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DYNAMIC BEHAVIORS OF A NON-SELECTIVE HARVESTING MAY COOPERATIVE SYSTEM INCORPORATING PARTIAL CLOSURE FOR THE POPULATIONS

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Abstract. A cooperative system of May type incorporating partial closure for the populations and non-selective harvesting is proposed and studied in this paper. The locally stability property of the equilibria are determined by analyzing the Jacobian matrix of the system about the equilibria. By using the comparison theorem of the differential equation, sufficient conditions which ensure the global attractivity of the boundary equilibria are obtained. By using the iterative method, we are able to show that the conditions which ensure the existence of the unique positive equilibrium is enough to ensure its global attractivity. Our study shows that the intrinsic growth rate and the fraction of the stocks for the harvesting plays crucial role on the dynamic behaviors of the system. Numeric simulations are carried out to show the feasibility of our results.

Keywords: cooperation; species; Lyapunov function; global stability.

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1. Introduction

Cooperation, one of the basic relationship between the species, has been studied by many scholars during the last decades, see [2]-[35] and the references cited therein. Topics such as

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the global attractivity of the positive equilibrium ([2]-[13], [22, 23]), the persistent property of the system ([14]-[30]), the existence and stability property of the positive periodic solution ([31]-[36]), the existence of the positive almost periodic solution ([8]), the influence of the feedback control variables ([14, 15, 17, 18, 19, 20, 21, 24, 27]), the influence of the stage structure([13],[33]), the influence of the harvesting([2, 3, 4]), the influence of the implusive([10]), the combine effect of the predator-prey-mutualist ([28, 29]) are investigated, and many excellent results are obtained.

May [2] suggested the following set of equations to describe a pair of mutualist:

$$\begin{aligned} \frac{dN_1}{dt} &= rN_1\Big[1-\frac{N_1}{K_1+\alpha N_2}\Big], \\ \frac{dN_2}{dt} &= rN_2\Big[1-\frac{N_2}{K_2+\beta N_1}\Big], \end{aligned} \tag{1.1}$$

where N_1,N_2 are the densities of the species, respectively. $r, K_i, \alpha, \beta, \ i=1,2$ are positive constants. The system admits an unique positive equilibrium (N_1^*,N_2^*), which is globally stable if $\alpha\beta<1$, and the system will "run away", with both populations growing unboundedly large if $\alpha\beta\geq 1$. To overcome the "run away" problem, May further considered the density restriction of the species and proposed the following system:

$$\begin{aligned} \dot{x} &= r_1x\Big[1-\frac{x}{K_1+\alpha_1 y}-\varepsilon_1 x\Big], \\ \dot{y} &= r_2y\Big[1-\frac{y}{K_2+\alpha_2 x}-\varepsilon_2 y\Big], \end{aligned} \tag{1.2}$$

where $r_i, K_i, \alpha_i, \varepsilon_i, \ i=1,2$ are positive constants. He showed that system (1.2) has a global stability equilibrium point. Since then, many scholars ([2, 3, 4]) also done works on this direction.

Based on the model (1.1) and (1.2), Wei and Li[2] proposed the following cooperative system with harvesting

$$\begin{aligned} \dot{x} &= x\Big(r_1-b_1x-\frac{a_1x}{y+k_1}\Big)-Eqx, \\ \dot{y} &= y\Big(r_2-b_2y-\frac{a_2y}{x+k_2}\Big), \end{aligned} \tag{1.3}$$

where x and y denote the densities of two populations at time t. The parameters r_1,r_2,a_1,a_2,b_1, b_2,k_1,k_2,E,q are all positive constants. By applying the comparison theorem of differential equations and constructing a suitable Lyapunov function, they obtained sufficient conditions which ensure the persistent and stability of the positive equilibrium, respectively. Xie, Chen and Xue[3] argued that the conditions in [2] is too complex, and by using the iterative method, they showed that

$$r_1 > Eq \tag{1.4}$$

is enough to ensure the system (1.3) admits a unique globally attractive positive equilibrium. This result greatly improve the main results of [2]. Recently, Chen, Wu and Xie[4] argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations, corresponding to system (1.3), they further proposed the following discrete cooperative model incorporating harvesting:

$$\begin{aligned} x(k+1) &= x(k)\exp\Big\{r_1 - Eq - b_1x(k) - \frac{a_1x(k)}{y(k)+k_1}\Big\},\\ y(k+1) &= y(k)\exp\Big\{r_2 - b_2y(k) - \frac{a_2y(k)}{x(k)+k_2}\Big\}, \end{aligned} \tag{1.5}$$

where $x(k), y(k)$ are the population density of the species x and y at k-generation. By using the iterative method and the comparison principle of difference equations, they also obtained a set of sufficient conditions which ensure the global attractivity of the interior equilibrium of the system. It bring to our attention that all of the paper [2]-[4] are considered the harvesting of the first species, without harvesting of the second species, this seems unrealistic, since generally speaking, in the harvesting process, human being will try to obtain as many resources as possible, with as little cost as possible.

On the other hand, as was pointed out by Chakraborty, Das and Kar[36], the study of resource-management including fisheries, forestry and wildlife management has great importance, it is necessary to harvest the population but harvesting should be regulated, such that both the ecological sustainability and conservation of the species can be implemented in a long run. Recently, Lin[37] investigated the dynamic behaviors of the following two species commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a_1 - b_1x + \frac{c_1y}{d_1+y^2}\Big) - q_1Emx,\\ \frac{dy}{dt} &= y(a_2 - b_2y) - q_2Emy, \end{aligned}$$

where $a_i, b_i, q_i, i = 1,2$ c_1, E, $m(0 < m < 1)$ and d_1 are all positive constants, where E is the combined fishing effort used to harvest and $m(0 < m < 1)$ is the fraction of the stock available for harvesting. His studied shows that depending on the range of the parameter m, the system may be collapse, or partial survival, or the two species could be coexist in a stable state. He also showed that if the system admits a unique positive equilibrium, then it is globally asymptotically stable. Recently, Chen[38] also studied the influence of non-selective harvesting to a Lotka-Volterra amensalism model incorporating partial closure for the populations, and he also founded that the dynamic behaviors of the system becomes complicated.

Stimulated by the works of [2]-[4], [36]-[38], in this paper, we will study the dynamic behaviors of the following non-selective harvesting May cooperative system incorporating partial closure for the populations

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y+k_1}\Big) - Eq_1 m x, \\ \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x+k_2}\Big) - Eq_2 m y, \end{aligned} \tag{1.6}$$

where x and y denote the densities of two populations at time t. The parameters r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q_i are all positive constants and have the same meaning as that of the system (1.3). E is the combined fishing effort used to harvest and $m(0 < m < 1)$ is the fraction of the stock available for harvesting.

We will try to give a thoroughly analysis of the dynamic behaviors of the above system. The paper is arranged as follows. We investigate the existence and locally stability property of the equilibria of system (1.2) in the next section. In section 3, By applying the differential inequality theory and the iterative method, we are able to investigate the global stability property of the boundary equilibrium and the positive equilibrium, respectively. Section 4 presents some numerical simulations concerning the stability of our model. We end this paper by a briefly discussion.

2. Local stability of the equilibria

The system always admits the boundary equilibrium $O(0,0)$.

If $r_2 > Emq_2$ holds, the system admits the boundary equilibrium $A(0,y_1)$, where $y_1 = \dfrac{k_2(r_2 - Emq_2)}{b_2 k_2 + a_2}$.

If $r_1 > Emq_1$ holds, the system admits the boundary equilibrium $B(x_1,0)$, where $x_1 = \dfrac{k_1(r_1 - Emq_1)}{b_1 k_1 + a_1}$.

If $r_1 > Emq_1$ and $r_2 > Emq_2$ hold, then the system admits a unique positive equilibrium

(x^*, y^*), x^* is the unique positive solution of the equation

$$A_1 x^2 + A_2 x + A_3 = 0, \tag{2.1}$$

where

$$\begin{aligned} A_1 &= b_1(r_2 - Emq_2) + b_1 b_2 k_1 + a_1 b_2, \\ A_2 &= -E^2 q_1 q_2 m^2 + E(q_1 r_2 + q_2 r_1 + b_2 k_1 q_1 - b_1 k_2 q_2 \\ &\quad + b_1 b_2 k_1 k_2 + a_1 b_2 k_2 + a_2 b_1 k_1 + b_1 k_2 r_2 - b_2 k_1 r_1 + a_1 a_2 - r_1 r_2, \\ A_3 &= (Emq_1 - r_1)\Big(k_2(r_2 - Emq_2) + a_2 k_1 + b_2 k_1 k_2\Big), \end{aligned}$$

and

$$y^* = \frac{(r_2 - Emq_2)(k_2 + x^*)}{b_2 k_2 + b_2 x^* + a_2}.$$

We shall now investigate the local stability property of the above equilibria.

The variational matrix of the system of Eq. (1.2) is

$$V(x,y) = \begin{pmatrix} B_1 & -\dfrac{a_1 x^2}{(y+k_1)^2} \\ \dfrac{a_2 y^2}{(x+k_2)^2} & B_2 \end{pmatrix}, \tag{2.2}$$

where

$$B_1 = r_1 - Eq_1 m - b_1 x - \frac{a_1 x}{y+k_1} - x\Big(b_1 + \frac{a_1}{y+k_1}\Big),$$
$$B_2 = r_2 - Eq_2 m - b_2 y - \frac{a_2 y}{x+k_2} - y\Big(b_2 + \frac{a_2}{x+k_2}\Big).$$

Theorem 2.1 *Assume that*

$$m > \max\left\{\frac{r_1}{Eq_1}, \frac{r_2}{Eq_2}\right\} \tag{2.3}$$

holds, then $O(0,0)$ is locally stable.

Proof. From (2.2) we could see that the Jacobian matrix of the system about the equilibrium point $O(0,0)$ is given by

$$\begin{pmatrix} r_1 - Emq_1 & 0 \\ 0 & r_2 - Emq_2 \end{pmatrix}. \tag{2.4}$$

The eigenvalues of the matrix are $\lambda_1 = r_1 - Emq_1, \lambda_2 = r_2 - Emq_2$. Hence, if $r < Emq_1$ and $s < Emq_2$ holds, then $\lambda_1 < 0, \lambda_2 < 0$, consequently $O(0,0)$ is locally stable. This ends the proof of Theorem 2.1.

Theorem 2.2 *Assume that*

$$\frac{r_2}{Eq_2} < m < \frac{r_1}{Eq_1} \tag{2.5}$$

holds, then $B(x_1,0)$ is locally stable.

Proof. From (2.2) we could see that the Jacobian matrix of the system about the equilibrium point $B(x_1,0)$ is given by

$$\begin{pmatrix} Emq_1 - r_1 & \dfrac{a_1(Emq_1 - r_1)^2}{(b_1k_1 + a_1)^2} \\ 0 & r_2 - Emq_2 \end{pmatrix}.$$

The eigenvalues of the matrix are $\lambda_1 = Emq_1 - r_1, \lambda_2 = r_2 - Emq_2$. Under the assumption (2.4), $\lambda_i < 0, i = 1,2$, and so, $B(x_1,0)$ is locally stable. This ends the proof of Theorem 2.2.

Theorem 2.3 *Assume that*

$$\frac{r_1}{Eq_1} < m < \frac{r_2}{Eq_2} \tag{2.6}$$

holds, then $A(0,y_1)$ is locally stable.

Proof. From (2.2) we could see that the Jacobian matrix of the system about the equilibrium point $A(0,y_1)$ is given by

$$\begin{pmatrix} r_1 - Emq_1 & 0 \\ \dfrac{a_2(Emq_2 - r_2)^2}{(b_2k_2 + a_2)^2} & Emq_2 - r_2 \end{pmatrix}. \tag{2.7}$$

Under the assumption (2.6), the two eigenvalues of the matrix satisfies $\lambda_1 = r_1 - Emq_1 < 0, \lambda_2 = Emq_2 - r_2 < 0$. consequently $A(0,y_1)$ is locally stable. This ends the proof of Theorem 2.3.

Theorem 2.4 *Assume that $m < \min\left\{\dfrac{r_2}{Eq_2}, \dfrac{r_1}{Eq_1}\right\}$ holds, then $C(x^*,y^*)$ is locally stable.*

Proof. Noting that the equilibrium point $C(x^*,y^*)$ satisfies the equation

$$\begin{aligned} r_1 - b_1x^* - \frac{a_1x^*}{y^* + k_1} - Eq_1m &= 0, \\ r_2 - b_2y^* - \frac{a_2y^*}{x^* + k_2} - Eq_2m &= 0, \end{aligned} \tag{2.8}$$

The Jacobian matrix about the equilibrium C is given by

$$\begin{pmatrix} -x^*\Big(b_1+\dfrac{a_1}{y^*+k_1}\Big) & \dfrac{(x^*)^2a_1}{(k_1+y^*)^2} \\ \dfrac{(y^*)^2a_2}{(k_2+x^*)^2} & -y^*\Big(b_2+\dfrac{a_2}{x^*+k_2}\Big) \end{pmatrix}. \tag{2.9}$$

The characteristic equation of (2.9) is

$$\Big[\lambda+x^*\Big(b_1+\frac{a_1}{y^*+k_1}\Big)\Big]\cdot\Big[\lambda+y^*\Big(b_2+\frac{a_2}{x^*+k_2}\Big)\Big]-\frac{(x^*)^2a_1}{(k_1+y^*)^2}\cdot\frac{(y^*)^2a_2}{(k_2+x^*)^2}=0,$$

which is equivalent to

$$\lambda^2+\Big[x^*\Big(b_1+\frac{a_1}{y^*+k_1}\Big)+y^*\Big(b_2+\frac{a_2}{x^*+k_2}\Big)\Big]+x^*y^*\Big(b_1b_2+b_1\frac{a_2}{x^*+k_2}+b_2\frac{a_1}{y^*+k_1}\Big)=0.$$

Therefore, the two eigenvalues of the above matrix satisfies

$$\lambda_1+\lambda_2=-x^*\Big(b_1+\frac{a_1}{y^*+k_1}\Big)-y^*\Big(b_2+\frac{a_2}{x^*+k_2}\Big)<0,$$

$$\lambda_1\cdot\lambda_2=x^*y^*\Big(b_1b_2+b_1\frac{a_2}{x^*+k_2}+b_2\frac{a_1}{y^*+k_1}\Big)>0.$$

Consequently,

$$\lambda_1<0,\ \lambda_2<0.$$

Hence, $C(x^*,y^*)$ is locally stable.

This ends the proof of Theorem 2.4.

3. Global attractivity

This section try to obtain some sufficient conditions which ensure the global asymptotical stability of the equilibria.

As a direct corollary of Lemma 2.2 of Chen[39], we have

Lemma 3.1. *If $a>0, b>0$ and $\dot{x}\geq x(b-ax)$, when $t\geq 0$ and $x(0)>0$, we have*

$$\liminf_{t\to+\infty} x(t)\geq\frac{b}{a}.$$

If $a>0, b>0$ and $\dot{x}\le x(b-ax)$, when $t\ge 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t)\le \frac{b}{a}.$$

Theorem 3.1

(1) Assume that

$$m>\max\left\{\frac{r_1}{Eq_1},\frac{r_2}{Eq_2}\right\} \tag{3.1}$$

holds, then $O(0,0)$ is globally attractive;

(2) Assume that

$$\frac{r_2}{Eq_2}<m<\frac{r_1}{Eq_1} \tag{3.2}$$

holds, then $B(x_1,0)$ is globally attractive;

(3) Assume that

$$\frac{r_1}{Eq_1}<m<\frac{r_2}{Eq_2} \tag{3.3}$$

holds, then $A(0,y_1)$ is globally attractive;

(4) Assume that

$$m<\min\left\{\frac{r_2}{Eq_2},\frac{r_1}{Eq_1}\right\} \tag{3.4}$$

holds, then $C(x^*,y^*)$ is globally attractive.

Proof.

(1) It follows from $m>\max\left\{\frac{r_1}{Eq_1},\frac{r_2}{Eq_2}\right\}$ that there exists enough small $\varepsilon>0$ such that

$$r_1-Eq_1m<-\varepsilon,\ \ r_2-Eq_2m<-\varepsilon. \tag{3.5}$$

From the first equation of system (1.6) and the positivity of the solution, by using (3.5), we have

$$\begin{aligned}\frac{dx}{dt} &= x\Big(r_1-b_1x-\frac{a_1x}{y+k_1}\Big)-Eq_1mx\\ &< (r_1-Eq_1m)x \\ &< -\varepsilon x,\end{aligned} \tag{3.6}$$

Hence

$$x(t)<x(0)\exp\{-\varepsilon t\}\to 0 \text{ as } t\to+\infty. \tag{3.7}$$

From the second equation of system (1.6) and the positivity of the solution, by using (3.5), we have

$$\begin{aligned}\frac{dy}{dt} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x+k_2}\Big) - Eq_2 my \\ &< (r_2 - Eq_2 m)y \\ &< -\varepsilon y,\end{aligned} \tag{3.8}$$

Hence

$$y(t) < y(0)\exp\{-\varepsilon t\} \to 0 \text{ as } t \to +\infty. \tag{3.9}$$

(2) By using the condition $m > \dfrac{r_2}{Eq_2}$, similarly to the analysis of (3.8)-(3.9), we have

$$y(t) \to 0 \text{ as } t \to +\infty. \tag{3.10}$$

For arbitrary enough small $\varepsilon > 0$, it follows from (3.10) that there exists a $T_1 > 0$, such that

$$y(t) < \varepsilon \text{ as } t > T_1.$$

For $t > T_1$, from the first equation of system (1.6), we have

$$\begin{aligned}\frac{dx}{dt} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y+k_1}\Big) - Eq_1 mx \\ &< x\Big(r_1 - b_1 x - \frac{a_1 x}{\varepsilon+k_1}\Big) - Eq_1 mx \\ &= x\Big(r_1 - Eq_1 m - (b_1 + \frac{a_1}{\varepsilon+k_1})x\Big).\end{aligned} \tag{3.10}$$

it follows from (3.10) and Lemma 3.1 that

$$\limsup_{t\to+\infty} x(t) \leq \frac{r_1 - q_1 Em}{b_1 + \dfrac{a_1}{\varepsilon+k_1}}. \tag{3.11}$$

On the other hand, from the first equation of system (1.6), we also have

$$\begin{aligned}\frac{dx}{dt} &> x\Big(r_1 - b_1 x - \frac{a_1 x}{k_1}\Big) - Eq_1 mx \\ &= x\Big(r_1 - q_1 Em - (b_1 + \frac{a_1 x}{k_1})x\Big),\end{aligned} \tag{3.12}$$

it follows from (3.12) and Lemma 3.1 that

$$\liminf_{t\to+\infty} x(t) \geq \frac{r_1 - q_1 Em}{b_1 + \dfrac{a_1}{k_1}}. \tag{3.13}$$

It follows from (3.11) and (3.13) that

$$\frac{r_1 - q_1 Em}{b_1 + \frac{a_1 x}{k_1}} \le \liminf_{t\to+\infty} x(t) \le \limsup_{t\to+\infty} x(t) \le \frac{r_1 - q_1 Em}{b_1 + \frac{a_1}{\varepsilon + k_1}}. \tag{3.14}$$

Since ε is any arbitrary small positive constants, setting $\varepsilon \to 0$ in (3.14) leads to

$$\lim_{t\to+\infty} x(t) = \frac{r_1 - q_1 Em}{b_1 + \frac{a_1}{k_1}} = \frac{k_1(r_1 - Emq_1)}{b_1 k_1 + a_1} = x_1.$$

(3) By using the condition $m > \frac{r_1}{Eq_1}$, similarly to the analysis of (3.5)-(3.7), we have

$$x(t) \to 0 \text{ as } t \to +\infty. \tag{3.15}$$

For arbitrary enough small $\varepsilon > 0$, it follows from (3.15) that there exists a $T_2 > 0$, such that

$$x(t) < \varepsilon \text{ as } t > T_2.$$

For $t > T_2$, from the second equation of system (1.6), we have

$$\begin{aligned} \frac{dy}{dt} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big) - Eq_2 my \\ &< y\Big(r_2 - b_2 y - \frac{a_2 y}{\varepsilon + k_2}\Big) - Eq_2 my \\ &= y\Big(r_2 - Eq_2 m - (b_2 + \frac{a_2}{\varepsilon + k_2})y\Big). \end{aligned} \tag{3.16}$$

It follows from (3.16) and Lemma 3.1 that

$$\limsup_{t\to+\infty} y(t) \le \frac{r_2 - q_2 Em}{b_2 + \frac{a_2}{\varepsilon + k_2}}. \tag{3.17}$$

On the other hand, from the second equation of system (1.6), we also have

$$\begin{aligned} \frac{dy}{dt} &> y\Big(r_2 - b_2 y - \frac{a_2 y}{k_2}\Big) - Eq_2 my \\ &= y\Big(r_2 - q_2 Em - (b_2 + \frac{a_2 y}{k_2})y\Big). \end{aligned} \tag{3.18}$$

It follows from (3.18) and Lemma 3.1 that

$$\liminf_{t\to+\infty} y(t) \ge \frac{r_2 - q_2 Em}{b_2 + \frac{a_2}{k_2}}. \tag{3.19}$$

It follows from (3.17) and (3.19) that

$$\frac{r_2-q_2Em}{b_2+\dfrac{a_2}{k_2}} \le \liminf_{t\to+\infty} y(t) \le \limsup_{t\to+\infty} y(t) \le \frac{r_2-q_2Em}{b_2+\dfrac{a_2}{\varepsilon+k_2}}. \tag{3.20}$$

Since ε is any arbitrary small positive constants, setting $\varepsilon\to 0$ in (3.20) leads to

$$\lim_{t\to+\infty} y(t) = \frac{r_2-q_2Em}{b_2+\dfrac{a_2}{k_2}} = \frac{k_2(r_2-Emq_2)}{b_2k_2+a_2} = y_1.$$

(4) By the first equation of system (1.6), we have

$$\dot{x}(t) \le x(t)(r_1-Eq_1m-b_1x(t)).$$

From Lemma 3.1, it follows that

$$\limsup_{t\to+\infty} x(t) \le \frac{r_1-Eq_1m}{b_1}. \tag{3.21}$$

Hence, for enough small $\varepsilon>0\Big(\varepsilon<\min\Big\{\dfrac{(r_1-Eq_1m)k_1}{k_1b_1+a_1},\dfrac{(r_2-Eq_2m)k_2}{k_2b_2+a_2}\Big\}\Big)$, it follows from (3.21) that there exists a $T_1'>0$ such that

$$x(t) < \frac{r_1-Eq_1m}{b_1}+\varepsilon \overset{\text{def}}{=} M_1^{(1)} \ \text{ for all } \ t>T_1'. \tag{3.22}$$

Similarly, for above $\varepsilon>0$, it follows from the second equation of system (1.6) that there exists a $T_1>T_1'$ such that

$$y(t) < \frac{r_2-Eq_2m}{b_2}+\varepsilon \overset{\text{def}}{=} M_2^{(1)} \ \text{ for all } \ t>T_1. \tag{3.23}$$

(3.23) together with the first equation of system (1.6) implies

$$\begin{aligned} \dot{x} &= x\Big(r_1-b_1x-\frac{a_1x}{y+k_1}\Big)-Eq_1mx \\ &\le x\Big(r_1-Eq_1m-b_1x-\frac{a_1x}{M_2^{(1)}+k_1}\Big) \ \text{ for all } \ t>T_1. \end{aligned} \tag{3.24}$$

Therefore, by Lemma 2.1, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{r_1-Eq_1m}{b_1+\dfrac{a_1}{M_2^{(1)}+k_1}}. \tag{3.25}$$

That is, for $\varepsilon > 0$ be defined by (3.21)-(3.22), there exists a $T_2' > T_1$ such that

$$x(t) < \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{M_2^{(1)} + k_1}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)} > 0 \ \text{ for all } \ t > T_2'. \tag{3.26}$$

It follows from (3.22) and the second equation of system (1.6) that

$$\begin{aligned} \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big) - Eq_2 m y \\ &\le y\Big(r_2 - Eq_2 m - b_2 y - \frac{a_2 y}{M_1^{(1)} + k_2}\Big) \end{aligned} \tag{3.27}$$

Therefore, by Lemma 3.1, we have

$$\limsup_{t \to +\infty} y(t) \le \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(1)} + k_2}}. \tag{3.28}$$

That is, for $\varepsilon > 0$ be defined by (3.22) and (3.23), there exists a $T_2 > T_2'$ such that

$$y(t) < \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(1)} + k_2}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)} > 0 \ \text{ for all } \ t > T_2. \tag{3.29}$$

From the first equation of system (1.6) and the positivity of $y(t)$,

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - Eq_1 m x \\ &\ge x\Big(r_1 - Eq_1 m - b_1 x - \frac{a_1 x}{k_1}\Big) \ \text{for all } \ t > T_2. \end{aligned} \tag{3.30}$$

Therefore, by Lemma 3.1, we have

$$\liminf_{t \to +\infty} x(t) \ge \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{k_1}}. \tag{3.31}$$

Hence, for $\varepsilon > 0$ be defined by (3.21)-(3.22), there exists a $T_3' > T_2$ such that

$$x(t) > \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{k_1}} - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)}, \ \text{ for all } \ t > T_3'. \tag{3.32}$$

Similarly, it follows from the second equation of system (1.6) that there exists a $T_3 > T_3'$ such that

$$y(t) > \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{k_2}} - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)}, \ \text{ for all } \ t > T_3. \tag{3.33}$$

(3.33) together with the first equation of system (1.6) implies that

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y+k_1}\Big) - Eq_1 m x \\ &\geq x\Big(r_1 - Eq_1 m - b_1 x - \frac{a_1 x}{m_2^{(1)}+k_1}\Big) \ \text{ for all } \ t > T_3. \end{aligned} \tag{3.34}$$

Therefore, by Lemma 3.1, we have

$$\liminf_{t\to+\infty} x(t) \geq \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{m_2^{(1)}+k_1}}. \tag{3.35}$$

That is, for $\varepsilon > 0$ be defined by (3.21)-(3.22), there exists a $T_4' > T_3$ such that

$$x(t) > \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{m_2^{(1)}+k_1}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_1^{(2)} > 0, \ \text{ for all } \ t > T_4'. \tag{3.36}$$

Similarly, by the second equation of system (1.6), for $\varepsilon > 0$ be defined by (3.21)-(3.22), there exists a $T_4 > T_4'$ such that

$$y(t) > \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{m_1^{(1)}+k_2}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)} > 0, \ \text{ for all } \ t > T_4. \tag{3.37}$$

Noting that $\dfrac{a_1}{M_2^{(1)}+k_1} > 0, \dfrac{a_2}{M_1^{(1)}+k_2} > 0$, it immediately follows that

$$\begin{aligned} M_1^{(2)} &= \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{M_2^{(1)}+k_1}} + \frac{\varepsilon}{2} < \frac{r_1 - Eq_1 m}{b_1} + \varepsilon = M_1^{(1)}; \\ M_2^{(2)} &= \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(1)}+k_2}} + \frac{\varepsilon}{2} < \frac{r_2 - Eq_2 m}{b_2} + \varepsilon = M_2^{(1)}. \end{aligned} \tag{3.38}$$

Also, since $m_1^{(1)} > 0, m_2^{(1)} > 0$, it follows that $\dfrac{a_1}{m_2^{(1)}+k_1} < \dfrac{a_1}{k_1}, \dfrac{a_2}{m_1^{(1)}+k_2} < \dfrac{a_2}{k_2}$, and so

$$\begin{aligned} m_1^{(2)} &= \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{m_2^{(1)}+k_1}} - \frac{\varepsilon}{2} > \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{k_1}} - \varepsilon = m_1^{(1)}; \\ m_2^{(2)} &= \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{m_1^{(1)}+k_2}} - \frac{\varepsilon}{2} > \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{k_2}} - \varepsilon = m_2^{(1)}. \end{aligned} \tag{3.39}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}, i=1,2, n=1,2,\cdots$, such that for $n \geq 2$

$$\begin{aligned} M_1^{(n)} &= \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{M_2^{(n-1)} + k_1}} + \frac{\varepsilon}{n}; \\ M_2^{(n)} &= \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(n-1)} + k_2}} + \frac{\varepsilon}{n}; \\ m_1^{(n)} &= \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{m_2^{(n-1)} + k_1}} - \frac{\varepsilon}{n}; \\ m_2^{(n)} &= \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{m_1^{(n-1)} + k_2}} - \frac{\varepsilon}{n}. \end{aligned} \tag{3.40}$$

Obviously,

$$m_i^{(n)} < x_i(t) < M_i^{(n)} \ \text{ for all } \ t \geq T_{2n}, \ \ i=1,2.$$

We claim that sequences $M_i^{(n)}, i=1,2$ are strictly decreasing, and sequences $m_i^{(n)}, i=1,2$ are strictly increasing. To proof this claim, we will carry out by induction. Firstly, from (3.38) and (3.39) we have

$$M_i^{(2)} < M_i^{(1)}, \ \ m_i^{(2)} > m_i^{(1)}, \ \ i=1,2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} < M_i^{(n-1)}, \ \ m_i^{(n)} > m_i^{(n-1)}, \ \ i=1,2. \tag{3.41}$$

Then

$$\frac{a_1}{M_2^{(n)} + k_1} > \frac{a_1}{M_2^{(n-1)} + k_1}, \ \ \frac{a_2}{M_1^{(n)} + k_2} > \frac{a_2}{M_1^{(n-1)} + k_2}. \tag{3.42}$$

From (3.42) and the expression of $M_i^{(n)}$, it immediately follows that

$$\begin{aligned} M_1^{(n+1)} &= \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{M_2^{(n)} + k_1}} + \frac{\varepsilon}{n+1} < \frac{r_1 - Eq_1 m}{b_1 + \dfrac{a_1}{M_2^{(n-1)} + k_1}} + \frac{\varepsilon}{n} = M_1^{(n)}, \\ M_2^{(n+1)} &= \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(n)} + k_2}} + \frac{\varepsilon}{n+1} < \frac{r_2 - Eq_2 m}{b_2 + \dfrac{a_2}{M_1^{(n-1)} + k_2}} + \frac{\varepsilon}{n} = M_2^{(n)}. \end{aligned} \tag{3.43}$$

Also, it follows from (3.41) that $m_i^{(n)} \geq m_i^{(n-1)}, \ \ i=1,2$. Then

$$\frac{a_1}{m_2^{(n)} + k_1} < \frac{a_1}{m_2^{(n-1)} + k_1}, \ \ \frac{a_2}{m_1^{(n)} + k_2} < \frac{a_2}{m_1^{(n-1)} + k_2}. \tag{3.44}$$

From (3.44) and the expression of $m_i^{(n)}$, it immediately follows that

$$\begin{aligned} m_1^{(n+1)} &= \frac{r_1 - Eq_1m}{b_1 + \dfrac{a_1}{m_2^{(n)} + k_1}} - \frac{\varepsilon}{n+1} > \frac{r_1 - Eq_1m}{b_1 + \dfrac{a_1}{m_2^{(n-1)} + k_1}} - \frac{\varepsilon}{n} = m_1^{(n)}, \\ m_2^{(n+1)} &= \frac{r_2 - Eq_1m}{b_2 + \dfrac{a_2}{m_1^{(n)} + k_2}} - \frac{\varepsilon}{n+1} > \frac{r_2 - Eq_1m}{b_2 + \dfrac{a_2}{m_1^{(n-1)} + k_2}} - \frac{\varepsilon}{n} = m_2^{(n)}. \end{aligned} \tag{3.45}$$

Therefore,

$$\lim_{t\to+\infty} M_1^{(n)} = \overline{x}, \ \lim_{t\to+\infty} M_2^{(n)} = \overline{y},$$
$$\lim_{t\to+\infty} m_1^{(n)} = \underline{x}, \ \lim_{t\to+\infty} m_2^{(n)} = \underline{y}$$

Letting $n \to +\infty$ in (3.40), we obtain

$$\begin{aligned} b_1\overline{x} + \frac{a_1\overline{x}}{\overline{y}+k_1} &= r_1 - Eq_1m, \\ b_2\overline{y} + \frac{a_2\overline{y}}{\overline{x}+k_2} &= r_2 - Eq_2m; \\ b_1\underline{x} + \frac{a_1\underline{x}}{\underline{y}+k_1} &= r_1 - Eq_1m, \\ b_2\underline{y} + \frac{a_2\underline{y}}{\underline{x}+k_2} &= r_2 - Eq_2m. \end{aligned} \tag{3.46}$$

(3.46) shows that $(\overline{x},\overline{y})$ and $(\underline{x},\underline{y})$ are positive solutions of the equations

$$\begin{aligned} b_1x + \frac{a_1x}{y+k_1} &= r_1 - Eq_1m, \\ b_2y + \frac{a_2y}{x+k_2} &= r_2 - Eq_2m, \end{aligned} \tag{3.47}$$

Already, we had showed in the previous section that under the assumption $r_1 > Eq_1m, r_2 > Eq_2m$, (3.47) has a unique positive solution $C(x^*,y^*)$. Hence, we conclude that

$$\overline{x} = \underline{x} = x^*, \ \ \overline{y} = \underline{y} = y^*,$$

that is

$$\lim_{t\to+\infty} x(t) = x^* \ \ \lim_{t\to+\infty} y(t) = y^*.$$

Thus, the unique interior equilibrium $C(x^*,y^*)$ is globally attractive.

This completes the proof of Theorem 3.1.

4. Numeric simulations

Now let's consider the following example.

Example 4.1. Consider the following May cooperative system incorporating partial closure for the populations

$$\begin{aligned} \dot{x} &= x\Big(2-x-\frac{2x}{y+1}\Big)-4\cdot\frac{2}{3}\cdot mx, \\ \dot{y} &= y\Big(2-y-\frac{2y}{x+1}\Big)-4\cdot\frac{3}{4}\cdot mx, \end{aligned} \tag{4.1}$$

here we choose $r_1 = r_2 = 2, b_1 = b_2 = 1, k_1 = k_2 = 1, a_1 = a_2 = 2, E = 4, q_1 = \frac{2}{3}, q_2 = \frac{3}{4}$. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q_i, m$ are all positive constants.

(1) Take $m = 0.8$, then

$$m > \max\left\{\frac{r_1}{Eq_1}, \frac{r_2}{Eq_2}\right\} = 0.75,$$

and so, from Theorem 3.1, $O(0,0)$ is globally attractive, see Fig.1, Fig. 2;

(2) Take $m = 0.7$, then

$$\frac{2}{3} = \frac{r_2}{Eq_2} < m < \frac{r_1}{Eq_1} = \frac{3}{4}$$

hold, then $B(0.04341686731, 0)$ is globally attractive, see Fig.3, Fig. 4;

(3) Take $m = 0.2$, then

$$m < \frac{r_2}{Eq_2} = \frac{2}{3}$$

and

$$m < \frac{r_1}{Eq_1} = \frac{3}{4}$$

hold, then $C(0.6596762984, 0.6349050103)$ is globally attractive, see Fig.5, Fig. 6;

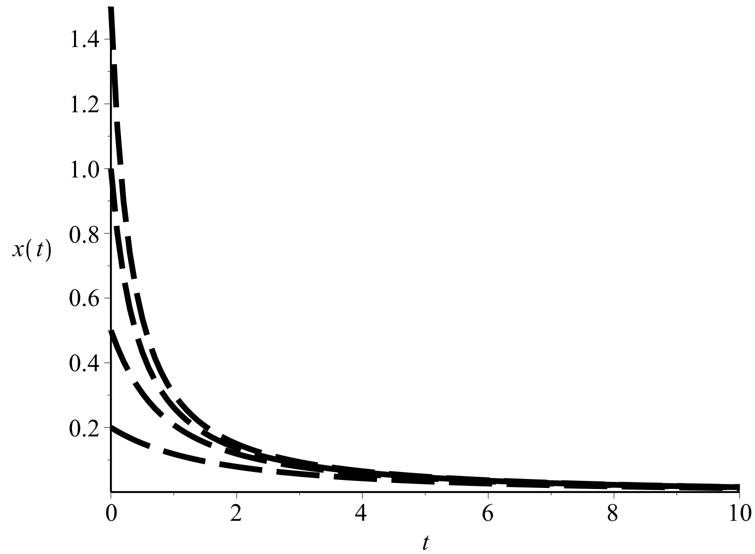

FIGURE 1. Dynamics behaviors of the first species in system (4.1). Here, we take the initial conditions $(x_1(0),x_2(0)) = (0.5,1.2),(1.5,1),(0.2,0.5)$ and $(1,0.6)$, $m = 0.8$, respectively.

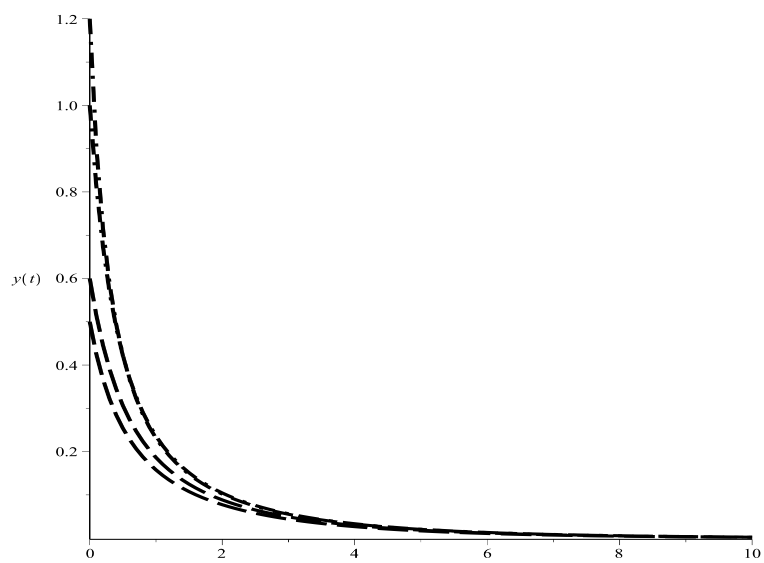

FIGURE 2. Dynamics behaviors of the second species in system (4.1). Here, we take the initial conditions $(x_1(0),x_2(0)) = (0.5,1.2),(1.5,1),(0.2,0.5)$ and $(1,0.6)$, $m = 0.8$, respectively.

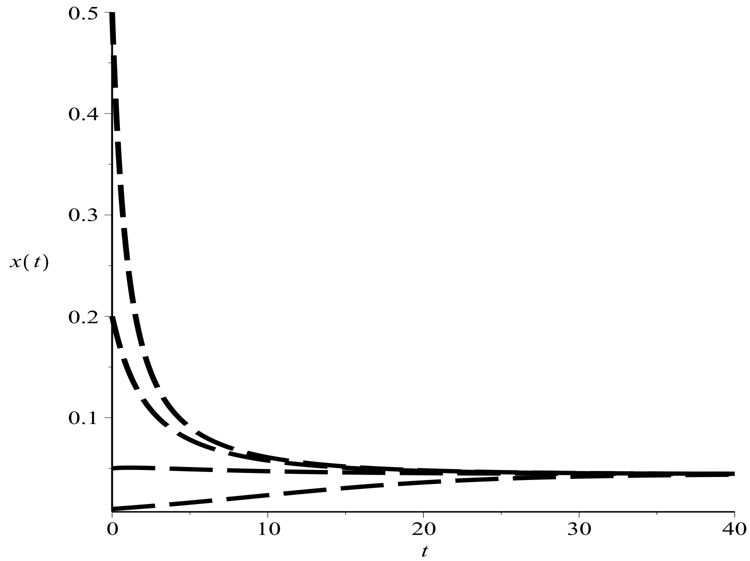

FIGURE 3. Dynamics behaviors of the first species in system (4.1). Here, we take the initial conditions $(x_1(0), x_2(0)) = (0.05, 1.2), (0.5, 1), (0.2, 0.5)$ and $(0.01, 0.6)$, $m = 0.7$, respectively.

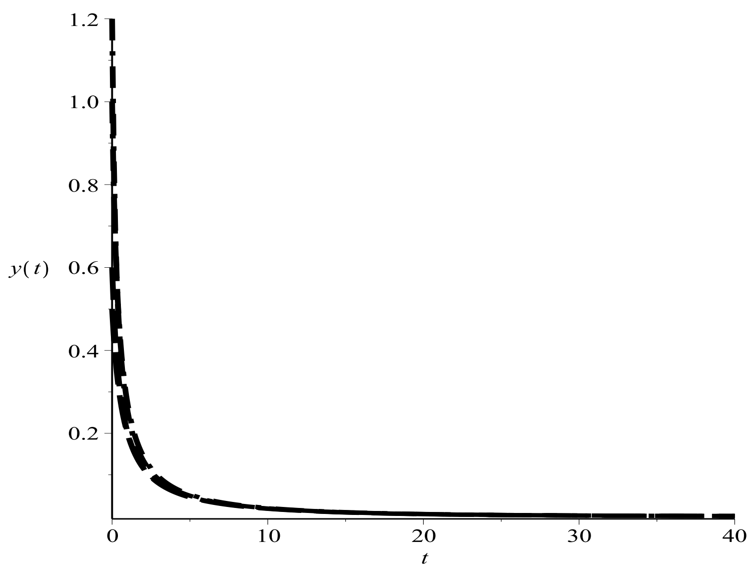

FIGURE 4. Dynamics behaviors of the second species system (4.1). Here, we take the initial conditions $(x_1(0), x_2(0)) = (0.05, 1.2), (0.5, 1), (0.2, 0.5)$ and $(0.01, 0.6)$, $m = 0.7$, respectively.

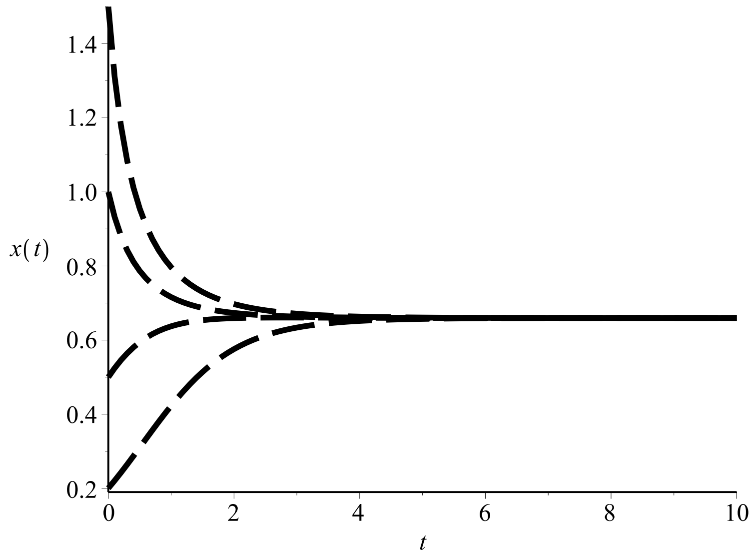

FIGURE 5. Dynamics behaviors of the first species in system (4.1). Here, we take the initial conditions $(x_1(0), x_2(0)) = (0.5, 1.2), (1.5, 1), (0.2, 0.5)$ and $(1, 0.6)$, $m = 0.2$, respectively.

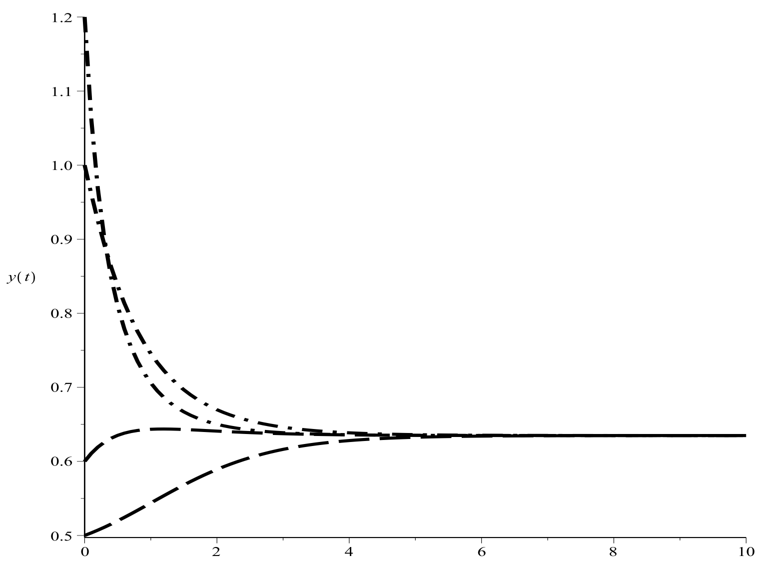

FIGURE 6. Dynamics behaviors of the second species in system (4.1). Here, we take the initial conditions $(x_1(0), x_2(0)) = (0.5, 1.2), (1.5, 1), (0.2, 0.5)$ and $(1, 0.6)$, $m = 0.2$, respectively.

5. Conclusion

Wei and Li[2] had considered the influence of the harvesting to the May cooperative system, however, they only considered the harvesting of the first species. In this paper, stimulated by the works of Chakraborty, Das, Kar[36], we propose the May cooperative system with both non-selective harvesting and partial closure for the populations, i. e., system (1.6).

Some interesting property about the system (1.6) and the influence of parameter m are obtained.

(1) Depending on the fraction of the stock available for harvesting, i. e., depending on the interval in which m is located,

$$m > \max\left\{\frac{r_1}{Eq_1}, \frac{r_2}{Eq_2}\right\}, \frac{r_2}{Eq_2} < m < \frac{r_1}{Eq_1}, \frac{r_1}{Eq_1} < m < \frac{r_2}{Eq_2}, m < \min\left\{\frac{r_2}{Eq_2}, \frac{r_1}{Eq_1}\right\},$$

the two species could be coexist in the long run, or some of the species is extinct, while the other one is permanent, or two of the species are both driven to extinction. That is, the fraction of the stock available for harvesting plays crucial role on the dynamic behaviors of the system. Obviously, those conditions are very simple and easily testified.

(2) Another amazing finding is that the conditions of Theorem 2.1 and 3.1 are independent of k_i and $a_i, i = 1, 2$. Though $k_i, a_i, i = 1, 2$ have influence on the final density of the both species, those parameters have no influence on the persistent property of the system. If the intrinsic growth rate of the species ($r_i, i = 1, 2$) are enough large, and the harvesting is limited to suitable area, then two species could survival in the long run.

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Conflict of Interests

The authors declare that there is no conflict of interests.

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