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IMPACT OF POLLUTION ON SARDINE, SARDINELLA, AND MACKEREL FISHERY: A BIOECONOMIC APPROACH

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Abstract. This paper studies a bioeconomic model of three species of small pelagic marine species in Moroccan coastal areas: Sardine, Sardinella and shark. The model combines competition and predation. Two areas are proposed, one is polluted and the other is not. The model combines a biological part describing the evolution of the biomass of stocks subjected to fishing mortality and an economic part explaining the mortality rate. We study the existence and stability of equilibrium states through eigenvalue analysis and the Routh-Hirwitz criterion, then introduce economic approaches to determine the effort needed to maximize the fishermen's income. Numerical simulations are performed. The objective of this paper is to study the impact of pollution on the existence, evolution of biomass and predation, fishing effort, catches, and profits.

Keywords: prey-predator model; Nash equilibrium; pollution rate; bioeconomic model; stability analysis.

2020 AMS Subject Classification: 93A30, 92D25, 92D40, 92B05, 91B76.

1. Introduction

Marine pollution is defined as the direct or indirect introduction of wastes, substances, or energy, including underwater sound sources of human origin, which results or is likely to result in adverse effects on living resources and marine ecosystems [1]. The consequences of marine

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pollution include a loss of biodiversity [2], risks to human health [3], impediments to marine activities such as fishing, tourism, and recreation [4], and reductions in the amenity value of the marine environment.

This pollution can stem from human activities in the catchment area, including industrial, agricultural, urban, or port origins, that reach the marine environment directly through discharges into the sea or indirectly through rivers [5]. However, the true impact of marine pollution on the environment is difficult to determine as the precise quantities of pollutants that reach the sea are not fully understood [6]. Further research is needed to deepen our knowledge on the transfer of pollutants within watersheds and their fate in transition zones.

Our study focuses on the Moroccan coasts, which, due to its geographical location between Europe and Africa, and between the Mediterranean and the Atlantic, offers a diverse range of ecosystems and marine species. The Mediterranean Sea is a highly productive area for fish, with numerous commercially important species caught in the region [7]. Fishing in the Mediterranean Sea is largely based on the exploitation of pelagic fish species, such as Sardine and Sardinella, and 450 species of fish in total are related to oceanic species found on the coasts of Portugal or Morocco.

We examine two different areas in the Mediterranean Sea: a polluted area (Area A) and a non-polluted area (Area B). Our focus is on the predator shark, and its relationship with small pelagic species such as Sardine and Sardinella, considering the negative impact of pollution on their existence, evolution, and exploitation. Within the framework of a differential equation-based prey-predator and competition model, our results demonstrate the importance of continuous monitoring of the marine environment to assess its health.

In this context, we can cite these works to demonstrate the importance of incorporating the effects of pollution in bioeconomic models in order to understand the impacts of human activities on marine ecosystems. In this work [8], the authors developed a bioeconomic model to examine the impacts of marine pollution on a fishery system. The model incorporated both biological and economic components, and the authors found that pollution can have significant impacts on the long-term sustainability of the fishery.

In [9], The authors developed a bioeconomic model to assess the impact of pollution on marine biodiversity conservation. The model incorporated economic and ecological variables, including the effects of fishing effort, pollution, and the interplay between predator and prey species.

In this work [10], the athors used a bioeconomic model to evaluate the impacts of marine pollution on fishing activities. The model considers the effects of pollution on fishing costs, revenue, and effort, and assesses the implications for fishing communities. We can cite also [11], where the authors study the predation interaction between phytoplankton and zooplankton under their exploitation in multi-fishing zones using a bioeconomic spatiotemporal discrete model. The entire domain is represented by a grid of colored cells, with two harvesting control strategies used to guarantee the survival of the organisms.

This work will study a bioeconomic model of three fish populations, Sardine, Sardinella, and shark, combining competition and predation. Our model is based on hypotheses that the three fish populations grow according to a logistic equation and that predators compete with each other for space and food. The bioeconomic model considers the negative effect of pollution on fishing effort, catches, fishermen's profits, and biomasses. In the first part of the work, we will determine the equilibrium points of the biological system and study their stabilities. In a subsequent part, we will introduce economic approaches to determine the effort necessary to maximize each fisherman's net economic income and perform numerical simulations to see the impact of pollution on fishing effort, catches, and profits.

2. Biological Model Description, Formulation, and Analysis

2.1. The mathematical model and the hypotheses. In this study we are interested in the study of three marine populations which are of the prey-predator type. The preys are distributed in two different zones: the first zone is a unpolluted zone A and the second a polluted zone B. These prey are the preferred prey of predators.

In the unpolluted zone A, the evolution of the biomass of prey in this zone is defined by x_A.They grow according to a logistic equation with growth rate r_1 and carrying capacity K_1.This population is preyed with the response rate α_1.

In the second area; the polluted area B, the evolution of the biomass of prey in this zone is defined by x_B. They grow according to a logistic equation with growth rate r_2 and carrying capacity K_2. This population is preyed with the response rate α_2. These preys die by the pollution of this zone by the rate δ. So it is clear that $r_2 > \delta$.

The evolution of the biomass of predators is defined by y.These predators feed on prey from both polluted and unpolluted areas where β_1, β_2 represent the rate of conversion to predators of prey in non polluted zone and polluted zone respectively. The coefficient d represents the natural mortality coefficient of the predator population. The parameters γ denote the coefficients of toxicity mortality by feeding on prey from the polluted area. So it is clear that $\alpha_2\beta_2 > \gamma, d < \alpha_1\beta_1 k_1$ and $\delta < \alpha_2\beta_2$.

Based on these given assumptions, we find the system that describes the evolution of the biomass of these three marine populations

$$\left\{\begin{array}{l} \dot{x}_A(t) = r_1 x_A(t)\left(1 - \frac{x_A(t)}{K_1}\right) - \alpha_1 x_A(t)\, y(t) \\ \dot{x}_B(t) = r_2 x_B(t)\left(1 - \frac{x_B(t)}{K_2}\right) - \alpha_2 x_B(t)\, y(t) - \delta x_B(t) \\ \dot{y}(t) = -d y(t) + \alpha_1\beta_1 x_A(t)\, y(t) + \alpha_2\beta_2 x_B(t)\, y(t) - \gamma y(t)\, x_B(t) \end{array}\right. \tag{1}$$

Subject to initial conditions: $x_A(0) > 0, x_B(0) > 0, y(0) > 0$.

All the parametres used in this model are assumed to be positive and all variables are not negative. Then the model proposed of the three populations can be rewritten as

$$\left\{\begin{array}{l} \dot{x}_A(t) = x_A(t)\, f_A(x_A(t), x_B(t), y(t)) \\ \dot{x}_B(t) = x_B(t)\, f_B(x_A(t), x_B(t), y(t)) \\ \dot{y}(t) = y(t) f(x_A(t), x_B(t), y(t)) \end{array}\right. \tag{2}$$

Note that

$$\left\{\begin{array}{l} f_A = r_1\left(1 - \frac{x_A(t)}{K_1}\right) - \alpha_1 y(t) \\ f_B = r_2\left(1 - \frac{x_B(t)}{K_2}\right) - \alpha_2 y(t) - \delta \\ f = -d + \alpha_1\beta_1 x_A(t) + \alpha_2\beta_2 x_B(t) - \gamma x_B(t) \end{array}\right. \tag{3}$$

The system (1) is defined in the field

$$\Omega = \{(x_A, x_B, y \in \mathbb{R} / x_A(0) > 0, x_B(0) > 0, y(0) > 0)\}$$

2.2. Positivity and boundedness of the solutions.

Theorem 1. All the solutions of the system (2.1) with the initial conditions are positive and bounded.

Proof. According to the system of equations (2.1) and the initial conditions we have

$$\begin{cases} x_A(t) = x_A(0)\exp\int_0^t f_A(x_A(\tau), x_B(\tau), y(\tau)) > 0 \\ x_B(t) = x_B(0)\exp\int_0^t f_B(x_A(\tau), x_B(\tau), y(\tau)) > 0 \\ y(t) = y(0)\exp\int_0^t f(x_A(\tau), x_B(\tau), y(\tau)) > 0 \end{cases} \tag{4}$$

So, All the solutions are positive.

2) We consider

$$\varphi = \beta_1\beta_2 x_A + \alpha_2\beta_2 x_B + \alpha_1\beta_1 y$$

its derivative with respect to time is given by

$$\begin{aligned} \frac{d\varphi}{dt} &= \beta_2\beta_1 i_1 x_1\left(1-\tfrac{x_1}{c_1}\right) + \beta_{13}\beta_{32} i_2 x_2\left(1-\tfrac{x_2}{c_2}\right) + \beta_{13}\beta_{23} i_3 x_3\left(1-\tfrac{x_3}{c_3}\right) \\ &- (\beta_{23}\beta_{31}\alpha_{12} + \beta_{32}\alpha_{21})x_1x_2 - \beta_{23}\beta_{31}(m_{1+}p_1)x_1 - \beta_{13}\beta_{32}(m_2+p_2)x_2 \\ &- \beta_{13}\beta_{23}(m_3+p_3)x_3 \\ &\leqslant \beta_{23}\beta_{31} i_1 x_1\left(1-\frac{x_1}{c_1}\right) + \beta_{13}\beta_{32} i_2 x_2\left(1-\frac{x_2}{c_2}\right) + \beta_{13}\beta_{23} i_3 x_3\left(1-\frac{x_3}{c_3}\right) \end{aligned}$$

For all $\eta > 0$, we have

$$\frac{d\varphi}{dt} + \eta\varphi(t) \leqslant c_1\beta_{23}\beta_{31}\frac{(i_1+\eta)^2}{4i_1} + c_2\beta_{13}\beta_{32}\frac{(i_2+\eta)^2}{4i_2} + c_3\beta_{13}\beta_{23}\frac{(i_3+\eta)^2}{4i_3}$$

Then, there exists $\varepsilon > 0$, with $\dfrac{d\varphi}{dt} + \eta\varphi(t) < \varepsilon$. By applying the theory of differential inequality, we obtain

$$0 < \varphi(t) \le \frac{\varepsilon}{\eta} + \left[\varphi(x_1(0), x_2(0), x_3(0) - \frac{\varepsilon}{\eta}\right] e^{-\eta t}$$

so,

$$0 < \lim_{t\to\infty}\varphi(t) \leqslant \frac{\varepsilon}{\eta}$$

Hence, all the solutions of the system with initial value in $\mathbb{R}_+^3$ are included in the following domain

$$\left\{(x_1, x_2, x_3) \in \mathbb{R}_+^3 / \ \varphi < \frac{\varepsilon}{\eta} + \zeta, \forall\zeta > 0\right\}$$

3. The Steady States of the System

We propose to study the existence of equilibrium states and the stability of the interior equilibrium point of our model [12].

3.1. Existence of different equilibrium points.

The equilibrium states of the system are solutions of the following system

$$\left\{ \begin{array}{l} r_1 x_A(t)\left(1-\frac{x_A(t)}{K_1}\right)-\alpha_1 x_A(t)\, y(t)=0 \\ r_2 x_B(t)\left(1-\frac{x_B(t)}{K_2}\right)-\alpha_2 x_B(t)\, y(t)-\delta x_B(t)=0 \\ -dy(t)+\alpha_1\beta_1 x_A(t)\, y(t)+\alpha_2\beta_2 x_B(t)\, y(t)-\gamma y(t)\, x_B(t)=0 \end{array} \right. \tag{5}$$

This system of equations has eight solutions

i: The trivial equilibrium point $P_1(0,0,0)$ and the axial equilibrium points

$$\left\{ \begin{array}{l} P_2(K_1,0,0), \\ P_3\left(0,-\frac{1}{r_2}(\delta K_2-K_2 r_2),0\right) \\ P_4\left(K_1,-\frac{1}{r_2}(\delta K_2-K_2 r_2),0\right) \end{array} \right.$$

ii: The equilibrium points in the plane (x_A,y) is $P_5\left(x_A^{(5)},0,y^{(5)}\right)$, where

$$\left\{ \begin{array}{l} x_A^{(5)}=\frac{d}{\alpha_1\beta_1} \\ y^{(5)}=\frac{1}{\alpha_1}r_1-\frac{d}{\alpha_1^2\beta_1 K_1}r_1 \end{array} \right.$$

iii: The equilibrium points in the plane (x_B,y) is $P_6\left(0,x_B^{(6)},y^{(6)}\right)$, where

$$\left\{ \begin{array}{l} x_B^{(6)}=-\frac{d}{\gamma-\alpha_2\beta_2} \\ y^{(6)}=-\frac{dr_2-\gamma\delta K_2+\gamma K_2 r_2+\delta\alpha_2\beta_2 K_2-\alpha_2\beta_2 K_2 r_2}{\alpha_2^2\beta_2 K_2-\gamma\alpha_2 K_2} \end{array} \right.$$

iv: The equilibrium points $P_7\left(x_A^{(7)},x_B^{(7)},y^{(7)}\right)$, where

$$\left\{ \begin{array}{l} x_A^{(7)}=K_1 \\ x_B^{(7)}=-\frac{d}{\gamma-\alpha_2\beta_2} \\ y^{(7)}=-\frac{dr_2-\gamma\delta K_2+\gamma K_2 r_2+\delta\alpha_2\beta_2 K_2-\alpha_2\beta_2 K_2 r_2}{\alpha_2^2\beta_2 K_2-\gamma\alpha_2 K_2} \end{array} \right.$$

v: The internal equilibrium point is $P_8\left(x_A^{(*)}, x_B^{(*)}, y^{(*)}\right)$, where

$$\begin{cases} x_A^{(*)} = \frac{\Delta_1}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ x_B^{(*)} = \frac{\Delta_2}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ y^{(*)} = -\frac{\Delta_3}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \end{cases}$$

with

$$\begin{aligned} \Delta_1 &= d\alpha_1K_1r_2-\gamma\delta\alpha_1K_1K_2+\gamma\alpha_1K_1K_2r_2-\gamma\alpha_2K_1K_2r_1+\alpha_2^2\beta_2K_1K_2r_1 \\ &+ \delta\alpha_1\alpha_2\beta_2K_1K_2-\alpha_1\alpha_2\beta_2K_1K_2r_2 \\ \Delta_2 &= d\alpha_2K_2r_1-\delta\alpha_1^2\beta_1K_1K_2+\alpha_1^2\beta_1K_1K_2r_2-\alpha_1\alpha_2\beta_1K_1K_2r_1 \\ \Delta_3 & \quad dr_1r_2+\gamma K_2r_1r_2-\gamma\delta K_2r_1+\delta\alpha_2\beta_2K_2r_1-\alpha_1\beta_1K_1r_1r_2-\alpha_2\beta_2K_2r_1r_2 \end{aligned}$$

The system of (3.1) has several solutions, but only one of them can give the coexistence of the biomass of the three species; this solution is the point $P_8\left(x_A^{(*)}, x_B^{(*)}, y^{(*)}\right)$

4. The Stability of the Steady States

The variational matrix of system (2.1) is as follow

$$J = \begin{bmatrix} J_{11} & 0 & -\alpha_1x_A \\ 0 & J_{22} & -\alpha_2x_B \\ \alpha_1\beta_1y & \alpha_2\beta_2y-\delta y & J_{33} \end{bmatrix}$$

where

$$\begin{cases} J_{11} = r_1\left(1-\frac{x_A}{K_1}\right)-\alpha_1y \\ J_{22} = r_2\left(1-\frac{x_B}{K_2}\right)-\alpha_2y(t)-\delta \\ J_{33} = -d+\alpha_1\beta_1x_A+\alpha_2\beta_2x_B-\gamma x_B \end{cases}$$

Proposition 1. *The steady state $P_1(0,0,0)$ is unstable.*

The variational matrix of system at the steady state $P_1(0,0,0)$ is

$$J_1 = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2-\delta & 0 \\ 0 & 0 & -d \end{bmatrix}$$

The eigenvalues of P_1 are

$$\begin{cases} \lambda_1 = r_1 > 0 \\ \lambda_2 = r_2 - \delta > 0 \\ \lambda_3 = -d < 0 \end{cases}$$

Proposition 2. *The steady state $P_2\,(K_1, 0, 0)$ is unstable as shown in Figure 1.*

The variational matrix of system at the steady state $P_2\,(K_1, 0, 0)$ is

$$J_2 = \begin{bmatrix} -r_1 & 0 & 0 \\ 0 & r_2 - \delta & 0 \\ 0 & 0 & -d + \alpha_1 \beta_1 k_1 \end{bmatrix}$$

The eigenvalues of P_2 are

$$\begin{cases} \lambda_1 = -r_1 < 0 \\ \lambda_2 = r_2 - \delta > 0 \\ \lambda_3 = -d + \alpha_1 \beta_1 k_1 > 0 \end{cases}$$

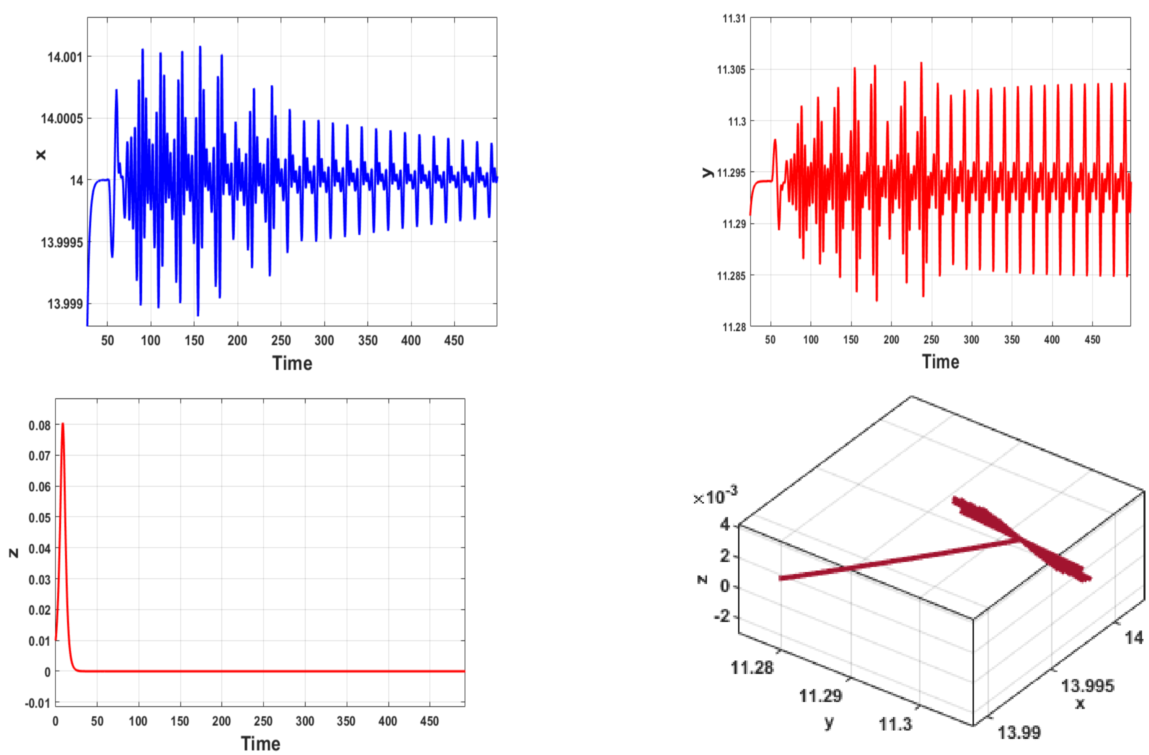

FIGURE 1. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 14$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.12$ and $d = 0.2$

Proposition 3. *The steady state $P_3\left(0,-\frac{1}{r_2}\left(\delta K_2-K_2r_2\right),0\right)$ is unstable, see Figure 2.*

The variational matrix of system at the steady state $P_3\left(0,-\frac{1}{r_2}\left(\delta K_2-K_2r_2\right),0\right)$ is

$$J_3=\begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2+2(\delta-r_2)-\delta & \frac{\alpha_2}{r_2}\left(\delta K_2-K_2r_2\right) \\ 0 & 0 & -d-\frac{1}{r_2}\left(\delta K_2-K_2r_2\right)\left(\alpha_2\beta_2-\gamma\right) \end{bmatrix}$$

The eigenvalues of P_3 are

$$\begin{cases} \lambda_1=r_1>0 \\ \lambda_2=-r_2+2\delta>0 \\ \lambda_3=-d-\frac{1}{r_2}\left(\delta K_2-K_2r_2\right)\left(\alpha_2\beta_2-\gamma\right)>0 \end{cases}$$

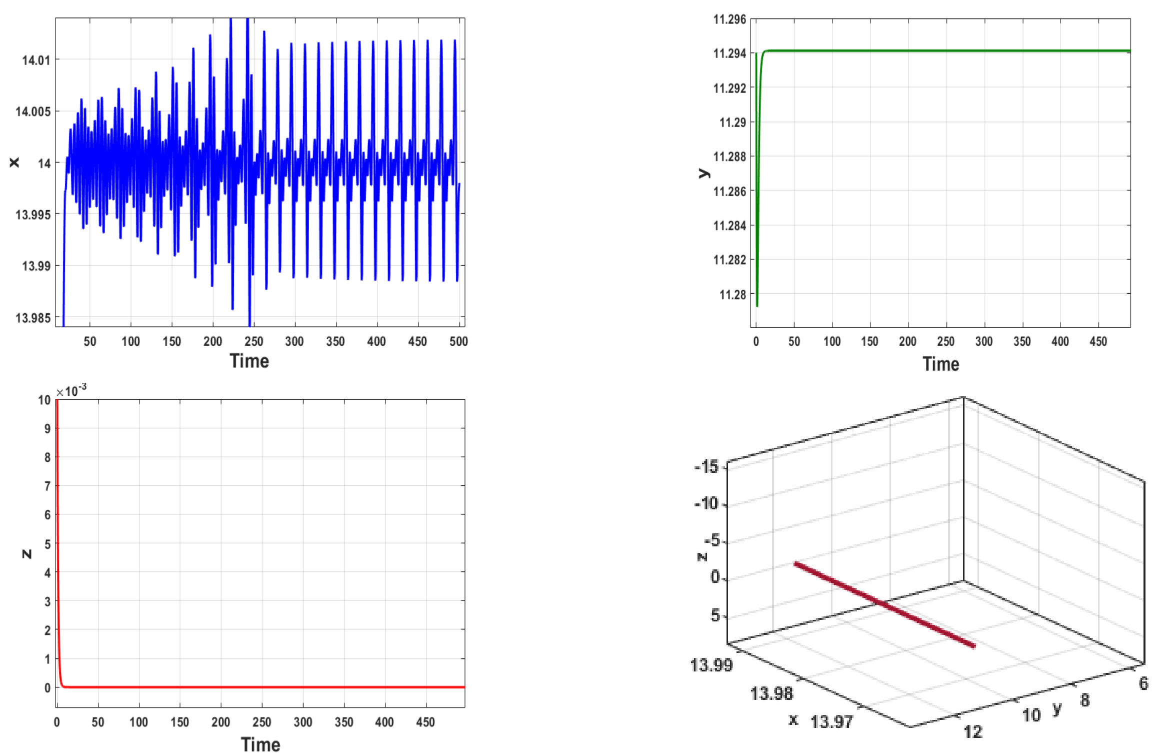

FIGURE 2. Dynamical behaviour and Phase portraits of the three populations for $r_1=0.8$, $r_2=0.85$, $K_1=14$, $K_2=12$, $\delta=0.05$, $\alpha_1=0.2$, $\alpha_2=0.3$, $\beta_1=0.18$, $\beta_2=0.2$, $\gamma=0.12$ and $d=0.2$

Proposition 4. *The steady state $P_4\left(K_1,-\frac{1}{r_2}\left(\delta K_2-K_2r_2\right),0\right)$ is stable, as shown in Figure 3.*

The variational matrix of system at the steady state $P_4\left(K_1, -\frac{1}{r_2}\left(\delta K_2 - K_2 r_2\right), 0\right)$ is

$$J_4 = \begin{bmatrix} -r_1 & 0 & -\alpha_1 k_1 \\ 0 & r_2 + 2(\delta - r_2) - \delta & \frac{\alpha_2}{r_2}\left(\delta K_2 - K_2 r_2\right) \\ 0 & 0 & -d + \alpha_1 \beta_1 k_1 - \frac{1}{r_2}\left(\delta K_2 - K_2 r_2\right)\left(\alpha_2 \beta_2 - \gamma\right) \end{bmatrix}$$

The eigenvalues of P_4 are

$$\begin{cases} \lambda_1 = -r_1 < 0 \\ \lambda_2 = -r_2 + \delta < 0 \\ \lambda_3 = -d + \alpha_1 \beta_1 k_1 - \frac{1}{r_2}\left(\delta K_2 - K_2 r_2\right)\left(\alpha_2 \beta_2 - \gamma\right) < 0 \end{cases}$$

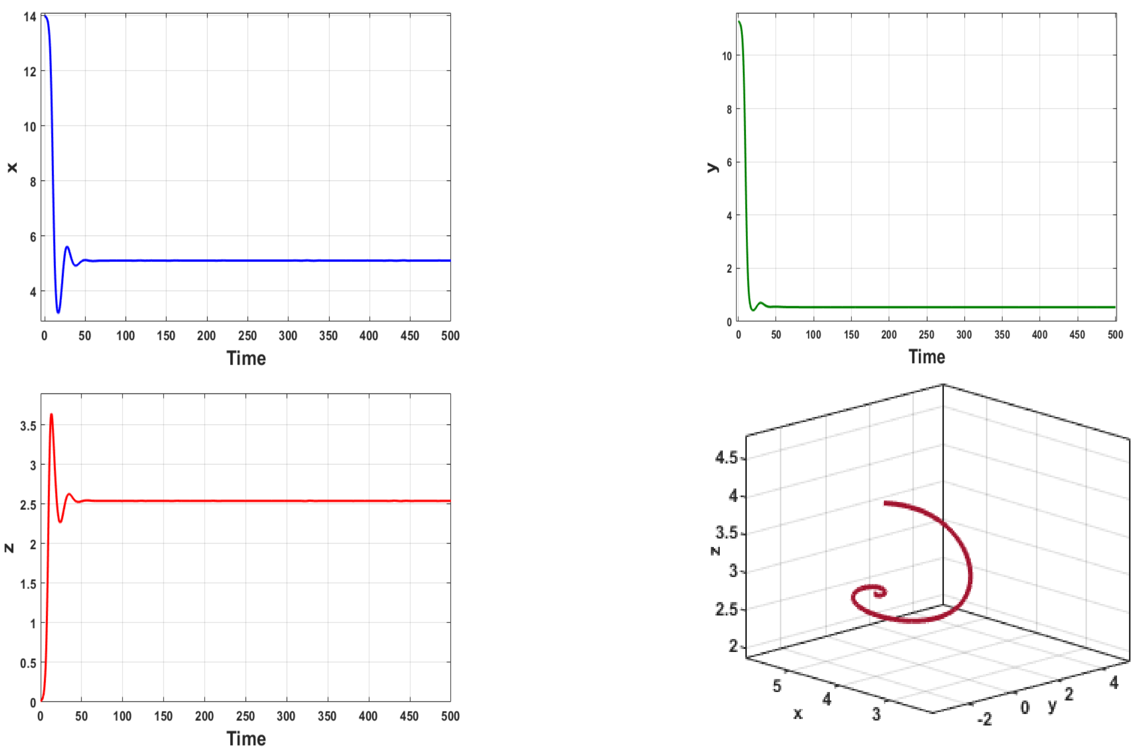

FIGURE 3. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 14$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.03$ and $d = 0.2$

Proposition 5. *The steady state* $P_5\left(x_A^{(5)}, 0, y^{(5)}\right)$*, where*

$$\begin{cases} x_A^{(5)} = \frac{d}{\alpha_1 \beta_1} \\ y^{(5)} = \frac{1}{\alpha_1} r_1 - \frac{d}{\alpha_1^2 \beta_1 K_1} r_1 \end{cases}$$

is unstable, see Figure 4.

The variational matrix of system at the steady state $P_5\left(x_A^{(5)}, 0, y^{(5)}\right)$, *where*

$$\begin{cases} x_A^{(5)} = \frac{d}{\alpha_1\beta_1} \\ y^{(5)} = \frac{1}{\alpha_1}r_1 - \frac{d}{\alpha_1^2\beta_1 K_1}r_1 \end{cases}$$

is

$$J_5 = \begin{bmatrix} -r_1(\frac{3d}{\alpha_1\beta_1 k_1}) & 0 & -\alpha_1\frac{d}{\alpha_1\beta_1} \\ 0 & r_2 - \alpha_2(\frac{1}{\alpha_1}r_1 - \frac{d}{\alpha_1^2\beta_1 K_1}r_1) - \delta & 0 \\ r_1(\beta_1 - \frac{d}{\alpha_1 K_1}) & \frac{1}{\alpha_1}r_1 - \frac{d}{\alpha_1^2\beta_1 K_1}r_1(\alpha_2\beta_2 - \gamma) & -2d \end{bmatrix}$$

The eigenvalues of P_5 *are*

$$\begin{cases} \lambda_1 = -r_1(\frac{3d}{\alpha_1\beta_1 k_1}) < 0 \\ \lambda_2 = r_2 - \alpha_2(\frac{1}{\alpha_1}r_1 - \frac{d}{\alpha_1^2\beta_1 K_1}r_1) - \delta > 0 \\ \lambda_3 = -2d < 0 \end{cases}$$

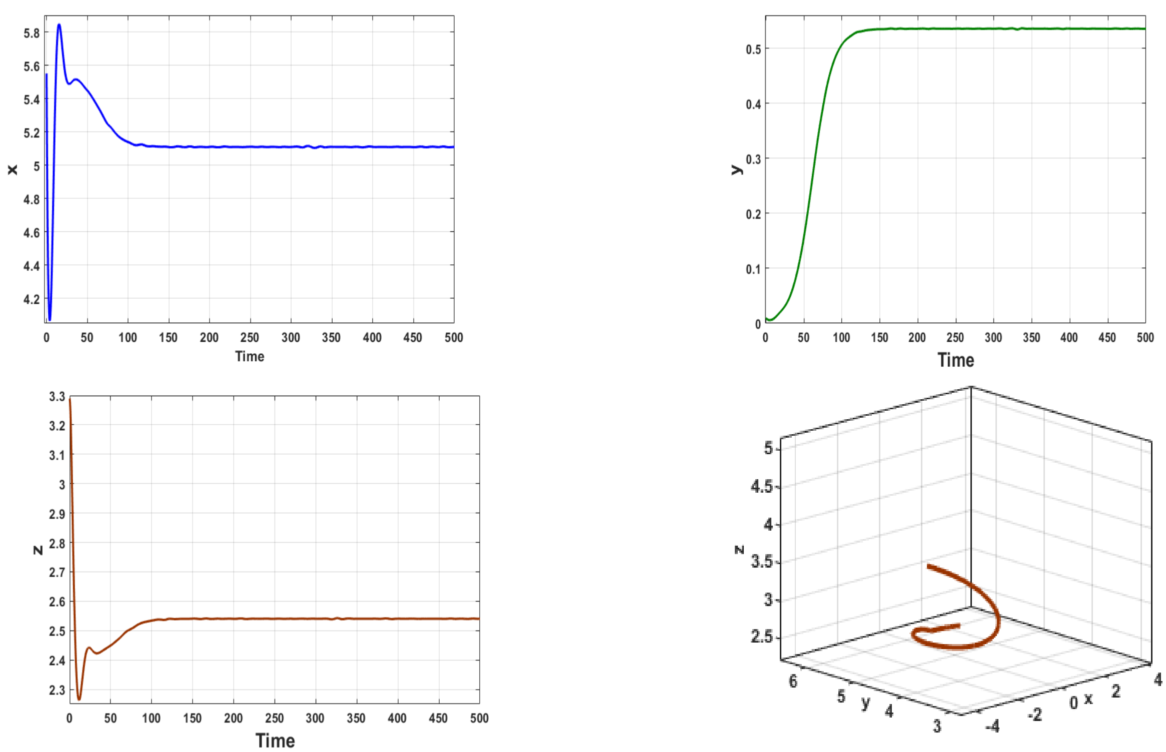

FIGURE 4. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 10$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.12$ and $d = 0.2$

Proposition 6. *The steady state* $P_6\left(0, x_B^{(6)}, y^{(6)}\right)$*, where*

$$\begin{cases} x_B^{(6)} = -\frac{d}{\gamma - \alpha_2\beta_2} \\ y^{(6)} = -\frac{dr_2 - \gamma\delta K_2 + \gamma K_2 r_2 + \delta\alpha_2\beta_2 K_2 - \alpha_2\beta_2 K_2 r_2}{\alpha_2^2\beta_2 K_2 - \gamma\alpha_2 K_2} \end{cases}$$

nothing can be concluded, see Figure 5.

The variational matrix of system at the steady state $P_6\left(0, x_B^{(6)}, y^{(6)}\right)$, where

$$\begin{cases} x_B^{(6)} = -\frac{d}{\gamma - \alpha_2\beta_2} \\ y^{(6)} = -\frac{dr_2 - \gamma\delta K_2 + \gamma K_2 r_2 + \delta\alpha_2\beta_2 K_2 - \alpha_2\beta_2 K_2 r_2}{\alpha_2^2\beta_2 K_2 - \gamma\alpha_2 K_2} \end{cases}$$

is:

$$J_6 = \begin{bmatrix} r_1 - \alpha_1 y^{(6)} & 0 & 0 \\ 0 & r_2(1 + \frac{2d}{k_2\gamma - \alpha_2\beta_2 K_2}) - \alpha_2 y^{(6)} - \delta & \alpha_2 \frac{d}{\gamma - \alpha_2\beta_2} \\ \alpha_1\beta_1 y^{(6)} & y^{(6)}(\alpha_2\beta_2 - \gamma) & 0 \end{bmatrix}$$

The eigenvalues of P_6 are

$$\begin{cases} \lambda_1 = r_1 - \alpha_1 y^{(6)} < 0 \\ \lambda_2 = r_2(1 + \frac{2d}{k_2\gamma - \alpha_2\beta_2 K_2} r_1) - \alpha_2 y^{(6)} - \delta > 0 \\ \lambda_3 = 0 \end{cases}$$

where $y^{(6)} = -\frac{dr_2 - \gamma\delta K_2 + \gamma K_2 r_2 + \delta\alpha_2\beta_2 K_2 - \alpha_2\beta_2 K_2 r_2}{\alpha_2^2\beta_2 K_2 - \gamma\alpha_2 K_2}$.

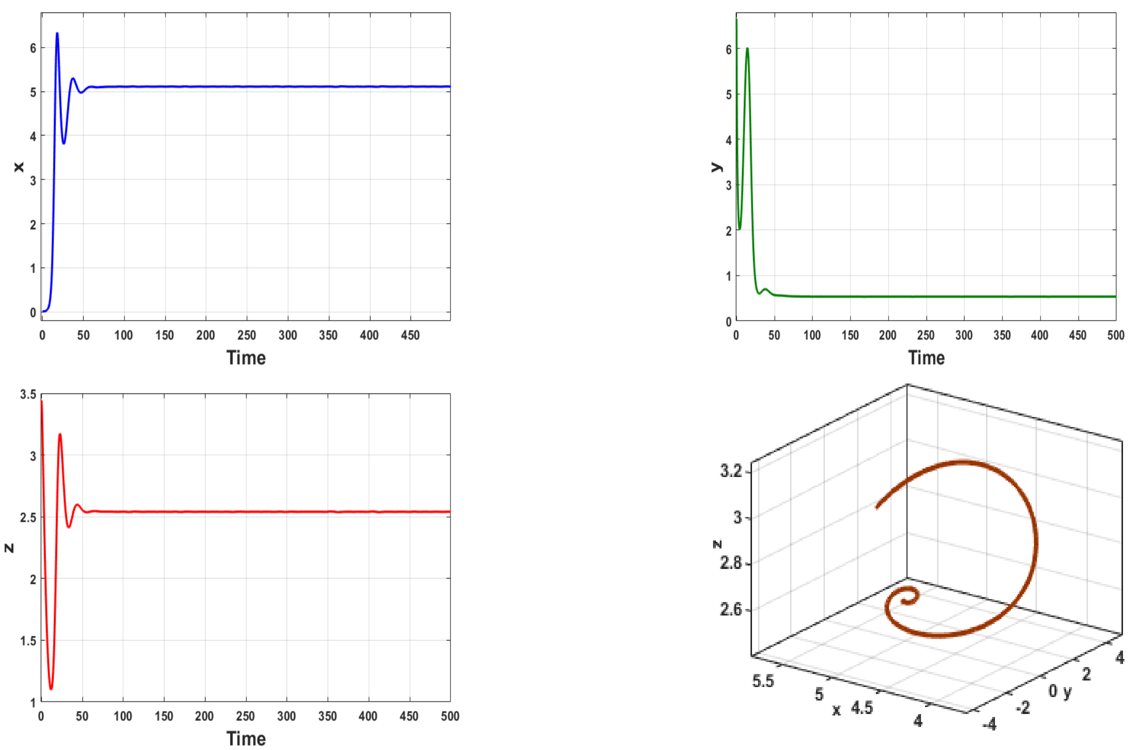

FIGURE 5. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 14$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.03$ and $d = 0.2$

Proposition 7. *The steady state* $P_7\left(x_A^{(7)}, x_B^{(7)}, y^{(7)}\right)$, *where*

$$\begin{cases} x_A^{(7)} = K_1 > 0 \\ x_B^{(7)} = -\frac{d}{\gamma - \alpha_2 \beta_2} > 0 \\ y^{(7)} = -\frac{dr_2 - \gamma\delta K_2 + \gamma K_2 r_2 + \delta \alpha_2 \beta_2 K_2 - \alpha_2 \beta_2 K_2 r_2}{\alpha_2^2 \beta_2 K_2 - \gamma \alpha_2 K_2} < 0 \end{cases}$$

is unstable, see Figure 6.

The variational matrix of system at the steady state $P_7\left(x_A^{(7)}, x_B^{(7)}, y^{(7)}\right)$, where

$$\begin{cases} x_A^{(7)} = K_1 \\ x_B^{(7)} = -\frac{d}{\gamma - \alpha_2 \beta_2} \\ y^{(7)} = -\frac{dr_2 - \gamma\delta K_2 + \gamma K_2 r_2 + \delta \alpha_2 \beta_2 K_2 - \alpha_2 \beta_2 K_2 r_2}{\alpha_2^2 \beta_2 K_2 - \gamma \alpha_2 K_2} \end{cases}$$

is:

$$J_7 = \begin{bmatrix} -r_1 - \alpha_1 y^{(7)} & 0 & -\alpha_1 k_1 \\ 0 & r_2(1+\frac{2d}{k_2\gamma-\alpha_2\beta_2K_2}) - \alpha_2 y^{(7)} - \delta & \alpha_2 \frac{d}{\gamma\alpha_2\beta_2} \\ \alpha_1\beta_1 y^{(7)} & y^{(7)}(\alpha_2\beta_2-\gamma) & \alpha_1\beta_1K_1 \end{bmatrix}$$

The eigenvalues of P_7 are

$$\begin{cases} \lambda_1 = -r_1 - \alpha_1 y^{(7)} < 0 \\ \lambda_2 = r_2(1+\frac{2d}{k_2\gamma-\alpha_2\beta_2K_2}) - \alpha_2 y^{(7)} - \delta > 0 \\ \lambda_3 = \alpha_1\beta_1K_1 > 0 \end{cases}$$

where $y^{(7)} = -\frac{dr_2-\gamma\delta K_2+\gamma K_2r_2+\delta\alpha_2\beta_2K_2-\alpha_2\beta_2K_2r_2}{\alpha_2^2\beta_2K_2-\gamma\alpha_2K_2}$.

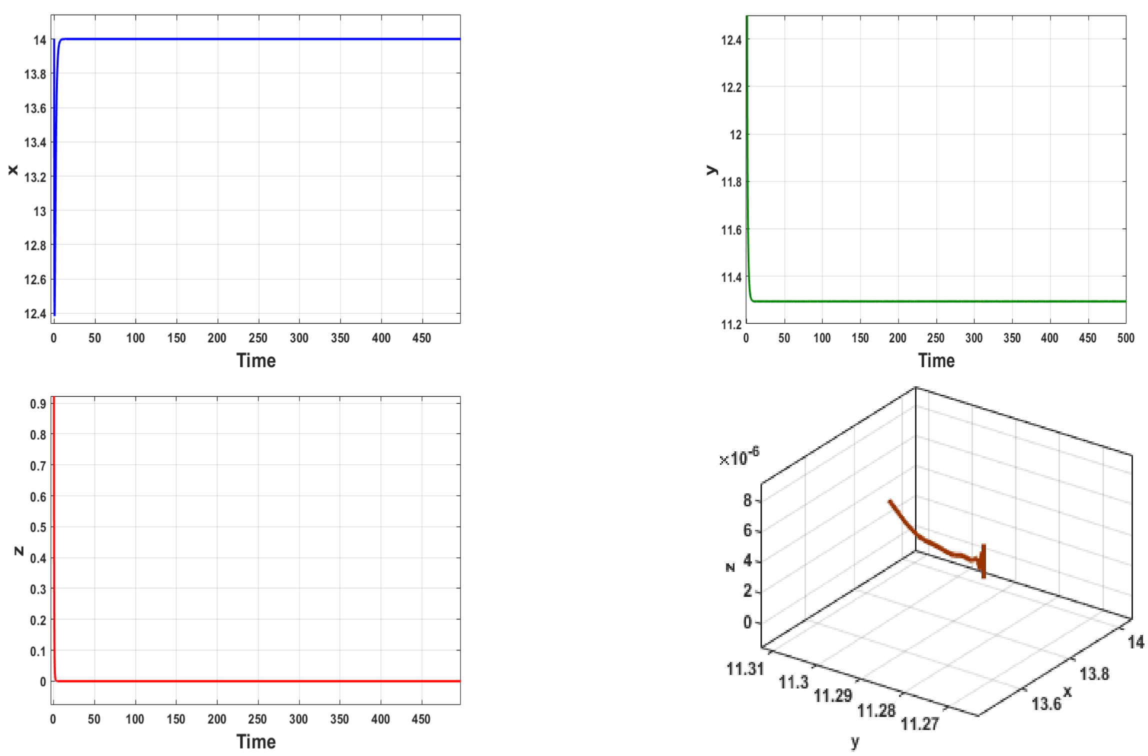

FIGURE 6. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 14$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.03$ and $d = 0.2$

Proposition 8. *The steady state* $P_8\left(x_A^{(*)}, x_B^{(*)}, y^{(*)}\right)$, *where*

$$\begin{cases} x_A^{(*)} = \frac{\Delta_1}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ x_B^{(*)} = \frac{\Delta_2}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ y^{(*)} = -\frac{\Delta_3}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \end{cases}$$

with

$$\begin{aligned} \Delta_1 &= d\alpha_1K_1r_2 - \gamma\delta\alpha_1K_1K_2 + \gamma\alpha_1K_1K_2r_2 - \gamma\alpha_2K_1K_2r_1 + \alpha_2^2\beta_2K_1K_2r_1 \\ &+ \delta\alpha_1\alpha_2\beta_2K_1K_2 - \alpha_1\alpha_2\beta_2K_1K_2r_2 \\ \Delta_2 &= d\alpha_2K_2r_1 - \delta\alpha_1^2\beta_1K_1K_2 + \alpha_1^2\beta_1K_1K_2r_2 - \alpha_1\alpha_2\beta_1K_1K_2r_1 \\ \Delta_3 &= dr_1r_2 + \gamma K_2r_1r_2 - \gamma\delta K_2r_1 + \delta\alpha_2\beta_2K_2r_1 - \alpha_1\beta_1K_1r_1r_2 - \alpha_2\beta_2K_2r_1r_2 \end{aligned}$$

is unstable.

The variational matrix of system at the steady state $P_8\left(x_A^{(*)}, x_B^{(*)}, y^{(*)}\right)$, is

$$J_8 = \begin{bmatrix} r_1(1-2\frac{x_A^{(*)}}{k_1}) - \alpha_1y^{(*)} & 0 & -\alpha_1x_A^{(*)} \\ 0 & r_1(1-2\frac{x_B^{(*)}}{k_2}) - \alpha_1y^{(*)} - \delta & -\alpha_2x_B^{(*)} \\ \alpha_1\beta_1y^{(*)} & y^{(*)}(\alpha_2\beta_2-\gamma) & (\alpha_2\beta_2-\gamma)x_B^{(*)} \end{bmatrix}$$

where

$$\begin{cases} x_A^{(*)} = \frac{\Delta_1}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ x_B^{(*)} = \frac{\Delta_2}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \\ y^{(*)} = -\frac{\Delta_3}{\alpha_1^2\beta_1K_1r_2+\alpha_2^2\beta_2K_2r_1-\gamma\alpha_2K_2r_1} \end{cases}$$

with

$$\Delta_1 = K_1\left(\alpha_1r_2\left(d+\gamma K_2\right) + \alpha_2\beta_2K_2\left(\delta\alpha_1+\alpha_2r_1\right)\right) - K_1K_2\left(\gamma\left(\delta\alpha_1+\alpha_2r_1\right) + \alpha_1\alpha_2\beta_2r_2\right)$$

$$\Delta_2 = \alpha_2K_2r_1\left(d-\alpha_1\beta_1K_1\right) - \alpha_1^2\beta_1K_1K_2\left(\delta-r_2\right)$$

$$\Delta_3 = \left(dr_1r_2 + \gamma K_2r_1r_2 - \gamma\delta K_2r_1 + \delta\alpha_2\beta_2K_2r_1 - \alpha_1\beta_1K_1r_1r_2 - \alpha_2\beta_2K_2r_1r_2\right)$$

In this cace, the characteristic polynomial is given by: $P(\lambda) = a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ where

$$\begin{cases} a_0 = 1 \\ a_1 = \frac{1}{k_1} r_1 x_A^* + \frac{1}{k_2} r_2 x_B^* - \frac{1}{y^*} > 0 \\ a_2 = \alpha_1^2 \beta_1 x_A^* y^* - \alpha_2 x_B^* y^* (\delta - \alpha_2\beta_2) - \frac{1}{y^*}\left(\frac{1}{k_1} r_1 x_A^* + \frac{1}{k_2} r_2 x_B^*\right) + \frac{1}{k_1 k_2} r_1 r_2 x_A^* x_B^* > 0 \\ a_3 = \frac{r_2\alpha_2}{k_2}(x_B^*)^2 y^* (\delta - \alpha_2\beta_2) + \left(\frac{r_1}{k_1} x_A^* + \frac{r_2}{k_2} x_B^*\right)\left(y^*\left(\alpha_1^2\beta_1 x_A^* - \alpha_2 x_B^* (\alpha_2\beta_2 - \delta)\right)\right) \\ -\alpha_1^2 \frac{\beta_1}{k_1} r_1 (x_A^*)^2 y^* - \frac{1}{k_1 k_2} r_1 r_2 x_A^* \frac{x_B^*}{y^*} > 0 \end{cases}$$

By using the conditions of stability of Routh-Hurwitz, one can proof that a_0, a_1, a_2, a_3 and $a_0 a - a_0 a_3$ are positive.

Then the interior equilibrium point $P_8\left(x_A^{(*)}, x_B^{(*)}, y^{(*)}\right)$ is locally asymptotically stable, as shown in the Figure 7.

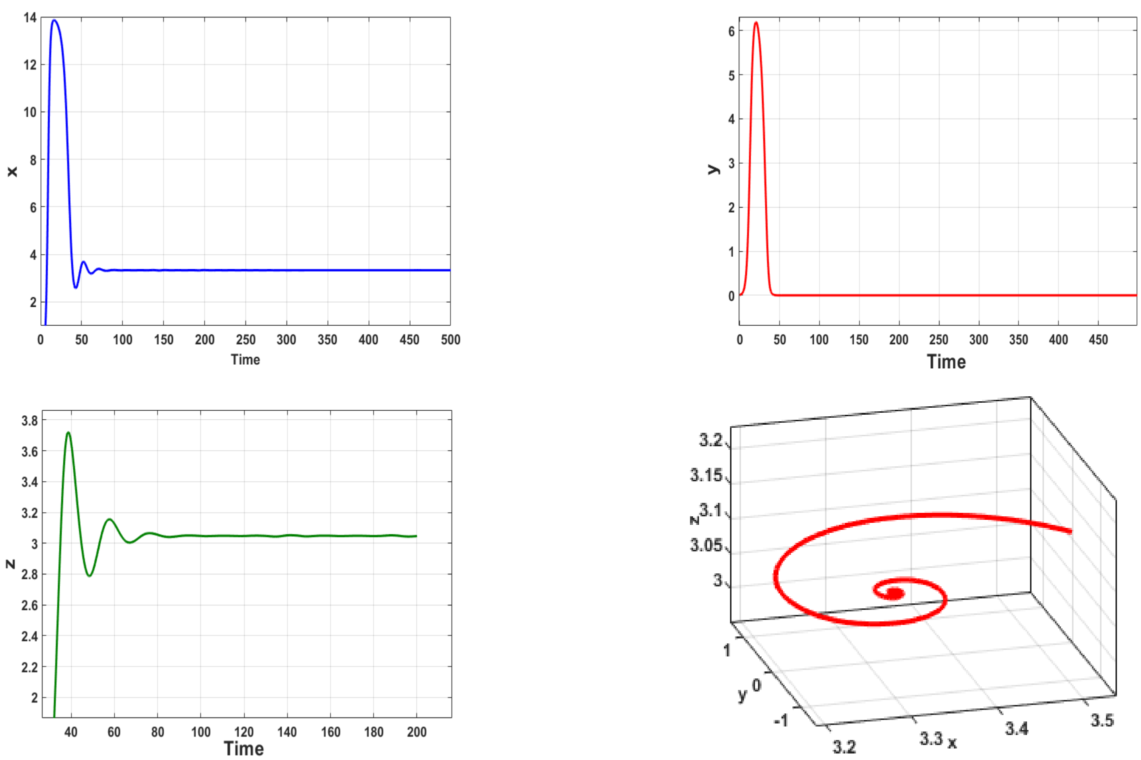

FIGURE 7. Dynamical behaviour and Phase portraits of the three populations for $r_1 = 0.8$, $r_2 = 0.85$, $K_1 = 10$, $K_2 = 12$, $\delta = 0.05$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.18$, $\beta_2 = 0.2$, $\gamma = 0.12$ and $d = 0.2$

5. PROFIT MAXIMISIZATION

The main objective of this part is to maximize the profits of the fishing fleets which exploit these marine species from their fishing effort (see [13]). For it, the three species proposed in this model are assumed to be caught by three fishing fleets. So the model becomes as follows

$$\begin{cases} \dot{x}_A(t) = r_1 x_A(t)\left(1 - \frac{x_A(t)}{K_1}\right) - \alpha_1 x_A(t)\, y(t) - q_1 E_1 x_A \\ \dot{x}_B(t) = r_2 x_B(t)\left(1 - \frac{x_B(t)}{K_2}\right) - \alpha_2 x_B(t)\, y(t) - \delta x_B(t) - q_2 E_2 x_B \\ \dot{y}(t) = -dy(t) + \alpha_1 \beta_1 x_A(t)\, y(t) + \alpha_2 \beta_2 x_B(t)\, y(t) - \gamma x_B y(t) - q_3 E_3 y \end{cases} \tag{2}$$

where q_i represents the catchability coefficient and $\left(E_{ij}\right)_{1 \le i,j \le 3}$represents the fishing effort deployed by fishing fleets to capture the species, it is defined as the product of fishing activity and fishing power.

The solution of the system (2) at bioeconomic equilibrium is given by

$$\begin{cases} x_A = b_{11}E_1 + b_{12}E_2 + b_{13}E_3 + x_A^* \\ x_B = b_{21}E_1 + b_{22}E_2 + b_{23}E_3 + x_B^* \\ y = b_{31}E_1 + b_{32}E_2 + b_{33}E_3 + y^* \end{cases}$$

where

$$\begin{cases} b_{11} = -\frac{\alpha_2 K_1 K_2 (\gamma - \alpha_2 \beta_2) q_1}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{12} = \frac{\alpha_1 K_1 K_2 (\gamma - \alpha_2 \beta_2) q_2}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{13} = -\frac{\alpha_1 K_1 q_3 r_2}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ x_A^* = \frac{d\alpha_1 K_1 r_2 + (\delta - r_2)(\alpha_1 K_1 K_2 (\alpha_2 \beta_2 - \gamma)) - \alpha_2 K_1 K_2 r_1 (\gamma - \alpha_2 \beta_2)}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{21} = -\frac{\alpha_1 \alpha_2 \beta_1 K_1 K_2 q_1}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{22} = \frac{\alpha_1^2 \beta_1 K_1 K_2 q_2}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{23} = -\frac{\alpha_2 K_2 q_3 r_1}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ x_B^* = \frac{K_2 \left(\alpha_2 r_1 (d - \alpha_1 \beta_1 K_1) - \alpha_1^2 \beta_1 K_1 (\delta - r_2)\right)}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{31} = \frac{\alpha_1 \beta_1 K_1 q_1 r_2}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{32} = -\frac{K_2 q_2 r_1 (\gamma - \alpha_2 \beta_2)}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ b_{33} = \frac{q_3 r_1 r_2}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \\ y^* = \frac{r_1((r_2 - \delta)(K_2(\alpha_2 \beta_2 - \gamma)) - d r_2 + \alpha_1 \beta_1 K_1 r_2)}{\alpha_1^2 \beta_1 K_1 r_2 - \alpha_2 K_2 r_1 (\gamma - \alpha_2 \beta_2)} \end{cases}$$

If we set $E=(E_1,E_2,E_3)^T, Y^*=(x_A^*,x_B^*,y^*)^T$ and $B=\left(-b_{ij}\right)_{1\leq i,j\leq 3}$ with $b_{ii}<0$ for $i=1,2,3$. Then we can write the solution of the system(2) in the following matrix form

$$X=-BE+X^*$$

We want to maximize the profits of the fishing fleets that exploit these marine species. According to Gordon, the profit formula is defined by

$$\Pi_i(E)=(TR)_i-(TC)_i$$

where $(TR)_i=p_i\times H_{ij}$ is the Total Revenue with p_i represents the price, $H_{ij}=q_jE_{ij}X_j$ is the catches of species j by fisherman i $(X_1=x_A,X_2=x_B,X_3=y)$, E_{ij} is the effort of the fisherman i to exploit the species j and $(TC)_i=\left\langle c^i,E^i\right\rangle$ is Total Cost with c^i a constant cost per unit of harvesting effort of the fisherman i. We note that $H_j=\sum_{j=1}^{3}H_{ij}$ the total catches of species j by all fishermen.

Therefore the final formula of Total Revenue

$$(TR)_i=p_1H_{i1}+p_2H_{i2}+p_3H_{i3}=\left\langle E^i,-pqBE^i+pqY^*-pqBE^j\right\rangle$$

We thus obtain the final formula of the profit of fisherman i is given by

$$\Pi_i(E)=\left\langle E^i,-pqBE^i+pqY^*-c^i-pqBE^j\right\rangle$$

In order to maximize the profits of the fishermen, we must first of all take into consideration the maintenance of the biodiversity of the three marine species, so we will assume that all the biomasses remain positive $Y=-BY+Y^*\geq 0$ i.e. for the fisherman i we must have $BE^i\leq BE^j-Y^*$.

Each of the three fleets tries to maximize their profits and achieve a fishing effort that is an optimal response to the effort of the other fishing fleets. And so, we have a Nash equilibrium situation where the strategy of each fishing fleet is optimal, taking into account the strategy of the other fishing fleets (see [14] and [15]). This problem can be translated mathematically into the following three problems:

The first fishing fleet must solve this problem $(P)_1$

$$(P)_1 = \begin{cases} & \max \Pi\left(E^1\right) = \left\langle E^1, -pqBE^1 + pqY^* - c^1 - pqBE^2 - pqBE^3 \right\rangle \\ \text{subject to} & BE^1 \leq -BE^2 - BE^3 + Y^* \\ & E^1 \geq 0 \\ & E^2, E^3 \text{ given} \end{cases}$$

The seconde fishing fleet must solve this problem $(P)_2$

$$(P)_2 = \begin{cases} & \max \Pi\left(E^2\right) = \left\langle E^2, -pqBE^2 + pqY^* - c^2 - pqBE^1 - pqBE^3 \right\rangle \\ \text{subject to} & BE^2 \leq -BE^1 - BE^3 + Y^* \\ & E^2 \geq 0 \\ & E^1, E^3 \text{ given} \end{cases}$$

The third fishing fleet must solve this problem $(P)_3$

$$(P)_3 = \begin{cases} & \max \Pi\left(E^3\right) = \left\langle E^3, -pqBE^3 + pqY^* - c^3 - pqBE^1 - pqBE^2 \right\rangle \\ \text{subject to} & BE^3 \leq -BE^1 - BE^2 + Y^* \\ & E^3 \geq 0 \\ & E^1, E^2 \text{ given} \end{cases}$$

The point $\left(E^1, E^2, E^3\right)$ is called the Nash equilibrium point if and only if E^1 is a solution of the problem $(P)_1$ for given E^3, E^2 and E^2 is a solution of the problem $(P)_2$ for given $E^1, E^3$and E^3 is a solution of the problem $(P)_3$ for given E^1, E^2.

In order to find our Nash equilibrium point we will use the essential conditions of Karush-Kuhn-Tucker (KKT). By applying these conditions to the first problem $(P)_1$ this will give us the existence of constants $u^1 \in \mathbb{R}^3_+$, $v^1 \in \mathbb{R}^3_+$ and $\mu^1 \in \mathbb{R}^3_+$ such that

$$(KKT)_1 \begin{cases} 2pqBE^1 - pqY^* + c^1 + pqBE^2 + pqBE^3 - u^1 - B^T \mu^1 = 0 \\ BE^1 + v^1 = -BE^2 - BE^3 + Y^* \\ \left\langle u^1, E^1 \right\rangle = \left\langle \mu^1, v^1 \right\rangle = 0 \end{cases}$$

By applying the conditions of (KKT) to the second problem $(P)_2$, this will give us the existence of constants $u^2 \in \mathbb{R}^3_+$, $v^2 \in \mathbb{R}^3_+$ and $\mu^2 \in \mathbb{R}^3_+$ such that

$$(KKT)_2 \begin{cases} 2pqBE^2 - pqY^* + c^2 + pqBE^1 + pqBE^3 - u^2 - B^T\mu^2 = 0 \\ BE^2 + v^2 = -BE^1 - BE^3 + Y^* \\ \langle u^2, E^2 \rangle = \langle \mu^2, v^2 \rangle = 0 \end{cases}$$

In the same way, by applying the essential conditions of (KKT) to $(P)_3$, this will give us the existence of constants $u^3 \in \mathbb{R}^3_+$, $v^3 \in \mathbb{R}^3_+$ and $\mu^3 \in \mathbb{R}^3_+$ such that

$$(KKT)_3 \begin{cases} 2pqBE^3 - pqY^* + c^3 + pqBE^1 + pqBE^2 - u^3 - B^T\mu^3 = 0 \\ BE^2 + v^3 \leq -BE^1 - BE^2 + Y^* \\ \langle u^3, E^3 \rangle = \langle \mu^3, v^3 \rangle = 0 \end{cases}$$

From the previous problems we get the following expressions

$$\begin{cases} u^1 = 2pqBE^1 - pqY^* + c^1 + pqBE^2 + pqBE^3 - B^T\mu^1 \\ u^2 = 2pqBE^2 - pqY^* + c^2 + pqBE^1 + pqBE^3 - B^T\mu^2 \\ u^3 = 2pqBE^3 - pqY^* + c^3 + pqBE^1 + pqBE^2 - B^T\mu^3 \\ v^1 = v^2 = v^3 = -BE^1 - BE^2 - BE^3 + Y^* \\ \langle u^i, E^i \rangle = \langle \mu^i, v^i \rangle = 0 \ \ \forall i = 1,2,3 \\ u^i, \mu^i, v^i \text{ and } E^i \geq 0 \quad \forall i = 1,2,3 \end{cases}$$

We have the scalar product of μ^i and v^i is zero and to maintain the biodiversity of the three marine species, it is natural to assume that all biomasses remain strictly positive, i.e. $Y^* > 0$ then $v^i > 0$ and $\mu^i = 0, \forall i = 1,2,3$. We note $v := v^1 = v^2 = v^3$ therefore the previous system become

$$\begin{cases} u^1 = 2pqBE^1 + pqBE^2 + pqBE^3 + c^1 - pqY^* \\ u^2 = pqBE^1 + 2pqBE^2 + pqBE^3 + c^2 - pqY^* \\ u^3 = pqBE^1 + pqBE^2 + 2pqBE^3 + c^3 - pqY^* \\ v = -BE^1 - BE^2 - BE^3 + Y^* \\ \langle u^i, E^i \rangle = \langle \mu^i, v \rangle = 0 \ \ \forall i = 1,2,3 \\ u^i, \mu^i, v^i \text{ and } E^i \geq 0 \quad \forall i = 1,2,3 \end{cases}$$

We can also write it in the following matrix form $w = NL + q$ where

$$w=\begin{pmatrix}u^1\\u^2\\u^3\\v\end{pmatrix},N=\begin{pmatrix}2pqB & pqB & pqB & B^T\\pqB & 2pqB & pqB & I\\pqB & pqB & 2pqB & I\\-B & -B & -B & I\end{pmatrix},L=\begin{pmatrix}E^1\\E^2\\E^3\\0\end{pmatrix}\text{ and } q=\begin{pmatrix}c^1-pqY^*\\c^2-pqY^*\\c^3-pqY^*\\Y^*\end{pmatrix}.$$

The generalized Nash equilibrium problem is equivalent to the following Linear Complementarity Problem $LCP(N,q)$: find vectors $w,L\in\mathbb{R}^{16}$such that

$$\begin{cases} w=NL+q\geq 0\\ L,w\geq 0\\ L^T w=0\end{cases}$$

The $LCP(N,q)$ has a unique solution for every q if and only if N is a $P-$matrix.

$\Delta_1=-2p_1q_1b_{11}>0$

$\Delta_2=4p^2q^2b_{11}b_{22}-b_{12}b_{21}>0$

$\Delta_3=b_{12}b_{31}b_{23}+b_{21}b_{13}b_{32}+8p^3q^3b_{11}b_{22}b_{33}-2pqb_{11}>0$

$\Delta_4 = 12p^4q^4b_{11}^2b_{22}b_{33}+2pqb_{11}b_{12}b_{31}b_{23}+2pqb_{11}b_{21}b_{13}b_{32}-3p^2q^2b_{11}^2b_{23}b_{32}-4p^2 q^2b_{11}b_{12}b_{21}b_{33}-4p^2q^2b_{11}b_{13}b_{22}b_{31}>0$

$\Delta_5=18p^5q^5b_{11}^2b_{22}^2b_{33}+4p^2q^2b_{11}b_{12}b_{22}b_{31}b_{23}+4p^2q^2b_{11}b_{21}b_{13}b_{22}b_{32}-6p^3q^3b_{11}b_{13}b_{22}^2b_{31}-6p^3q^3b_{11}^2b_{22}b_{23}b_{32}-8p^3q^3b_{11}b_{12}b_{21}b_{22}b_{33}>0$

$\Delta_6 = 27p^6q^6b_{11}^2b_{22}^2b_{33}^2+8p^3q^3b_{11}b_{21}b_{13}b_{22}b_{32}b_{33}+8p^3q^3b_{11}b_{12}b_{22}b_{31}b_{23}b_{33}-12p^4q^4b_{11}b_{12}b_{21}b_{22}b_{33}^2-12p^4q^4b_{11}b_{13}b_{22}^2b_{31}b_{33}-12p^4q^4b_{11}^2b_{22}b_{23}b_{32}b_{33}>0$

$\Delta_7 = 36p^7q^7b_{11}^3b_{22}^2b_{33}^2-18p^5q^5b_{11}^2b_{12}b_{21}b_{22}b_{33}^2-18p^5q^5b_{11}^2b_{13}b_{22}^2b_{31}b_{33}-16p^5q^5b_{11}^3 b_{22}b_{23}b_{32}b_{33}+12p^4q^4b_{11}^2b_{12}b_{22}b_{31}b_{23}b_{33}+12p^4q^4b_{11}^2b_{21}b_{13}b_{22}b_{32}b_{33}>0$

$\Delta_8 = 48p^8q^8b_{11}^3b_{22}^3b_{33}^2-24p^6q^6b_{11}^2b_{13}b_{22}^3b_{31}b_{33}-24p^6q^6b_{11}^3b_{22}^2b_{23}b_{32}b_{33}-27p^6q^6b_{11}^2 b_{12}b_{21}b_{22}^2b_{33}^2+18p^5q^5b_{11}^2b_{12}b_{22}^2b_{31}b_{23}b_{33}+18p^5q^5b_{11}^2b_{21}b_{13}b_{22}^2b_{32}b_{33}>0$

$\Delta_9 = 64p^9q^9b_{11}^3b_{22}^3b_{33}^3-36p^7q^7b_{11}^2b_{12}b_{21}b_{22}^2b_{33}^3-36p^7q^7b_{11}^2b_{13}b_{22}^3b_{31}b_{33}^2-36p^7q^7b_{11}^3 b_{22}^2b_{23}b_{32}b_{33}^2+27p^6q^6b_{11}^2b_{12}b_{22}^2b_{31}b_{23}b_{33}^2+27p^6q^6b_{11}^2b_{21}b_{13}b_{22}^2b_{32}b_{33}^2>0$

$\Delta_{10} = 32p^8q^8b_{11}^3b_{22}^3b_{33}^3+16p^8q^8b_{11}^4b_{22}^3b_{33}^3+64p^9q^9b_{11}^3b_{22}^3b_{33}^3-12p^7q^7b_{11}^3b_{12}^2b_{22}^2b_{33}^3-12p^7q^7b_{11}^3b_{13}^2b_{22}^3b_{33}^2+16p^8q^8b_{11}^3b_{12}^2b_{22}^2b_{33}^3+16p^8q^8b_{11}^3b_{13}^2b_{22}^3b_{33}^2-60p^7q^7b_{11}^2b_{12}b_{21}b_{22}^2b_{33}^3-12p^7q^7b_{11}^3b_{12}b_{21}b_{22}^2b_{33}^3-18p^6q^6b_{11}^3b_{22}^2b_{23}b_{32}b_{33}^2-60p^7q^7b_{11}^2b_{13}b_{22}^3b_{31}b_{33}^2-9p^6q^6b_{11}^4b_{22}^2b_{23}b_{32}b_{33}^2-12p^7q^7b_{11}^3b_{13}b_{22}^3b_{31}b_{33}^2-36p^7q^7b_{11}^3b_{22}^2b_{23}b_{32}b_{33}^2+$

$45p^6q^6b_{11}^2b_{12}b_{22}^2b_{31}b_{23}b_{33}^2 + 45p^6q^6b_{11}^2b_{21}b_{13}b_{22}^2b_{32}b_{33}^2 + 9p^6q^6b_{11}^3b_{12}b_{13}b_{22}^2b_{23}b_{33}^2 + 9p^6q^6b_{11}^3b_{12}b_{13}b_{22}^2b_{32}b_{33}^2 + 9p^6q^6b_{11}^3b_{12}b_{22}^2b_{31}b_{23}b_{33}^2 + 9p^6q^6b_{11}^3b_{21}b_{13}b_{22}^2b_{32}b_{33}^2 - 12p^7q^7b_{11}^3b_{12}b_{13}b_{22}^2b_{23}b_{33}^2 - 12p^7q^7b_{11}^3b_{12}b_{13}b_{22}^2b_{32}b_{33}^2 > 0$

$\Delta_{11} > 0$

$\Delta_{12} > 0$

Since the matrix N of our problem is $P-$matrix, we can deduce that the linear complementarity problem $LCP(N,q)$ admits one and only one solution. The solution is given by

$$\begin{cases} E^1 = \frac{1}{4}B^{-1}\left(Y^* - \frac{c^1}{pq}\right) \\ E^2 = \frac{1}{4}B^{-1}\left(Y^* - \frac{c^2}{pq}\right) \\ E^3 = \frac{1}{4}B^{-1}\left(Y^* - \frac{c^3}{pq}\right) \end{cases}$$

where

$$B^{-1} = \begin{bmatrix} -\frac{1}{K_1q_1}r_1 & 0 & -\frac{\alpha_1}{q_1} \\ 0 & -\frac{1}{K_2q_2}r_2 & -\frac{\alpha_2}{q_2} \\ \alpha_1\frac{\beta_1}{q_3} & -\frac{1}{q_3}(\gamma - \alpha_2\beta_2) & 0 \end{bmatrix}$$

Finally, we obtain the fishing effort that maximizes the profit of the first fisherman for caching the prey population in non-polluted zone as follow

$$E_{11} = \frac{1}{4}\left(\frac{r_1}{K_1q_1}\left(\frac{c^1}{p_1q_1} - x_A^*\right) + \frac{\alpha_1}{q_1}\left(\frac{c^1}{p_3q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the first fisherman for caching the prey population in polluted zone as follow

$$E_{12} = \frac{1}{4}\left(\frac{r_2}{K_2q_2}\left(\frac{c^1}{p_2q_2} - x_B^*\right) + \frac{\alpha_2}{q_2}\left(\frac{c^1}{p_3q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the first fisherman for caching the predator population as follow

$$E_{13} = \frac{1}{4}\left(\alpha_1\frac{\beta_1}{q_3}\left(\frac{c^1}{p_1q_1} - x_A^*\right) + \frac{\gamma - \alpha_2\beta_2}{q_3}\left(\frac{c^1}{p_2q_2} - x_B^*\right)\right)$$

the fishing effort that maximizes the profit of the second fisherman for caching the prey population in non-polluted zone as follow

$$E_{21} = \frac{1}{4}\left(\frac{r_1}{K_1q_1}\left(\frac{c^2}{p_1q_1} - x_A^*\right) + \frac{\alpha_1}{q_1}\left(\frac{c^2}{p_3q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the second fisherman for caching the prey population in polluted zone as follow

$$E_{22} = \frac{1}{4}\left(\frac{r_2}{K_2 q_2}\left(\frac{c^2}{p_2 q_2} - x_B^*\right) + \frac{\alpha_2}{q_2}\left(\frac{c^2}{p_3 q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the second fisherman for caching the predator population as follow

$$E_{23} = \frac{1}{4}\left(\alpha_1 \frac{\beta_1}{q_3}\left(x_A^* - \frac{c^2}{p_1 q_1}\right) + \frac{\gamma - \alpha_2\beta_2}{q_3}\left(\frac{c^2}{p_2 q_2} - x_B^*\right)\right)$$

the fishing effort that maximizes the profit of the third fisherman for caching the prey population in non-polluted zone as follow

$$E_{31} = \frac{1}{4}\left(\frac{r_1}{K_1 q_1}\left(\frac{c^3}{p_1 q_1} - x_A^*\right) + \frac{\alpha_1}{q_1}\left(\frac{c3}{p_3 q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the third fisherman for caching the prey population in polluted zone as follow

$$E_{32} = \frac{1}{4}\left(\frac{r_2}{K_2 q_2}\left(\frac{c^3}{p_2 q_2} - x_B^*\right) + \frac{\alpha_2}{q_2}\left(\frac{c3}{p_3 q_3} - y^*\right)\right)$$

the fishing effort that maximizes the profit of the third fisherman for caching the predator population as follow

$$E_{33} = \frac{1}{4}\left(\alpha_1 \frac{\beta_1}{q_3}\left(x_A^* - \frac{c^3}{p_1 q_1}\right) + \frac{\gamma - \alpha_2\beta_2}{q_3}\left(\frac{c^3}{p_2 q_2} - x_B^*\right)\right)$$

6. Numerical Simulation

In this part, we will see the impact of pollution on the profits of fishermen, their fishing effort as well as on the catches made by these fishermen.

According to Figure 8 which represents the evolution of catches according to the rate of pollution, where the bars in blue represent the catches made of the first species, the bars in orange represent the catches made by the fishing fleets of the second species and the bars in gray are the catches of the third species, while the yellow bars are the total catches of the three species made by the fishing fleets.

For the catches of the first species is higher followed by the catches of the second species then the catches of the third species. For the catches of the first species, we note that the catches are almost stable with a small variation which is almost negligible even if the pollution rate increases.

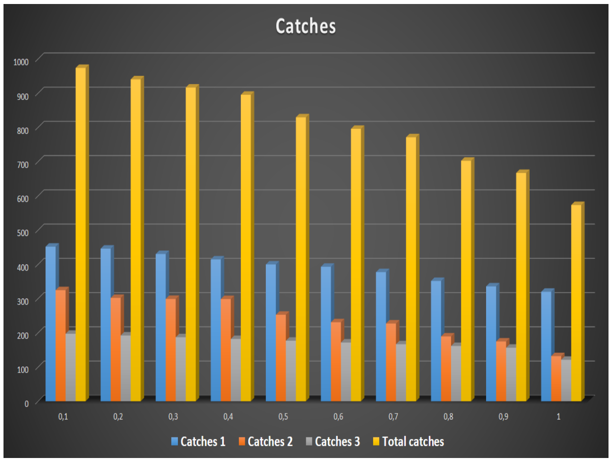

FIGURE 8. Evolution of catches in relation to pollution rate

For the catches of the second species, we notice that from the value 0.1 to the value 0.4 of the pollution rate, the catches are almost stable with a very small decrease and from the pollution rate equal to 0.5 up to the value 1 we notice a large reduction in catches.
For the captures of the third species, we note that from the value 0.1 to the value 0.4 of the pollution rate, the captures are almost stable and from the pollution rate equal to 0.5 to the value 1 we notice a large decrease in the level of catches of this species. Thus, the total catches of these three marine species are also decreasing.

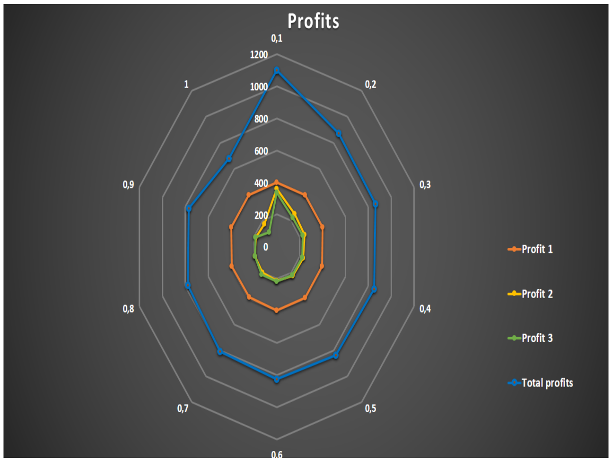

FIGURE 9. Evolution of profits according to the variation of the pollution rate

For the profits, we obtained the results mentioned in Figure 9, where the orange lines represent the profits made by the exploitation of the first species, the yellow line represents the profits made by the exploitation of the second species while the green line the profits made by the exploitation of the third species and the last blue line represents the total profit from the exploitation of the three marine species. For the profits made by the exploitation of the first species and which is in an unpolluted area, we note that the profits are almost the same and are stable. For the profits made by the exploitation of the second species and which is in a polluted area, we notice that from the pollution rate equal to 0.1 to 0.4, a decrease then from the value 0.5 to the value 1 we notice a very large decrease in profits. The same applies to the profits made by the exploitation of the third species; which is the predator of the two previous marine species, we notice at first that the profits are almost stable from the value 0.1 up to the value 0.4 with a very small decrease and after this value 0.4, the profits start to decrease very quickly each time the level of pollution increases.

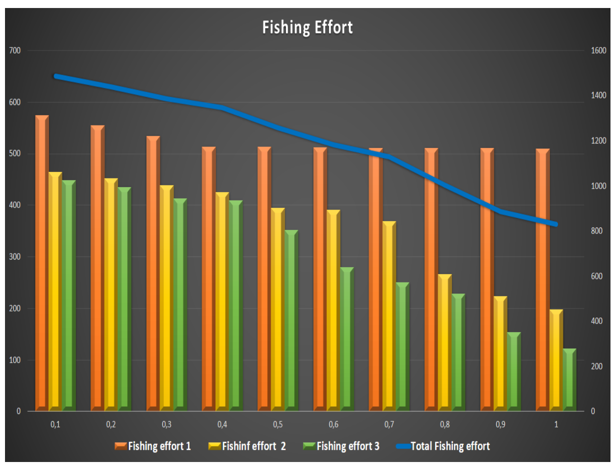

FIGURE 10. Evolution of the fishing effort according to the variation of the pollution rate.

We now move on to the fishing effort deployed by fishermen to capture these three marine species where the orange bars represent the fishing effort deployed to capture the first species, the yellow bars represent the fishing effort deployed to capture the second species and the green bars represent the fishing effort deployed to catch the first species while the blue curve represents the total fishing effort as shown in Figure 10. We note that the fishing effort deployed to catch the first species is almost the same for all values of the pollution rate. For the fishing effort deployed to catch the second species, we notice that from the pollution rate equal to 0.1 up to the value 0.4 the effort was almost stable then from the value 0.5 up to the value 1 we notice a large decrease in fishing effort. Similarly for the fishing effort deployed to capture the third species, we note that the effort was stable from the value 0.1 up to the value 0.4 and after this value we notice a large decrease in the level of the fishing effort. The explanation that can be given for the results obtained in Figures 8, 9 and 10 is that for the first species which is found in an unpolluted environment, the fishing effort, the catches and the profits have not changed since this area is not polluted and this species has no direct interaction with other species found in a polluted area. For the second species which is in a polluted area and which is the prey of the third species, where the latter feeds on the two species the first which is found in an unpolluted area and the second in a polluted area, the second species after a certain higher value this species begins to be reduced by this pollution and even to die which directly affects the predator which is gradually reduced and its biomass too. This reduction in the level of the second and third species thus implies a reduction in the level of fishing effort because pollution makes marine species more vulnerable to capture. This reduction in the level of fishing effort thus implies a reduction in the level of captures and therefore a reduction in the level of captures since this-Pollution has a negative effect on the seabed and the entire ecosystem because it can kill these species or cause diseases that can be transmitted to humans.

7. Conclusion

In this work, we conclude that the effect of pollution on fishery resources is significant, particularly for both polluted and unpolluted marine areas. Our bioeconomic model, which considers the interplay between competition and predation among three fish populations (Sardine, Sardinella, and Shark) and the impact of pollution on fishing effort, catches, fishermen's profits, and biomasses, is proposed as a tool to study this issue. Our results showed the importance of controlling the exploitation of this marine population in ensuring their sustainability, and highlighted the critical role that pollution plays in affecting the mortality rates of fishery resources in both polluted and unpolluted marine areas also has a significant impact on fishing effort, catches, and profits in the fishery industry.

Conflict of Interests

The authors declare that there is no conflict of interests.

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