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SUPPRESSING TACTICS BY OPTIMAL STERILE RELEASE PROGRAM AGAINST WILD MOSQUITOES

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Abstract. The sterile insect technique (SIT) offers a promising strategy for reducing wild mosquitoes in nature. But many factors cause incomplete sterility in a sterile release program. This article develops a mosquitoes model involving pulsed releasing sterile mosquitoes with gender structure and partial residual fecundity. Firstly, we devote to eliminate the wild mosquitoes by chronical period control. Next the impulsive control tactics with alterable pulse times and releasing amounts in finite time restraining wild mosquitoes are established. To acquire optimal strategy, the corresponding impulsive optimal problem is translated into continuous parameter selection problem by means of the transformation of time-scaling and time translation. Lastly, simulations based on gradient descent and genetic algorithm are given. Results indicate that pulsed releasing sterile mosquitoes greatly suppress the wild mosquitoes. Moreover, hybrid control is superior to the amount control and periodic impulsive control.

Keywords: SIT; optimal control; impulsive release; gradient descent; genetic algorithm.

2010 AMS Subject Classification: 93c27, 65K99.

1. Introduction

Sterile insect technique (SIT), in which radiation-sterilized males are released into the field to mate with wild females thereby preventing them from producing viable offspring [1, 2], as successfully restrained field insect species [3]. Recently, SIT has been used against mosquitoes and

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the corresponding mathematical model are formulated to simulate the evolution of mosquitoes [4, 5, 6, 7, 8]. In these models, the released sterile males are suppose to be 100% sterile. Practically, complete sterilization cannot be achieved for some objectiv reasons (technical problems, such as lower irradiation doses, environmental conditions or others) [9]. So it is essential to study partial sterility in the control process.

Inspired by a Aedes albopictus model with the partial residual fecundity [1, 10, 11], we firstly study the perennial and periodic impulsive releasing sterile males strategy. Then due to the high cost of breeding and experimenting with mosquitoes in captivity, we research the short-term and optimal impulsive releasing tactics. Denoting by $M(t)$ the wild male, by $F(t)$ female, and by $S(t)$ sterile male, P.A. Bliman in [1] proposed SIT models as following

$$(1) \qquad \begin{cases} \frac{dM(t)}{dt} = r\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_M M(t), \\ \frac{dF(t)}{dt} = (1-r)\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_F F(t), \\ \frac{dS(t)}{dt} = \Lambda - \mu_S S(t) \end{cases}$$

where μ_M, μ_F and μ_S represent mortalities of $M(t)$, $F(t)$ and $S(t)$, respectively. r is the sex ratio; ρ is the average number of eggs laid per female mosquito per day; Λ is the sustained release of sterile males; β is the competitive effect between the mosquitoes; ε is the residual fertility with $0 \le \varepsilon < 1$ and $\varepsilon = 0$ meaning the complete sterility; γ is the competitiveness index of sterile males with $0 < \gamma \le 1$ while $\gamma = 1$ meaning sterile males have the equally competitive with wild males.

Considering the basic number of offspring for both female and male populations, i.e. the average number of wild offspring that a mosquito produce per day, we obtain

$$(2) \qquad \mathcal{N}_F := \frac{(1-r)\rho}{\mu_F},\ \mathcal{N}_M := \frac{r\rho}{\mu_M}.$$

The outline of the paper is organized as follows. In Section 2, the wild mosquitoes control model with periodic pulse releasing sterile males are established and analysed; In Section 3, the wild mosquitoes control model with alterable pulse times and releasing amounts are built and the corresponding impulsive optimal problem are constituted and translated into continuous parameter selection problem; In Section 4, simulations and optimal control strategies based on gradient descent and genetic algorithm are given; Finally, a simple conclusion is list.

2. Model With Periodic Pulse Controls

In the above model (1), converting continuous releases into pulsed releases leads to a new interaction model for mosquito populations in terms of sex structure:

$$(3)\quad \begin{cases} \left.\begin{aligned} &\frac{dM(t)}{dt} = r\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_M M(t), \\ &\frac{dF(t)}{dt} = (1-r)\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_F F(t), \\ &\frac{dS(t)}{dt} = -\mu_S S(t), \end{aligned}\right\} t \neq i\tau,\ t\in(0,T] \\ S(i\tau^+) = S(i\tau)+\Lambda,\ t = i\tau,\ i = 1,2,\cdots \end{cases}$$

where $M(0) = M^0, F(0) = F^0, S(0) = S^0$. Suppose that τ and Λ denote the period and amount of releasing sterile male mosquitoes, respectively.

Let $X(t) = (M(t),F(t),S(t))^T$ is an arbitrary solution of (3). Then $X(t)$ is continuous between every two adjacent pulses and $X(i\tau^+) = \lim_{\varepsilon\to 0} X(i\tau+\varepsilon)(\varepsilon > 0)$ exists. Thus, for the smooth characteristics of the functions in the first three equations, the existence and uniqueness of the solution of (3) are guaranteed. Referring to [1, 7], we infer to the positivity and boundedness of the solution of system (3), as well as the existence and global asymptotic stability of bounded periodic solutions of the system (3).

Theorem 2.1 *The solution $X(t)$ of the system (3) with non-negative initial conditions is always non-negative, and there exists a constant $K > 0$ such that each component of $X(t)$ is less than K for $t \geq 0$.*

Theorem 2.2 *System (3) has a wild mosquito population-eradication periodic solution $(0,0,\widetilde{S}(t))$ being locally asymptotically stable where $\widetilde{S}(0) = \Lambda/(1-e^{-\mu_S\tau})$ and*

$$(4)\quad \widetilde{S}(t) = \widetilde{S}(0)e^{-\mu_S(t-i\tau)},\ t\in(i\tau,(i+1)\tau], i = 0,1,2.\cdots$$

Theorem 2.3 *Assume that*

$$\Lambda > \Lambda_{crit} \triangleq \frac{2(cosh(\mu_S\tau)-1)}{\tau\mu_S}\frac{(1-\varepsilon)\mathcal{N}_F}{\gamma(1-\varepsilon\mathcal{N}_F)e\beta}$$

then the wild mosquito population-eradication periodic solution $(0,0,\widetilde{S}(t))$ of system (3) is globally asymptotically stable.

Theorem 2.3 provides a control tactics to eliminate the wild mosquitoes by chronical and periodic releasing.

3. Optimal Mosquitoes Control by Impulsive Releasing Sterile Males

The stable wild mosquito population-eradication periodic solution means that the sterile males can suppress effectively wild mosquitos under impulsive controls. However, we have not considered the cost of various controls which is an important problem in practice. Hence, we devote this section to minimizing the amount of wild mosquitos at the terminal time with minimum control cost against fertile mosquitoes by impulsive releasing sterile mosquitoes. The optimal control problem can be solved by the time-scale conversion technology, that is, the time range is divided into several sub-intervals [12]. Then, the original switching time point is converted into the pre-fixed switching time point of the extended system. Therefore, the optimal problem is the same as the optimal parameter selection [13].

Assume that the amount of sterile males Λ_i is released at t_i in finite time $[0,T]$ for $i = 1,2,\cdots n-1$. Then we optimize the releasing time and amount of sterile insects which is called hybrid control strategy. Taking the releasing interval and the amount as controlled variables to be controlled, then a finite-time control system is proposed as follows

$$\left\{\begin{array}{l}\left.\begin{array}{l}\frac{dM(t)}{dt} = r\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_M M(t),\\ \frac{dF(t)}{dt} = (1-r)\rho\frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)}e^{-\beta(M(t)+F(t))} - \mu_F F(t),\\ \frac{dS(t)}{dt} = -\mu_S S(t),\end{array}\right\} t \neq t_i,\ t \in (0,T],\\ S(t_i{}^+) = S(t_i) + \Lambda_i,\ t = t_i,\ i = 1,\ldots,n-1,\end{array}\right. \tag{5}$$

with initial conditions

$$M(0) = M^0, F(0) = F^0, S(0) = S^0, \tag{6}$$

Here $S(t_i{}^+) = \lim_{\varepsilon\to 0^+} S(t_i+\varepsilon)$, and T is terminal time. Let $t_i - t_{i-1} = \tau_i$. Assume that releasing time t_i and amount of the sterile insect Λ_i satisfies:

$$\begin{array}{l}0 = t_0 \le t_1 \le \cdots \le t_{n-1} \le t_n = T,\\ c_i \le \tau_i \le d_i,\ i = 1,\ldots,n,\ \sum_{i=1}^{n}\tau_i = T,\end{array} \tag{7}$$

$$a_i \le \Lambda_i \le b_i, \tag{8}$$

where a_i, b_i, c_i and d_i are given and non-negative constants.

Denote vectors $\tau = (\tau_1, \cdots, \tau_n)^T$ and $\Lambda = (\Lambda_1, \cdots, \Lambda_{n-1})^T$. Our optimal control problem can now be expressed as:

(P1) *Given the system (5) with the initial condition (6), find the parameter vector pair* (τ, Λ) *meeting (7) and (8) such that the cost function*

$$J(\tau, \Lambda) = M(T) + F(T) + c\sum_{i=1}^{n-1} \Lambda_i \tag{9}$$

is minimized, where c is the cost of per sterile mosquito.

Using the transformation of time scaling and time translation [8], the problem (**P1**) can be transformed into an equivalent optimal parameter selection problem, which can then be solved based on the gradient. Define an indicator function of I:

$$\chi_I(s) = \begin{cases} 1, & \text{if } s \in I, \\ 0, & \text{otherwise }. \end{cases}$$

Then construct the transformation from $t \in [0, T]$ to $s \in [0, n]$ [8, 14] as follows

$$\frac{dt(s)}{ds} = \sum_{i=1}^{n} \tau_i \chi_{(i-1,i)}(s) \triangleq v(s) \text{ with } t(0) = 0, \tag{10}$$

which maps the releasing moment $0, \tau_1, \tau_1 + \tau_2, \cdots, \sum_{i=1}^{n-1} \tau_i, T$, to the time point $s = 0, 1, \cdots, n$. So system (5) reduces to

$$\begin{cases} \left. \begin{array}{l} \frac{dM(s)}{ds} = v(s)\{r\rho \frac{F(s)(M(s)+\varepsilon\gamma S(s))}{M(s)+\gamma S(s)} e^{-\beta(M(s)+F(s))} - \mu_M M(s)\}, \\ \frac{dF(s)}{ds} = v(s)\{(1-r)\rho \frac{F(s)(M(s)+\varepsilon\gamma S(s))}{M(s)+\gamma S(s)} e^{-\beta(M(s)+F(s))} - \mu_F F(s)\}, \\ \frac{dS(s)}{ds} = -v(s)\mu_S S(s), \end{array} \right\} s \in (0, n], \\ S(i^+) = S(i) + \Lambda_i, \ i = 1, \ldots, n-1. \end{cases} \tag{11}$$

with initial conditions (6).

In the meantime, the cost function (9) becomes to

$$\tilde{J}(\tau,\Lambda) = M(n) + F(n) + c\sum_{i=1}^{n-1}\Lambda_i. \tag{12}$$

Then problem (**P1**) turns into:

($\tilde{\mathbf{P}}\mathbf{1}$) *Given the system (11) with the initial condition (6), find the parameter vector pair* (τ,Λ) *meeting (7) and (8) such that the cost function (12) is minimized.*

However, this problem is still difficult to solve. So we will introduce time translation transformation. For $i = 1,\dots,n$, define

$$M_i(s) = M(s+i-1),\ F_i(s) = M(s+i-1),\ S_i(s) = S(s+i-1),\ \pi_i(s) = t(s+i-1). \tag{13}$$

Then, system (10) and (11) as well as initial condition (6) are reduced to

(14)

$$\begin{cases} \left.\begin{aligned} &\frac{dM_i(s)}{ds} = \tau_i\{r\rho\frac{F_i(s)(M_i(s)+\varepsilon\gamma S_i(s))}{M_i(s)+\gamma S_i(s)}e^{-\beta(M_i(s)+F_i(s))} - \mu_M M_i(s)\} \triangleq f_1^i(s),\\ &\frac{dF_i(s)}{ds} = \tau_i\{(1-r)\rho\frac{F_i(s)(M_i(s)+\varepsilon\gamma S_i(s))}{M_i(s)+\gamma S_i(s)}e^{-\beta(M_i(s)+F_i(s))} - \mu_F F_i(s)\} \triangleq f_2^i(s),\\ &\frac{dS_i(s)}{ds} = -\tau_i\mu_S S_i(s) \triangleq f_3^i(s),\\ &\dot{\pi}_i(s) = \tau_i,\ i = 1,\dots,n, \end{aligned}\right\} s \in (0,1],\\ S_j(0) = S_{j-1}(1) + \Lambda_{j-1},\ j = 2,\dots,n. \end{cases}$$

with

$$M_1(0) = M(0) = M^0, F_1(0) = F(0) = F^0, S_1(0) = S(0) = S^0. \tag{15}$$

Then cost function (12) and problem ($\tilde{\mathbf{P}}\mathbf{1}$) becomes respectively

$$\hat{J}(\tau,\Lambda) = M(1) + F(1) + c\sum_{i=1}^{n-1}\Lambda_i, \tag{16}$$

($\hat{\mathbf{P}}\mathbf{1}$) *Given the system (14) with the initial condition (15), find the parameter vector pair* (τ,Λ) *meeting (7) and (8) such that the cost function (16) is minimized.*

Thus ($\hat{\mathbf{P}}\mathbf{1}$) is a parameter selection problem which can be solved by normal optimization method. According to the Theorem 6.1 in [15], define corresponding Hamiltonian functions H_i

for $i = 1, \ldots, n$,

(17)

$$H_i(s, M_i(s), F_i(s), S_i(s), \lambda^i(s), \tau, \Lambda) = \begin{pmatrix} \lambda_1^i(s) & \lambda_2^i(s) & \lambda_3^i(s) \end{pmatrix} \begin{pmatrix} f_1^i(s) & f_2^i(s) & f_3^i(s) \end{pmatrix}^T,$$

where $\lambda^i(s) = (\lambda_1^i(s), \lambda_2^i(s), \lambda_3^i(s))$ is the corresponding costate determined by the following costate equations

$$\begin{aligned}
\dot{\lambda}_1^i(s) &= -\tau_i \Bigg\{ \lambda_1^i(s) \left(r\rho \frac{F_i(\gamma S_i - \varepsilon\gamma S_i)}{(M_i + \gamma S_i)^2} e^{-\beta(M_i+F_i)} - \beta r\rho \frac{F_i(M_i + \varepsilon\gamma S_i)}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} - \mu_M \right) \\
&\quad + \lambda_2^i(s) \left((1-r)\rho \frac{F_i(\gamma S_i - \varepsilon\gamma S_i)}{(M_i + \gamma S_i)^2} e^{-\beta(M_i+F_i)} - \beta(1-r)\rho \frac{F_i(M_i + \varepsilon\gamma S_i)}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} \right) \Bigg\},
\end{aligned}$$

$$\begin{aligned}
\dot{\lambda}_2^i(s) &= -\tau_i \Bigg\{ \lambda_1^i(s) \left(r\rho \frac{M_i + \varepsilon\gamma S_i}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} - \beta r\rho \frac{F_i(M_i + \varepsilon\gamma S_i)}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} \right) \\
&\quad + \lambda_2^i(s) \left((1-r)\rho \frac{M_i + \varepsilon\gamma S_i}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} - \beta(1-r)\rho \frac{F_i(M_i + \varepsilon\gamma S_i)}{M_i + \gamma S_i} e^{-\beta(M_i+F_i)} - \mu_F \right) \Bigg\},
\end{aligned}$$

$$\begin{aligned}
\dot{\lambda}_3^i(s) &= -\tau_i \Bigg\{ \lambda_1^i(s) \left(r\rho \frac{F_i\varepsilon\gamma(M_i + \gamma S_i) - F_i(M_i + \varepsilon\gamma S_i)\gamma}{(M_i + \gamma S_i)^2} e^{-\beta(M_i+F_i)} \right) \\
&\quad + \lambda_2^i(s) \left((1-r)\rho \frac{F_i\varepsilon\gamma(M_i + \gamma S_i) - F_i(M_i + \varepsilon\gamma S_i)\gamma}{(M_i + \gamma S_i)^2} e^{-\beta(M_i+F_i)} \right) - \lambda_3^i(s)\mu_S \Bigg\},
\end{aligned}$$

with boundary conditions

$$\begin{cases} \lambda_1^n(1) = 0, \lambda_2^n(1) = 0, \lambda_3^n(1) = 1, \\ \lambda_1^i(1) = \lambda_1^{i+1}(0), \lambda_2^i(1) = \lambda_2^{i+1}(0), \lambda_3^i(1) = \lambda_3^{i+1}(0), \ i = 1, \ldots, n-1. \end{cases} \tag{18}$$

Denote

$$y(s) = \begin{pmatrix} F(s), M(s), S(s) \end{pmatrix}^T, \ y_i(0) = \psi^{i-1}(y_{i-1}(1), \Lambda_{i-1}).$$

Hence, according to the system (14), for $i = 2, \ldots, n$ we get

$$\psi^{i-1}(y_{i-1}(1), \Lambda_{i-1}) = \begin{pmatrix} M_{i-1}(1), F_{i-1}(1), S_{i-1}(1) + \Lambda_{i-1} \end{pmatrix}^T.$$

Therefore, the corresponding gradient of the cost function (16) relative to the parameters τ_j and Λ_k are determined as

$$\begin{aligned}\nabla_{\tau_j} J(\tau,\Lambda) &= \int_0^1 \sum_{i=1}^{n} \frac{\partial H_i(s, M_i(s), F_i(s), S_i(s), \lambda_i(s), \tau, \Lambda)}{\partial \tau_j} ds \\ &= \int_0^1 \left\{ \lambda_1^j \left(\frac{r\rho F_j (M_j + \varepsilon\gamma S_j)}{M_j + \gamma S_j} e^{-\beta(M_j+F_j)} - \mu_M M_j \right) \right. \\ &\quad \left. + \lambda_2^j \left(\frac{(1-r)\rho F_j (M_j + \varepsilon\gamma S_j)}{M_j + \gamma S_j} e^{-\beta(M_j+F_j)} - \mu_F F_j \right) - \lambda_3^j \mu_S S_j \right\} ds\end{aligned} \tag{19}$$

for $j = 1, 2, \cdots n$, and

$$\begin{aligned}\nabla_{\Lambda_k} \hat{J}(\tau,\Lambda) &= c + \sum_{i=1}^{n-1} \lambda^{i+1}(0)^T \frac{\partial \psi^i(y_i(1), \Lambda_i)}{\partial \Lambda_k} \\ &= c + \begin{pmatrix} \lambda_1^{k+1}(0) & \lambda_2^{k+1}(0) & \lambda_3^{k+1}(0) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ &= c + \lambda_3^{k+1}(0)\end{aligned} \tag{20}$$

for $k = 1, 2, \cdots n-1$.

Specially, if sterile insects are released n times at regular intervals τ on $[0,T]$, then $n\tau = T$. Thus the released amounts of sterile insects Λ_i meeting constraint (8) are optimally selected. So we get a amount control model:

$$\begin{cases} \left.\begin{aligned} &\frac{dM(t)}{dt} = r\rho \frac{F(t)(M(t)+\varepsilon\gamma M_S(t))}{M(t)+\gamma S(t)} e^{-\beta(M(t)+F(t))} - \mu_M M(t), \\ &\frac{dF(t)}{dt} = (1-r)\rho \frac{F(t)(M(t)+\varepsilon\gamma S(t))}{M(t)+\gamma S(t)} e^{-\beta(M(t)+F(t))} - \mu_F F(t), \\ &\frac{dS(t)}{dt} = -\mu_S S(t), \end{aligned}\right\} t \neq i\tau,\ t \in (0,T] \\ S(i\tau^+) = S(i\tau) + \Lambda_i,\ t = i\tau,\ i = 1, 2, \cdots, n-1. \end{cases} \tag{21}$$

with initial conditions (6). Our optimal control problem can now be formulated:

(P2) *Given a system (21), under the initial condition (6), find a vector of release parameters for sterile insects Λ fulfilling (8) such that the cost function*

$$J(\Lambda) = M(T) + F(T) + c \sum_{i=1}^{n-1} \Lambda_i \tag{22}$$

is minimized.

Similarly, after the time scaling transform and time translation, system (21) reduces to

(23)
$$\begin{cases} \left.\begin{aligned} &\frac{dM_i(s)}{ds} = f_1^i(s) = \tau\{r\rho\frac{F_i(s)(M_i(s)+\varepsilon\gamma S_i(s))}{M_i(s)+\gamma S_i(s)}e^{-\beta(M_i(s)+F_i(s))} - \mu_M M_i(s)\}, \\ &\frac{dF_i(s)}{ds} = f_2^i(s) = \tau\{(1-r)\rho\frac{F_i(s)(M_i(s)+\varepsilon\gamma S_i(s))}{M_i(s)+\gamma S_i(s)}e^{-\beta(M_i(s)+F_i(s))} - \mu_F F_i(s)\}, \\ &\frac{dS_i(s)}{ds} = f_3^i(s) = -\tau\mu_S S_i(s), \\ &\dot{\pi}_i(s) = \tau, \ , i = 1,\dots,n, \end{aligned}\right\} s \in (0,1], \\ S_j(0) = S_{j-1}(1) + \Lambda_{j-1}, \ j = 2,\dots,n. \end{cases}$$

and (15). And the ultimate cost function is

$$\hat{J}(\Lambda) = M_n(1) + F_n(1) + c\sum_{i=1}^{n-1}\Lambda_i. \tag{24}$$

According the definition of Hamiltonian function in (17), then the costate equations with boundry conditions and the gradient of (24) on Λ_k are obtained.

4. Compute and Simulation

Table 1. Parameter values for the system (1)[1].

Parameter	Value	Unit
r	0.5	-
ρ	6.66	day^{-1}
μ_M, μ_F, μ_S	1/13, 1/15, 1/8.5	day^{-1}
β	3.026×10^{-4}	-
γ	0.91	-
ε	0.015	-

The genetic algorithm (GA) is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Nowadays the toolbox of genetic algorithm is used in Matlab to solve the continuous optimization problems. In this section, we will apply gradient descent (GD) and genetic algorithm (GA) to solve the optimal control strategy for suppressing wild mosquitoes.

Some parameters derive from reference [2] in which sterile mosquitoes were released by an open-release field trial in Guangzhou with the highest dengue transmission rate in China. In our simulation, taking days as the unit of time, the total optimal control time is 63 days, that is, $T = 63$. Assume that the sterile mosquitoes are released $N-1 = 8$ times in variable amounts. Take the production and release cost of a sterile mosquito as $c = 0.0002$. Other parameter values shown in Table 1 derive from [1]. Then, assuming that sterile mosquitoes are free, the equilibria of wild mosquitoes are $\widehat{M} = 5999$ and $\widehat{F} = 6923$ for model (1) so that the total amount of wild mosquitoes is up to 12923. We take them as the initial levels in the following simulations. Moreover, according to the expression of Λ_{crit} in section 2, the critical value is 2.284×10^5. Thus sterile mosquitoes are released eight times and 2.284×10^5 every time. So the total amount of sterile mosquitoes reach 1.827×10^6 and the cost value and wild mosquitoes at the terminal time are 1584.22 and 1219, respectively. In addition, in the following simulation, we restrict $1 \le \tau_i \le 10$, and $\sum_{i=1}^{n} \tau_i = T$ and set $1\times10^4 \le \Lambda_i \le 3\times10^5$.

4.1. Numerical simulation of gradient descent (GD).

Referring to the algorithm in [8, 16], the optimization strategies were solved numerically using gradient descent method on the Matlab.

Simulation 1. Optimal release time interval and release amount

Taking the initial impulsive control time intervals as $\tau_i = 7$ for $i = 1,2,\cdots,8$ and starting with a initial release amount $\Lambda_0 = (2.284\times10^5, 2.284\times10^5, 2.284\times10^5, 2.284\times10^5, 1\times10^4, 1.65\times10^5, 2.284\times10^5, 2.284\times10^5)$, we obtain that the cost value $J_0 = 1631.96$ and the total wild mosquito population $M(T)+F(T) = 1323$ at $T = 63$. Then we solve the optimal problem and obtain a set of optimal release intervals

$$\tau^* = (1, 7.2761, 7.2663, 7.3544, 7.4387, 7.5012, 7.5581, 7.6052, 10)$$

and optimized release amounts

$$\Lambda^* = (1.778\times10^5, 1.873\times10^5, 2.119\times10^5, 2.257\times10^5, 2.328\times10^5, 2.368\times10^5, 2.372\times10^5, 2.179\times10^5)$$

which are described by Figure 1 (a). Accordingly the minimum cost is $J^* = 1438.28$ while the total wild mosquito population at the terminal time is $M^*(T)+F^*(T) = 1093$.

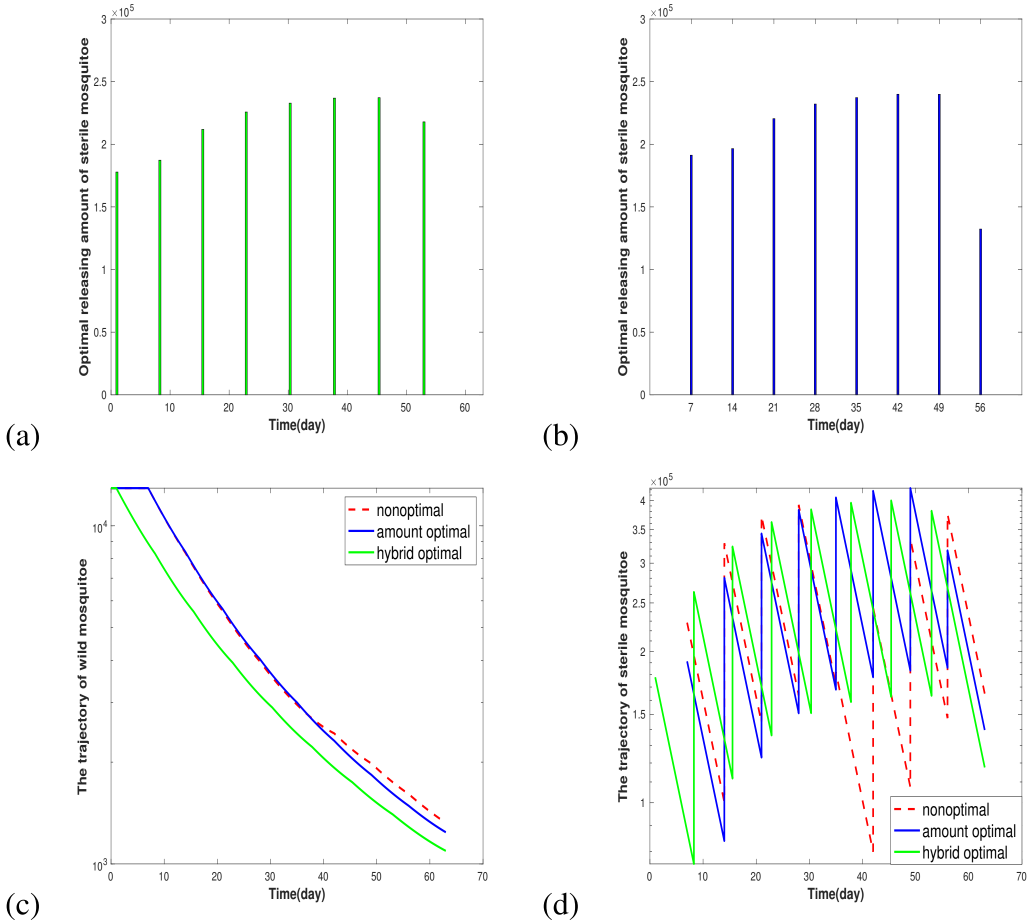

FIGURE 1. Comparison of two optimal control strategies. (a) Release strategy of the hybrid optimal control. (b) Release strategy of the optimal amount control. (c) Comparisons of total wild mosquitoes population under different controls. (d) Comparisons of sterile mosquitoes under different controls.

Simulation 2. Optimal release amount at fixed moment

We take the same initial conditions as the optimized control above, then obtain the optimal release amount

$$\Lambda^* = (1.913\times10^5, 1.965\times10^5, 2.204\times10^5, 2.321\times10^5, 2.3723\times10^5, 2.399\times10^5, 2.3985\times10^5, 1.323\times10^5)$$

which is displayed in Figure 1 (b). The corresponding cost value $J^* = 1578.35$, while the total amount of wild mosquitoes at terminal time is $M^*(T)+F^*(T) = 1240$.

Comparison of two optimal control strategies shows that the hybrid control is more advantageous. In addition, it is found that wild mosquito populations continue to decline as the amount of releases increases. However, even if the release reaches the upper limit, the wild mosquitoes cannot be eliminated either in the short term.

4.2. Numerical simulation of genetic algorithm(GA).

Choosing the same initial conditions as the above simulation, we use the genetic algorithm to solve two optimal control strategies. Then for the hybrid control, optimal release time interval and amount respectively are

$$\tau^* = (1,1,1,10,10,10,10,10,10)$$

and

$$\Lambda^* = (1.805\times10^5, 1.302\times10^5, 1.117\times10^5, 2.019\times10^5, 2.692\times10^5, 2.341\times10^5, 2.578\times10^5, 2.533\times10^5).$$

Further, for the amount control, optimal release amount is

$$\Lambda^* = (1.886\times10^5, 2.195\times10^5, 2.142\times10^5, 2.24\times10^5 2.388\times10^5, 2.373\times10^5, 2.452\times10^5, 1.332\times10^5).$$

The corresponding results of both algorithms are list in the Table 2.

Table 2: Comparison of different control tactics and algorithms.

Type of control	Cost value	Amount of wild mosquitoes at the terminal time	Total amount released
Without control	–	12922	0
Non-optimal control	1631.96	1323	1.545×10^6
Periodic pulsed control	1584.22	1219	1.827×10^6
GD for amount control	1578.35	1240	1.690×10^6
GD for hybrid control	1438.28	1093	1.727×10^6
GA for amount control	1578.55	1238	1.701×10^6
GA for hybrid control	1474.87	1147	1.639×10^6

Our numerical simulations show that impulsive controls radically reduce the level of wild mosquitos. Futhermore, the results of both optimization algorithms are similar and the result of hybrid control is superior to the amount control and periodic impulsive control in terms of the minimal objective function, the lower releasing amount and the lesser wild mosquitos at the terminal time. Even so, there is no significant difference among various impulsive control measures in which sterile mosquitoes are released eight times.

5. Conclusion

SIT is an environmentally friendly and species-specific control tool for suppressing Aedes albopictus. According to Theorem 3, wild mosquitoes will become extinct after chronical SIT control. But it is not realistic so that the impulsive control tactics with alterable pulse times and releasing amounts in finite time restraining wild mosquitoes are established. The corresponding impulsive optimal problem are constituted and translated into continuous parameter selection problem by means of the transformation of time-scaling and time translation. Then simulations and optimal control strategies based on gradient descent and genetic algorithm are given. Simulations indicate that hybrid control is superior to the amount control and periodic impulsive control in terms of the objective function, the total release amount and the amount of wild mosquitos at the terminal time.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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