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ANALYTIC NUMERIC SOLUTION OF CORONAVIRUS (COVID-19) PANDEMIC MODEL IN FRACTIONAL - ORDER

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Abstract. In this paper, we consider the coronavirus (COVID-19) pandemic model. The fractional ordinary differential equations were defined in the sense of the Caputo derivative. Adams-type predictor-corrector method with $\alpha \in [0,1]$ is employed to compute an approximation to the solution of the model of fractional order. The obtained results are compared with the results by Atangana Baleanu derivative method. Basic reproduction number, R_0, affects the model behaviour. We used R_0 to establish the stability and existence conditions at the equilibrium points. The results obtained show that the method is highly applicable and also an efficient approach for solving fractional ordinary differential equations of such order.

Keywords: coronavirus; Caputo derivative; fractional-order; stability analysis.

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1. Introduction

The study of epidemiology, which involves the transmission of diseases within a population has recently gained more attention among researchers in various fields. The outbreak of coronavirus (also known as COVID-19) in 2019, which is still spreading globally, Ebola outbreak in 2014 and the SARS outbreak in 2003 had even led to more advanced research in this area. Many models for various infectious diseases have been mathematically developed in order to study the dynamical process of pandemic. The models are able to integrate realistic features of disease spreading. Historically, a simple deterministic model was studied by Kermack and McKendrick in 1927, which is referred to as susceptible-infected-recovered (SIR) model. This model divides the populations into three states, which are susceptible, S, infected, I, and recovered (removed), R, respectively. The susceptible individuals are assumed to become infected with a rate of transmission which is proportional to the fraction of infected individuals in the overall population (fully mixed approximation) and infected individuals recover at a constant rate.

Just of recent, intensive research on mathematical epidemiology is being carried out to develop more realistic pandemic models. Example of such are [1, 2, 3, 4] , which are concern on modeling the dynamic of novel corona virus. The corona virus was first detected in Wuhan, the capital city of Hubei province in the People's Republic of China on December, 2019. However, there are yet to be effective vaccines for the treatment of this novel virus. It is sad to not that the virus has more than one hundred and eighty-two thousand death toll, over two million confirmed cases and more than seven hundred thousand recovered rate, globally as at $22th$ April, 2020. The Covid-19 has continued to spread to other parts of the world, including the African continent with over twenty-four thousand confirmed cases, over six thousand recoveries and over one thousand death. It is noted that the appearance of the virus is between 2 days to 2 weeks, which has given the world a body shake-up. United States of America, Spain, Italy, German, China. France, Iran and United Kingdom are among the countries reporting with high cases of the virus outbreak in the world.

However, advises have been given to all and sundry to frequently sanitize their hands and avoid contact with face and high-contact surface to prevent the spread of COVID−19 while emphasizing that COVID−19 is not a death sentence if timely and adequate measures are taken.

In this research direction, physicists and mathematicians are among the academic players that contributes to the knowledge of mathematical epidemiology. These group of scholars have continuously worked on modeling various pandemic outbreak. A major part of these epidemiological research is focus on the rate-based differential-equation models, i.e. compartmental models on completely mixing population.

In recent years, the research in infectious modeling had been shifted to fractional differential equations (FDEs) model from ordinary differential equations (ODEs). Various fractional epidemic models have been studied [6, 7, 8, 23, 11, 10].

Fractional calculus, which is an important branch of mathematics was born in 1695. It was investigated that when describing some unconventional phenomena in engineering applications and physical science, including biological system [13, 21, 21], finance system [14], just to mention a few, fractional calculus has the superiority accuracy than the integer-order . On the other hand, the well-known Caputo fractional derivative (defined by Michele Caputo in 1967) and famous Riemann-Liouville fractional integral are the main subjects of many studies in fractional calculus [15, 16, 17]. The research work in this area is under a huge development which includes; the study of theory of fractional calculus [18], [19], efficient numerical schemes [20, 8, 9]and application on physical problem [12]. In addition to that, the fractional derivative is commonly applied to increase the stability region of various systems, which is more suitable than integer order [21, 25].

[2] introduced a conceptual model for Covid-19 in Wuhan with the consideration of governmental actions and reactions of various individual. The structure of the model is concise, and effectively captures the course of the virus outbreak. The authors further gave brief explanations on sympathetic trends of the outbreak.

More so, [28] presented a fractional derivative epidemic modelling, by deriving an infectivity SIR model of fractional-order from a stochastic process incorporating a time-since-infection dependence on the infectivity of the individuals. They incorporated a fractional derivative into the SIR model recovery rate in order to incorporate the effects of chronic infection, by concluding that based on the system parameters, the fractional model allows both endemic, long time equilibrium state, and disease-free, but are unable to identify any disease process using the required power law properties of the infectivity which gives rise to the fractional order in the epidemic model.

2. Model Description

More recently, [22] published a paper describing the modeling and dynamics of the novel Covid-19 (2019-nCoV). The model gave a short explanation among categories of interactions. First interaction is between the bats and the unknown hosts, which may be the wild animal. Then, the second interaction is between the people and their interaction with the market's seafood, which is the infections reservoir. The major source of infection as described in the model is the seafood when these agents (bats and the unknown hosts) releases the infection on the items (seafood, such as crayfish, fish just to mentioned a few). Thus, when the people buy the infected items from the market, there is tendency for the people to be infected by the virus either as symptomatically or asymptomatically. The model was reduced with the idea that the market's seafood has the large ability source to infect individual that come to transact in the market. The author reduced the model with ignoring the interaction between the bats and the hosts as follows; The model is given in the system (1)

$$\begin{aligned}
\frac{dS_p}{dt} &= \Pi_p - \mu_p S_p - \frac{\eta_p S_p\left(\psi A_p + I_p\right)}{N_p} - \eta_w S_p M,\\
\frac{dE_p}{dt} &= \frac{\eta_p S_p\left(\psi A_p + I_p\right)}{N_p} + \eta_w S_p M - \left(1-\theta_p\right) w_p E_p - \theta_p \rho_p E_p - \mu_p E_p,\\
\frac{dI_p}{dt} &= \left(1-\theta_p\right) w_p E_p - \left(\tau_p + \mu_p\right) I_p,\\
\frac{dA_p}{dt} &= \theta_p \rho_p E_p - \left(\tau_{ap} + \mu_p\right) A_p,\\
\frac{dR_p}{dt} &= \tau_{ap} A_p + \tau_p I_p - \mu_p R_p,\\
\frac{dM}{dt} &= -\pi M + \varpi_p A_p + Q_p I_p.
\end{aligned} \tag{1}$$

With $S(0) = S_0$, $E(0) = E_0$, $I(0) = I_0$, $A(0) = A_0$, $R(0) = R_0$, $M(0) = M_0$.

The parameters interpretations are as follows: N is the total population of the people, S_p is the susceptible, E_p is the exposed, I_p is the infected (symptomatic), A_p is the asymptotically infected, and R_p is the recovered/removed people. M represents the market. Π_p is the birth rate, μ_p is the natural death rate and η_p is given as the transmission's disease coefficient between infected and susceptible individuals. η_p represents recruitment coefficient. ψ represents the transmissible multiple of infected (symptomatic) and asymptotically infected. Given $\psi \in [0,1]$, which means when $\psi = 0$, no transmissible multiple will exists and thus vanish, and if $\psi = 1$, it shows that similar procedure will take place like symptomatic infection. θ_p represents the proportion of asymptomatic infection. ϖ_p and p_p are the transmission rates of infected that have completed the incubation period, respectively. Further more θ_p and τ_p are the recovery or removal rate of both infected (symptomatic), asymptotically infected, respectively. Q_p is the contribution of the virus to market by infected (symptomatic), while ϖ_p is the contribution of the virus to market by asymptotically infected and π is the removing rate of virus from market

We have the following definitions of fractional integral-order and the Caputo fraction derivative-order:

Definition 2.1. *The fractional integral with fractional order $\beta \in \Re^t$ of function $x(t), t > 0$ is defined as:*

$$I^{\beta} x(t) = \int_0^t \frac{(t-s)^{\beta-1} x(s)}{\Gamma(\beta)} \mathrm{d}s \tag{2}$$

where $t = t_0$ is the initial time and $\Gamma(\beta)$ is the Euler's gamma function.

Definition 2.2. *The Caputo fractional derivative with order $\alpha \in n-1, n$ of function $x(t), t > 0$ is defined as:*

$$cD_t{}^{\alpha} x(t) = I^{n-\alpha} D^n x(t), D_t = \frac{d}{dt}. \tag{3}$$

3. Stability Analysis of Fractional Order System

The stability analysis (local), which is established on stability theory of fractional-order system is examined in this section. It is noted that the equilibrium point of fractional order is the same with the integer counterpart but their condition is quite different from each other. For integer order, the equilibrium point is not stable when the eigenvalue is non-negative, while that of the fractional order can still be stable even when the eigenvalue is non-negative.

Theorem 3.1. *[Stability Analysis] The necessary and sufficient condition for Caputo fractional derivative to be locally asymptotically stable, with system* (5) *where* $\alpha \in (0,1]$ *is if and only if* λ_i *of the Jacobian,* $\frac{\partial}{\partial y} f(t,y)$, *computed at the equilibrium points is satisfied by* $|\arg \lambda_i| > \frac{\alpha\pi}{2}$, $i = 1,2,3,4,5,6$.

Proof. Consider the following commensurate fraction-order system:

$$cD_t{}^{\alpha} y_i(t) = f(t, y_i(t)), y_i(t_o) = y_0 \tag{4}$$

where $cD_t{}^{\alpha}$ is the Caputo fractional derivative with order $\alpha \in (0,1]$.

In order to evaluate the equilibrium points, let put

$$cD_t{}^{\alpha} y_i(t) = 0 \Rightarrow f_i(f_1{}^{eqn}, f_2{}^{eqn}, f_3{}^{eqn}, f_4{}^{eqn}, f_5{}^{eqn}, f_6{}^{eqn}) = 0. \tag{5}$$

for which we can get the equilibrium points $f_1{}^{eqn}, f_2{}^{eqn}, f_3{}^{eqn}, f_4{}^{eqn}, f_5{}^{eqn}, f_6{}^{eqn}$.

Now to evaluate the asymptotic stability, let consider the system $cD_t{}^{\alpha} f(x) = f(x,y)$ in the sense of Caputo and to find the asymptotic stability, let $y_i(t) = y_i{}^{eqn} \varepsilon_i(t)$. The equilibrium point $(f_1{}^{eqn}, f_2{}^{eqn}, f_3{}^{eqn}, f_4{}^{eqn}, f_5{}^{eqn}, f_6{}^{eqn})$ is locally asymptotically stable if the eigenvalues of the Jacobian

$$\begin{bmatrix}
\frac{\partial(f_1)}{\partial(y_1)} & \frac{\partial(f_1)}{\partial(y_2)} & \frac{\partial(f_1)}{\partial(y_3)} & \frac{\partial(f_1)}{\partial(y_4)} & \frac{\partial(f_1)}{\partial(y_5)} & \frac{\partial(f_1)}{\partial(y_6)} \\
\frac{\partial(f_2)}{\partial(y_1)} & \frac{\partial(f_2)}{\partial(y_2)} & \frac{\partial(f_2)}{\partial(y_3)} & \frac{\partial(f_2)}{\partial(y_4)} & \frac{\partial(f_2)}{\partial(y_5)} & \frac{\partial(f_2)}{\partial(y_6)} \\
\frac{\partial(f_3)}{\partial(y_1)} & \frac{\partial(f_3)}{\partial(y_2)} & \frac{\partial(f_3)}{\partial(y_3)} & \frac{\partial(f_3)}{\partial(y_4)} & \frac{\partial(f_3)}{\partial(y_5)} & \frac{\partial(f_3)}{\partial(y_6)} \\
\frac{\partial(f_4)}{\partial(y_1)} & \frac{\partial(f_4)}{\partial(y_2)} & \frac{\partial(f_4)}{\partial(y_3)} & \frac{\partial(f_4)}{\partial(y_4)} & \frac{\partial(f_4)}{\partial(y_5)} & \frac{\partial(f_4)}{\partial(y_6)} \\
\frac{\partial(f_5)}{\partial(y_1)} & \frac{\partial(f_5)}{\partial(y_1)} & \frac{\partial(f_5)}{\partial(y_1)} & \frac{\partial(f_5)}{\partial(y_1)} & \frac{\partial(f_5)}{\partial(y_1)} & \frac{\partial(f_5)}{\partial(y_1)} \\
\frac{\partial(f_6)}{\partial(y_1)} & \frac{\partial(f_6)}{\partial(y_2)} & \frac{\partial(f_6)}{\partial(y_3)} & \frac{\partial(f_6)}{\partial(y_4)} & \frac{\partial(f_6)}{\partial(y_5)} & \frac{\partial(f_6)}{\partial(y_6)}
\end{bmatrix}$$

evaluated at the equilibrium point is satisfied by $\left|\arg\lambda_{1,2,3,4,5,6}\right| > \frac{\alpha\pi}{2}$ [25, 26, 29, 30]. □

The above integer-order derivatives of the system (1) is replaced by fractional derivatives of order $0 < \alpha \leq 1$ in the sense of Caputo as follows:

$$
\begin{aligned}
cD_t{}^{\alpha}S_p(t) &= \Pi_p - \mu_p S_p - \frac{\eta_p S_p(\psi A_p + I_p)}{N_p} - \eta_w S_p M,\\
cD_t{}^{\alpha}E_p(t) &= \frac{\eta_p S_p(\psi A_p + I_p)}{N_p} + \eta_w S_p M - (1-\theta_p)\, w_p E_p - \theta_p\rho_p E_p - \mu_p E_p,\\
cD_t{}^{\alpha}I_p(t) &= (1-\theta_p)\, w_p E_p - (\tau_p + \mu_p) I_p,\\
cD_t{}^{\alpha}A_p(t) &= \theta_p\rho_p E_p - (\tau_{ap} + \mu_p) A_p,\\
cD_t{}^{\alpha}R_p(t) &= \tau_{ap} A_p + \tau_p I_p - \mu_p R_p,\\
cD_t{}^{\alpha}M(t) &= -\pi M + \varpi_p A_p + Q_p I_p.
\end{aligned}
\tag{6}
$$

where $0 < \alpha \leq 1$. All the parameters are positive constants.

We use R_0 to establish the stability and existence conditions of both disease-free and endemic for the equilibrium points, which is the number of people that one sick person will infect (on average).

There are two equilibria in the system (6) when equating them to zero, namely, the existence of disease-free equilibrium, E^0 point, and the endemic equilibrium, E^e points.

3.1. Disease-free equilibrium, E^0. At this subsection, we demonstrate the asymptotic stability of the disease-free, E^0. when $R_0 < 1$. The basic reproduction number, R_0, of the system (6) as defined by [22] is

$$
R_0 = \frac{\theta_p\rho_p(\mu_p+\tau_p)(\pi\psi\mu_p\eta_w + w_p\Pi_p\eta_p) + (1-\theta_p)\, w_p(\tau_{ap}+\mu_p)(\Pi_p Q_p\eta_w + \pi\eta_p\mu_p)}{\Pi_p(\mu_p+\tau_p)(\tau_{ap}+\mu_p)(\theta_p(\rho_p - w_p)\mu_p + \mu_p + W_p)},
$$

The disease-free equilibrium is:

$$
E^0 = (S^* = \frac{\Pi_p}{\mu_p}, E^* = 0, I^* = 0, A^* = 0, R^* = 0, M^* = 0) \tag{7}
$$

System (6) at E^0 is asymptotically stable if after obtaining the Jacobian matrix and it's eigenvalues are satisfied by using

$$|\arg \lambda_i| > \frac{\alpha\pi}{2}, \tag{8}$$

which was described in Theorem (3.1). This ensures that the E^0 is locally asymptotically stable if $R_0 < 1$, or otherwise unstable when $R_0 > 1$.

However, condition in system (8) is satisfied for the disease-free equilibrium, E_{E^0}, as given in the Theorem (3.1)

Theorem[section]

Theorem 3.2. *[disease-free equilibrium] A sufficient condition for the system* (6) *to be locally asymptotically stable at* E^0 *is if and only if*

$$R_0 = \frac{\theta_p\rho_p(\mu_p+\tau_p)(\Pi\psi\mu_p\eta_w + w_p\Pi_p\eta_p) + (1-\theta_p)w_p(\tau_{ap}+\mu_p)(\Pi_pQ_p\eta_w + \pi\eta_p\mu_p)}{\Pi_p(\mu_p+\tau_p)(\tau_{ap}+\mu_p)(\theta_p(\rho_p-w_p)\mu_p+\mu_p+W_p)} < 1. \tag{9}$$

Proof. To prove Theorem 3.2, it would be sufficient to show that all eigenvalues of Jacobian of (6) at E^0 have a negative real part. Hence, we have

$$\begin{bmatrix} -\mu_p & 0 & -\frac{\Pi_p\eta_p}{\mu_pNu_p} & 0 & 0 & -\frac{\eta_w\Pi_p}{\mu_p} \\ 0 & -(1-\theta_p)W_p-\theta_p\rho_p-\mu_p & \frac{\Pi_p\eta_p}{\mu_pNu_p} & 0 & 0 & \frac{\eta_w\Pi_p}{\mu_p} \\ 0 & (1-\theta_p)W_p & -\tau_p-\mu_p & 0 & 0 & 0 \\ 0 & \theta_p\rho_p & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_p & 0 & -\mu_p & 0 \\ 0 & 0 & Q_p & 0 & 0 & -u \end{bmatrix}$$

Then for

$$S_{eqn}, E_{eqn}, I_{eqn}, A_{eqn}, R_{eqn}, M_{eqn} = (S^* = \frac{\Pi_p}{\mu_p}, E^* = 0, I^* = 0, A^* = 0, R^* = 0, M^* = 0)$$

we find that

$$A = \begin{bmatrix} -\mu_p & 0 & -\frac{\eta_p \Pi_p}{\mu_p N u_p} & 0 & 0 & -\frac{\eta_w \Pi_p}{\mu_p} \\ 0 & -(1-\theta_p) W_p - \theta_p \rho_p - \mu_p & \frac{\eta_p \Pi_p}{\mu_p N u_p} & 0 & 0 & \frac{\eta_w \Pi_p}{\mu_p} \\ 0 & (1-\theta_p) W_p & -\tau_p - \mu_p & 0 & 0 & 0 \\ 0 & \theta_p \rho_p & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_p & 0 & -\mu_p & 0 \\ 0 & 0 & Q_p & 0 & 0 & -u \end{bmatrix},$$

and its eigenvalues has

$$[-\mu_p], \qquad [-\mu_p]. \tag{10}$$

It follows that Equation (10) is less than zero, which implies that $R_0 < 0$ and satisfy the condition in Equation (8). Hence, the eigenvalues of the system (6) is always negative (due to all the parameters are positive). So (6) is locally asymptotically stable. Then the disease-free equilibrium, E^0, is locally asymptotically stable. Conversely, it becomes unstable when

$$R_0 = \frac{\theta_p \rho_p (\mu_p + \tau_p)(\pi \psi \mu_p \eta_w + w_p \Pi_p \eta_p) + (1-\theta_p) w_p (\tau_{ap} + \mu_p)(\Pi_p Q_p \eta_w + \pi \eta_p \mu_p)}{\Pi_p (\mu_p + \tau_p)(\tau_{ap} + \mu_p)(\theta_p (\rho_p - w_p) \mu_p + \mu_p + W_p)} > 1. \tag{11}$$

□

3.2. Endemic equilibrium, E^e**.** According to the system (6), we obtain the endemic points,E^e, by solving the quadratic equation , $\lambda^6 + A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E$. We define $E^e = (S_p^*, E_p^*, I_p^*, A_p^*, R_p^*, M^*)$ as the endemic point of system (6). We give more results on the endemic equilibrium in the next section.

4. Experimental Simulation Calculation

The Adams-type predictor-corrector method is applied for all simulations. It has been introduced in [24], and further investigated in [23, 27, 25], which is an implicit numerical method. The Adams-type predictor-corrector scheme gives the error-free means of solving a problem with a sensible and logical choice of the time step [23]. This scheme can also be applied to different numerical works like [31] and [32], just to mention few

To illustrate the stability of the fractional epidemic model as in Equation (6), we choose the following parameters $\Pi_p = 107644.22451$, $\mu_p = 0.01302252898$, $\eta_p = 0.05$, $x = 0.02$, $\eta_w = 0.000001231$, $\theta_p = 0.1243, W_p = 0.00047876, \rho_p = 0.005, \tau_p = 0.09871, \tau_{ap} = 0.854302, Q_p = 0.000398, \varpi_p = 0.001, u = 0.01$, and $p_p = 8266000$ with the following initial values $(S_p, E_p, I_p, A_p, R_p, M) = (8065518, 200000, 282, 200, 0, 50000)$ [22]. By direct solving, and using Maple 18 software, it can be shown that the equilibrium points of the fractional-order model (6) are,

$$E_1(S_{p_1}, E_{p_1}, I_{p_1}, A_{p_1}, R_{p_1}, M_1) = (0.265999997 10^6, 0, 0, 0, 0, 0),$$

and

$$E_2(S_{p_2}, E_{p_2}, I_{p_2}, A_{p_2}, R_{p_2}, M_1) = (5.651243575 10^{7,} -4.467596802 10^7, -1.676361008 10^5,$$

$$-32013.52343, -3.370818116 10^6, -8204.596258).$$

Hence, the Jacobian matrix for the corresponding equilibrium point (R_1, I_1) is given as

$$J = \begin{bmatrix} H^* & 0 & -6.049^{-9} S & 0 & 0 & +1.231^{-6} S \\ 1.210^{-10} A + 6.049^{-9} I - 1.231^{-6} M & +1.406^{-2} & 6.049^{-10} S & 0 & 0 & 1.231^{6} S \\ 0 & 1.925^{-4} & +1.117^{-1} & 0 & 0 & 0 \\ 0 & 6.215^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.871^{-2} & 0 & +1.302^{-2} & 0 \\ 0 & 0 & 3.98^{-4} & 0 & 0 & -0.01 \end{bmatrix}$$

where $H^* = 1.303^{-2} + 1.210^{-10} A + 6.049^{-9} I + 1.231^{-6} M$

and its eigenvalues for disease-free, E^0 are

$$\lambda_i = \begin{bmatrix} -0.0130225289800000 + 0.0I \\ -0.111776330077532 + 0.0I \\ 1.202177465 \times 10^{-17} + 0.0I \\ -0.0165618396943470 + 0.0I \\ -0.0130225289800000 + 0.0I \\ -0.00745763833812104 + 0.0I \end{bmatrix}$$

that of the endemic, E^e are

$$\lambda_i = \begin{bmatrix} -0.112003453058485 + 0.0I \\ 0.00536130750693023 + 0.0I \\ 5.981499334 \times 10^{-17} + 0.0I \\ -0.0183894177579582 + 0.0I \\ -0.0130225289800000 + 0.0I \\ -0.0126690330704869 + 0.0I \end{bmatrix}$$

while the characteristic equation of the fractional pandemic model as in Equation (6) is:

$$P(\lambda) = \lambda^6 + 0.2233925961\,\lambda^5 + 0.01567525179\,\lambda^4 + 0.0003930329343\,\lambda^3$$
$$+0.000004072131855\,\lambda^2 + 0.00000001494421577\,\lambda$$

Therefore, the argument $\left|\arg \lambda_{1,2,3,4,5,6}\right|$ of matrix J at $\alpha = 0.8$ fall with the range of values, 3.141592654. The values of $|\arg \lambda_1|$ of the $(S_p, E_p, I_p, A_p, R_p, M)$ points is said to be stable and the system gives the asymptotically stable because all the eigenvalues fulfill $|\arg \lambda_1| > \frac{\alpha\pi}{2}$. That is, $|\arg \lambda_1| = 3.141592654 > 2.000000000 = \frac{\alpha\pi}{2}$.

Also, by direct calculation, it is easy to show that

$$R_0 = \frac{\theta_p \rho_p (\mu_p + \tau_p)(\pi\psi\mu_p\eta_w + w_p\Pi_p\eta_p) + (1-\theta_p) w_p (\tau_{ap} + \mu_p)(\Pi_p Q_p \eta_w + \pi\eta_p\mu_p)}{\Pi_p (\mu_p + \tau_p)(\tau_{ap} + \mu_p)(\theta_p(\rho_p - w_p)\mu_p + \mu_p + W_p)}$$

= 0.0001737293975, which obtained result are in agreement and compatible with Theorem 3.2, (disease-free equilibrium), where $R_0 < 0.0001737293975$. This implies that the conditions for asymptotically stable and existence as discussed above are satisfied. It indicates that the spread of a disease depends on the contact rates with infected individual within the population. Basic reproduction number, R_0, which is the number of people that one sick person will infect (on average) also affects the model behaviour. We used R_0 to establish the existence and stability conditions at the equilibrium points. For pandemic processes, this parameter determines a threshold whenever $R_0 > 1$, a typical infective gives rise, on average, to more than one secondary infection, leading to pandemic. Otherwise, when $R_0 < 1$, then, infective typically give rise (on average) to less than one secondary infection, and in this case, the the prevalence of infection cannot increase.

Figure 1 displays the comparison result by Atangana Baleanu derivative with Figure $(a)-(b)$ in [22] with Caputo derivative in both integer and fractional forms. Caputo derivative technique is very applicable and also this is a better and efficient approach for the solving fractional ordinary differential equations of such order.

Figure 2 shows the dynamics of corona virus model with different values of individual in a particular time, t (day) in a stable endemic equilibrium when parameters are taken as $\Pi_p = 107644.22451$, $\mu_p = 0.01302252898$, $\eta_p = 0.05$, $x = 0.02$, $\eta_w = 0.000001231$, $\theta_p = 0.1243, W_p = 0.00047876, \rho_p = 0.005, \tau_p = 0.09871, \tau_{ap} = 0.854302, Q_p = 0.000398, \varpi_p = 0.001, u = 0.01$, and $p_p = 8266000$. With the initial [22], while Figure 4 displayed the reactions of subgroups population in system (6).

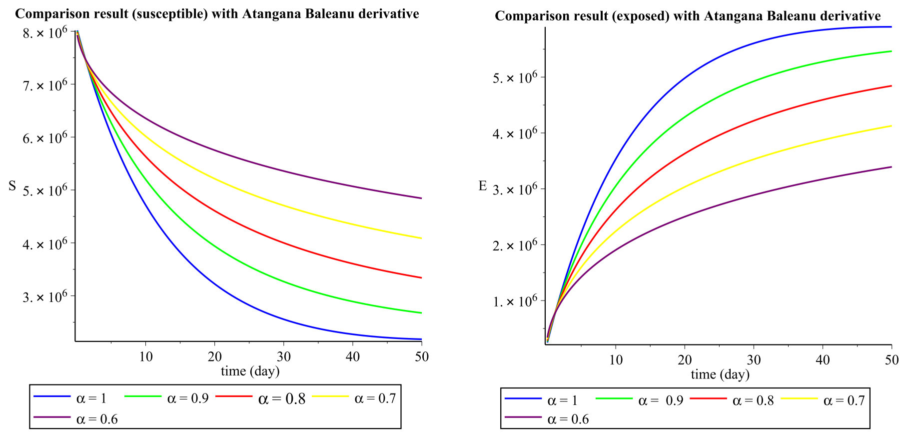

FIGURE 1. The comparison result by Atangana Baleanu derivative for Figure $(a)-(b)$ in [22] with Caputo derivative.

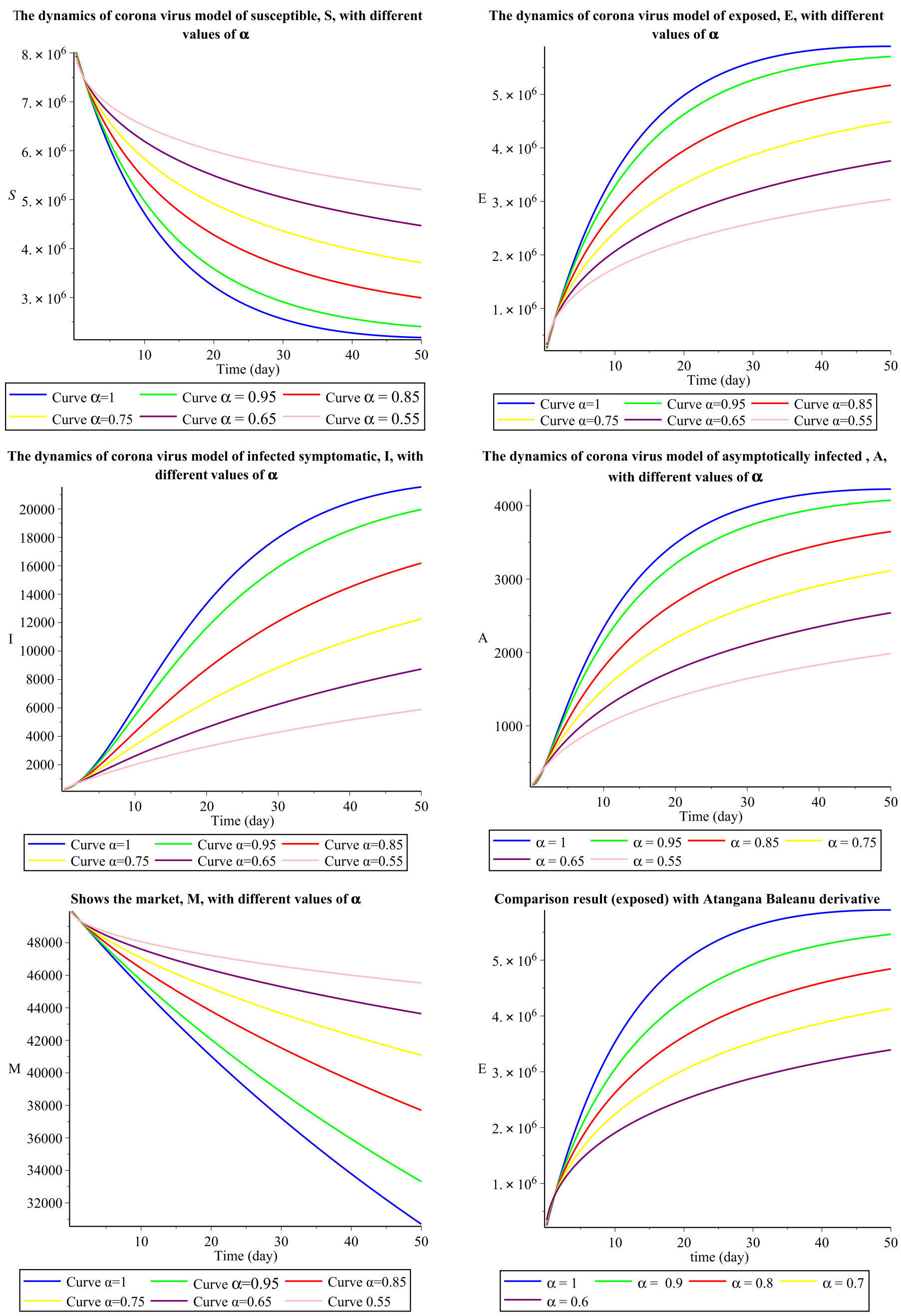

FIGURE 2. The total time (day) susceptible, exposed, infected symptomatic, asymptotically infected, recovered, and market population in system (6).

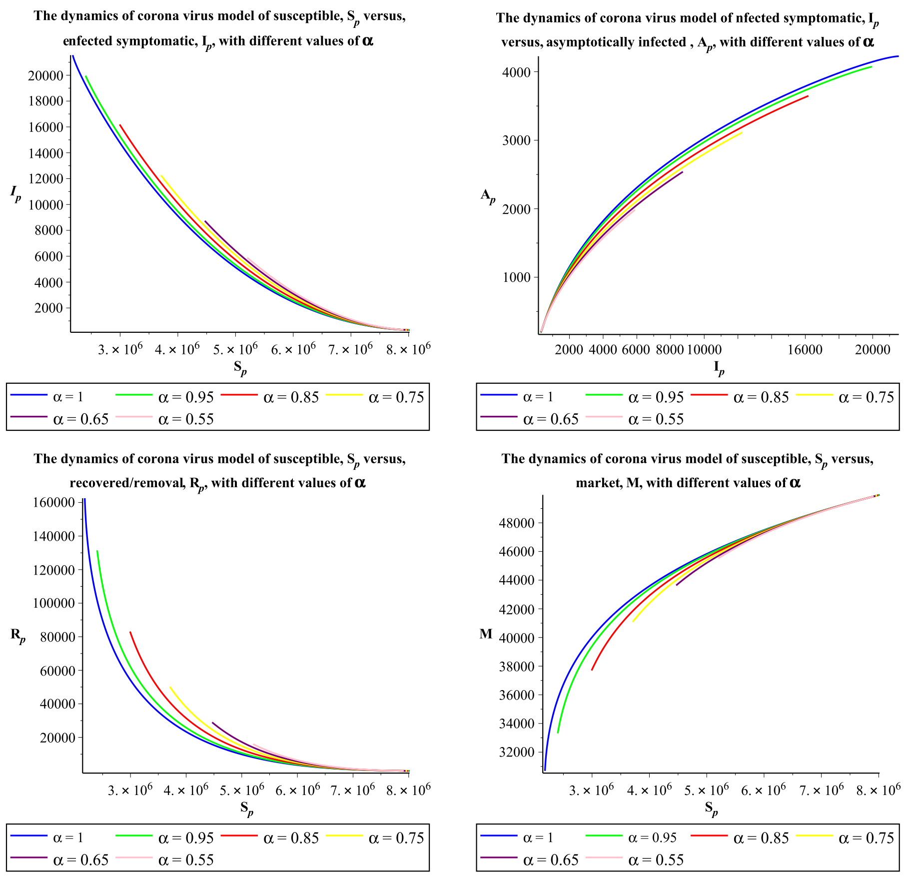

FIGURE 3. Shows the reactions of subgroups population in system (6).

5. Conclusion

In this paper, we have considered the coronavirus (COVID-19) pandemic model. The fractional ordinary differential equations were defined in the sense of the Caputo derivative. Adams-type predictor-corrector method with $\alpha \in (0,1]$ is applied to obtain an approximation to the solution of the model of fractional order. The results obtained by the proposed scheme are compared with that of Atangana Baleanu derivative method in integer both fractional order forms. Basic reproduction number, R_0, affects the model behaviour. We used R_0 to establish the existence and stability conditions at the equilibrium points. The proposed method is very applicable and can also be used as a alternate approach for the solving fractional ordinary differential equations of such order. We see the tool as the recommended tools for modeling in science and engineering.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] G. Bareikis, A. Mačiulis, Modeling the Dirichlet distribution using multiplicative functions, Nonlinear Anal., Model. Control. 25 (2) (2020), 282-300.

[2] Q. Lin, S. Zhao, D. Gao, et al. A conceptual model for the coronavirus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action, Int. J. Infect. Dis. 93 (2020), 211–216.

[3] Z. He, What further should be done to control COVID-19 outbreaks in addition to cases isolation and contact tracing measures? BMC Med. 18 (2020), 80.

[4] L. Wynants, B. Van Calster, G.S. Collins, et al. Prediction models for diagnosis and prognosis of covid-19: systematic review and critical appraisal, BMJ. 369 (2020), m1328.

[5] M.A. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, Alex. Eng. J. 59 (2020), 2379–2389.

[6] A. Atangana, J.F. Botha, A generalized groundwater flow equation using the concept of variable-order derivative, Bound. Value Probl. 2013 (2013), 53.

[7] E. Ahmed, A.S. Elgazzar, On fractional order differential equations model for nonlocal epidemics, Physica A., Stat. Mech. Appl. 379 (2007), 607–614.

[8] K. Al-Khaled, Numerical solution of time-fractional partial differential equations using Sumudu decomposition method. Rom. J. Phys. 60 (1-2) (2015), 99-110.

[9] P. Rahimkhani, Y. Ordokhani, E. Babolian, A new operational matrix based on Bernoulli wavelets for solving fractional delay differential equations, Numer. Algor. 74 (2017), 223–245.

[10] R. Jan, M.A. Khan, P. Kumam, P. Thounthong, Modeling the transmission of dengue infection through fractional derivatives, Chaos Soliton Fractals. 127 (2019), 189–216.

[11] C.N. Angstmann, A.M. Erickson, B.I. Henry, A.V. McGann, J.M. Murray, J.A. Nichols, Fractional Order Compartment Models, SIAM J. Appl. Math. 77 (2017), 430–446.

[12] M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel. Prog. Fract. Differ. Appl. 1 (2015), 73-85

[13] Q. Sun, M. Xiao, B. Tao, G. Jiang, J. Cao, F. Zhang, C. Huang, Hopf bifurcation analysis in a fractional-order survival red blood cells model and PD^{α} control, Adv. Differ. Equ. 2018 (2018), 10.

[14] J. Ma, W. Ren, Complexity and Hopf Bifurcation Analysis on a Kind of Fractional-Order IS-LM Macroeconomic System, Int. J. Bifurcation Chaos. 26 (2016), 1650181.

[15] I. Podlubny, Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications, Academic Press, San Diego, Vol. 198, 1998.

[16] D. Baleanu, J.A. Tenreiro Machado, C. Cattani, M.C. Baleanu, X.-J. Yang, Local Fractional Variational Iteration and Decomposition Methods for Wave Equation on Cantor Sets within Local Fractional Operators, Abstr. Appl. Anal. 2014 (2014), 535048.

[17] R. Herrmann, Fractional Calculus: An Introduction for Physicists, 2nd ed., World Scientific, 2014.

[18] K. Diethelm, N.J. Ford, Analysis of Fractional Differential Equations, J. Math. Anal. Appl. 265 (2002), 229–248.

[19] D. Baleanu, S. Rezapour, H. Mohammadi, Some existence results on nonlinear fractional differential equations, Proc. R. Soc. A. 371 (2013), 20120144.

[20] A.H. Bhrawy, T.M. Taha, J.A.T. Machado, A review of operational matrices and spectral techniques for fractional calculus, Nonlinear Dyn. 81 (2015), 1023–1052.

[21] D. Rostamy, E. Mottaghi, Stability analysis of a fractional-order epidemics model with multiple equilibriums, Adv. Differ. Equ. 2016 (2016), 170.

[22] M.A. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, Alex. Eng. J. 59 (2020), 2379–2389.

[23] I. Ameen, P. Novati, The solution of fractional order epidemic model by implicit Adams methods, Appl. Math. Model. 43 (2017), 78–84.

[24] K. Diethelm, N.J. Ford, A.D. Freed, A Predictor-Corrector Approach for the Numerical Solution of Fractional Differential Equations. Nonlinear Dyn. 29 (2002), 3–22.

[25] H.A.A. El-Saka, The fractional-order SIR and SIRS epidemic models with variable population size. Math. Sci. Lett. 2 (2013), 195-200.

[26] E. Ahmed, A.M.A. El-Sayed, H.A.A. El-Saka, Equilibrium points, stability and numerical solutions of fractional-order predator–prey and rabies models, J. Math. Anal. Appl. 325 (2007), 542–553.

[27] Owoyemi, Abiodun Ezekiel and Sulaiman, Ibrahim Mohammed and Mamat, Mustafa and Olowo, Sunday Ezekiel. Stability and Bifurcation Analysis in A Fractional-Order Epidemic Model with Sub-Optimal Immunity, Nonlinear Incidence and Saturated Recovery Rate. Int. J. Appl. Math. in press.

[28] C.N. Angstmann, B.I. Henry, A.V. McGann, Discretization of Fractional Differential Equations by a Piecewise Constant Approximation, ArXiv:1605.01815 [Physics]. (2016).

[29] D. Matignon, Stability results for fractional differential equations with applications to control processing, Proceedings Computational Engineering in Systems and Application multiconference, IMACS, IEEE-SMC, Lille, France, July 1966, pp. 963–968.

[30] J. Singh, D. Kumar, M.A. Qurashi, D. Baleanu, A new fractional model for giving up smoking dynamics, Adv. Differ. Equ. 2017 (2017), 88.

[31] Y.T. Toh, C. Phang, J.R. Loh, New predictor-corrector scheme for solving nonlinear differential equations with Caputo-Fabrizio operator, Math. Meth. Appl. Sci. 42 (2019), 175–185.

[32] Y.X. Ng, C. Phang, Computation of Stability Criterion for Fractional Shimizu–Morioka System Using Optimal Routh–Hurwitz Conditions, Computation. 7 (2019), 23.