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A MATHEMATICAL MODEL FOR JIGGERS INFESTATION INCORPORATING EFFECTS OF MEDIA CAMPAIGNS

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Abstract. Jiggers infestation remains a problem especially in areas with limited or no interventions. In this paper, a mathematical model that incorporates media campaigns is proposed with the aim of investigating the potential role awareness through media campaigns on jiggers infestation dynamics. We introduce a class of those that are aware in which the awareness does not completely protect individuals from jiggers. The model analysis is presented in terms of the basic reproduction number. Bifurcation analysis reveals that the model has an intrinsic backward bifurcation whenever the parameter that accounts for the proportion of larvae that develop into adult female fleas involved in jiggers transmission is included. Sensitivity analysis performed and the results suggest that the effective infestation contact rate, as well as the rate at which the larvae develop into adult fleas are the main mechanisms that fuel jiggers infestation. The implications of the result to public health and the management of jiggers infestations are discussed.

Keywords: jiggers; stability; media campaigns; awareness; simulations

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1. INTRODUCTION

Jiggers are small pin-head-sized chigoe fleas found in sandy terrains of warm, dry climates. It hides in the crevices and cracks found on the floors, walls of dwellings and items like furniture and it feeds on warm blooded hosts including man, cats, dogs, rats, pigs, cattle and sheep. The female flea feeds by burrowing into the skin of its host. The abdomen becomes enormously enlarged between the second and third segments so that the flea forms a round sac with the shape and size of a pea. The impregnated female Tunga embeds itself in the skin under the toe-nails and fingernails of humans, where the resultant sores may fill with pus and become infected. After two weeks, over 100 eggs are released through the exposed skin opening and fall to the ground. The flea then dies and is slowly sloughed by the host's skin. The eggs hatch in the dust within 3-4 days. In 21-28 days, they go through their larval and pupal stages and become adults. The complete life cycle of a Tungapenetrans lasts for about a month [1].

The evaluation of the effectiveness of control strategies is critical in mathematical modeling [4]. Apart from the classical models governing the spread of infectious diseases as a result of interactions between the susceptibles and the infected, other disease control activities that play a pivotal role need to be included. These activities include media campaigns, vaccination, treatment among others. In particular, media campaigns play a vital role in influencing both the individual's behavior towards the disease and policy. It is the media awareness programs that make people know about a given outbreak of a disease in order to take the necessary measures and precautions. In case of jiggers infestations, precautions such as wearing protective masks, stopping the sharing of pins, practicing good hygiene, getting vaccinated, quarantine of infested individuals among others is necessary in order to decrease the chances of one being infected. For a disease such as jiggers infestation, the evaluation of the effectiveness of media campaigns as a control strategy has the potential of influencing policy and the management of the disease.

While a lot has been done to sensitise people about jiggers infestation through media campaigns, very little effort has been made to model the effects of media campaigns on jiggers infestation. However models on media campaigns on infectious diseases have been done by many researchers, see for instance: [12], who modelled the impact of information transmission on epidemic outbreaks through the diffusion of health information disseminated as a result of the presence of a disease; [16], who investigated the impact of media/psychological on multiple outbreaks of emerging infectious diseases**;** [20], who investigated the impact of media coverage using a compartmental model; [2], developed a non linear mathematical model in order to asses the impact of creating awareness by the media on the spread of vector borne diseases and [3], who modelled the role of awareness programs by media on spread of an infectious disease.

The role of media in highlighting anti-jiggers campaigns in Murang'a County, Kenya was investigated in [14] without using mathematical modelling. The study intended to establish ways of creating awareness about the role of media in highlighting anti-jiggers campaigns which would contribute to the improvement of living standards of the infested and affected people. The study established that media should give prominence to investigation of house hold activities of community health extensions workers (CHEWS) and community health workers (CHWS). In [5], the effects of jiggers infestation on agricultural productivity was investigated. It was found out that 67.35% of farmers in Murarandia division of Murang'a County are infested with jiggers. They recommended Murang'a County governance to create awareness about effects of jiggers infestation through media campaigns.

In the presence of an infestation, media campaigns have the potential of creating a class of people who are aware and therefore change their behavior with regards to the management of the infestation. In this paper we consider a model for effects of media campaigns on jiggers infestation with the aim of evaluating the impact of media campaigns on the dynamics of jiggers infestation. The results have implications on policy and disease management.

This work is arranged as follows. In Section 2 we formulate the model and in Section 2.4 we effectuate analysis of the model by finding out important thresholds such as the reproduction number R_0 and different equilibria of the model. We then express the stability of equilibria and carry out bifurcation analysis. In Section 3, we carry out numerical simulations which include, sensitivity analysis, parameter variation which are used to establish the influence of some important parameters on the effect of media campaigns on jiggers infestation. Section 4 concludes the paper.

2. The model

2.1. Model formulation.

We formulate a deterministic model comprising of the human population and the jiggers and its developmental stages. The human population is categorized into four compartments such that at any time $t > 0$, there are susceptible humans, S_u, who are not aware of jiggers infestation, susceptible humans who are aware of the jiggers infestation, S_a, infested humans with the parasite, I, and those who would have recovered after treatment R. Thus the size of the human population is given by

$$N(t) = S_u(t) + S_a(t) + I(t) + R(t).$$

The flea cycle is categorized into three compartments such that at any time $t > 0$, there are $E(t)$, eggs released from those infested by the adult female fleas, the combined stages of the larvae and pupa stages, $L(t)$, and the adult female flea, $F(t)$. The combination of the larvae and pupa stages is motivated by the fact that the crucial dynamics are driven by the interaction of the hman population and the fleas. Further, we let $M(t)$ be the cumulative density of awareness by media campaigns at any time t. Such formulation has been done in [2, 3]. We assume that growth of the media campaigns are proportional to the number of cases of infested humans. Also, the reduction of media campaigns as a result of social and psychological effects is included in the modeling process, as media campaigns uptake often wanes with time.

The rate at which individuals enter the susceptible population through births and immigration is given by Π. Through the burrowing of the female flea into the susceptible humans' skin who are not aware of the jiggers infestation, a susceptible individual becomes infested and moves to the infested compartment $I(t)$. The unaware susceptible humans become aware of jiggers infestations at a rate η which measures the effects of disseminated information in raising awareness resulting in individuals moving into the aware compartment S_a. We assume that awareness does not provide total protection from infestation. We thus assume that individuals who 'ignore' the media campaigns still become infested and then move to the infested compartment at a rate $(1-\sigma)\beta$, where the constant σ measures the efficacy of the media campaigns in protecting aware individuals and β the effective contact rate, i.e the contacts that result in infestation per unit time. Awareness reduces the risk of infestation by a factor $\sigma \in (0,1)$ so that if $\sigma = 0$ the campaigns are as good as not being there and if $\sigma = 1$ the campaigns are 100% effective in preventing infestations. The generation of new infestations is modeled by the expressions $\beta F S_u$ and $(1-\sigma)\beta F S_a$ as the susceptible humans (both aware and unaware) are infested by the adult female fleas. Once an individual is infested, recovery is possible in the presence of treatment at a rate γ. This could be made possible by removing the fleas from their cavity using sterile instruments followed by thorough cleaning and covering the remaining crater with topical antibiotic to prevent secondary infestation. Recovery from infestation does not provide protection from further infection, hence individuals in the recovered class can move back to the susceptible class at a rate ω. Individuals in each class die naturally at a rate μ. Given that m represents the growth rate of density of awareness programs, which is assumed to be proportional to the number of infested humans, we model the growth for media campaigns by a linear function mI. It is important to state that different functions can be proposed for the growth of the media campaigns. We can think of the logistic growth function or a saturating function of the Michaelis Menten type, if we assume the media campaigns are limited in growth over time, which sound more plausible. We however assume a linear function for mathematical tractability and leave the other functions for future studies. Also, we let ϕ represent the depletion rate of media campaigns due to ineffectiveness, social and psychological barriers. Thus as time progresses, some media campaigns do not influence people.

Fleas' eggs are released from individuals infested, that is those in class I at a rate τ. We assume that from each infested individual, N_i eggs are released. The eggs released onto the ground hatch into a larvae and then pupa, of which we combine the two stages so that the development takes place at a rate ρ. This is due to the fact that, separating them does not necessarily give a modelling advantage but rather increases the equation numbers and also control of the infestation is by larvicides and not at pupal level. The eggs die at a rate ν_e. The larvae develop into adult fleas at a rate δ and die at a rate ν_l. We assume that a proportion ε of the larvae develop into adult fleas. The development of the adult fleas is modeled by a saturation function $\dfrac{\varepsilon\delta L}{1+L}$ where $\varepsilon\delta$ is the maximum number of female adult fleas that will eventually burrow into the skin of the human susceptible's skin. However the adult female fleas die naturally at a rate ν_f. The model diagram is depicted in Figure 1.

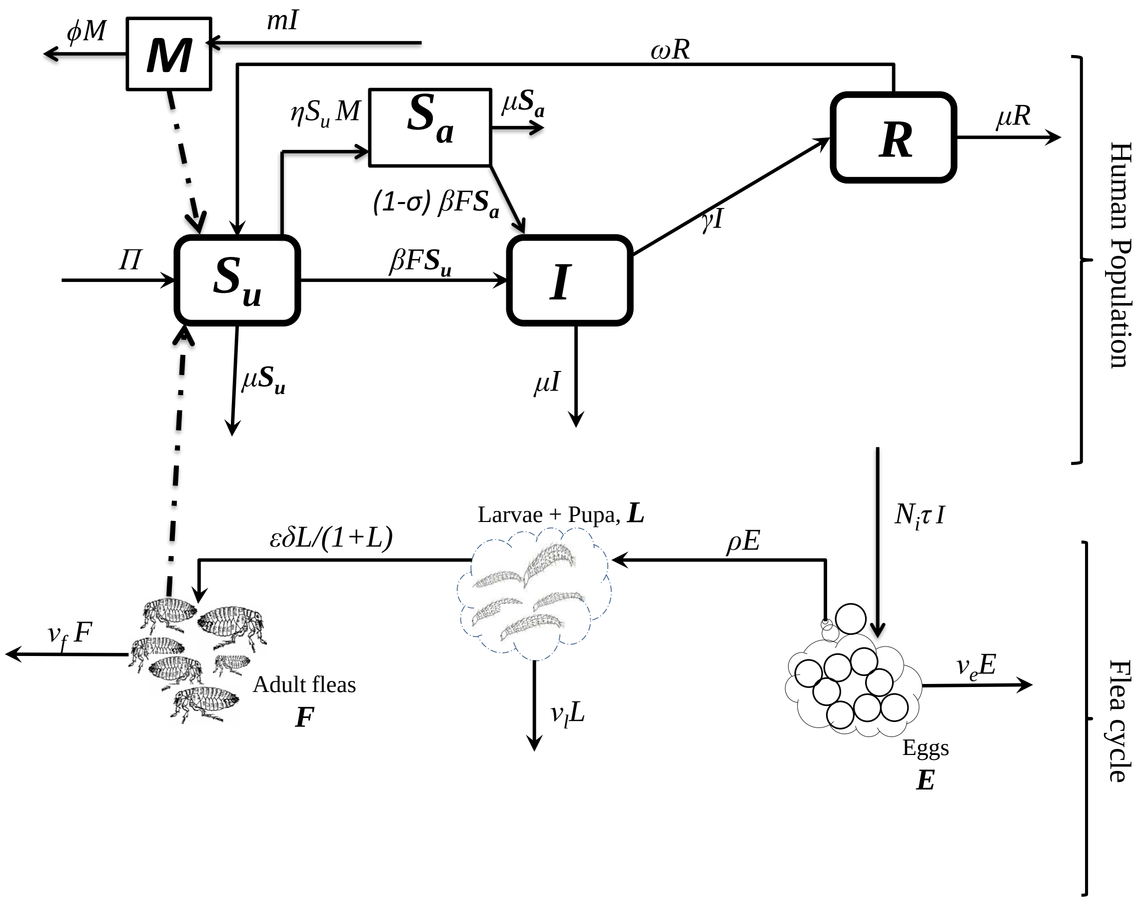


FIGURE 1. The model diagram for jiggers infestation incorporating effects of media campaigns.

2.2. Parameter estimation. Since not so much has been done mathematically to model jiggers infestation, parameters that relate to transmission and progression rates are difficult to find. In this section, we assumed majority of the parameters using information collected on the populations in Kenya. Some have been acquired from the literature mostly from [1] and also with reference to the population infested with jiggers in Murang'a County in Kenya. Some of the demographic parameters used in our model simulation are described as follows:

- The demographic data released in [7], estimated life expectancy at birth to be 63.52 years in 2014 and 63.4 years in 2015. This can then be estimated from 50 to 70 years. Thus the natural death rate of humans is estimated as $0.0000291 \leq \mu \leq 0.0000548$ per day.
- According to [7], the average birth rate in Kenya was estimated to be 28.27 births per 1000 population in the year 2014 and 26.4 per 1000 population in 2015. Therefore the birthrate is estimated to be $0.0000775 \leq \Pi \leq 0.0000937$ per day.
- From [1], it takes 2-4 days for the eggs to hatch on the ground. Therefore we approximate the rate at which the eggs develop into larvae to be, $0.25 \leq \rho \leq 0.5$ per day. After the eggs have hatched onto the ground, it takes 3-4 weeks for them to go through the larvae and pupa stage to become adults. Thus the rate at which the larvae develops into a pupa can also be estimated as $0.036 \leq \delta \leq 0.048$ per day.
- A proportion p of the infested persons become chronically infested. We thus consider $0 \leq p \leq 1$.
- The natural death rates of the vector, that is, $\nu_e, \nu_l$and ν_f can be approximated from the life cycle of the flea given in [1] as follows; $0.003 \leq \nu_e \leq 0.02, 0.038 \leq \nu_l \leq 0.081$ and $0.06 \leq \nu_f \leq 0.1$ per day.

The rest of the parameters assumed are presented in Table 1.

TABLE 1. Description and estimation of parameter values used in the model. The parameter values are given per day.

Par	Description	Range (per day)	Point-value	Source
Π	Recruitment rate of human population	$\frac{0.0264}{365}-\frac{0.0283}{365}$	$\frac{0.0268}{365}$	[7]
β	Effective infestation rate	0–2	0.08	Assumed
γ	Rate at which infested humans recover after treatment	0–1	0.65	Assumed
η	Dissemination of awareness	0–1	0.35	Assumed
μ	Natural death rate for humans	$\frac{0.014}{365}-\frac{0.02}{365}$	$\frac{0.017}{365}$	[7]
δ	Rate at which larvae develop into adult fleas	0.036–0.048	0.042	[1]
ε	Proportion of larvae that develop into adult female fleas that are involved in jiggers transmission	0–1	0.068	Assumed
ν_f	Natural death rate of the adult female flea	0.06–0.1	0.089	[1]
ν_l	Natural death rate of the larvae	0.038–0.081	0.058	[1]
ν_e	Natural death rate of the eggs	0.003–0.02	0.0158	[1]
ρ	Rate at which the eggs develop into the combined pupa and larvae stage	0.2– 0.5	0.353	[1]
τ	Rate of egg production from infested humans by adult female fleas	0.2– 0.5	0.312	[1]
ϕ	Depletion rate of awareness programs	0–1	0.068	Assumed
m	Growth rate of density of awareness programs	0–1	0.089	Assumed
ω	Rate at which the recovered humans become susceptible to jiggers infestation	0–2	0.053	Assumed
N_i	Number of eggs released per a jiggers from infested humans	0–100	90	Assumed
σ	Efficacy of media campaigns	0–1	0.22	Assumed

2.3. Model equations. Given the the model description in Figure 1 and assumptions made, the following non-linear first order ordinary differential equations are derived.

$$\left.\begin{aligned}
\frac{dS_u}{dt} &= \Pi + \omega R - \beta F S_u - \eta S_u M - \mu S_u,\\
\frac{dS_a}{dt} &= \eta S_u M - (1-\sigma)\beta F S_a - \mu S_a,\\
\frac{dI}{dt} &= \beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma)I,\\
\frac{dR}{dt} &= \gamma I - (\mu+\omega)R,\\
\frac{dE}{dt} &= N_i \tau I - (\nu_e + \rho)E,\\
\frac{dL}{dt} &= \rho E - \left(\nu_l + \frac{\varepsilon\delta}{1+L}\right)L,\\
\frac{dF}{dt} &= \frac{\varepsilon\delta L}{1+L} - \nu_f F,\\
\frac{dM}{dt} &= mI - \phi M,
\end{aligned}\right\} \tag{1}$$

subject to the following initial conditions

$$S_u(0) > 0, S_a(0) > 0, I(0) \geq 0, R(0) \geq 0, E(0) \geq 0, L(0) \geq 0, F(0) \geq 0. \tag{2}$$

2.4. Positivity and boundedness of solutions. For the jiggers infestation model system (1) to be epidemiologically meaningful, it is important to prove that all its state variables are non-negative for all time. This is to mean that solutions of the model system (1) with non-negative initial data, will remain non-negative for all time $t > 0$.

Theorem 1. *Let the initial data be given as in* (2)*. Then the solutions of the model system* (1) *are nonnegative for all* $t > 0$*. Furthermore,*

$$\limsup_{t\to\infty} N(t) \leq \frac{\Pi}{\mu}, \quad \limsup_{t\to\infty} E(t) \leq \frac{\Lambda}{\nu_e+\rho}, \quad \limsup_{t\to\infty} L(t) \leq \frac{J}{\nu_e+\varepsilon\delta} \text{ and } \limsup_{t\to\infty} F(t) \leq \frac{\gamma}{\nu_f},$$

where $J = \frac{\Pi}{\mu} - N(0)$ *for* $\frac{\Pi}{\mu} \geq N(0)$.

Proof. Let $t_1 = \sup\{t > 0 : (S_u, S_a, I, R, E, L, F, M) > 0 \in [0,t]\}$. Thus, $t_1 > 0$. It follows from the first equation of the model system (1) that

$$\frac{dS_u}{dt} < \Pi - \beta F S_u - \eta S_u M - \mu S_u,$$

which can be rewritten as

$$\frac{dS_u}{dt}\left\{S_u(t)\exp\left[\left(\int_0^{t_1}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu t\right)\right]\right\}<\Pi\exp\left[\left(\int_0^{t_1}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu t\right)\right].$$

Hence

$$S_u(t_1)\exp\left[\left(\int_0^{t_1}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu t_1\right)\right]-S_u(0)<\int_0^{t_1}\Pi\exp\left[\left(\int_0^{p}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu p\right)\right]dp$$

so that

$$S_u(t)<S_u(0)\exp\left[-\left(\int_0^{t_1}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu t_1\right)\right]$$
$$+\exp\left[-\left(\int_0^{t_1}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu t_1\right)\right]\left[\int_0^{t_1}\Pi\exp\left[\left(\int_0^{p}(\beta F(\zeta)+\eta M(\zeta))d\zeta+\mu p\right)\right]dp\right]>0.$$

Similarly, it can be shown that S_a, I, R, E, L, F, M are all positive for all $t>0$. □

Theorem 2. *The region* $\Omega \in \Re^8_+$ *given by*

$$\Omega=\left\{(S_u,S_a,I,R,E,L,F,M)\in\Re^8_+ : N\le\max\left\{N(0),\frac{\Pi}{\mu}\right\},\quad E\le\max\left\{E(0),\frac{\Lambda}{\nu_e+\rho}\right\},\right.$$
$$\left.L\le\max\left\{L(0),\frac{J}{\nu_e+\varepsilon\delta}\right\},\quad F\le\max\left\{F(0),\frac{\gamma}{\nu_f}\right\}\right\},$$

is positively invariant for the model system (1) *with nonnegative initial conditions in* $\Re^8_+$.

Proof. The rate of change of the total population is obtained by adding the human and gthe jiggers components of the model system (1) to give

$$\frac{dN}{dt}=\Pi-\mu N,\quad \frac{dE}{dt}=\Lambda-(\nu_e+\rho)E,\quad \frac{dL}{dt}=J-(\nu_e+\varepsilon\delta)L \text{ and } \frac{dF}{dt}=\gamma-\nu_f F. \tag{3}$$

Following the work in [8, 10, 19] it can be shown that the solutions of the expressions in (3) are respectively, given by

(4)

$$\left.\begin{array}{l} N(t)\le N(0)e^{-\mu t}+\frac{\Pi}{\mu}\left(1-e^{-\mu t}\right),\quad E(t)\le E(0)e^{-(\nu_e+\rho)t}+\frac{\Lambda}{\nu_e+\rho}\left(1-e^{-(\nu_e+\rho)t}\right),\\ L(t)\le L(0)e^{-(\nu_e+\varepsilon\delta)t}+\frac{J}{(\nu_e+\varepsilon\delta)}\left(1-e^{-(\nu_e+\varepsilon\delta)t}\right),\quad F(t)\le F(0)e^{-\nu_f t}+\frac{\gamma}{\nu_f}\left(1-e^{-\nu_f t}\right).\end{array}\right\}$$

Taking the population described by $N(t)$ in (4), there are two possible scenarios in studying the behaviour of $N(t)$. In the first scenario, we consider $N(0)>\frac{\Pi}{\mu}$ so that, at time $t=0$, the right-hand side (RHS) of the expression in (4) experiences the largest possible value of $N(0)$. That is, $N(t)\le N(0)$ for all time $t\ge 0$. In the second scenario, we consider $N(0)<\frac{\Pi}{\mu}$, so that the largest possible value of the RHS of (4) approaches $\frac{\Pi}{\mu}$ as time t approaches infinity. Thus, $N(t) \leq \frac{\Pi}{\mu}$ for all time $t \geq 0$. From these two scenarios, we conclude that $N(t) \leq \max\left\{N(0), \frac{\Pi}{\mu}\right\}$ for all time $t \geq 0$. Similar approach can be used to describe the behaviour of $E(t)$, $L(t)$ and $F(t)$ so that we respectively get

$$E \leq \max\left\{E(0), \frac{\Lambda}{\nu_e + \rho}\right\}, \quad L \leq \max\left\{L(0), \frac{J}{\nu_e + \varepsilon\delta}\right\}, \quad F \leq \max\left\{F(0), \frac{\gamma}{\nu_f}\right\}.$$

Thus, the region Ω is positively invariant. Hence, it is sufficient to consider the dynamics of the flow generated by (1) in Ω. In this region, the system is epidemiologically and mathematically well-posed [10, 11]. Thus, every solution of the system (1) with initial conditions in Ω remains in Ω for all $t > 0$. □

MODEL ANALYSIS

2.5. Basic reproduction number. Let $\mathscr{E}^0 = (S_u^0, S_a^0, I^0, R^0, E^0, L^0, F^0, M^0)$ be the jiggers free steady state (JFS) of the system (1). At $\mathscr{E}^0$, the sub classes $S_a(t), I(t), R(t), E(t), L(t), F(t)$ and $M(t)$ are equal to zero, hence we obtain, $S_u^0 = \dfrac{\Pi}{\mu}$. The JFS point ($\mathscr{E}^0$) for the system (1) is therefore given by

$$\mathscr{E}^0 = \left\{\frac{\Pi}{\mu}, 0, 0, 0, 0, 0, 0, 0\right\}. \tag{5}$$

Thus, following the description in [15], the reproduction number R_0 is defined as the average number of the secondary cases of infection arising from a single primary case of infection in an entirely susceptible population. The reproduction number is used to predict whether the epidemic spreads or dies out.

Adopting the notation in [15], the matrices that represent new cases of infections and transfer/transition are respectively given by

$$\mathscr{F} = \begin{pmatrix} 0 & 0 & 0 & \frac{\beta\Pi}{\mu} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathscr{V} = \begin{pmatrix} \mu + \gamma & 0 & 0 & 0 \\ -N_i\tau & (\nu_e + \rho) & 0 & 0 \\ 0 & -\rho & \delta\varepsilon + \nu_l & 0 \\ 0 & 0 & -\varepsilon\delta & \nu_f \end{pmatrix}.$$

Therefore, the basic reproduction number R_0 is given as the spectral radius of $\mathscr{F}\mathscr{V}^{-1}$. That is

$$R_0 = \beta\left(\frac{\Pi}{\mu}\right)\left(\frac{N_i\tau}{\mu+\gamma}\right)\left(\frac{\varepsilon\delta}{\nu_l+\varepsilon\delta}\right)\left(\frac{\rho}{\rho+\nu_e}\right)\left(\frac{1}{\nu_f}\right). \tag{6}$$

From the basic reproduction number, R_0, it can be clearly seen that $\frac{1}{\mu+\gamma}$ is the duration of stay in class I, $\frac{\rho}{\rho+\nu_e}$ is the fraction of eggs that become pupa/larvae, $\frac{\delta\varepsilon}{\delta\varepsilon+\nu_l}$ is the fraction of larvae/pupa that become adult flea, $\frac{1}{\nu_f}$ is the life expectancy of the female fleas.
From Theorem 2 in [15], we have the following result.

Theorem 3. *The JFE of the system of equations* (1) *is locally asymptotically stable when* $R_0 < 1$ *and unstable otherwise.*

This general result has been reviewed in [15] and thus not proved again here to avoid redundancy. The theorem implies that jiggers infestation will disappear from the community when R_0 is kept below one if the initial sizes of the subpopulations of system (1) are in the basin of attraction of the jiggers free equilibrium.

2.6. Global stability of the jiggers free equilibrium points. To investigate the global stability of the jiggers free equilibrium, we construct Lyapunov functions. We define a candidate Lyapunov function as

$$L(I,E,L,F) = \Phi_1 I + \Phi_2 E + \Phi_3 L + \Phi_4 F, \tag{7}$$

where Φ_1, Φ_2, Φ_3 and Φ_4 are non-negative constants to be determined. It follows that the derivative of (7) is given by

$$\begin{aligned}
\frac{dL}{dt} &= \Phi_1\frac{dI}{dt} + \Phi_2\frac{dE}{dt} + \Phi_3\frac{dL}{dt} + \Phi_4\frac{dF}{dt},\\
&= \Phi_1\left[\beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma)I\right] + \Phi_2\left[N_i\tau I - (\nu_e+\rho)E\right]\\
&\quad + \Phi_3\left[\rho E - \left(\nu_l + \frac{\varepsilon\delta}{1+L}\right)L\right] + \Phi_4\left[\frac{\varepsilon\delta L}{1+L} - \nu_f F\right],\\
&\le \Phi_1\left[\beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma)I\right] + \Phi_2\left[N_i\tau I - (\nu_e+\rho)E\right]\\
&\quad + \Phi_3\left[\rho E - (\nu_l+\varepsilon\delta)L\right] + \Phi_4\left[\varepsilon\delta L - \nu_f F\right],\\
&= \left[\Phi_2 N_i\tau - \Phi_1(\mu+\gamma)\right]I + \left[\Phi_3\rho - \Phi_2(\nu_e+\rho)\right]E + \left[\Phi_4\varepsilon\delta - \Phi_3(\nu_l+\varepsilon\delta)\right]L + \left[\Phi_1\frac{\beta\Pi}{\mu} - \Phi_4\nu_f\right]F.
\end{aligned}$$

In order to determine the non-negative coefficients Φ_1, Φ_2, Φ_3 and Φ_4, we set the coefficients of E, I and F to zero and solve to obtain

$$\Phi_1 = \frac{(\nu_e+\rho)(\nu_l+\varepsilon\delta)\nu_f\mu}{\beta\Pi\varepsilon\delta}, \quad \Phi_2 = \rho, \quad \Phi_3 = \nu_e+\rho, \quad \Phi_4 = \frac{(\nu_e+\rho)(\nu_l+\varepsilon\delta)}{\varepsilon\delta}. \tag{8}$$

Substituting the coefficients obtained in (8) into the time derivative of (7), we get

$$\frac{dL}{dt} \leq \beta\Pi\varepsilon\delta[R_0-1]L. \tag{9}$$

From (9), it can be clearly seen that when $R_0 \leq 1$, $\frac{dL}{dt} \leq 0$, with equality at $R_0 = 1$. Furthermore, $\frac{dL}{dt} = 0$ if and only if $E = I = F = 0$. Thus, the largest compact invariant set in $\left\{(S_u, S_a, I, R, E, L, F, M) \in \Omega : \frac{dL}{dt} = 0\right\}$, when $R_0 \leq 1$ is the singleton $\mathscr{E}^0$. Hence, $\mathscr{E}^0$ is the only steady state when $R_0 \leq 1$. Using LaSalle Invariance Principle [13], this implies that $\mathscr{E}^0$ is globally attractive in Ω if $R_0 \leq 1$. The epidemiological implication of jiggers free equilibrium being globally asymptotically stable is that jiggers epidemic will be eliminated from the community if the threshold quantity R_0 is decreased to and/or maintained at a value below one.

2.7. The jiggers persistent equilibria. Within the context of our jiggers infestation model the jiggers persistent equilibrium refers to a state when jiggers infestation is maintained over for long time scales in the community. Therefore, we let $\mathscr{E}^* = (S_u^*, S_a^*, I^*, R^*, E^*, L^*, F^*, M^*)$ represent the jiggers persistent equilibrium points of the model system (1). Solving the equations in (1) at steady states by equating the left hand side to zero and expressing all other state variables in terms of I^*, we obtain

$$\begin{aligned}
S_u^* &= \frac{\phi\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)(\Pi(\mu+\omega)+\gamma\omega I^*)}{(\mu+\omega)\left(\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)(\mu\phi+\eta m I^*)+\beta\delta N_i\rho\tau\varepsilon I^*\phi\right)},\\
S_a^* &= \frac{\eta m\nu_f^2 I^*(\nu_e+\rho)^2(\delta\varepsilon+\nu_l)^2(\Pi(\mu+\omega)+\gamma\omega I^*)}{(\mu+\omega)\left(\mu\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)+\beta\delta N_i\rho(1-\sigma)\tau\varepsilon I^*\right)Q_1}, \quad R^* = \frac{\gamma I^*}{\mu+\omega},\\
E^* &= \frac{\tau I^* N_i}{\nu_e+\rho}, \quad L^* = \frac{\rho\tau I^* N_i}{(\nu_e+\rho)(\delta\varepsilon+\nu_l)}, \quad F^* = \frac{\delta N_i\rho\tau I^*\varepsilon}{\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)}, \quad M^* = \frac{m I^*}{\phi},
\end{aligned} \tag{10}$$

where $Q_1 = \left(\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)(\mu\phi+\eta m I^*)+\beta\delta N_i\rho\tau\varepsilon\phi I^*\right)$. Substituting (10) into the third equation of system (1) and simplifying the following equation is obtained

$$h(I^*) = (A_2 I^{*2} + A_1 I^* + A_0) = 0, \tag{11}$$

where

$$\left.\begin{aligned}
A_2 &= \beta\delta\mu N_i\rho\tau\varepsilon(1-\sigma)(\gamma+\mu+\omega)\left(\eta m\nu_f\left(\nu_e+\rho\right)\left(\delta\varepsilon+\nu_l\right)+\beta\delta N_i\rho\tau\varepsilon\phi\right),\\
A_1 &= \beta\delta N_i\rho\tau\varepsilon\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)(\mu\phi(\omega(-\sigma(\gamma+\mu)+\gamma+2\mu)+\mu(2-\sigma)(\gamma+\mu))\\
&\quad +\eta m\Pi(1-\sigma)(\mu+\omega))-\beta^2\delta^2N_i^2\Pi\rho^2(1-\sigma)\tau^2\varepsilon^2\phi(\mu+\omega)\\
&\quad -\eta\mu m(\gamma+\mu)\nu_f^2(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\varepsilon+\nu_l\right)^2,\\
A_0 &= \mu^2\phi\nu_f^2(\gamma+\mu)(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\varepsilon+\nu_l\right)^2(1-R_0).
\end{aligned}\right\}$$

The polynomial (11) can be analysed to investigate the existence of multiple equilibria when the basic reproduction number is below unity. To analyse the possible number of positive solutions to the polynomial (11), we proceed as follows: The roots to the polynomial (11) is obtained by the quadratic formula given by

$$I^* = \frac{-A_1 \pm \sqrt{A_1^2-4A_2A_0}}{2A_2}. \tag{12}$$

It is easy to see that $A_0 > 0$ when $R_0 < 1, A_0 = 0$ when $R_0 = 1$ and $A_0 < 1$ when $R_0 > 1$. When $A_0 < 0, \Delta = A_1^2 - 4A_2A_0 > 0$ and the polynomial (11) has a unique positive solution. This implies that the system (1) has a unique jiggers persistent equilibrium. On the other hand, when $R_0 < 1$, then $A_0 > 0$ and by adding a condition that $A_1 < 0$ and $\Delta > 0$, we obtain two positive real roots implying existence of two positive equilibria. If $R_0 = 1$, then $A_0 = 0$ and there exists a unique non-zero solution of the polynomial (11) which is positive if and only if $A_1 < 0$. Thus, we have the following results on the existence of equilibria of the model system (1).

Theorem 4.

(i) *System* (1) *always has a disease-free equilibrium* $\mathscr{E}^0$.

(ii) *If* $R_0 > 1$, *then model system* (1) *has a unique jiggers persistent equilibrium* $\mathscr{E}^*$.

(iii) *Has no jiggers persistent equilibrium if* $R_0 < R_0^c$ *where* R_0^c *is referred to as critical* R_0. *Note that* R_0^c *is given by*

$$R_0^c = 1 - \frac{A_1^2}{4A_2\mu^2\phi\nu_f^2(\gamma+\mu)(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\varepsilon+\nu_l\right)^2}.$$

The above expression for R_0^c is obtained after setting discriminant $\Delta = 0$ and making R_0 the subject of the relation.

(iv) *Has two jiggers persistent equilibria for some parameter values of $R_0^c < R_0 < 1$. In this range, one jiggers persistent equilibrium and the jiggers free equilibrium are locally stable.*

(v) *Has one positive equilibrium for $R_0 = 1$ provided $A_1 < 0$ and $\Delta > 0$, otherwise there is no positive equilibrium.*

(vi) *Has no jiggers persistent equilibrium otherwise.*

The epidemiological implication of Theorem 4 item (iv) is that, jiggers infestation may still persist even if $R_0 < 1$.

Conclusion (iv) of Theorem 4 indicates that a backward bifurcation may occur for values of R_0 when $R_0^c < R_0 < 1$. To describe the local stability of the jiggers persistent equilibrium as well as the direction at $R_0 = 1$, we will use the theorem, remark, and corollary that are based on the Center Manifold Theory (CMT) as explained in [6]. The conditions under which backward bifurcation exists are followed from the Theorem 4.1 proven in [6]. In order to apply the Center Manifold Theory (CMT), we make the following changes to the state variables, we let $S_u = x_1, S_a = x_2, I = x_3, R = x_4, E = x_5, L = x_6, F = x_7, M = x_8$. The system (1) can now be written in the form $\frac{dx}{dt} = f(x)$, where $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$. The system (1) therefore becomes

$$
\left.\begin{aligned}
\dot{x_1} &= \Pi + \omega x_4 - \beta x_7 x_1 - \eta x_1 x_8 - \mu x_1, \\
\dot{x_2} &= \eta x_1 x_8 - (1-\sigma)\beta x_7 x_2 - \mu x_2, \\
\dot{x_3} &= \beta x_7 x_1 + (1-\sigma)\beta x_7 x_2 - (\mu+\gamma) x_3, \\
\dot{x_4} &= \gamma x_3 - (\mu+\omega) x_4, \\
\dot{x_5} &= N_i \tau x_3 - (\nu_e + \rho) x_5, \\
\dot{x_6} &= \rho x_5 - \left(\nu_l + \frac{\varepsilon\delta}{1+x_6}\right) x_6, \\
\dot{x_7} &= \frac{\varepsilon\delta x_6}{1+x_6} - \nu_f x_7, \\
\dot{x_8} &= m x_3 - \phi x_8.
\end{aligned}\right\} \tag{13}
$$

The basic reproduction number of the system (1) is as given by (6).

Suppose, we choose $\theta = \beta$ as the bifurcation parameter so that when $\mathscr{R}_0 = 1$, we have

$$\theta = \frac{\mu(\gamma+\mu)\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)}{\delta\Pi\rho\tau\varepsilon N_i}. \tag{14}$$

The Jacobian matrix J of the linearised system (13) at the JFE $\mathscr{E}^0$ and for $\beta = \theta$ given by

$$J = \begin{pmatrix} -\mu & 0 & 0 & \omega & 0 & 0 & \frac{-\beta\Pi}{\mu} & \frac{\eta\Pi}{\mu} \\ 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\mu+\gamma) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i\tau & 0 & -(\nu_e+\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho & -\nu_l+\varepsilon\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon\delta & -\nu_f & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 & -\phi \end{pmatrix}$$

, has a simple zero eigenvalue. The left eigen vector of J, $v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$ and the right eigen vector $w = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_1)$, both associated to the eigen value zero, are associated to the solutions of the system

$$\left.\begin{aligned} Jw &= [0,0,0,0,0,0,0,0]' \\ vJ &= [0,0,0,0,0,0,0,0]' \\ vw &= 1 \end{aligned}\right\} \tag{15}$$

Thus, it enables us to use the Center Manifold Theory to analyse the stability of the system (13) near $\beta = \theta$. Therefore a right eigenvector w associated with zero eigenvalue has components

$$\begin{aligned} & w_1 = -\frac{\left(\mu^2\phi(\gamma+\mu+\omega)+\eta m\Pi(\mu+\omega)\right)}{\mu^2\phi(\mu+\omega)}, \quad w_2 = \frac{\eta m\Pi}{\mu^2\phi}, \quad w_3 = 1, \quad w_4 = \frac{\gamma}{\mu+\omega}, \\ & w_5 = \frac{\tau N_i}{\nu_e+\rho}, \quad w_6 = \frac{\rho\tau N_i}{(\nu_e+\rho)(\delta\varepsilon+\nu_l)}, \quad w_7 = \frac{\delta\rho\tau\varepsilon N_i}{\nu_f(\nu_e+\rho)(\delta\varepsilon+\nu_l)}, \quad w_8 = \frac{m}{\phi}. \end{aligned} \tag{16}$$

Similarly, the corresponding left eigenvector v associated with zero eigenvalue has components

$$\begin{aligned} v_1 =& v_2 = v_4 = v_8 = 0, \quad v_3 = 1, \quad v_5 = \frac{\gamma+\mu}{\tau N_i}, \\ v_6 =& \frac{(\gamma+\mu)(\nu_e+\rho)}{\rho\tau N_i}, \quad v_7 = \frac{(\gamma+\mu)(\nu_e+\rho)(\delta\varepsilon+\nu_l)}{\delta\rho\tau\varepsilon N_i}. \end{aligned} \tag{17}$$

We now compute **a** and **b** as outlined in [6]. From the system (13), the non-zero partial derivatives of $f(x)$ associated with **a** are given by

$$\frac{\partial f_2}{\partial x_1 \partial x_7} = \theta. \tag{18}$$

Thus, the expression for **a** is given by

$$\begin{aligned} \mathbf{a} =& \quad v_3 w_1 w_7 \frac{\partial f_2}{\partial x_1 \partial x_7}, \\ =& \quad -\left(\frac{\left(\mu^2\phi(\gamma+\mu+\omega)+\eta m\Pi(\mu+\omega)\right)}{\mu^2\phi(\mu+\omega)}\frac{\delta\varepsilon}{\nu_f}\right) w_6 < 0. \end{aligned} \tag{19}$$

We finally compute the value of **b**. The non-zero partial derivatives of $f(x)$ associated with **b** is given by

$$\frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}. \tag{20}$$

Therefore the expression for **b** is given by

$$\mathbf{b} = v_3 w_7 \frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}\frac{\delta\varepsilon}{\nu_f} w_6 > 0. \tag{21}$$

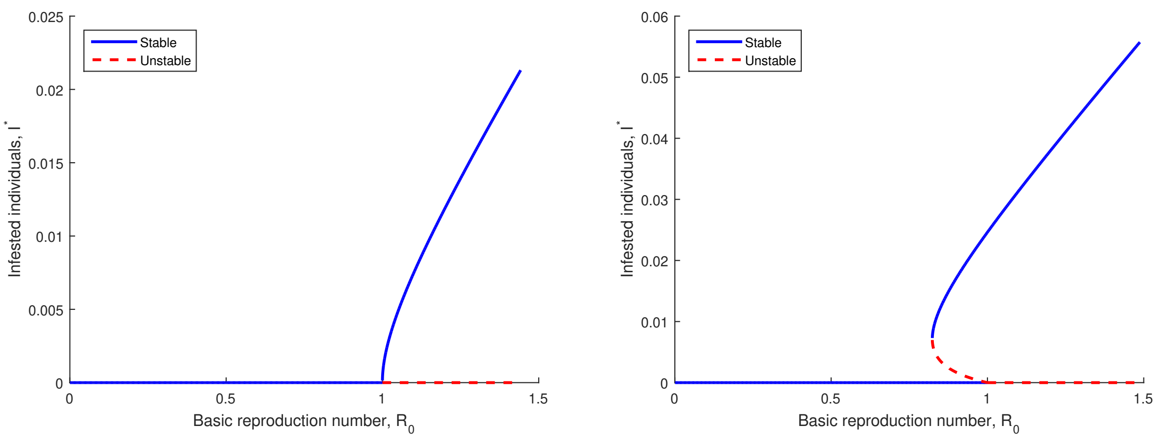
The direction of the bifurcation is determined by the signs of **a** and **b**. Obviously, $\mathbf{b} > 0$. Thus, the direction of bifurcation is determined by the following conditions

$$\begin{cases} \text{if } \mathbf{a} > 0, \text{ then the bifurcation is backward} \\ \text{or} \\ \text{if } \mathbf{a} < 0, \text{ then the bifurcation is forward} \end{cases}$$

Since, $\mathbf{a} < 0$ and $\mathbf{b} > 0$, from item 4 in [6] we conclude that model (1) undergoes a forward bifurcation at $R_0 = 1$.

Remark 1. *The backward bifurcation also provides some information on the jiggers persistent equilibrium of the model system* (1)*. For example, we obtain that the jiggers persistent equilibrium $\mathscr{E}_1^*$ is locally asymptotically stable and the jiggers persistent equilibrium $\mathscr{E}_2^*$ is unstable. In fact, the numerical example in Figure 2a show that there are only two equilibria, stable jiggers persistent equilibrium when $R_0 > 1$ and jiggers free equilibrium which is stable when $R_0 < 1$. In Figure 2b there is exitence of both jiggers free equilibrium (stable) and jiggers persistent equilibrium (unstable) when $R_0 < 1$ and stable jiggers persistent equilibrium when $R_0 > 1$.*

We present the bifurcation diagrams in Figure (2) for different values of γ. The model has a forward transcritical bifurcation whereas for large values of γ a backward bifurcation is clearly evident.



(A) (B)

FIGURE 2. Forward bifurcation in (a) for $\gamma = 0.145$ and backward bifurcation in (b) for $\gamma = 0.24$. The other parameters were fixed at the following values; $\Pi = 5, \tau = 0.0021, \sigma = 0.001, m = 0.45, \rho = 0.0013, \mu = 0.0012, \delta = 0.25, N_i = 10, \varepsilon = 0.023, \nu_f = 0.89, \nu_e = 0.3, \nu_l = 0.045, \phi = 0.016, \omega = 0.3, \eta = 0.01$.

3. MAIN RESULTS

NUMERICAL SIMULATIONS

Before carrying out the numerical simulations, we focus on the sensitivity analysis of the model parameters to the model outputs.

3.1. Sensitivity analysis. In order to identify critical inputs of our jiggers infestation epidemic model and gain insights on how input uncertainty influences model outcome, sensitivity analysis is conducted [17]. To accompolish this, Latin hypercube sampling (LHS) technique is employed. This technique provides a comprehensive method of assessing model sensitivity to parameters over multidimensional parameter space. One of the merits of LHS technique is that it requires fewer samples of parameters than simple random sampling to achieve the same accuracy [17]. The technique works in combination with the partial rank correlation coefficient (PRCC) which estimates the sign and strength of the relationship that exists between each model parameter and any specified output variable [9, 18]. The PRCC values are bounded between 1 and -1, with a PRCC value close to 1 or -1 indicating very strong positive (or negative) correlation. The results are presented in Figure 3.

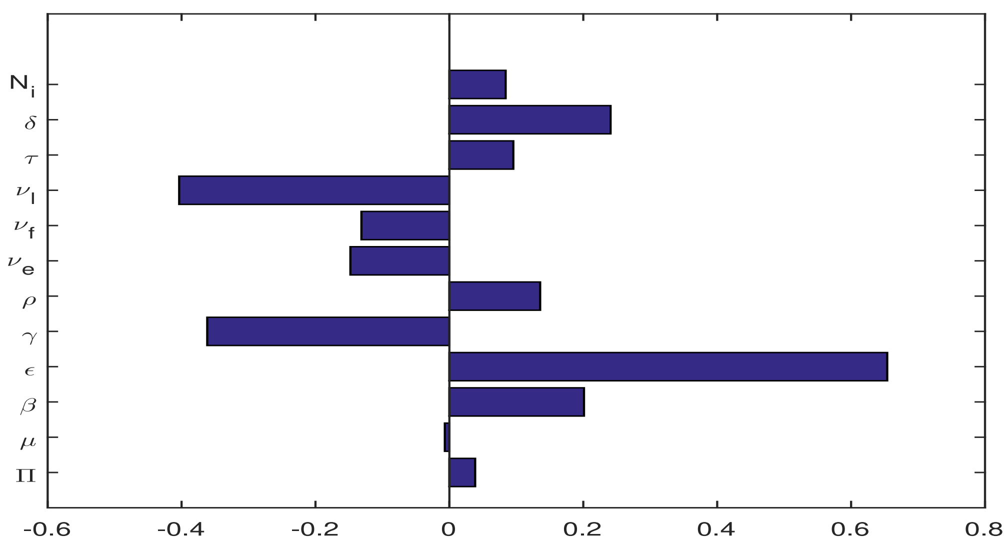


FIGURE 3. Sensitivity analysis of reproduction number R_0. Distribution of R_0 values obtained from Latin Hypercube Sampling for parameters as in Section 2.2, with 10000 simulations. The parameter values and ranges used are presented in Table 1.

The results shown in Figure 3 indicate that the dominant parameter contributing to the most variability of R_0 is ε, the proportion of larvae that develop into adult female fleas involved jiggers development within an individual. The effective infestation contact rate β as well as the rate at which the larvae develop into adult fleas, δ, are also influential parameters in the variability of R_0. The parameters ε, β and δ have the potential to make jiggers infestation increased. On the other hand, the rate at which infested humans recover after treatment, γ and the natural death rate, ν_l parameters have the potential of controlling the infestation when increased. The results suggest that public health efforts should focus primarily on increasing ν_l and γ. This can be achieved, for example, through a permanent program of screening and spraying of affected areas and treating the infested individuals. Finally, the control of the parameters ε, β and δ can also be significant in reducing the transmission. For this reason, efforts to prohibit the development of larvae into adult female fleas responsible for jiggers infestation and health education are an essential component to control these parameters. Although in real life situations these strategies are often difficult to implement, their benefits can be considerable.

3.2. Simulation results. In this section, numerical simulations of the model system (1) are presented. The findings in Figure 4a show that increasing γ, the rate at which infested humans recover after treatment leads to the reduction in the number of infested individuals. Similarly, increasing σ that is the efficacy of media campaigns, the number of infested individuals reduces as shown in Figure 4b. Thus, for eradication of jiggers infestation, the treatment and the media campaigns should be enhanced. High value of the said parameters imply enough treatment for a large population of infested individuals, thus not favouring a situation where there will always be infested individuals within the community.

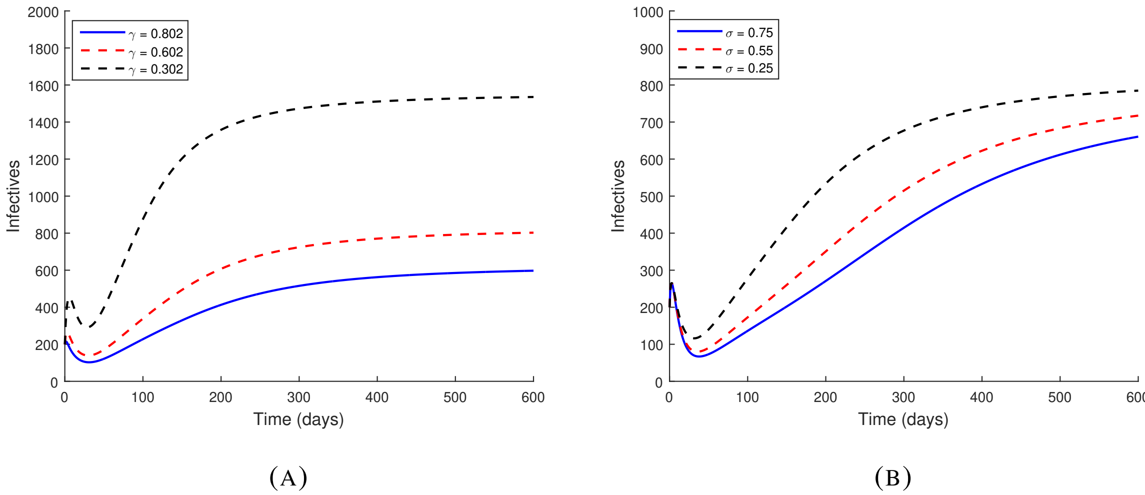
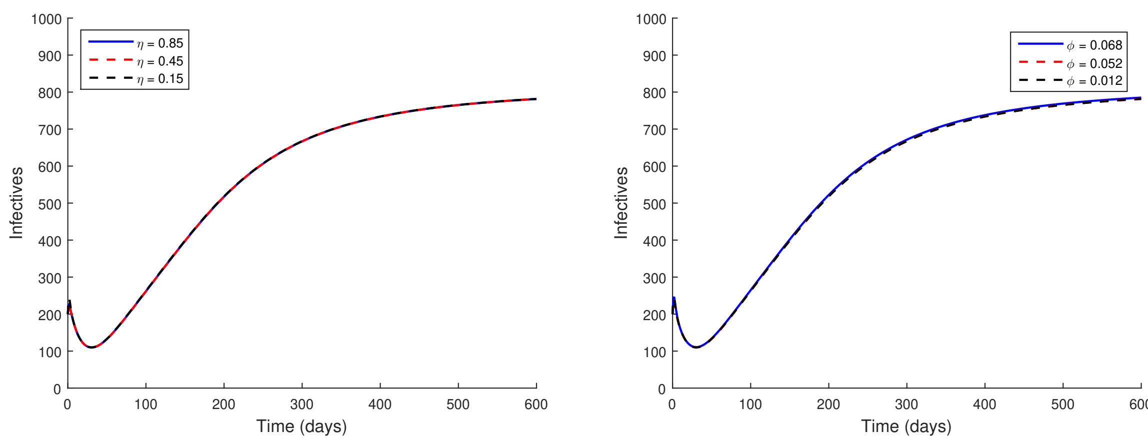


FIGURE 4. Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 1. Figure 4a is produced as a result of variation in rate at which infested humans recover after treatment (γ) while Figure 4b is produced as a result of variation in efficacy of media campaigns (σ).

It is seen in Figures 5a and 5b that η, dissemination of awareness rate and ϕ, depletion rate of awareness programs have very negligible effect on the number of infested individuals. The results are indicative of the fact that provided the efficacy of the media campaigns and treatment remain high, the community will always be cognisant of the need to fight against jiggers infestation. These results are attributed to depletion of the infested individuals which reduce the likelihood of getting new infections following treatment and high media campaigns efficacy.



(A) (B)

FIGURE 5. Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 1. Figure 5a is produced as a result of variation in dissemination of awareness (η) while Figure 5b is produced as a result of variation in depletion rate of awareness programs (ϕ).

4. CONCLUSION

In this study we formulated a jiggers infestation epidemic model with media campaigns. The basic reproduction R_0, which plays a key role in the prediction of disease persistence or extinction is computed. In the presented jiggers infestation epidemic model, the analytical results are indicative of the fact that indeed R_0 is a threshold. The jiggers infestation model exhibits the phenomenon of backward bifurcation where a jiggers-free equilibrium and two nontrivial equilibria coexist even if the basic reproduction number, R_0, is below one. The appearance of backward bifurcation indicates that it is not sufficient to decrease the basic reproduction number below unity for the containment of jiggers infestation. Therefore, to effectively control jiggers infestation, one has to reduce the basic reproduction number R_0 below another threshold referred to as the critical value of the basic reproduction number R_0^c. Lyapunov direct method has been used to show the global stability of the jiggers free equilibrium in the absence of backward bifurcation. Moreover, sensitivity analysis using Latin Hypercube Sampling (LHS) results indicate that the effective infestation contact rate and the rate at which the larvae develop into adult fleas are parameters that contribute to persistence of jiggers infestation epidemic in the community. The simulation results are indicative of the fact that for eradication of jiggers infestation, the treatment and the media campaigns should be enhanced. The results obtained in this paper have important implications in the management of jiggers.

As much as a lot of research has been done on jiggers infestation, majority of it has been based on social perspective and very little on mathematical modelling. From the social perspective it was not possible to get the exact parameters stimulating jiggers infestation which was made possible through our work. From mathematical modelling side, none had been done on effects of media campaigns and more so incorporating sensitivity analysis. This paper therefore provides this information and a clear guidance to the policy makers on the exact action to take before the jigger menace spreads out.

However the model formulated in this paper is not without short comings. In fact lack of sufficient data on how many humans recover out of media campaigns limited the numerical analysis and interpretation. We dependent mostly on literature data and estimated values. If this was available then the model could be fit to data and therefore provide more exact analysis.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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