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NUMERICAL SIMULATIONS OF INLAND FISHERIES MATHEMATICAL MODELS

CHOIRUL BASIR[1,*], ASEP K. SUPRIATNA[2,*], SUKONO[2], JUMADIL SAPUTRA[3]

[1]Doctoral Program of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia

[2]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia

[3]Departement of Economics, Faculty of Business and Social Development, Universiti Malaysia Terengganu, Kuala Nerus 21030, Malaysia

Abstract: This research focuses on numerical simulations of the prey-predator mathematical model in inland fisheries. The problem of excessive harvesting results in unsustainable harvesting so fishermen will spend capital for restocking the fish population. Mathematical models can provide ideal harvesting effort calculations to obtain a sustainable harvesting strategy to reduce the cost of harvesting in the following period. Two mathematical models were created in this paper. The first model considered a condition in which there was no interaction between prey and predator, and the second model considered a condition in which there was interaction between prey and predator. The first model assumed that a juvenile fish population (x1) was introduced and grew into native adult fish (x2) and the native adult fish reproduced spawn fish (x3). The second model behaves like the first but adds a predatory fish population, and assumes it interacts with the introduced juvenile stock fish population (x1) and the spawn fish population (x3). Dynamic analysis of the mathematical models is carried out to obtain the equilibrium points and their stability analysis. Numerical simulations of both models were done using the Euler method to show that the

*Corresponding authors
E-mail addresses: choirul21001@mail.unpad.ac.id, a.k.supriatna@unpad.ac.id
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maximum sustainable yield effort (EMSY) value influences the number of fish populations in inland fisheries because harvesting effort will maintain sustainable harvesting. The simulation suggests that if the harvesting effort is above the 2EMSY value, the fish population will approach the equilibrium point (0,0,0) in the first model and (0,0,0,0) in the second model. However, if the harvesting effort is less than or the same as the 2EMSY value, the fish population will approach the equilibrium point when adult fish and spawn fish exist in both models. In conclusion, determining harvesting efforts following the EMSY value is expected to support sustainable harvesting so that the economy of inland fishery can be sustainable.

Keywords: predator-prey; mathematical model; inland fisheries; MSY; numerical simulations.

2020 AMS Subject Classification: 34C25, 34D20, 92D25, 92D40.

1. INTRODUCTION

Fisheries are an integral part of the blue economy and demand thorough research across multiple scientific disciplines to ensure sustainable management [1], [2], [3], [4], [5], [6]. The study of species interactions within their habitats allows for a deeper understanding of mathematical modeling in sustainable resource management in inland fisheries. In these environments, predator fish species will prey on other fish to survive, resulting in a dynamic predation process where both predator and prey fish coexist [7], [8]. The sizes of both predator and prey populations are influenced by the intensity of predation, which is determined by whether predatory fish have reached their predation threshold and the time required for digestion before engaging in subsequent predation [9], [10], [11], [12].

In a mathematical model, differential equations can provide an overview of population dynamics over time since they represent the dynamics of the number of species populations relative to time variables [13], [14]. The dynamic analysis of the model will also provide a complete understanding, allowing for stability analysis of the produced conditions [15], [16]. Mathematical models can provide information on maximum sustainable yield (MSY) [17], enabling fishermen to determine ideal harvesting conditions to obtain sustainable yield [18], [19], [20]. In a sustainable harvesting condition, fishermen benefit economically by minimizing the need for fish stocking without decreasing the intensity of harvesting [9], [21], [22]. The traditional concept, which includes continuous harvesting without considering the sustainability of future harvests, can be abandoned and replaced with a concept adhering to MSY [17], [23], [25].

Mathematical model requires precise numerical simulations and visualization in order to provide in-depth information on the calculation and visualization process [25], [26]. Numerical simulations may be carried out using Mathematical model requires precise numerical simulations and visualization to provide in-depth information on the calculation and visualization process [25], [26]. Numerical simulations can use different methods, including the Euler, Runge Kutta, and many others. The results from the numerical simulations and visualization can provide alternative options for fishermen to minimize their loss of economically valuable fish.

2. Materials and Methods

The process of developing a mathematical model, the dynamic analysis of the model [27], [28], and numerical simulations using Euler's method were conducted to show and visualize population numbers over time in graphical form.

2.1 Mathematical Model

The first model was constructed in a simplified form engaging introduced juvenile fish stock (x_1), which would then develop into native adult fish (x_2) and reproduce as spawn fish (x_3). Both fishes, the juvenile fish stock (x_1) and the spawn fish (x_3) have the same species but different origins. The introduced juvenile fish stock (x_1) was introduced into the habitat from external sources, while spawn fish (x_3) were the offspring of breeding native adult fish (x_2) within the habitat. Specifically, there was no interaction with predator fish (y) in this first mathematical model, as shown in Figure 1.

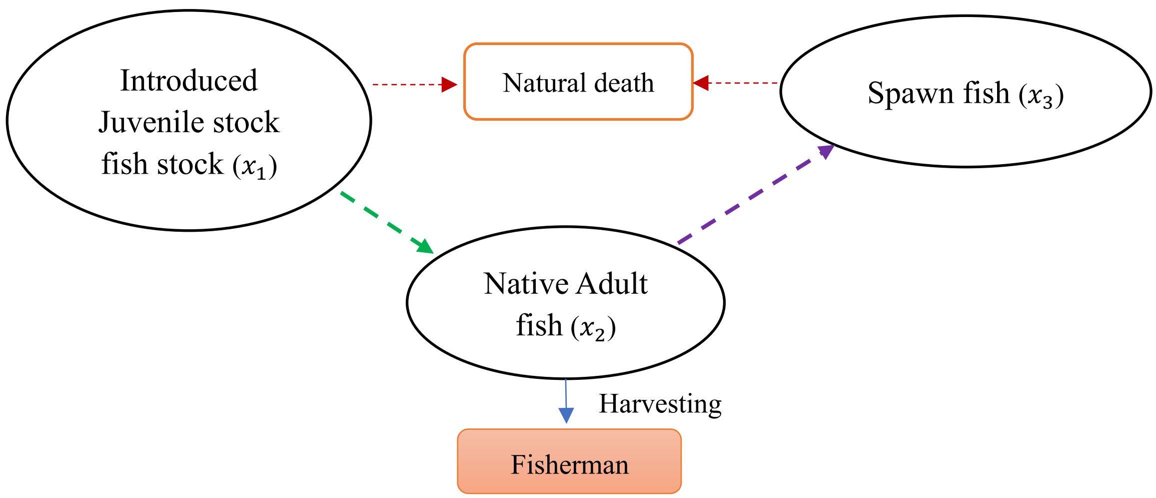


Figure 1. The first model fish species interaction diagram

The first mathematical model covered the harvesting of fish in the reservoir (x_2) without interaction between juvenile stocked fish (x_1) and the fish spawn (x_3) with predatory fish. This scenario could happen in various form, including using a cage system land fisheries to separate prey and predator, thereby preventing interactions [29]. The parameters used included $m, p, r, K,$ and e and they were all positive.

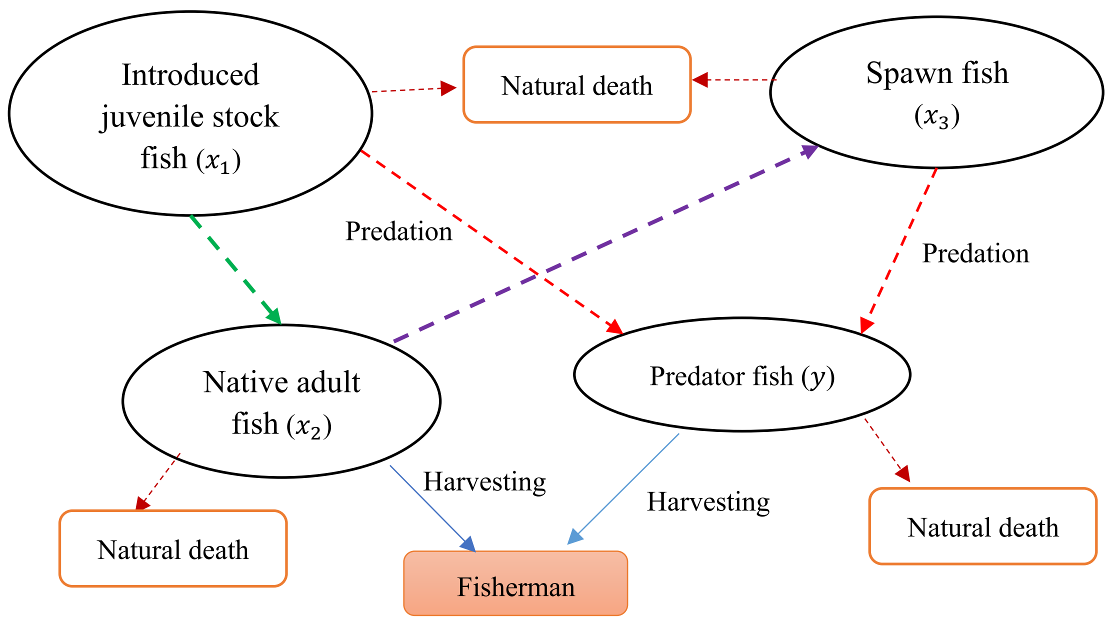


Figure 2. The second model fish species interaction diagram

The second mathematical model consisted of harvesting native adult fish (x_2) and predator fish (y) in the reservoir, with interactions occurring between introduced juvenile fish stock (x_1) and spawn fish (x_3) with predator fish (y). The parameters used consisted of $\beta_x, b_x, m, p, r, K, e, C_x, \mu,$ and δ and they were all positive.

Table 1. Mathematical Model

The First Model	**The Second Model**
$f_1 = \frac{dx_1}{dt} = -mx_1 - px_1$	$g_1 = \frac{dx_1}{dt} = -\frac{\beta_x x_1 y}{1 + b_x x_1} - mx_1 - px_1,$
$f_2 = \frac{dx_2}{dt} = px_3 - \frac{r{x_2}^2}{K} - ex_2 + px_1$	$g_2 = \frac{dx_2}{dt} = px_3 - \frac{r{x_2}^2}{K} - ex_2 + px_1,$
$f_3 = \frac{dx_3}{dt} = rx_2 - mx_3 - px_3.$	$g_3 = \frac{dx_3}{dt} = rx_2 - \frac{\beta_x x_3 y}{1 + b_x x_3} - mx_3 - px_3,$
	$g_4 = \frac{dy}{dt} = \frac{C_x \beta_x x_1 y}{1 + b_x x_1} + \frac{C_x \beta_x x_3 y}{1 + b_x x_3} - \mu y - \delta y^2 - ey.$

As shown in Table 1, the mathematical model assumed that there was only one fish stocking period. To ensure a repeated stocking period, further investigations were required. Therefore, this research aimed to optimize harvesting in order to reduce the stocking period to minimize fishermen's expenses [3].

Table 2. Description of symbols and parameters

Symbol	Description	Unit
x_1	Introduced juvenile stock fish population	Ton
x_2	Native adult fish population	Ton
x_3	Spawn fish population	Ton
y	Predator fish population	Ton
m	Death rate of stocked fish	$time^{-1}$
r	Intrinsic growth rate of fish	$time^{-1}$
K	*Carrying capacity*	Ton
p	Fish stocking rate	$time^{-1}$
e	Fishing rate	$time^{-1}$
β_x	Predation coefficient of stocked fish or fish spawned by predator	$(Ton \times time)^{-1}$
b_x	Half saturation coefficient/predation saturation level in predatory fish	Ton^{-1}
C_x	The conversion coefficient of the number of calories required by predator for each prey	-
μ	Death rate of predatory fish	$time^{-1}$
δ	The rate of predator competition for prey	$time^{-1}$

2.2. EQUILIBRIUM POINTS

The equilibrium points of the first and second mathematical models are presented in Table 1. Since there was only one stocking period, introduced juvenile fish stock (x_1) would be depleted, resulting equilibrium points of $E(0, x_2{}^*, x_3{}^*)$ and $T(0, x_2{}^*, x_3{}^*,\ y^*)$ for the first and second models, respectively. The resulting equilibrium points must meet the conditions of $x_1 \geq 0, x_2 \geq 0,\ x_3 \geq 0$, and $y \geq 0$ [27]. Therefore, the equilibrium point that did not meet the requirements would not be used in this study. It was discovered that the equilibrium points for both the first and second models are the same.

Table 3. The Eligible Equilibrium Points

No.	Mathematical Model	Equilibrium Points	Condition
1.	The First Model	$E_0(0,0,0)$	-
2.	The First Model	$E_1\left(0, K\left(\frac{pr-e(m+p)}{r(m+p)}\right), K\left(\frac{pr-e(m+p)}{(m+p)^2}\right)\right)$	$pr-e(m+p)\geq 0$
3.	The Second Model	$T_0(0,0,0,0)$	-
4.	The Second Model	$T_1\left(0, K\left(\frac{pr-e(m+p)}{r(m+p)}\right), K\left(\frac{pr-e(m+p)}{(m+p)^2}\right), 0\right)$	$pr-e(m+p)\geq 0$

2.3. JACOBIAN MATRIX OF THE EQUILIBRIUM POINTS

The Jacobian matrices of the models are created to determine the stability of the equilibrium points.

Table 4. Jacobian Matrix of Mathematical Models

No.	Mathematical Models	Equilibrium Points
1.	The First Model	$J_{E_0}=\begin{bmatrix}-m-p & 0 & 0\\ p & -e & p\\ 0 & r & -m-p\end{bmatrix}$
2.	The First Model	$J_{E_1}=\begin{bmatrix}-m-p & 0 & 0\\ p & -\frac{2pr}{(m+p)}-3e & p\\ 0 & r & -m-p\end{bmatrix}$
3.	The Second Model	$J_{T_0}=\begin{bmatrix}-m-p & 0 & 0 & 0\\ p & -e & p & 0\\ 0 & r & -m-p & 0\\ 0 & 0 & 0 & -\mu-e\end{bmatrix}$
4.	The Second Model	$J_{T_1}=\begin{bmatrix}-(m+p) & 0 & 0 & 0\\ p & \left(\frac{2pr-3e(m+p)}{(m+p)}\right) & p & 0\\ 0 & r & -(m+p) & 0\\ 0 & 0 & 0 & \frac{C_x\beta_x K(pr-e(m+p))}{(m+p)^2+b_x K(pr-e(m+p))}-\mu-e\end{bmatrix}$

2.4. STABILITY OF THE EQUILIBRIUM POINTS

In both the first and second models, the stability analysis of the equilibrium points were carried out using the Jacobian matrices by investigating the eigenvalues (λ) of the matrices, i.e.

solving the equation $|A - \lambda I| = 0$ [27].

Table 5. The Stability of The Equilibrium points

No.	Mathematical Model	Equilibrium Points	Condition
1.	The First Model	$E_0(0,0,0)$	Stable
2.	The First Model	$E_1\left(0, K\left(\frac{pr - e(m+p)}{r(m+p)}\right), K\left(\frac{pr - e(m+p)}{(m+p)^2}\right)\right)$	Stable
3.	The Second Model	$T_0(0,0,0,0)$	Stable
4.	The Second Model	$T_1\left(0, \frac{prK - eK(m+p)}{r(m+p)}, \frac{prK - eK(m+p)}{(m+p)^2}, 0\right)$	Stable

Definition 2.1 [27]. The stability of an equilibrium point is fulfilled when all the eigenvalues are negative.

For the first mathematical model, at the equilibrium point $E_0(0,0,0)$, the eigenvalues are

$\lambda_1 = -m - p$,

$\lambda_2 = -\frac{1}{2}\left(e + m + p + \sqrt{e^2 - 2em - 2ep + m^2 + 2mp + p^2 + 4pr}\right)$, and

$\lambda_3 = -\frac{1}{2}\left(e + m + p - \sqrt{e^2 - 2em - 2ep + m^2 + 2mp + p^2 + 4pr}\right)$

For the first mathematical model, at the equilibrium point $E_1\left(0, K\left(\frac{pr-e(m+p)}{r(m+p)}\right), K\left(\frac{pr-e(m+p)}{(m+p)^2}\right)\right)$ the eigenvalues are

$\lambda_1 = -m - p$,

$\lambda_2 = \frac{-\left((m+p+3e)(m+p) + 2pr\right) - \sqrt{D^*}}{2(m+p)}$, and

$\lambda_3 = \frac{-\left((m+p+3e)(m+p) + 2pr\right) - \sqrt{D^*}}{2(m+p)}$

with $D^* = \left(m + p + 3e + \frac{2pr}{(m+p)}\right)^2 - 4\left(\frac{2pr}{(m+p)} + 3e\right)(m+p) + 4pr$.

$\lambda_3 = \frac{C_x \beta_x K(pr - e(m+p))}{(m+p)^2 + b_x K(pr - e(m+p))} - \mu - e$, with m, p, μ, and e are positive constants. Hence, the conditions for the stability are $\lambda_1 < 0, \lambda_2 < 0$ and $\lambda_3 < 0$ and:

$e > \frac{2pr}{3(m+p)}$ and $\left(C_x\beta_x K - b_x K(\mu+e)\right)\left(pr - e(m+p)\right) - (\mu+e)(m+p)^2 < 0.$

2.5. MSY

Further, we calculate the MSY value to determine the ideal conditions for sustainable harvesting [17], [23], [25]. Harvesting was only carried out on mature prey populations, i.e., the native adult fish (x_2) and predator fish (y), while introduced juvenile fish stock (x_1) and spawn fish (x_3) were left un-harvested.

Table 6. The Effort and MSY of the Models

No.	Mathematical Models	Equilibrium Points	E_{MSY}	MSY
1.	The First Model	$E_0(0,0,0)$	-	-
2.	The First Model	$E_1\left(0, K\left(\frac{pr-e(m+p)}{r(m+p)}\right), K\left(\frac{pr-e(m+p)}{(m+p)^2}\right)\right)$	$\left(\frac{pr}{2(m+p)}\right)$	$\frac{p^2rK}{4(m+p)^2}$
3.	The Second Model	$T_0(0,0,0,0)$	-	-
4.	The Second Model	$T_1\left(0, K\left(\frac{pr-e(m+p)}{r(m+p)}\right), K\left(\frac{pr-e(m+p)}{(m+p)^2}\right), 0\right)$	$\left(\frac{pr}{2(m+p)}\right)$	$\frac{p^2rK}{4(m+p)^2}$

The E_{MSY} value represented a sustainable harvesting effort to obtain the MSY. It this the effort that maintaining a viable population condition. The calculation of the MSY value E_{MSY} could be followed as outlined in the [30], [31].

3. RESULT AND DISCUSSIONS

The general form of Euler's method was obtained from the first model.

$$x_1(t+0.1) = x_1(t) + \Delta t \times \{-mx_1(t) - px_1(t)\}$$

$$x_2(t+0.1) = x_2(t) + \Delta t \times \left\{px_3(t) - \frac{r\left(x_2(t)\right)^2}{K} - ex_2(t) + px_1(t)\right\}$$

$$x_3(t+0.1) = x_3(t) + \Delta t \times \{rx_2(t) - mx_3(t) - px_3(t)\}$$

The general form of Euler's method was obtained on the second model.

$$x_1(t+0.1) = x_1(t) + \Delta t \times \left\{-\frac{\beta_x x_1(t)y(t)}{1+b_x x_1(t)} - mx_1(t) - px_1(t)\right\}$$

$$x_2(t+0.1) = x_2(t) + \Delta t \times \left\{ px_3(t) - \frac{r\big(x_2(t)\big)^2}{K} - ex_2(t) + px_1(t) \right\}$$

$$x_3(t+0.1) = x_3(t) + \Delta t \times \left\{ rx_2(t) - \frac{\beta_x x_3(t)y(t)}{1+b_x x_3(t)} - mx_3(t) - px_3(t) \right\}$$

$$x_3(t+0.1) = y(t) + \Delta t \times \left\{ \frac{C_x\beta_x x_1(t)y(t)}{1+b_x x_1(t)} + \frac{C_x\beta_x x_3(t)y(t)}{1+b_x x_3(t)} - \mu y(t) - \delta\big(y(t)\big)^2 - ey(t) \right\}$$

3.1 EULER'S METHOD FOR THE FIRST MODEL

The Euler's method was used in the first model to carry out numerical simulations of mathematical model. From the first model, the formula for the Euler's method was derived, starting from $t = 0$. The initial value used was $x_1(0) = 8$, $x_2(0) = 0$, and $x_3(0) = 0$, with the parameters being $p = 8, r = 0.7, m = 0.2$, and $K = 207.522$ in accordance with the research criteria [32]. For the harvest effort value, the value used was $e_{MSY} = 0{,}34$.

Introduced juvenile stock fish population (x_1)

$$x_1(t+0.1) = x_1(t) + \Delta t \times \{-mx_1(t) - px_1(t)\}$$
$$x_1(0+0.1) = x_1(0) + \Delta t \times \{-mx_1(0) - px_1(0)\}$$
$$x_1(0.1) = 8 + (0.1-0) \times \{-(0.2\times 8) - (8\times 8)\}$$
$$x_1(0.1) = 8 + (0.1) \times (-65.6)$$
$$x_1(0.1) = 8 - 6.56$$
$$x_1(0.1) = 1.44$$

Native adult fish population (x_2)

$$x_2(t+0.1) = x_2(t) + \Delta t \times \left\{ px_3(t) - \frac{r\big(x_2(t)\big)^2}{K} - ex_2(t) + px_1(t) \right\}$$

$$x_2(0+0.1) = x_2(0) + \Delta t \times \left\{ px_3(0) - \frac{r\big(x_2(0)\big)^2}{K} - ex_2(0) + px_1(0) \right\}$$

$$x_2(0.1) = 0 + (0.1-0) \times \left\{ (8)(0) - \frac{0.7\times(0)^2}{207.522} - (0.34\times 0) + (8\times 8) \right\}$$

$$x_2(0.1) = 0 + (0.1) \times 64$$
$$x_2(0.1) = 0 + 6.4$$
$$x_2(0.1) = 6.4$$

Spawn fish population (x_3)

$$x_3(t+0.1) = x_3(t) + \Delta t \times \{rx_2(t) - mx_3(t) - px_3(t)\}$$

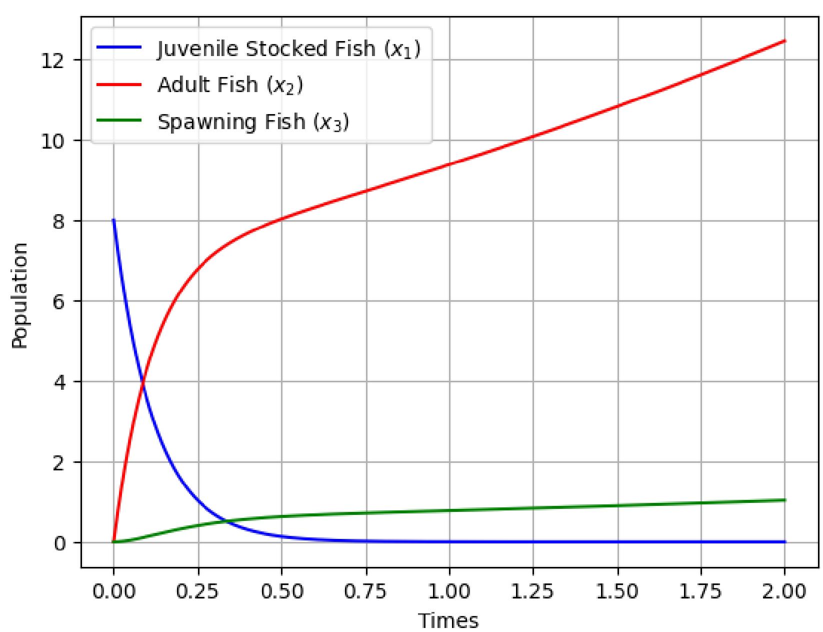
$x_3(0+0.1) = x_3(0) + \Delta t \times \{rx_2(0) - mx_3(0) - px_3(0)\}$

$x_3(0.1) = 0 + (0.1-0) \times \{(0.7 \times 0) - (0.2 \times 0) - (8 \times 0)\}$

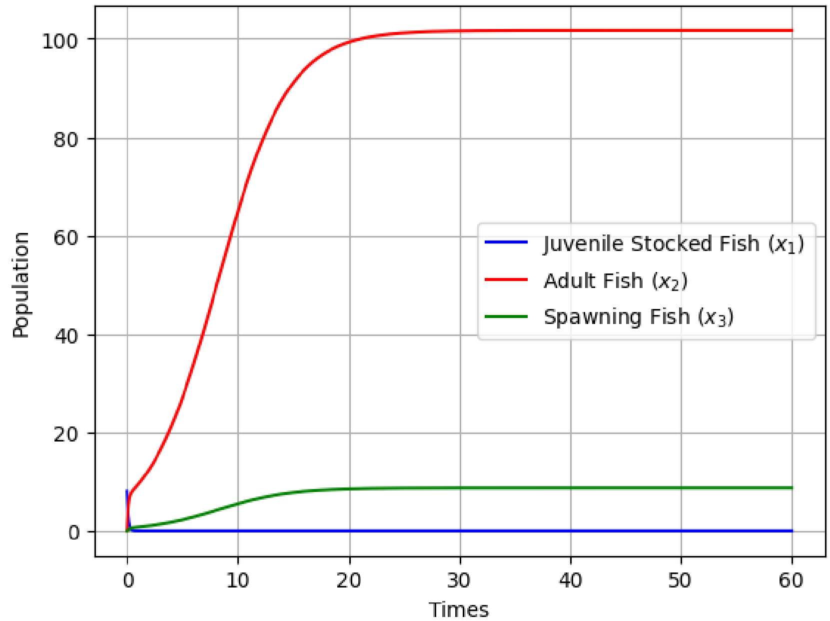
$x_3(0.1) = 0$

Table 7. All population of fish with Euler's Method on the first model

T	$e_{MSY} = \dfrac{pr}{2(m+p)}$	$x_1(t)$	$x_2(t)$	$x_3(t)$
0	0.34	8	0	0
0.1	0.34	1.44	6.40	0
0.2	0.34	0.26	7.10	0.45
0.3	0.34	0.05	7.16	0.58
0.4	0.34	0.01	7.16	0.61
0.5	0.34	0.00	7.14	0.61
0.6	0.34	0.00	7.13	0.61
0.7	0.34	0.00	7.11	0.61
0.8	0.34	0.00	7.10	0.61
0.9	0.34	0.00	7.08	0.61
1	0.34	0.00	7.06	0.60
1.1	0.34	0.00	7.05	0.60
⋮	⋮	⋮	⋮	⋮
⋮	0.34	⋮	⋮	⋮



(a) $0 \leq t \leq 2$



(b) $0 \leq t \leq 60$

Figure 1. Visualization of Numerical simulations of the first model for all populations

The species population in the first mathematical model showed that introduced juvenile stock fish (x_1) population would approach zero. The populations of native adult fish (x_2) and spawn fish (x_3) tended to remain stable at a certain point.

3.2. Euler's Method for Second Model

The Euler's method was used in the first model to carry out numerical simulations of mathematical model. From the second model, the formula for the Euler method was derived, starting from $t = 0$. The initial value used were $x_1(0) = 8$, $x_2(0) = 0$, $x_3(0) = 0$, and $y(0) = 1$, with the parameters being $p = 8, r = 0.7, m = 0.2$, and $K = 207.522$ in accordance with the research requirements [32]. The assumed parameters were $\beta_x = 0.2, b_x = 0.1, C_x = 0.001, \mu = 0.01$, and $\delta = 0.001$, and the harvest effort value used was $e_{MSY} = 0{,}34$.

Introduced Juvenile Stock fish population (x_1)

$$x_1(t + 0.1) = x_1(t) + \Delta t \times \left\{-\frac{\beta_x x_1(t)y(t)}{1 + b_x x_1(t)} - mx_1(t) - px_1(t)\right\}$$

$$x_1(0 + 0.1) = x_1(0) + \Delta t \times \left\{-\frac{\beta_x x_1(0)y(0)}{1 + b_x x_1(0)} - mx_1(0) - px_1(0)\right\}$$

$$x_1(0.1) = 8 + (0.1 - 0) \times \left\{-\frac{(0.2 \times 8 \times 1)}{1 + (0.1 \times 8)} - (0.2 \times 8) - (8 \times 8)\right\}$$

$$x_1(0.1) = 8 + (0.1) \times \{-0.89 - 1.6 - 64\}$$

$x_1(0.1) = 8 + (0.1) \times (-66.49)$

$x_1(0.1) = 8 - 6.65$

$x_1(0.1) = 1.35$

Native adult fish population (x_2)

$$x_2(t+0.1) = x_2(t) + \Delta t \times \left\{ px_3(t) - \frac{r\big(x_2(t)\big)^2}{K} - ex_2(t) + px_1(t) \right\}$$

$$x_2(0+0.1) = x_2(0) + \Delta t \times \left\{ px_3(0) - \frac{r\big(x_2(0)\big)^2}{K} - ex_2(0) + px_1(0) \right\}$$

$$x_2(0.1) = 0 + (0.1-0) \times \left\{ (8)(0) - \frac{0.7 \times (0)^2}{207.522} - (0.34 \times 0) + (8 \times 8) \right\}$$

$x_2(0.1) = 0 + (0.1) \times 64$

$x_2(0.1) = 0 + 6.40$

$x_2(0.1) = 6.40$

Spawn fish population (x_3)

$$x_3(t+0.1) = x_3(t) + \Delta t \times \left\{ rx_2(t) - \frac{\beta_x x_3(t) y(t)}{1 + b_x x_3(t)} - mx_3(t) - px_3(t) \right\}$$

$$x_3(0+0.1) = x_3(0) + \Delta t \times \left\{ rx_2(0) - \frac{\beta_x x_3(0) y(0)}{1 + b_x x_3(0)} - mx_3(0) - px_3(0) \right\}$$

$$x_3(0.1) = 0 + (0.1-0) \times \left\{ (0.7 \times 0) - \frac{(0.2 \times 0 \times 1)}{1 + (0.1 \times 0)} - (0.2 \times 0) - (8 \times 0) \right\}$$

$x_3(0.1) = 0$

Predator fish population (y)

$$x_3(t+0.1) = y(t) + \Delta t \times \left\{ \frac{C_x \beta_x x_1(t) y(t)}{1 + b_x x_1(t)} + \frac{C_x \beta_x x_3(t) y(t)}{1 + b_x x_3(t)} - \mu y(t) - \delta\big(y(t)\big)^2 - ey(t) \right\}$$

$$x_3(0+0.1) = y(0) + \Delta t \times \left\{ \frac{C_x \beta_x x_1(0) y(0)}{1 + b_x x_1(0)} + \frac{C_x \beta_x x_3(0) y(0)}{1 + b_x x_3(0)} - \mu y(0) - \delta\big(y(0)\big)^2 - ey(0) \right\}$$

$$x_3(0.1) = 1 + (0.1-0) \times \left\{ \frac{0.0016}{1.8} + 0 - (0.01 \times 1) - 0.001(1)^2 - (0.34 \times 1) \right\}$$

$x_3(0.1) = 1 + (0.1) \times (0.00089 + 0 - 0.01 - 0.001 - 0.34)$

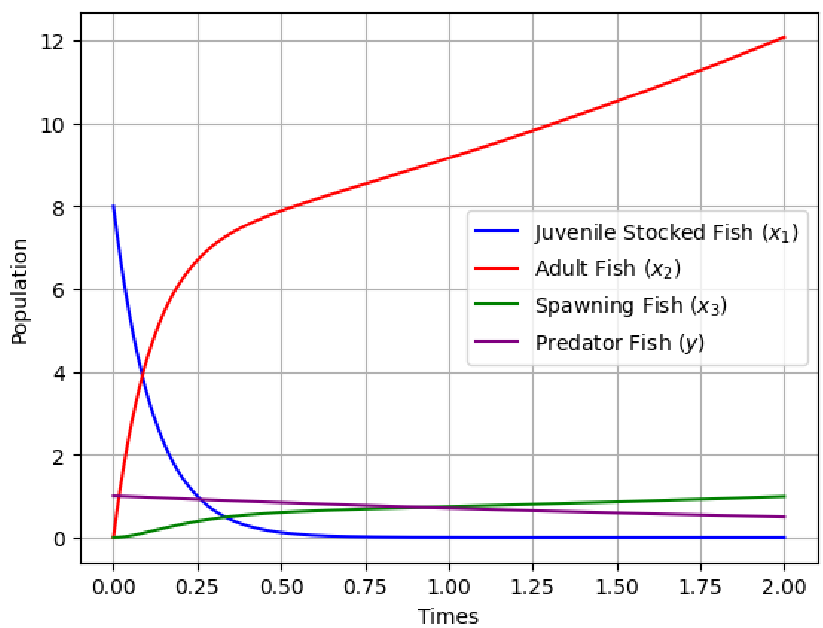
$x_3(0.1) = 1 - 0.035$

$x_3(0.1) = 0.965$

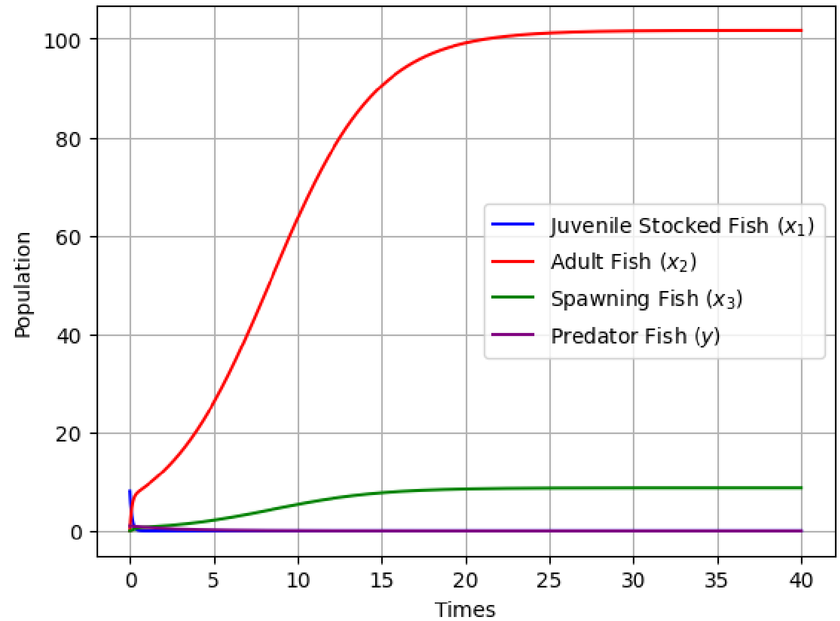
Numerical calculations using the Euler's method were shown in Table 8.

Table 8. The all population of fish with Euler's Method on the second model

T	$e_{MSY} = \dfrac{pr}{2(m+p)}$	$x_1(t)$	$x_2(t)$	$x_3(t)$	$y(t)$
0	0.34	8	0	0	1
0.1	0.34	1.40	6.40	0.00	0.96
0.2	0.34	0.24	7.07	0.45	0.93
0.3	0.34	0.04	7.12	0.57	0.90
0.4	0.34	0.01	7.11	0.59	0.87
0.5	0.34	0.00	7.08	0.59	0.84
0.6	0.34	0.00	7.06	0.59	0.81
0.7	0.34	0.00	7.04	0.59	0.78
0.8	0.34	0.00	7.02	0.59	0.75
0.9	0.34	0.00	7.00	0.59	0.73
1	0.34	0.00	6.97	0.59	0.70
1.1	0.34	0.00	6.95	0.59	0.68
⋮	⋮	⋮	⋮	⋮	⋮
⋮	0.34	⋮	⋮	⋮	⋮



(a) $0 \leq t \leq 2$



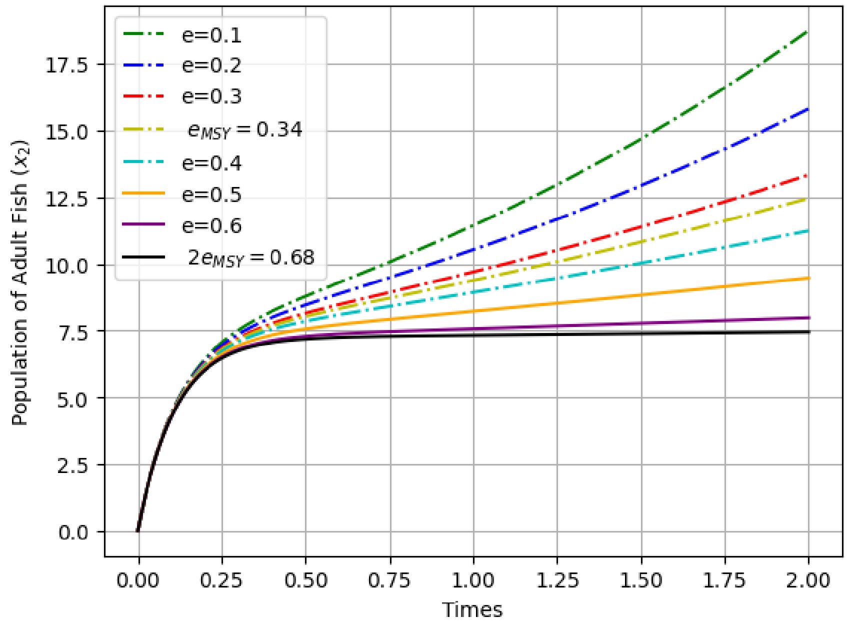
(b) $0 \leq t \leq 40$

Figure 2. Numerical simulations second model of the all population

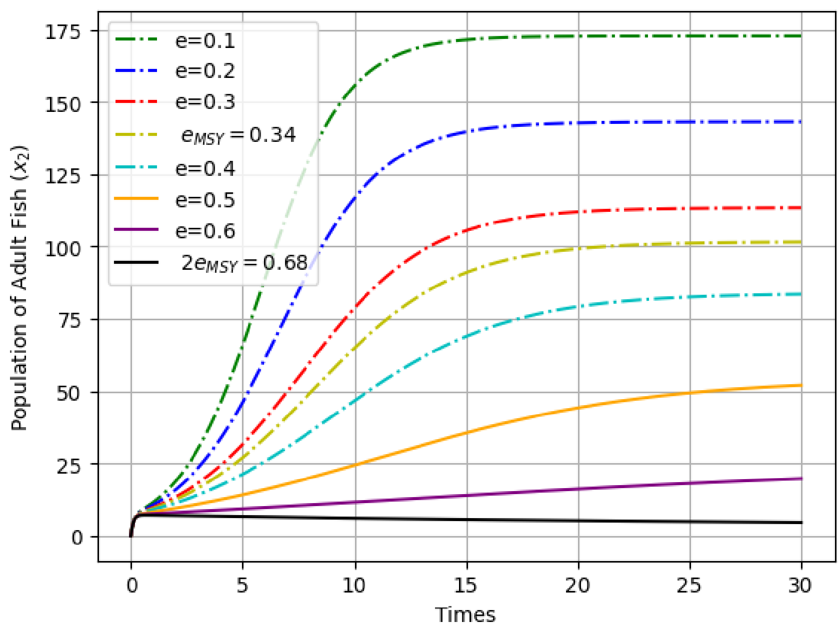
The species population in the first mathematical model showed that introduced juvenile stock fish (x_1) and predator fish (y) would approach zero. The populations of native adult fish (x_2) and spawn fish (x_3) tended to remain stable at a certain point.

3.3. NUMERICAL SIMULATIONS ON EFFORT OF HARVESTING

In this numerical simulation, a numerical simulation will be displayed based on harvesting effort. Harvesting when following the e_{MSY} value, below and above the e_{MSY} value will be visible in the visualization of the numerical simulation results. The two mathematical models created produce two similar conditions, namely the existing population is a population of adult fish (x_2) and spawning fish (x_3) so that for visualization the population is displayed on the fish population. If the harvesting effort used is less than $2e_{MSY}$ then the population will follow the equilibrium point with the existing population, but if the harvesting effort is more than $2e_{MSY}$ then the population will follow the equilibrium point with the population going towards zero.



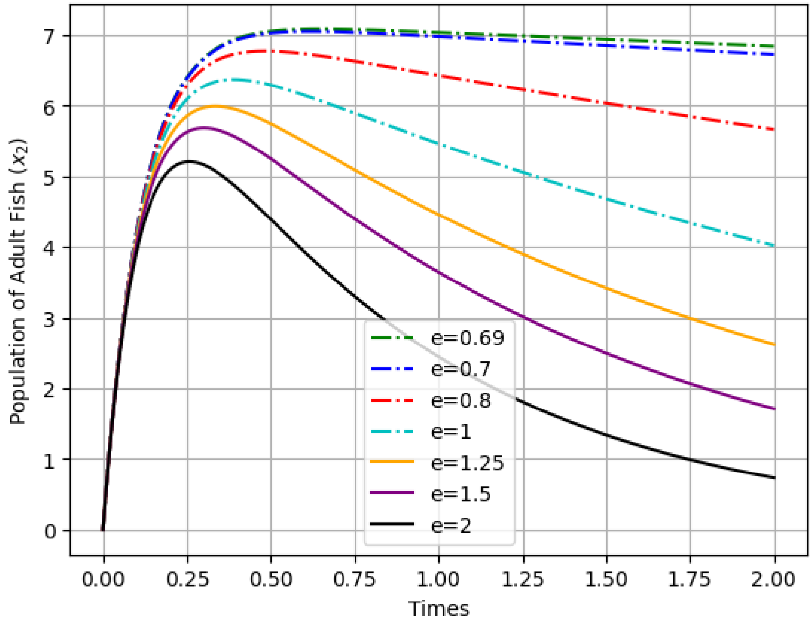
(a) $0 \leq t \leq 2; 0.1 \leq e \leq 0.68$



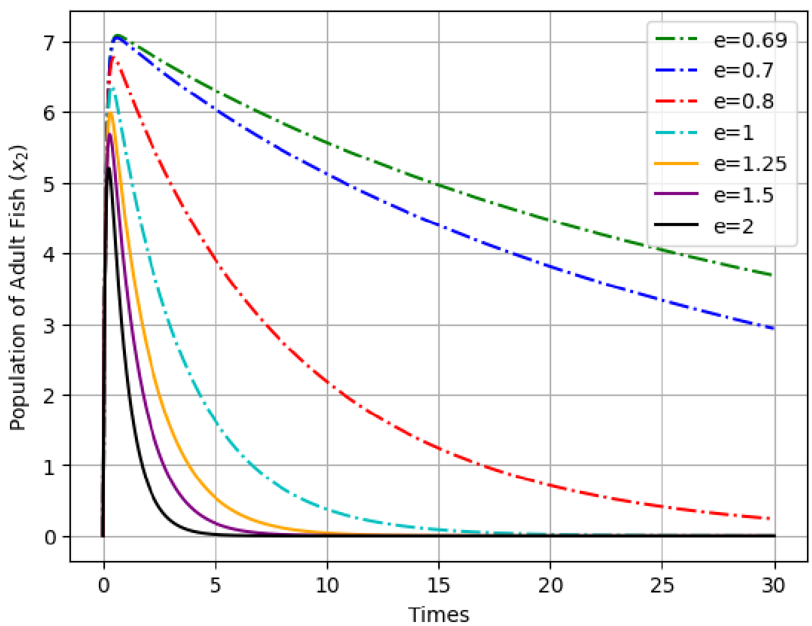
(b) $0 \leq t \leq 30; 0.1 \leq e \leq 0.68$

Figure 3. Numerical simulations population of native adult fish (x_2)

In figure 3, fish harvesting when it is less than e_{MSY} tends to stabilize the population of native adult fish (x_2). This condition results in the population of native adult fish (x_2) remaining in the system. Visualization shows that at a value of $0.1 \leq e \leq 0.68$ the trend of the graph will follow the equilibrium point $E_1(0, x_2{}^*, x_3{}^*)$ or the equilibrium point $T_1(0, x_2{}^*, x_3{}^*, 0)$.



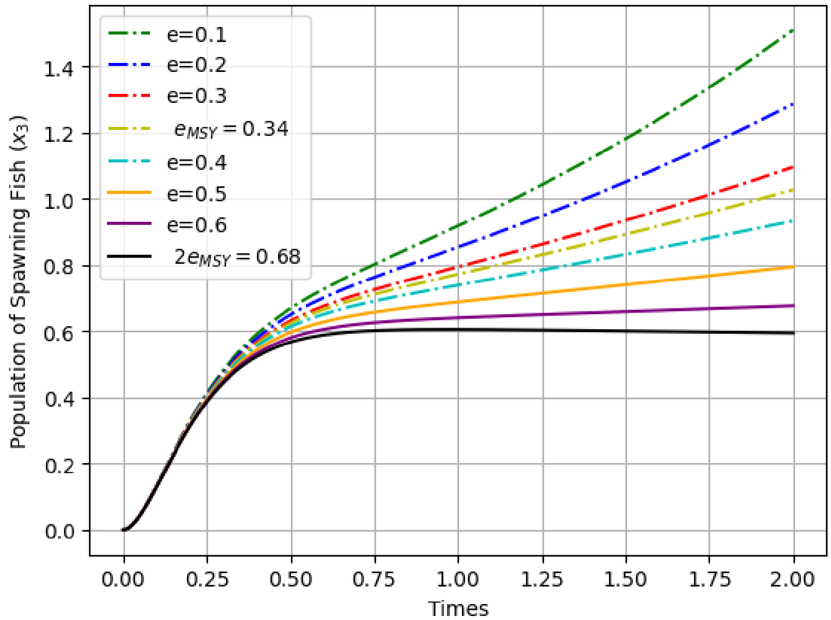
(a) $0 \leq t \leq 2; 0.69 \leq e \leq 2$



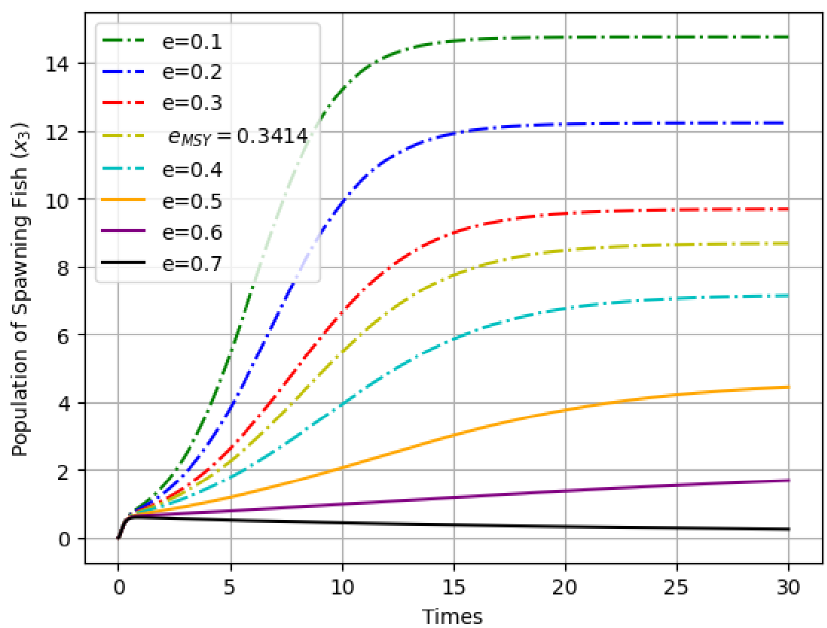
(b) $0 \leq t \leq 30; 0.69 \leq e \leq 2$

Figure 4. Numerical simulations population of native adult fish (x_2)

The population of native adult fish (x_2) was reduced to zero, ensuring the population was depleted in the system. The condition was not expected because it would continuously require introduced juvenile stock fish (x_1).



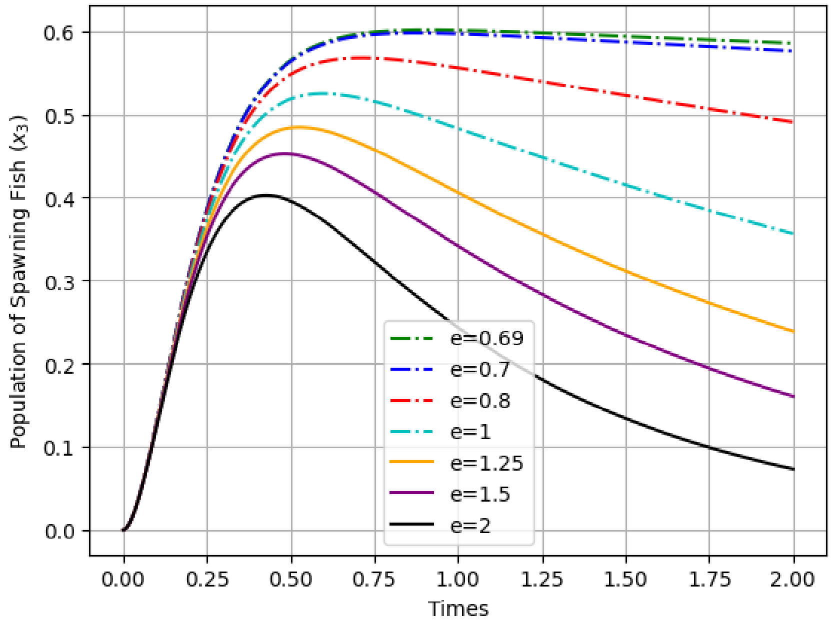
(a) $0 \leq t \leq 2; 0.1 \leq e \leq 0.68$



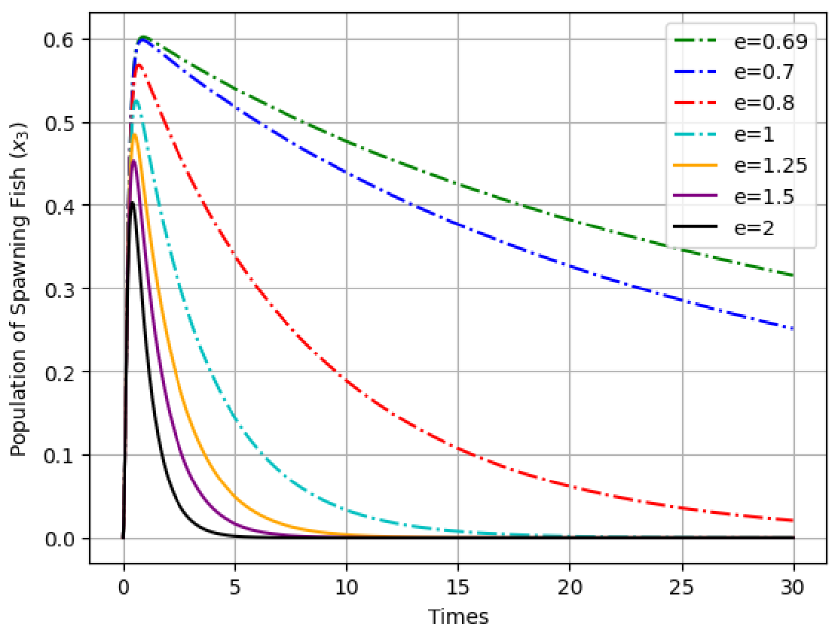
(b) $0 \leq t \leq 30; 0.1 \leq e \leq 0.7$

Figure 5. Numerical simulations population of spawn fish (x_3)

In figure 5, fish harvesting when it is less than $2e_{MSY}$ tends to stabilize the population of spawning fish (x_3). This condition results in the population of spawning fish (x_3) remaining in the system.



(a) $0 \leq t \leq 2; 0.69 \leq e \leq 2$



(b) $0 \leq t \leq 30; 0.69 \leq e \leq 2$

Figure 6. Numerical simulations population of spawn fish (x_3)

The population of spawn fish (x_3) was reduced to zero, ensuring that the population would run out in the system. The condition was not expected because it would further require introduced juvenile stock fish (x_1).

4. CONCLUSIONS

Numerical simulations in this research show that the effort for maximum sustainable yield value (E_{MSY}) produced from the two mathematical models has the same effect on sustainable harvesting. Harvesting effort above the value of $2E_{MSY}$ will make the population decline and approach the zero equilibrium point. Meanwhile, if the harvesting effort is less than or the same as 2EMSY, then the adult fish and spawn fish populations will continue to exist. In this case, the adult fish population can be harvested at the level of the MSY, while the spawn fish population will still develop into adult fish without re-stocking. This condition generates the sustainable harvesting that we are looking for. The model here assumed that prey stocking takes place only once. The effect of repeated stocking is worth analyzing. Among other directions of studies in the future are the effect of recruitment delay time to maturity [33] and the inclusion of fleet size of ships in the harvesting effort [34], [35].

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CONFLICT OF INTERESTS

The authors declared that there was no conflict of interest.

REFERENCES

[1] C.W. Clark, The worldwide crisis in fisheries: economic models and human behavior, Cambridge University Press, 2006.

[2] C. Basir, A.K. Supriatna, Sukono, et al. Prey–predator mathematics model for fisheries insurance calculations in the search of optimal strategies for inland fisheries management: a systematic literature review, Sustainability 15 (2023), 12376. https://doi.org/10.3390/su151612376.

[3] C. Vaillant-Tenzer, Mathematics of sustainable and profitable management of fisheries resources, preprint, (2021). https://doi.org/10.31219/osf.io/gfney.

[4] S. De Silva, S. Funge-Smith, A review of stock enhancement practices in the inland water fisheries of Asia, Deakin University, (2005). https://hdl.handle.net/10536/DRO/DU:30010336.

[5] L. Abraham, K. Barnett, M.L. Baskett, et al. Effects of fishing, species interactions, and climate on populations and communities: insights for ecosystem-based fisheries management, Dissertation, 2003.

[6] L. Jiménez-Badillo, "Application of holistic and analytical models for the management of tilapia fisheries in reservoirs, Hidrobiológica, 14 (2004), 61-68.

[7] T.K. Kar, B. Ghosh, Bifurcations and feedback control of a stage-structure exploited prey-predator system, Int. J. Environ. Sci. Technol. 2 (2010), 131–141.

[8] H. Herwartz, Modelling interaction patterns in a predator-prey system of two freshwater organisms in discrete time: an identified structural VAR approach, Stat. Methods Appl. 31 (2022), 63–85. https://doi.org/10.1007/s10260-021-00564-8.

[9] A. Supriatna, Optimal harvesting for a predator–prey metapopulation, Bull. Mathe. Biol. 60 (1998), 49–65. https://doi.org/10.1006/bulm.1997.0005.

[10] A.K. Supriatna, H.P. Possingham, Harvesting a two-patch predator-prey metapopulation, Nat. Resour. Model. 12 (1999), 481–498. https://doi.org/10.1111/j.1939-7445.1999.tb00023.x.

[11] D. Das, T.K. Kar, Marine reserve and its consequences in a predator-prey system for ecotourism and fishing, Int. J. Math. Model. Numer. Optim. 11 (2021), 20-36. https://doi.org/10.1504/IJMMNO.2021.111720.

[12] B. Ghosh, T.K. Kar, Sustainable use of prey species in a prey–predator system: Jointly determined ecological thresholds and economic trade-offs, Ecol. Model. 272 (2014), 49–58. https://doi.org/10.1016/j.ecolmodel.2013.09.013.

[13] E. Kralles, Mathematical modeling of fish populations in Lake Ontario using differential equations, Master's Theses, State University of New York, 2018.

[14] D.S. Jones, M. Plank, B.D. Sleeman, Differential equations and mathematical biology, Chapman and Hall/CRC, 2009. https://doi.org/10.1201/9781420083583.

[15] R.N. Clark, Control system dynamics, Cambridge University Press, Cambridge, 1996.

[16] D.K. Arrowsmith, C.M. Place, Dynamical systems, Springer, Dordrecht, 1992. https://doi.org/10.1007/978-94-011-2388-4.

[17] A.K. Supriatna, Maximum sustainable yield for marine metapopulation governed by coupled generalised logistic equations, J. Sustain. Sci. Manag. 7 (2012), 201–206.

[18] S.A. Levin, T.G. Hallam, L.J. Gross, eds., Applied Mathematical Ecology, Springer, Berlin, Heidelberg, 1989. https://doi.org/10.1007/978-3-642-61317-3.

[19] C.S. Chou, A. Friedman, Introduction to mathematical biology, Springer, Cham, 2016.

https://doi.org/10.1007/978-3-319-29638-8.

[20] M.H.A. Biswas, M.R. Hossain, M.K. Mondal, Mathematical modeling applied to sustainable management of marine resources, Procedia Eng. 194 (2017), 337–344. https://doi.org/10.1016/j.proeng.2017.08.154.

[21] A. Daci, Fish harvesting models and their applications in a reservoir in Saranda, Albania, J. Multidiscip. Eng. Sci. Technol. 3 (2016), 5279-5282.

[22] M.F. Laham, I.S. Krishnarajah, J.M. Shariff, Fish harvesting management strategies using logistic growth model, Sains Malays. 41 (2012), 171-177.

[23] T.K. Kar, B. Ghosh, Impacts of maximum sustainable yield policy to prey–predator systems, Ecol. Model. 250 (2013), 134–142. https://doi.org/10.1016/j.ecolmodel.2012.11.015.

[24] H. Husniah, A.K. Supriatna, Marine biological metapopulation with coupled logistic growth functions: The MSY and quasi MSY, AIP Conf. Proc. 1587 (2014), 51–56. https://doi.org/10.1063/1.4866532.

[25] M. Thrusfield, R. Christley, H. Brown, et al. Mathematical modelling, in: Veterinary Epidemiology, Wiley, 2018. https://doi.org/10.1002/9781118280249.ch23.

[26] R.W. Shonkwiler, J. Herod, Mathematical biology: an introduction with maple and Matlab, Springer, New York, 2009. https://doi.org/10.1007/978-0-387-70984-0.

[27] L. Perko, Differential equations and dynamical systems, Springer, New York, 2001. https://doi.org/10.1007/978-1-4613-0003-8.

[28] C.W. Clark, M. Mangel, eds., Dynamic state variable models in ecology: methods and applications, Oxford University Press, New York, 2000.

[29] B. Ghosh, D. Pal, T.K. Kar, et al. Biological conservation through marine protected areas in the presence of alternative stable states, Math. Biosci. 286 (2017), 49–57. https://doi.org/10.1016/j.mbs.2017.02.004.

[30] M.B. Schaefer, A study of the dynamics of the fishery for yellowfin tuna in the eastern tropical Pacific Ocean. Bull. Inter-Amer. Trop. Tuna Comm. 2 (1957), 245–285.

[31] M. Schaefer, Some aspects of the dynamics of populations important to the management of the commercial marine fisheries, Bull. Math. Biol. 53 (1991), 253–279. https://doi.org/10.1016/S0092-8240(05)80049-7.

[32] S. Triharyuni, D. Aldila, A. Aisyah, et al. Model penebaran ikan nila di Waduk Malahayu, Brebes, Jawa Tengah, J. Penelit. Perikan. Indones. 25 (2019), 161. https://doi.org/10.15578/jppi.25.3.2019.161-168.

[33] A.K. Supriatna, G.N. Tuck, H.P. Possingham, On the exploitation of a two-patch metapopulation with delayed juvenile recruitment and predation, J. Indones. Math. Soc. 8 (2003), 139-150.

[34] H. Husniah, A.K. Supriatna, Optimal number of fishing fleet for a sustainable fishery industry, in: 2015 International Conference on Technology, Informatics, Management, Engineering & Environment (TIME-E), IEEE, Samosir, Toba Lake, Indonesia, 2015: pp. 18–24. https://doi.org/10.1109/TIME-E.2015.7389741.

[35] H. Husniah, A.K. Supriatna, Optimal number of fishing fleet for a sustainable fishery industry with a generalized logistic production function, in: 2015 International Conference on Industrial Engineering and Systems Management (IESM), IEEE, Seville, Spain, 2015: pp. 546–554. https://doi.org/10.1109/IESM.2015.7380211.