady state is studied by using Routh Hurwitz criterion. In Section 4, we determining the fishing effort that maximizes the fishermen's net economic revenues by computing a generalized Nash equilibrium problem and solving a linear complementarity problem. In section 5, we discuss the influence of the catchability coefficient on the net economic revenue. Finally, we give a conclusion in Section 6.

2. Mathematical model

In this section, we propose to define a bioeconomic model of three fish populations; i.e prey (x), predator 1 (y_1) and predator 2 (y_2) with the competition between the first predator and the second predator.

2.1. Basic model

The following system is considered to model the evolution of the biomasses of the three fish populations

$$\begin{cases} \dfrac{dx}{dt} = x\left(r_1 - k_1 x\right) - \alpha x y_1 - \beta x y_2 \\[2ex] \dfrac{dy_1}{dt} = y_1\left(r_2 - k_2 y_1\right) + \bar{\alpha} x y_1 - \delta_1 y_1 y_2 \\[2ex] \dfrac{dy_2}{dt} = y_2\left(r_3 - k_3 y_2\right) + \bar{\beta} x y_2 - \delta_2 y_1 y_2 \end{cases} \tag{1}$$

subject to initial condition

$$x(0) > 0,\ y_1(0) > 0,\ y_2(0) > 0. \tag{2}$$

Here $x(t)$ is the biomass of prey, $y_1(t)$ is the biomass of predator 1 and $y_2(t)$ is the biomass of predator 2 at time t. $\left(r_j\right)_{j=1,2,3}$ are the net growth rates associated with x, y_1 and y_2, respectively. $\left(k_j\right)_{j=1,2,3}$are implicitly related to the carrying capacities of prey, predator 1 and predator 2, respectively. α and β are the rates of changes of the prey population in response to the presence of predator 1 (y_1) and predator 2 (y_2), respectively; i.e. they are the predation rate coefficients of predator 1 (y_1) and predator 2 (y_2), respectively. $\bar{\alpha}$ and $\bar{\beta}$ are the rates of changes in the predator

1 (y_1) and predator 2 (y_2), respectively; i.e. they are the conservation rates of prey into predator 1 (y_1) and predator 2 (y_2), respectively. δ_1 and δ_2 are the coefficients of competition; precisely, δ_1 represent the influence of predator 2 on predator 1, and δ_2 represent the influence of predator 1 on predator 2. All parameters are assumed to be positive and all variables nonnegative.

2.2. Proposed model

Now, we consider an extend version of the model (1) by assuming that the three populations prey, predator 1 and predator 2 are being harvested with three different agencies and therefore the total effort E_1, E_2 and E_3 dedicated to prey, predator 1 and predator 2 populations are different which results different catchability coefficients q_1, q_2 and q_3, respectively.

The proposed model in the presence of harvesting of prey, predator 1 and predator 2 is given by

$$\begin{cases} \dfrac{dx}{dt} = x(r_1 - k_1 x) - \alpha x y_1 - \beta x y_2 - q_1 E_1 x = x f_1(x, y_1, y_2) \\ \\ \dfrac{dy_1}{dt} = y_1(r_2 - k_2 y_1) + \bar{\alpha} x y_1 - \delta_1 y_1 y_2 - q_2 E_2 y_1 = y_1 f_2(x, y_1, y_2) \\ \\ \dfrac{dy_2}{dt} = y_2(r_3 - k_3 y_2) + \bar{\beta} x y_2 - \delta_2 y_1 y_2 - q_3 E_3 y_2 = y_2 f_3(x, y_1, y_2) \end{cases} \tag{3}$$

subject to initial condition

$$x(0) > 0,\ y_1(0) > 0,\ y_2(0) > 0. \tag{4}$$

Here

$$\begin{aligned} f_1 &= r_1 - k_1 x - \alpha y_1 - \beta y_2 - q_1 E_1 \\ f_2 &= r_2 - k_2 y_1 + \bar{\alpha} x - \delta_1 y_1 - q_2 E_2 \\ f_3 &= r_3 - k_3 y_2 + \bar{\beta} x - \delta_2 y_1 - q_3 E_3 \end{aligned}$$

The system (4) is defined on the set $\Omega = \left\{(x, y_1, y_2) \in \mathbb{R}^3 / x \geq 0, y_1 \geq 0, y_2 \geq 0\right\}$.

2.3. Positivity and boundedness of solutions

Theorem *All solutions* $(x(t), y_1(t), y_2(t))$ *of the system* (3) *with the initial condition* (4) *are positive for all* $t \geq 0$, *and are uniformly bounded.*

Proof. (i) By system of equations (3) with initial condition (4) we have

$$\begin{cases} x(t) = x(0)exp\left(\int_0^t f_1(x(\tau), y_1(\tau), y_2(\tau)\right)d\tau > 0 \\ y_1(t) = y_1(0)exp\left(\int_0^t f_2\left(x(\tau), y_1(\tau), y_2(\tau)\right)\right)d\tau > 0 \\ y_2(t) = y_2(0)exp\left(\int_0^t f_3\left(x(\tau), y_1(\tau), y_2(\tau)\right)\right)d\tau > 0. \end{cases}$$

Therefore, all solutions starting from an interior of the first octant remain in it for all future time.

(ii) We consider

$$\varphi(t) = \bar{\alpha}\bar{\beta}x(t) + \alpha\bar{\beta}y_1(t) + \bar{\alpha}\beta y_2(t)$$

The time derivative along the solutions of the system (3) is

$$\begin{aligned} \frac{d\varphi}{dt} &= \bar{\alpha}\bar{\beta}x(r_1 - k_1x) + \alpha\bar{\beta}y_1(r_2 - k_2y_1) + \bar{\alpha}\beta y_2(r_3 - k_3y_2) \\ &\quad -\alpha\bar{\beta}\delta_1y_1y_2 - \bar{\alpha}\beta\delta_2y_1y_2 - \bar{\alpha}\bar{\beta}q_1E_1x - \alpha\bar{\beta}q_2E_2y_1 \\ &\quad -\bar{\alpha}\beta q_3E_3y_2 \\ &\leq \bar{\alpha}\bar{\beta}x(r_1 - k_1x) + \alpha\bar{\beta}y_1(r_2 - k_2y_1) + \bar{\alpha}\beta y_2(r_3 - k_3y_2) \end{aligned}$$

For each $\eta > 0$, we have

$$\begin{aligned} \frac{d\varphi}{dt} + \eta\varphi(t) &\leq \bar{\alpha}\bar{\beta}x(r_1 - k_1x) + \alpha\bar{\beta}y_1(r_2 - k_2y_1) + \bar{\alpha}\beta y_2(r_3 - k_3y_2) \\ &\quad +\eta\bar{\alpha}\bar{\beta}x + \eta\alpha\bar{\beta}y_1 + \eta\bar{\alpha}\beta y_2 \\ &\leq \frac{\bar{\alpha}\bar{\beta}}{4k_1}(r_1+\eta)^2 + \frac{\alpha\bar{\beta}}{4k_2}(r_2+\eta)^2 + \frac{\bar{\alpha}\beta}{4k_3}(r_3+\eta)^2 \end{aligned}$$

So, the right-hand side is positive, then it is bounded for all $(x(t), y_1(t), y_2(t)) \in \mathbb{R}^3_+$. Therefore, we find a $\varepsilon > 0$ with $\frac{d\varphi}{dt} + \eta\varphi(t) < \varepsilon$. Applying the theory of differential inequality [18] we obtain

$$0 < \varphi(t) \leq \frac{\varepsilon}{\eta} + \left[\varphi(x(0); y_1(0); y_2(0)) - \frac{\varepsilon}{\eta}\right]e^{-\eta t}$$

Then

$$0 < \lim_{t\to\infty}\varphi(t) \leq \frac{\varepsilon}{\eta}$$

Hence, all solutions of (3) initiating from $\mathbb{R}^3_+$ are confined in the region

$$\left\{(x_1;x_2;y)\in\mathbb{R}^3_+/\ \varphi<\frac{\varepsilon}{\eta}+\zeta;\ \text{for any } \zeta>0\right\}$$

this proves the result. □

3. Existence of steady states

In this section, we propose to study the existence of various steady states and the stability of interior equilibrium point.

3.1. Existence of various steady states

The steady states of the system (3) are the solutions of the following system

$$\begin{cases} r_1-q_1E_1-k_1x-\alpha y_1-\beta y_2=0 \\ \\ r_2-q_2E_2-k_2y_1+\bar{\alpha}x-\delta_1y_2=0 \\ \\ r_3-q_3E_3-k_3y_2+\bar{\beta}x-\delta_2y_1=0 \end{cases} \tag{5}$$

The system (3) has eight steady states.

(i) The trivial steady state $S_0=(0,0,0)$ and the axial steady states

$$S_1=(\frac{r_1-q_1E_1}{k_1},0,0),\ S_2=(0,\frac{r_2-q_2E_2}{k_2},0) \text{ and } S_3=(0,0,\frac{r_3-q_3E_3}{k_3}).$$

(ii) The boundary steady state in xy_1-plane given by

$$\begin{aligned} S_{xy_1} &= (\bar{x},\bar{y}_1,0) \\ &= \left(\frac{k_2\left(r_1-q_1E_1\right)-\alpha\left(r_2-q_2E_2\right)}{k_1k_2+\alpha\bar{\alpha}},\frac{k_1\left(r_2-q_2E_2\right)+\bar{\alpha}\left(r_1-q_1E_1\right)}{k_1k_2+\alpha\bar{\alpha}},0\right). \end{aligned}$$

(iii) The boundary steady state in xy_2-plane given by

$$\begin{aligned} S_{xy_2} &= (\hat{x},0,\hat{y}_2) \\ &= \left(\frac{k_3\left(r_1-q_1E_1\right)-\beta\left(r_3-q_3E_3\right)}{k_1k_3+\beta\bar{\beta}},0,\frac{k_1\left(r_3-q_3E_3\right)+\bar{\beta}\left(r_1-q_1E_1\right)}{k_1k_3+\beta\bar{\beta}}\right). \end{aligned}$$

(iv) The boundary steady state in y_1y_2-plane given by

$$\begin{aligned} S_{y_1y_2} &= (0,\tilde{y}_1,\tilde{y}_2) \\ &= \left(0,\frac{k_3\left(r_2-q_2E_2\right)-\delta_1\left(r_3-q_3E_3\right)}{k_2k_3-\delta_1\delta_2},\frac{k_2\left(r_3-q_3E_3\right)-\delta_2\left(r_2-q_2E_2\right)}{k_2k_3-\delta_1\delta_2}\right). \end{aligned}$$

(v) The unique interior steady state $S=(x,y_1,y_2)$ with

$$\begin{cases} x = & a_{11}E_1+a_{12}E_2+a_{13}E_3+x^* \\ y_1 = & a_{21}E_1+a_{22}E_2+a_{23}E_3+y_1^* \\ y_2 = & a_{31}E_1+a_{32}E_2+a_{33}E_3+y_2^* \end{cases}$$

where

$$\begin{aligned} a_{11} = &\ \left[q_1\left(\delta_1\delta_2-k_2k_3\right)\right]/\Delta \\ a_{12} = &\ \left[q_2\left(\alpha k_3-\delta_2\beta\right)\right]/\Delta \\ a_{13} = &\ \left[q_3\left(\beta k_2-\delta_1\alpha\right)\right]/\Delta \\ x^* = &\ \left[r_1\left(k_2k_3-\delta_1\delta_2\right)+r_2\left(\beta\delta_2-\alpha k_3\right)+r_3\left(\alpha\delta_1-\beta k_2\right)\right]/\Delta \end{aligned}$$

$$\begin{aligned} a_{21} = &\ \left[q_1\left(\delta_1\bar{\beta}-\bar{\alpha}k_3\right)\right]/\Delta \\ a_{22} = &\ \left[-q_2\left(\beta\bar{\beta}+k_1k_3\right)\right]/\Delta \\ a_{23} = &\ \left[q_3\left(\bar{\alpha}\beta+\delta_1k_1\right)\right]/\Delta \\ y_1^* = &\ \left[r_2\left(\beta\bar{\beta}+k_1k_3\right)+r_1\left(\bar{\alpha}k_3-\bar{\beta}\delta_1\right)-r_3\left(\bar{\alpha}\beta+\delta_1k_1\right)\right]/\Delta \end{aligned}$$

$$\begin{aligned} a_{31} = &\ \left[q_1\left(\bar{\alpha}\delta_2-\bar{\beta}k_2\right)\right]/\Delta \\ a_{32} = &\ \left[q_2\left(\delta_2k_1+\alpha\bar{\beta}\right)\right]/\Delta \\ a_{33} = &\ \left[q_3\left(\alpha\bar{\alpha}-k_1k_2\right)\right]/\Delta \\ y_2^* = &\ \left[r_3\left(k_1k_2+\alpha\bar{\alpha}\right)+r_1\left(\bar{\beta}k_2-\bar{\alpha}\delta_2\right)+r_2\left(\delta_2k_1+\alpha\bar{\beta}\right)\right]/\Delta \end{aligned}$$

$$\Delta = \quad k_1k_2k_3-\delta_1\delta_2k_1+\beta\bar{\beta}k_2+\alpha\bar{\alpha}k_3-\alpha\delta_1\bar{\beta}-\beta\bar{\alpha}\delta_2$$

Therefore, one can remark that the interior steady state solution can be written in the matrix form

$$S=-AE+S^*$$

where $A=(-a_{ij})_{1\le i,j\le 3}$, $E=(E_1,E_2,E_3)^T$ and $S^*=(x^*,y_1^*,y_2^*)^T$.

3.2. Local stability of the interior steady state

We remark that the system (3) have eight solutions, but only one of them can give the coexistence of the three populations, in this case the biomasses of the three populations are strictly positive, this solution is the interior steady state $S=(x,y_1,y_2)$ which is feasible if $k_1>\max\{\alpha,\beta\}$, $k_2>\max\{\delta_1,\bar{\alpha}\}$ and $k_3>\max\{\bar{\beta},\delta_2\}$. In the following, we provide the local stability of the interior steady state applying the Routh Hurwitz criterion.

The variational matrix of the system at the steady state $S=(x,y_1,y_2)$ is

$$J(S)=\begin{bmatrix} J_{11} & -\alpha x & -\beta x \\ \bar{\alpha} y_1 & J_{22} & -\delta_1 y_1 \\ \bar{\beta} y_2 & -\delta_2 y_2 & J_{33} \end{bmatrix}$$

where

$$\begin{cases} J_{11}=r_1-q_1E_1-2k_1x-\alpha y_1-\beta y_2 \\ J_{22}=r_2-q_2E_2-2k_2y_1+\bar{\alpha}x-\delta_1 y_2 \\ J_{33}=r_3-q_3E_3-2k_3y_2+\bar{\beta}x-\delta_2 y_1 \end{cases}$$

Using the fact that by (5) we have

$$\begin{cases} r_1-q_1E_1-2k_1x-\alpha y_1-\beta y_2=-k_1x \\ r_2-q_2E_2-2k_2y_1+\bar{\alpha}x-\delta_1 y_2=-k_2y_1 \\ r_3-q_3E_3-2k_3y_2+\bar{\beta}x-\delta_2 y_1=-k_3y_2 \end{cases}$$

The characteristic polynomial of the variational matrix is

$$P(\lambda)=\rho_0\lambda^3+\rho_1\lambda^2+\rho_1\lambda+\rho_3$$

where

$$\begin{cases} \rho_0 = 1 \\ \\ \rho_1 = k_1 x + k_2 y_1 + k_3 y_2 \\ \\ \rho_2 = x y_1 \left(k_1 k_2 + \alpha\bar{\alpha}\right) + x y_2 \left(k_1 k_2 + \beta\bar{\beta}\right) + y_1 y_2 \left(k_2 k_3 + \delta_1 \delta_2\right) \\ \\ \rho_3 = x y_1 y_2 \left[\beta\left(k_2\bar{\beta} - \bar{\alpha}\delta_2\right) + k_3\left(k_1 k_2 + \alpha\bar{\alpha}\right) - \delta_1\left(k_2\delta_2 + \alpha\bar{\alpha}\right)\right] \end{cases}$$

Following the conditions of existence it is easy to show that ρ_0, ρ_1, ρ_2, ρ_3 and $\rho_1\rho_2 - \rho_0\rho_3$ are positive. Therefore, based upon the Routh-Hurwitz criterion it can be concluded that the steady state point $S = (x, y_1, y_2)$ is locally asymptotically stable.

4. Maximization of fishermen's economic revenues

The main purpose of this section is to determine the effort that maximize the net economic revenue of each fisherman.

4.1. Net Economic Revenue

According to Gordon's economic theory [19]

$$\text{Net Economic Revenue } (NER) = \text{Total Revenue } (TR) - \text{Total Cost } (TC)$$

where the Total Revenue $(TR)_i$ and Total Cost $(TC)_i$ of fisherman i in the system (3) are given by

$$\begin{cases} (TR)_i = & \langle E^{(i)}, qp(S^* - A\sum\limits_{i=1}^{n} E^{(i)})\rangle \\ (TC)_i = & \langle c^{(i)}, E^{(i)}\rangle \end{cases}$$

where $E^{(i)} = (E_{i1}, E_{i2}, E_{i3})^T$ is the vector effort must provide by fisherman i to catch the three populations; $c^{(i)}$ is the constant cost per unit of harvesting; and p is the price per unit harvested biomass.

While the economic profit of fisherman i is equal to the net economic revenue (NER), then we obtain the following equation

$$\begin{aligned}(NER)_i &= (TR)_i - (TC)_i \\ &= \langle E^{(i)}, -pqAE^{(i)} + pqS^* - c^{(i)} - \sum_{j=1, j\neq i}^{n} pqAE^{(j)} \rangle\end{aligned}$$

Let us add that the bioeconomic model is meaningful if and only if the biomasses of the three populations are strictly positive $S = S^* - AE \geq S_0 > 0$. Hence, for fisherman i we must have $AE^{(i)} \leq S^* - \sum_{j=1, j\neq i}^{n} AE^{(j)}$.

4.2. Generalized Nash Equilibrium Problem

To determine the effort that maximizes the NER of each fisherman we must solve the following generalized Nash equilibrium problem. Note that this problem exists when there is no unilateral profitable deviation from any of the fishermen involved.

$$(P_i)\begin{cases} \max\,(NER)_i = & \langle E^{(i)}, -pqAE^{(i)} + pqS^* - c^{(i)} - \sum_{j=1, j\neq i}^{n} pqAE^{(j)} \rangle \\ \text{subject to} & \\ & AE^{(i)} \leq S^* - \sum_{j=1, j\neq i}^{n} AE^{(j)} \\ & E^{(i)} \geq 0 \\ & E^{(j)} \text{ given for } \ 1 \leq j \neq i \leq n \end{cases}$$

By definition, the point $(E^{(1)}, \ldots, E^{(n)})$ is called a generalized Nash equilibrium point if and only if $E^{(i)}$ is a solution of problem (P_i) for $E^{(j)}$ $(1 \leq j \neq i \leq n)$ given.

According to the essential conditions of Karush-Kuhn-Tucker if $E^{(i)}$ is a solution of the problem (P_i) then, there exist constants $u^{(i)} \in \mathbb{R}^3_+$, $v^{(i)} \in \mathbb{R}^3_+$ and $m^{(i)} \in \mathbb{R}^3_+$ such that

$$\begin{cases} 2pqAE^{(i)} + c^{(i)} - pqS^* + \sum_{j=1, j\neq i}^{n} pqAE^{(j)} - u^{(i)} + A^T m^{(i)} = 0 \\ AE^{(i)} + v^{(i)} = S^* \sum_{j=1, j\neq i}^{n} AE^{(j)} \\ \langle u^{(i)}, E^{(i)} \rangle = \langle m^{(i)}, v^{(i)} \rangle = 0 \end{cases}$$

which leads to the following system

$$\begin{cases} u^{(1)} = 2pqAE^{(1)} + c^{(1)} - pqS^* + \sum\limits_{j=2}^{n} pqAE^{(j)} + A^T m^{(1)} \\ u^{(2)} = 2pqAE^{(2)} + c^{(2)} - pqS^* + \sum\limits_{j=1, j\neq 2}^{n} pqAE^{(j)} + A^T m^{(2)} \\ \vdots \\ u^{(n)} = 2pqAE^{(n)} + c^{(n)} - pqS^* + \sum\limits_{j=1}^{n-1} pqAE^{(j)} + A^T m^{(n)} \\ v^{(1)} = -AE^{(1)} - \sum\limits_{j=2}^{n} AE^{(j)} + S^* \\ v^{(2)} = -AE^{(2)} - \sum\limits_{j=1, j\neq 2}^{n} AE^{(j)} + S^* \\ \vdots \\ v^{(n)} = -AE^{(n)} - \sum\limits_{j=1}^{n-1} AE^{(j)} + S^* \\ \langle u^{(i)}, E^{(i)} \rangle = \langle m^{(i)}, v^{(i)} \rangle = 0 \qquad \forall i = 1, \ldots, n \\ \\ E^{(i)}, u^{(i)}, m^{(i)}, v^{(i)} \geq 0 \qquad \forall i = 1, \ldots, n \end{cases}$$

As the scalar product of $(m^{(i)})_{i=1,\ldots,n}$ and $(v^{(i)})_{i=1,\ldots,n}$ is zero, and $v := v^{(1)} = \ldots = v^{(n)} > 0$ (To maintain the biodiversity of fish populations, it is natural to assume that all biomasses remain strictly positive, that is $x > 0$, $y_1 > 0$ and $y_2 > 0$) so $m^{(i)} = 0$ for all $i = 1, \ldots, n$. Hence, we have the following expressions

$$\begin{cases} u^{(1)} = 2pqAE^{(1)} + c^{(1)} - pqS^* + \sum\limits_{j=2}^{n} pqAE^{(j)} \\ u^{(2)} = 2pqAE^{(2)} + c^{(2)} - pqS^* + \sum\limits_{j=1, j\neq 2}^{n} pqAE^{(j)} \\ \vdots \\ u^{(n)} = 2pqAE^{(n)} + c^{(n)} - pqS^* + \sum\limits_{j=1}^{n-1} pqAE^{(j)} \\ v = -AE^{(i)} - \sum\limits_{j=1, j\neq i}^{n} AE^{(j)} + S^* \\ \langle u^{(i)}, E^{(i)} \rangle = 0 \qquad \forall i = 1, \ldots, n \\ \\ E^{(i)}, u^{(i)}, v \geq 0 \qquad \forall i = 1, \ldots, n \end{cases}$$

Or in matrix form $W = MZ + B$, where

$$W = \begin{pmatrix} u^{(1)} \\ u^{(2)} \\ \vdots \\ u^{(n)} \\ v \end{pmatrix}, \; M = \begin{bmatrix} 2pqA & pqA & \dots & pqA & A^T \\ pqA & 2pqA & \ddots & \vdots & I \\ \vdots & \ddots & \ddots & pqA & \vdots \\ pqA & & pqA & 2pqA & \vdots \\ -A & \dots & \dots & -A & I \end{bmatrix}, \; Z = \begin{pmatrix} E^{(1)} \\ E^{(2)} \\ \vdots \\ E^{(n)} \\ 0 \end{pmatrix}$$

and $B = \begin{pmatrix} c^{(1)} - pqS^* \\ c^{(2)} - pqS^* \\ \vdots \\ c^{(n)} - pqS^* \\ S^* \end{pmatrix}$

4.3. Linear Complementarity Problem

The previous generalized Nash equilibrium problem is equivalent to the following Linear Complementarity Problem $LCP(M,B)$:

Find vectors $Z, W \in \mathbb{R}^{3(n+1)}$ such that

$$\begin{cases} W = MZ + B \geq 0 \\ \\ Z, W \geq 0 \\ \\ Z^T W = 0. \end{cases}$$

Lemma *$LCP(M,B)$ has a unique solution for every B if and only if M is a P-matrix.*

Proof. See Cottle [20] and Murty [21]. □

Let us add that a matrix A is called P-matrix if the determinant of every principal submatrix of A is positive (see [20]). The class of P-matrices generalizes many important classes of matrices, such as positive definite matrices, M-matrices, and inverse M-matrices, and arises in applications. Note that each matrix symmetric positive definite is P-matrix, but the reverse is not always true.

Hence, the linear complementarity problem $LCP(M,B)$ has one and only one solution since the matrix M is P-matrix, which confirm the existence and uniqueness of the generalized Nash equilibrium solution

$$E^{(i)} = \frac{1}{(n+1)} (pqA)^{-1} \left(pqk_i - c^{(i)} \right)$$

5. Discussion

What is the influence of the catchability coefficients on the net economic revenue?

We take as a case of study two fishermen who catch the three fish populations having the following characteristics

$$k_1 = 20,\ k_2 = 10,\ k_3 = 5,\ r_1 = 5,\ r_2 = 4,\ r_3 = 3,\ \alpha = 0.9,\ \beta = 0.7,\ \bar{\alpha} = 0.5,$$
$$\bar{\beta} = 0.4,\ \delta_1 = 0.2,\ \delta_2 = 0.3,\ p_1 = 10,\ p_2 = 12,\ p_3 = 20,\ c_1 = 0.01,\ c_2 = 8.10^{-3},$$
$$q_1 = 0.99,\ q_2 = 0.5,\ q_3 = 0.8$$

In the following, we will discover how changes in catchability coefficients can affect the net economic revenue of each fisherman taking into account the conservation of the biodiversity.

According to [17] in theory, the catchability coefficient is related to fishing effort and fishing mortality by relation $q = \frac{F}{E}$ where q represent catchability coefficient, F represent fishing mortality and E represent fishing effort. Based on this relation, one can notice that when q decreases then E increases. The following numerical simulations will confirm this result.

q_1	q_2	q_3	E_1	E_2	H_1	H_2	$(NER)_1$	$(NER)_2$
0.99	0.5	0.8	2.8	2.9	9.274	9.271	69.01	68.94
0.1	0.05	0.08	28.39	28.40	9.39	9.37	83.45	83.13
0.02	0.006	0.007	280.8	281.5	10.4	10.2	127.9	127
0.003	0.0006	0.0008	2490.6	2559.5	20.8	18.1	189.7	179.4

Table 1. Catchability influence on the fishing effort, catches and NER

According to the table 1, we note that a decrease in the level of catchability leads to an increase in the fishing effort level and consequently an increase in catches level which leads to an increase in net economic revenue as well.

Therefore, in order to achieve maximum profit taking into account the conservation of the resources, it is sufficient to have a small catchability coefficient.

6. Conclusion

In this paper, we have study a bioconomic model for several fishermen who catch fish populations. The first fish population is a prey of the second and the third one; the second and the third fish populations are predators of the first one and competes with each other for space or food. Using the generalized Nash equilibrium problem we have maximize the net economic revenue of each fisherman at biological equilibrium. To show the influence of catchability on the net economic revenues of fishermen we have carried out some numerical simulations.

Conflict of Interests

The authors declare that there is no conflict of interests.

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