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IMPROVING STATISTICAL PROCESS CONTROL FOR WATER QUALITY IN CATFISH FARMING WITH ROBUST EWMA AND ALTERNATIVE ESTIMATORS

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Abstract: Water pH plays a crucial role in catfish (Clarias sp.) aquaculture, as pH imbalance can disrupt dissolved oxygen levels, weaken the fish's immune system, and affect overall growth. Continuous monitoring is essential to maintain optimal water quality. The Exponentially Weighted Moving Average (EWMA) control chart is commonly used for pH monitoring, but its performance deteriorates when data exhibit non-normality. We propose a robust EWMA control chart incorporating interquartile range (IQR) and Biweight estimators to address this limitation and enhance sensitivity in non-normal environments. This study analyses pH data from catfish nursery tanks at the Bina Bersama Fishery Group in Makassar, Indonesia, collected from May 28, 2024, to June 27, 2024, across 10 samples. We evaluate the control charts using different smoothing parameters ($\lambda = 0.01$, 0.5, and 1) and assess their performance based on Average Run Length (ARL). The results indicate that the Robust EWMA control chart with the IQR estimator outperforms the Biweight estimator, achieving an ARL of 1 at a more minor process shift (0.2) compared to 0.5 for Biweight. The optimal configurations are obtained at $\lambda = 0.01$, with $\mu_0 = 7.8287$ and $\sigma = 0.039$ for IQR, and $\sigma = 0.055$ for Biweight. These findings suggest that the IQR-based Robust EWMA control chart is more effective in detecting small shifts in pH, providing a superior monitoring tool for ensuring stable water quality in catfish aquaculture.

Keywords: robust EWMA; interquartile range; biweight; water quality monitoring; catfish aquaculture; control chart.
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1. INTRODUCTION

Process quality is a key determinant of business competitiveness in today's globalized economy. Implementing Total Quality Management (TQM) ensures high-quality standards while enhancing productivity and customer loyalty [1]. Research has demonstrated that quality improvement is directly linked to company performance, which in turn drives customer satisfaction and business sustainability [2]. Various statistical quality control techniques have been developed to monitor and maintain process quality, among which control charts serve as a fundamental tool for detecting variations in production processes [3].

The Shewhart control chart is one of the most widely used statistical process control methods for detecting large shifts in process parameters [4]. However, its limited sensitivity to small process changes makes it less effective in certain industrial settings. To address this limitation, the Exponentially Weighted Moving Average (EWMA) control chart has been introduced, which enhances sensitivity by incorporating exponentially weighted averages of past observations [5]. This approach allows for the early detection of small deviations in process parameters, making it a preferred choice for continuous process monitoring. Despite its advantages, the EWMA control chart relies on the assumption of normality, making it vulnerable to outliers and deviations from standard distributional assumptions.

In real-world industrial applications, data often exhibit non-normal distributions and contain outliers, which can compromise the effectiveness of conventional control charts such as Shewhart and EWMA [6]. The presence of outliers can lead to misleading conclusions, reducing the accuracy of process monitoring efforts. To address these challenges, the Robust Exponentially Weighted Moving Average (Robust EWMA) control chart has been developed as an alternative. This method retains the fundamental structure of EWMA while incorporating robust estimators to mitigate the effects of outliers, thereby providing more reliable performance in monitoring non-normally distributed data.

The effectiveness of the Robust EWMA control chart largely depends on the choice of robust estimators. Among the various estimators available, the Interquartile Range (IQR) and Biweight estimators have been widely recognized for their ability to improve process monitoring accuracy [7]. The IQR estimator is particularly useful in handling moderate outliers in near-normal data distributions, whereas the Biweight estimator is more effective in dealing with extreme outliers and highly variable distributions [7]. By leveraging these estimators, the Robust EWMA control chart enhances the resilience of process monitoring against irregularities in industrial data.

While statistical quality control methods are extensively used in manufacturing, they have also been applied in healthcare, environmental management, and agriculture [8]. One critical application is water quality monitoring in aquaculture, particularly in catfish farming, where water pH stability plays a crucial role in fish health, growth, and overall productivity [9]. Water pH imbalance can lead to physiological stress, metabolic inefficiencies, and increased susceptibility to disease, ultimately affecting yield quality and profitability [10]. The Bina Bersama Fisheries Group, located in Bumi Tamalanrea Permai (BTP), Makassar, is actively engaged in catfish farming, supplying fish to various local food businesses. However, water pH in cultivation ponds often fluctuates due to environmental factors such as weather changes, raw water quality, and pollution, making standard EWMA control charts ineffective in consistently detecting water quality deviations. Given the high variability and frequent occurrence of outliers in pH data, a Robust EWMA control chart presents a more effective solution for ensuring optimal water quality in aquaculture.

Several studies have explored the robustness of the Robust EWMA control chart in handling data distribution abnormalities and outliers. Prior research has demonstrated that Robust EWMA is effective in maintaining sensitivity to process shifts while mitigating the effects of non-normality [10]. Furthermore, studies comparing different robust estimators in EWMA control charts have found that robust methods improve detection accuracy for out-of-control conditions [11]. Notably, [7] showed that the Biweight estimator provides faster detection of process anomalies while maintaining stability in complex data conditions. Despite these advancements, limited research has focused on applying the Robust EWMA control chart to water quality monitoring in aquaculture. Given that pH fluctuations significantly impact fish health and farm productivity, there is a pressing need to develop a more resilient monitoring system to enhance decision-making in fish farming operations.

This study aims to analyze the application of the Robust EWMA control chart in monitoring water pH stability in catfish farming ponds. Specifically, this research seeks to develop a Robust EWMA model incorporating IQR and Biweight estimators for monitoring non-normally distributed pH data, evaluate the performance of the Robust EWMA control chart using Average Run Length (ARL) metrics to assess its effectiveness in detecting process variations, and compare the sensitivity of different robust estimators (IQR vs. Biweight) in improving the monitoring of pH fluctuations in Bina Bersama Fisheries Group's catfish ponds in Makassar. By applying the Robust EWMA method, this study aims to improve water quality monitoring, ensuring optimal conditions for catfish farming. The results are expected to contribute not only to quality control research in aquaculture but also to broader applications of robust statistical process control methods.

2. PRELIMINARIES

The data used in this study are primary data from observations at the Bina Bersama Fisheries Group in Makassar City. Observations were made from May 28, 2024 - June 27, 2024. The data consisted of 10 samples of catfish nursery tanks with 30 observations. The research variable used in this study is the pH content of catfish cultivation water with daily sampling. The structure of the research data used is shown in Table 1. In addition, the robust estimators used in the REWMA control chart are IQR and Biweight. After determining the estimator used, the next step is to determine the value of the weighting parameter. This study's REWMA control chart weighting parameters are λ=0.01,0.5, and 1. Furthermore, the REWMA control chart control limit constant value is set at L=3 to ensure appropriate control limits.

Statistical Process Control (SPC) has undergone significant development and is widely applied in various industrial sectors to maintain the stability and consistency of production processes. Conceptually, a control chart is a statistical tool used to monitor the performance of a process based on the measurement of certain variables, such as mean or variance, with the primary objective of identifying random or causally determined process variations. Therefore, control charts not only facilitate the identification of problems in the process and enable the implementation of corrective actions before the problem develops further but also play an essential role in improving operational efficiency by reducing unwanted variations [13].

TABLE I

DATA ON THE PH CONTENT OF CATFISH FARMING WATER

Subgrup	**Nursery basin sample (j)**									
(i)	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	8	7.9	7.7	8	6.7	8	7.9	8.2	8	8
2	7.9	7.85	7.9	8	7.9	7.9	7.4	7.95	8.15	7.9
3	7.9	7.7	7.8	8	6.8	8	7.9	8.3	8.1	7.9
4	7.9	7.8	7.8	7.95	7.9	7.9	7.4	7.9	8.15	7.9
5	8	7.8	7.7	8	6.8	7.9	7.9	8.1	8	8
6	7.85	7.8	7.85	8	7.9	7.9	7.4	7.95	8.15	7.85
7	7.85	7.85	7.8	8	7.9	7.9	7.4	8	8.15	7.85
8	7.9	7.9	7.75	8	7.9	7.9	7.35	8	8.15	7.9
9	7.85	7.85	7.9	8	7.9	7.95	7.4	8	8.15	7.85
10	6.8	7.9	7.7	8	6.8	7.9	7.9	8	8	6.8
11	7.9	7.7	7.9	8	6.4	7.9	8	8.1	8	7.9
12	7.9	7.85	7.75	8	7.9	7.9	7.4	7.95	8.15	7.9
13	7.85	7.85	7.85	7.95	7.9	7.9	7.4	7.95	8.15	7.85
14	7.8	7.85	7.8	7.95	7.9	7.9	7.4	7.95	8.15	7.8
15	7.8	7.8	7.8	7.9	6.8	7.8	7.8	6.8	7.9	7.8
16	7.9	7.8	7.8	7.9	6.8	7.8	7.9	8.1	8	7.9
17	8	7.7	7.9	8	6.6	7.8	7.9	8.2	8	8
18	7.9	7.9	7.9	8	7.9	7.9	7.3	7.95	8.15	7.9
19	7.2	7.8	7.9	7.9	7.2	7.9	7.9	8.2	8.1	7.2
20	7.9	7.8	7.8	7.9	7.9	7.9	7.4	7.9	8.1	7.9
21	7.9	7.9	7.8	8	7.9	7.9	7.4	7.95	8.15	7.9
22	7.9	7.8	7.8	7.9	6.8	7.9	7.9	6.8	8	7.9
23	6.7	7.8	7.8	7.9	6.8	7.8	7.8	8.1	8	6.7
24	7.9	7.85	7.8	7.95	7.9	7.95	7.4	8	8.1	7.9
25	7.8	7.85	7.8	7.9	7.9	7.9	7.35	7.85	8.15	7.8
26	8	7.8	7.8	7.9	6.8	7.8	7.8	8.2	8	8
27	8	7.9	7.9	8	6.8	7.9	8	8.2	8.1	8
28	7.8	7.8	7.8	7.95	7.9	7.9	7.4	7.85	8.15	7.8
29	6.6	7.8	7.8	7.9	6.8	8	7.8	8.4	7.9	6.6
30	7.9	7.9	7.9	8	6.6	8	7.9	8.4	8	7.9

1) *Normality Test:* Normality testing is an important stage in statistical analysis to determine whether data follows a normal distribution. One frequently used method is the Kolmogorov-Smirnov test, which compares the empirical cumulative distribution of the sample with the theoretical cumulative distribution. This test is based on the maximum difference between the sample cumulative distribution and the hypothesized distribution, as described by Kolmogorov in his important paper in 1933 [14].

The statistic used in this test is D_n, Which is calculated as the maximum value of the absolute difference between the empirical cumulative distribution $(S_n(x))$ and the theoretical cumulative distribution $(F_0(x))$ with the D_n value compared to a critical value to determine whether the data comes from the hypothesized distribution [15]. If D_nIf greater than the critical value, the null hypothesis that the data follows a normal distribution will be rejected [16]. The formula used in the Kolmogorov-Smirnov test is as follows:

$$D_n = \underset{-\infty < x < \infty}{sup} |S_n(x) - F_0(x)| \tag{1}$$

where $S_n(x)$ is the empirical cumulative distribution of the sample, $F_0(x)$ is the hypothesized theoretical cumulative distribution and D_n is the test statistic representing the maximum difference.

2) *Robust Exponentially Weighted Moving Average:* Robust estimators are one of the commonly used statistical approaches when the assumption of normality distribution is not met. This estimator is a better alternative to other traditional control map methods. This method is able to produce more accurate results with increased statistical power and high sensitivity, but remains efficient if the normality assumption is met [17]. This Robust EWMA method utilises two robust estimators, namely IQR and Biweight to produce control maps that are more resistant to outliers and abnormal data variations.

The IQR estimator is a variability measure based on splitting the data into quartiles. The IQR equation can be defined as follows [18]:

$$IQR = Q_3 - Q_1 \tag{2}$$

The value of Q_3 is the 3rd quartile and Q_1 is the 1st quartile obtained by solving the integrals $0.75 = \int_{-\infty}^{Q_3} f(x)dx$ and $0.25 = \int_{-\infty}^{Q_1} f(x)dx$. Then, for an unbiased estimator of σ value from IQR is $\hat{\sigma} = \frac{\overline{IQR}}{d_n}$ with $\overline{IQR} = \frac{1}{n}\sum_{i=1}^{n} IQR_i$ and d_n is the correction factor for sample n with $d_n = 1{,}3121$ [18]. The UCL, CL, LCL, and W_i statistics for the Robust Exponentially Moving Average control map using the IQR estimator can be calculated using the following equation:

$$UCL = \mu_0 + L\frac{\overline{IQR}}{d_n\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}} \tag{3}$$

$$CL = W_0 = \mu_0 \tag{4}$$

$$LCL = \mu_0 - L\frac{\overline{IQR}}{d_n\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}} \tag{5}$$

$$W_i = \lambda\bar{X}_i + (1-\lambda)W_{i-1} \tag{6}$$

where W_i is the plot statistic of the IQR estimator REWMA control chart at time i, λ is the smoothing parameter, $\bar{X}_i$ is the sample mean at time i, and W_{i-1} is the plot statistic of the IQR estimator REWMA control chart at the previous time.

Furthermore, the Biweight estimator is a type of M-estimate estimator that reduces the influence of outlier data by giving smaller weights to observations that are far from the centre of the distribution [7]. This estimator uses a bisquare weight function, which gives zero weight to values that are outside a certain range. The process of calculating the biweight estimator is iterative, starting with an initial estimate (often the median), and then updating based on weights calculated from the data [19]. The equation for the Biweight estimator is shown as follows [7]:

$$Biweight_i = \frac{n}{\sqrt{n-1}}\frac{\sqrt{\sum_{|U_i|<1}\left(x_j - T_i\right)^2(1-U_i^2)^4}}{\left|\sum_{|U_i|<1}\left(x_j - U_i^2\right)(1-5U_i^2)\right|} \tag{7}$$

with U_i is obtained using the following equation:

$$U_i = (x_j - T_i)/(c \cdot MAD_i) \tag{8}$$

Value of x_j is the jth sample, T_i is the median of the sample at the i-th time, c is the scale constant with $c = 9$ and MAD is the (Median Absolute Deviation) obtained from the following equation [20]:

$$MAD_i = 1{,}4826\left[Median\left|x_{i,j} - Median\left(x_{i,j}\right)\right|\right] \tag{9}$$

Then, the unbiased estimator of σ from Biweight is $\hat{\sigma} = \overline{Biweight}$. The UCL, LCL, CL, and Y_i statistics of the Robust Exponentially Weighted Moving Average control map using the Biweight estimator can be calculated using the following equation:

$$UCL = \mu_0 + L\frac{\overline{Biweight}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{10}$$

$$CL = W_0 = \mu_0 \tag{11}$$

$$LCL = \mu_0 - L\frac{\overline{Biweight}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{12}$$

$$Y_i = \lambda \bar{X}_i + (1-\lambda)Y_{i-1} \tag{13}$$

The variable Y_i is the plot statistic of the Biweight estimator REWMA control map at time i, λ is the smoothing parameter, $\bar{X}_i$ is the sample mean at time i, Y_{i-1} is the plot statistic of the IQR estimator REWMA control map at the previous time.

3) Average Run Length: ARL is the average number of samples required for a control chart to signal out-of-control. ARL is an important measure in assessing the performance of statistical control charts, because the faster a change is detected, the more effective the control chart is in reducing process errors. ARL is used to assess performance in detecting small deviations in process parameters while maintaining a balance between detection sensitivity and false alarm rate. In addition, ARL allows evaluation of a control chart's robustness to unusual distributions, especially for industries that rely heavily on quality control [13].

Generally, three procedures are used to obtain the run length distribution of the EWMA graph with or without assuming the estimated parameters, including the Integral Equation, Markov Chain Approach, and Monte Carlo Simulation [21]. There are 2 types of ARL, namely the ARL_0 notation for the in-control state and ARL_1 for the out-of-control state expressed in the following equation:

$$ARL_0 = \frac{1}{P(reject\ H_0|H_0\ true)} = \frac{1}{\alpha} \tag{14}$$

$$ARL_1 = \frac{1}{1-P(accept\ H_0|H_0\ false)} = \frac{1}{1-\beta} \tag{15}$$

with α is a type I error, which means rejecting H_0 when the hypothesis is true, the process is in an uncontrolled state when, in reality, the process is in a controlled state. Then, β is a type II error, which means accepting H_0 when the hypothesis is false, the process is in a controlled condition when, in reality, the process is in an uncontrolled state.

3. MAIN RESULTS

A. Testing the assumption of normality distribution on water pH data.

Testing the assumption of normality distribution on water pH data is carried out to ensure data conformity with the non-normality assumptions required in the Robust EWMA method. The normality test is performed using the Kolmogorov-Smirnov test which compares the cumulative distribution of sample data with the theoretical normal distribution. Normality testing on the pH content data of catfish farming pond water using the Kolmogorov-Smirnov test can be seen as follows:

Hypothesis:

H_0 : Data is normally distributed

H_1 : Data is not normally distributed

Testing Criteria: If the $p-value > \alpha$ with $(\alpha = 0.05)$, then H_0 is accepted.

Test Statistics: The test was conducted using SPSS software, which is shown in Table 2:

TABLE II

KOLMOGOROV-SMIRNOV TEST

α	$p-value$
0.05	0.000

Table 2 shows the normality test results on the pH content data of catfish farming pond water using the Kolmogorov-Smirnov test. The resulting p-value is then compared with the value determined previously. Because the $p-value < \alpha$ or $0.000 < 0.05$, it can be concluded that H_0 is rejected and H_1 is accepted, which means that the data on the pH content of catfish farm pond water is not normally distributed. The data is then analyzed using the Robust EWMA method.

B. *Establishment of Robust Exponentially Weighted Moving Average Control Map with IQR and Biweight Estimator*

This REWMA method utilizes two robust estimators, namely IQR and Biweight, to produce a control map that is more resistant to outliers and abnormal data variations. The plot statistics of W_i for a value of $\lambda = 0.01$ are as follows:

$$\begin{aligned}
W_1 &= \lambda \bar{x}_1 + (1-\lambda)W_{1-1} \\
&= \lambda \bar{x}_1 + (1-\lambda)W_0 \\
&= (0.01)(7.8) + (1-0.01)(7.8287) \\
&= 7.8284
\end{aligned}$$

$\vdots$

$$\begin{aligned}
W_{30} &= \lambda \bar{x}_{30} + (1-\lambda)W_{30-1} \\
&= \lambda \bar{x}_{30} + (1-\lambda)W_{29} \\
&= (0.01)(7.87) + (1-0.01)(7.8279) \\
&= 7.8283
\end{aligned}$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.8370$$

$$CL = 7.8287$$

$$LCL = 7.8203$$

The REWMA control CHART with IQR estimator constructed using the control limits and W_i plot statistics can be seen in Figure 1:

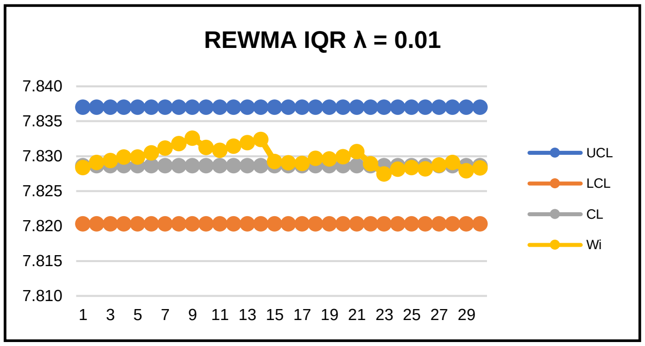

Fig.1 REWMA Control Chart with IQR Estimator for $\boldsymbol{\lambda = 0.01}$

Figure 1 shows the REWMA control chart with IQR estimator for $\lambda = 0.01$. Based on the control chart, it is obtained that all W_i plot points are in a state of in control. This indicates that the pH content of catfish farming in Bina Bersama Fishery Group of Makassar City is in a statistically controlled state. Furthermore, the plot statistics of W_i for a value of $\lambda = 0.05$ are as follows:

$$\begin{aligned} W_1 &= \lambda \bar{x}_1 + (1-\lambda) W_{1-1} \\ &= \lambda \bar{x}_1 + (1-\lambda) W_0 \\ &= (0.5)(7.8) + (1-0.5)(7.8287) \\ &= 7.8143 \end{aligned}$$

$$\vdots$$

$$\begin{aligned} W_{30} &= \lambda \bar{x}_{30} + (1-\lambda) W_{30-1} \\ &= \lambda \bar{x}_{30} + (1-\lambda) W_{29} \\ &= (0.5)(7.87) + (1-0.5)(7.784) \\ &= 7.8269 \end{aligned}$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.8967$$

$$CL = 7.8287$$

$$LCL = 7.7607$$

The REWMA control chart with IQR estimator constructed using the control limits and W_i plot statistics can be seen in Figure 2:

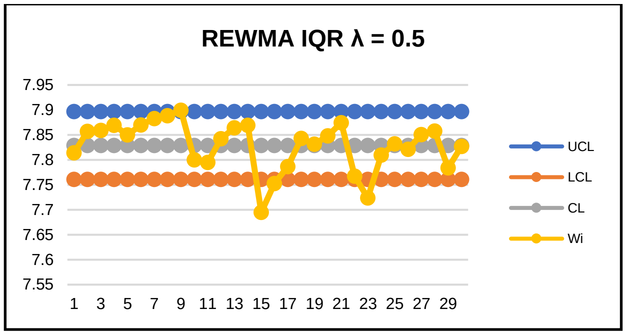

Fig. 2 REWMA Control Chart with IQR Estimator for $\boldsymbol{\lambda = 0.5}$

Figure 2 shows the REWMA control chart with IQR estimator for $\lambda = 0.5$. Based on the control v, 4 W_i plot points are out of control, including plot points 9, 15, 16, and 23. This indicates that the pH content of catfish farming in the Bina Bersama Fisheries Group of Makassar City is in a statistically uncontrolled state. Furthermore, the plot statistics of W_i for a value of $\lambda = 1$ are as follows:

$$
\begin{aligned}
W_1 &= \lambda\bar{x}_1 + (1-\lambda)W_{1-1} \\
&= \lambda\bar{x}_1 + (1-\lambda)W_0 \\
&= (1)(7.8) + (1-1)(7.8287) \\
&= 7.8
\end{aligned}
$$

$$
\begin{aligned}
W_{30} &= \lambda\bar{x}_{30} + (1-\lambda)W_{30-1} \\
&= \lambda\bar{x}_{30} + (1-\lambda)W_{29} \\
&= (1)(7.87) + (1-1)(7.71) \\
&= 7.87
\end{aligned}
$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.9465$$
$$CL = 7.8287$$
$$LCL = 7.7109$$

The REWMA control chart with IQR estimator constructed using the control limits and W_i plot statistics can be seen in Figure 3:

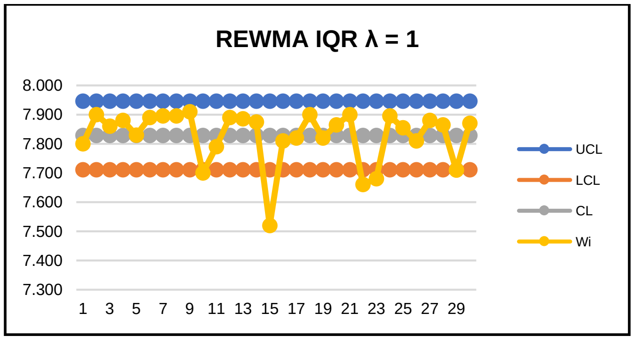

Fig. 3 REWMA Control Chart with IQR Estimator for $\boldsymbol{\lambda = 1}$

Figure 3 shows the REWMA control chart with IQR estimator for $\lambda = 1$. Based on the control chart, 5 W_i plot points are out of control, including plot points 10, 15, 22, 23, and 29. This indicates that the pH content of catfish farming in the Bina Bersama Fishery Group of Makassar City is in a statistically uncontrolled state. Furthermore, for the Robust EWMA method for the Biweight estimator, the plot statistics of Y_i for the value of $\lambda = 0.01$ can be calculated as follows:

$$\begin{aligned} Y_1 &= \lambda \bar{x}_1 + (1-\lambda) Y_{1-1} \\ &= \lambda \bar{x}_1 + (1-\lambda) Y_0 \\ &= (0.01)(7.8) + (1-0.01)(7.8287) \\ &= 7.8284 \end{aligned}$$

⋮

$$\begin{aligned} Y_{30} &= \lambda \bar{x}_{30} + (1-\lambda) Y_{30-1} \\ &= \lambda \bar{x}_{30} + (1-\lambda) Y_{29} \\ &= (0.01)(7.87) + (1-0.01)(7.8279) \\ &= 7.8283 \end{aligned}$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.8405$$
$$CL = 7.8287$$
$$LCL = 7.8168$$

The REWMA control chart with Biweight estimator constructed using the control limits and Y_i plot statistics can be seen in Figure 4:

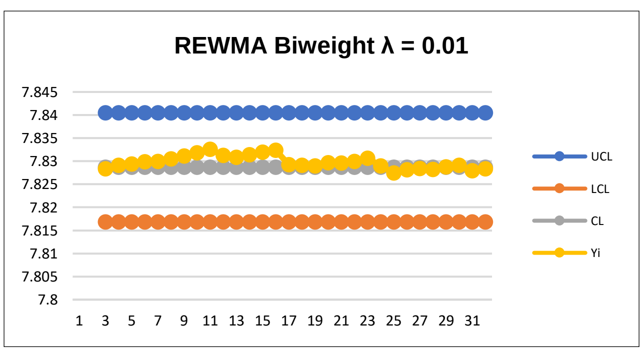

Fig. 4 REWMA Control Chart with Biweight Estimator for $\boldsymbol{\lambda = 0.01}$

Figure 4 shows the REWMA control chart with the Biweight estimator for $\lambda = 0.01$. Based on the control chart, it is obtained that all Y_i plot points are in a state of in control. This indicates that the pH content of catfish farming in Bina Bersama Fishery Group of Makassar City is in a statistically controlled state. Furthermore, the plot statistics of Y_i for a value of $\lambda = 0.5$ are as follows:

$$Y_1 = \lambda\bar{x}_1 + (1-\lambda)Y_{1-1}$$
$$= \lambda\bar{x}_1 + (1-\lambda)Y_0$$
$$= (0.5)(7.8) + (1-0.5)(7.8287)$$
$$= 7.8143$$

⋮

$$Y_{30} = \lambda\bar{x}_{30} + (1-\lambda)Y_{30-1}$$
$$= \lambda\bar{x}_{30} + (1-\lambda)Y_{29}$$
$$= (0.5)(7.87) + (1-0.5)(7.7839)$$
$$= 7.8269$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.9252$$
$$CL = 7.8287$$
$$LCL = 7.7322$$

The REWMA control map with Biweight estimator constructed using the control limits and Y_i plot statistics can be seen in Figure 5:

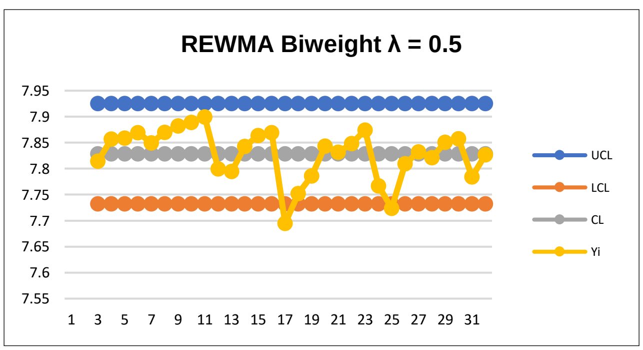

Fig. 5 REWMA Control Chart with Biweight Estimator for $\boldsymbol{\lambda = 0.5}$

Figure 5 shows the REWMA control chart with the Biweight estimator for $\lambda = 0.5$. Based on the control chart, there are 2 Y_i plot points that are out of control, namely plot points 15 and 23. This indicates that the pH content of catfish farming in the Bina Bersama Fishery Group of Makassar City is statistically uncontrolled. Furthermore, the plot statistics of Y_i for a value of $\lambda = 1$, are as follows:

$$Y_1 = \lambda x_1 + (1-\lambda)Y_{1-1}$$
$$= \lambda x_1 + (1-\lambda)Y_{\bar{0}}$$
$$= (1)(7.8) + (1-1)(7.8287)$$
$$= 7.8$$

⋮

$$Y30 = \lambda\bar{x}30 + (1-\lambda)Y30{-}1$$
$$= \lambda\bar{x}_{30} + (1-\lambda)Y_{29}$$
$$= (1)(7.87) + (1-1)(7.71)$$
$$= 7.87$$

Furthermore, the UCL, CL, and LCL values are as follows:

$$UCL = 7.9958$$
$$CL = 7.8287$$
$$LCL = 7.6615$$

The REWMA control chart with Biweight estimator constructed using the control limits and Y_i plot statistics can be seen in Figure 6:

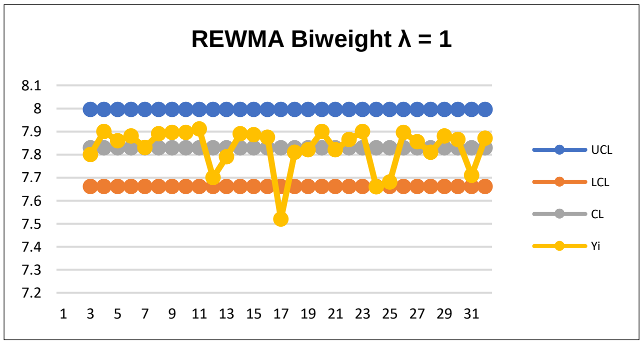

Fig. 6 REWMA Control Chart with Biweight Estimator for $\lambda = 1$

Figure 6 shows the REWMA control chart with the Biweight estimator for $\lambda = 1$. Based on the control chart, there are 2 Y_i plot points that are out of control, namely plot points 15 and 22. This indicates that the pH content of catfish farming in the Bina Bersama Fishery Group of Makassar City is statistically uncontrolled.

C. *Comparison of Robust Exponentially Weighted Moving Average Control Chart Performance with IQR and Biweight Estimators Based on Average Run Length Value*

The obtained REWMA control chart cannot effectively evaluate the performance of the control chart. Therefore, the Average Run Length (ARL) metric measures the sensitivity level. Based on the ARL_1 value, the most optimal control chart, has a lower ARL value during the process shift. The weighting values used in the ARL1 calculation are $\lambda = 0.01$, $\lambda = 0.5$, and $\lambda = 1$ using a shift value of $k = 0$ to 1. ARL values on REWMA control charts with other IQR estimators can be seen in Table 3:

TABLE III

ARL VALUE OF REWMA CONTROL CHART WITH IQR ESTIMATOR

k	ARL of REWMA Control Chart with IQR Estimator		
	$\lambda = 0.01$	$\lambda = 0.5$	$\lambda = 1$
0	370.398	370.398	370.398
0.01	178.238	364.998	368.582
0.02	56.904	349.614	363.229
0.03	20.707	326.411	354.611
0.04	8.923	298.212	343.144
0.05	4.529	267.777	329.344
0.06	2.680	237.331	313.769
0.07	1.822	208.396	296.977
0.08	1.398	181.849	279.486
0.09	1.184	158.077	261.748
0.1	1.078	137.136	244.138
0.2	1	35.162	109.967
0.3	1	11.439	49.610
0.4	1	4.780	24.171
0.5	1	2.5195	12.825
0.6	1	1.632	7.402
0.7	1	1.253	4.634
0.8	1	1.091	3.134
0.9	1	1.028	2.279
1	1	1.007	1.772

Table 3 shows that the ARL value of the REWMA control chart using the IQR estimator for $\lambda = 0.01$ is lower than the ARL values for $\lambda = 0.5$, and $\lambda = 1$. The decrease in ARL value indicates that the number of samples required to detect the out of control signal is also reduced. Thus, the smaller the ARL value, the more sensitive the control map is to identify out of control signals in a process. Thus, it can be concluded that the REWMA control chart with IQR estimator at $\lambda = 0.01$ shows superior performance in detecting changes in pH content in catfish farming at Bina Bersama Fisheries Group Makassar City. Meanwhile, the ARL values on REWMA control charts with other Biweight estimators can be seen in Table 4:

TABLE IV

ARL VALUE OF REWMA CONTROL CHART WITH BIWEIGHT ESTIMATOR

$\boldsymbol{k}$	**ARL of REWMA Control Chart with Biweight Estimator**		
	$\boldsymbol{\lambda = 0.01}$	$\boldsymbol{\lambda = 0.5}$	$\boldsymbol{\lambda = 1}$
0	370.398	370.398	370.398
0.01	337.039	369.852	370.216
0.02	263.134	368.221	369.670
0.03	188.841	365.531	368.763
0.04	131.349	361.825	367.500
0.05	91.060	357.157	365.888
0.06	63.711	351.597	363.935
0.07	45.217	345.223	361.650
0.08	32.617	338.121	359.046
0.09	23.929	330.381	356.135
0.1	17.858	322.097	352.931
0.2	2.330	227.720	308.426
0.3	1.122	147.534	253.139
0.4	1.004	94.044	200.075
0.5	1	60.688	155.224
0.6	1	40.032	119.665
0.7	1	27.074	92.320
0.8	1	18.786	71.552
0.9	1	13.374	55.827
1	1	9.765	43.895

Table 4 shows that the ARL value of the REWMA control chart using the Biweight estimator for $\lambda = 0.01$ is lower than the ARL values for $\lambda = 0.5$, and $\lambda = 1$. The decrease in ARL value indicates that the number of samples required to detect the out of control signal is also reduced. Thus, the smaller the ARL value, the more sensitive the control map is to identify out of control signals in a process. Thus, it can be concluded that the REWMA control chart with the Biweight estimator at $\lambda = 0.01$ shows superior performance in detecting changes in pH content in catfish farming at Bina Bersama Fisheries Group Makassar City.

The IQR and Biweight estimators each show the same optimal weight value of $\lambda = 0.01$. A comparison of the optimum ARL value for each estimator is made to see the most effective estimator for detecting process changes. The following ARL value comparison chart for the Robust EWMA control map with IQR and Biweight estimators is shown in Figure 7 below:

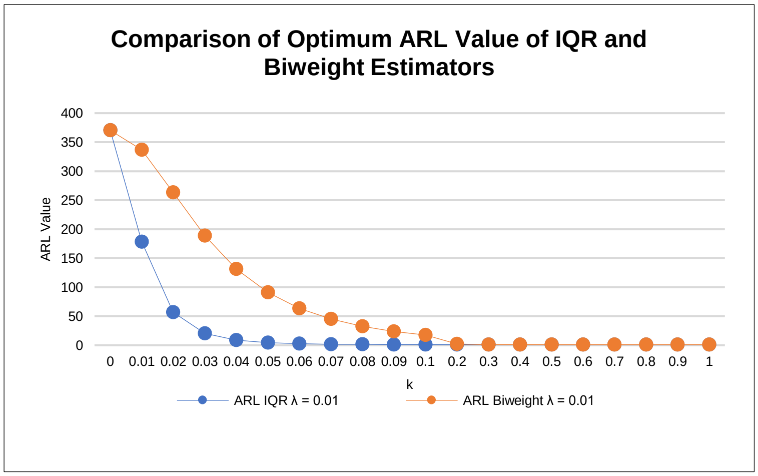

Fig. 7 Comparison of Optimum ARL Value of IQR and Biweight Estimators

Figure 7 shows the ARL value comparison between the IQR and Biweight estimators on the Robust EWMA control chart with $\boldsymbol{\lambda = 0.01}$. At $\boldsymbol{k = 0}$, both estimators have the same ARL value of $\mathbf{370.398}$. However, as k increases, the IQR estimator shows better performance, with a more sensitive ARL in detecting out of control signals than Biweight, especially at shifts between 0.02 to $\mathbf{0.2}$. In this range, the ARL of the IQR estimator decreases significantly, while the ARL of Biweight is still slower to reach the optimal value close to 1. Based on the graph, the best ARL (the lowest ARL value) is obtained from the IQR estimator which shows higher sensitivity in detecting small shifts in the process compared to the Biweight estimator.

4. CONCLUSION

This study demonstrates that the Robust EWMA control chart with the IQR estimator is a more effective method for monitoring small changes in water quality in catfish farming compared to the Biweight estimator. With higher sensitivity in detecting pH shifts, this approach enables early detection of potential water quality issues that could negatively impact fish health and productivity. For business operators in aquaculture, particularly catfish farmers, the implementation of this method offers several advantages: first, Increased Productivity and Higher Yield Quality; its Faster and more accurate pH detection allows for early corrective actions, preventing fish stress that could slow growth and increase mortality rates, second Operational Cost Efficiency. This More precise pH monitoring reduces the excessive use of chemical pH stabilizers, lowering production costs and optimizing resource utilization. Third, Ensuring Business Sustainability. Maintaining stable water conditions helps preserve fish health, reduces disease risk, and minimizes mass mortality, which can significantly impact long-term business viability. Fourth Data-Driven decision-making: By leveraging more accurate statistical approaches, farmers gain data-driven insights into water management, leading to better and more informed decision-making and Enhanced Market Competitiveness, producing high-quality and sustainable yields improves business reputation in both domestic and export markets, where food safety and quality standards are becoming increasingly stringent.

Thus, implementing Robust EWMA with the IQR estimator provides academic benefits in developing advanced water quality monitoring techniques and delivers tangible business advantages by enhancing efficiency, productivity, and profitability in catfish farming.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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