Available online at http://scik.org

Commun. Math. Biol. Neurosci. 2025, 2025:86

https://doi.org/10.28919/cmbn/9327

ISSN: 2052-2541

FOURIER SERIES NONPARAMETRIC REGRESSION ESTIMATOR FOR MODELING STATUS OF UNMET NEED IN EAST JAVA PROVINCE IN 2023

MUHAMMAD ZULFADHLI, I NYOMAN BUDIANTARA*, VITA RATNASARI, AFIQAH SAFFA SURIASLAN

Department of Statistics, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia

Abstract: In recent years, the Estimator of Fourier Series Nonparametric Regression (FSNR) for quantitative data has generated a lot of attention. In practice, though, there is frequently a correlation between predictor and response, with categorical data serving as the response. Only certain techniques are used in some of the methodologies created today to address the health case of qualitative response data. No previous study can handle health data using FSNR estimator. This study presents a novel approach FSNR estimator with response variables in the form of categorical data specifically within the context of public health research. The research methods used are theoretical and application studies. The FSNR estimators method for categorical data assumes a relationship between the logit function and predictor variables that has a repeating pattern. The Newton-Raphson technique and MLE were used to obtain the FSNR estimators. To apply this method, we used application data status of unmet need in East Java Province in 2023. The unmet need rate in East Java was quite high, reaching 12.97 percent, while the target is 11.74 percent. Because of the lower deviance value, greater AUC, and Press's Q values, the results show that the FSNR offers much superior estimation results and accuracy for data applications.

Keywords: categorical data; Fourier series; Fourier series nonparametric regression; binary logistic regression; unmet need.

2020 AMS Subject Classification: 62P10.

---

*Corresponding author

E-mail address: i_nyoman_b@statistika.its.ac.id

Received May 01, 2025

1. INTRODUCTION

When the regression curve's function exhibits a Fourier Series pattern, FSNR can be used to ascertain the link between the predictors and response variable. It is expected that the nonparametric regression curve falls into a particular function space. The data were to form their own estimates independently. The nonparametric regression method is hence very adaptable. Smoothing techniques can be used to execute the nonparametric approach based on observed data. The Spline estimator is one of several smoothing methods [1], Fourier Series estimator [2], Wavelet estimator [3], Kernel estimator [4], and Local Polynomial [5].

Data with varying patterns that rely on knot points are subjected to spline estimators [1]. The local polynomial estimator has been utilized in nonparametric regression with two or more response variables in order to decrease its asymptotic variance and bias characteristics [6]. Wavelet estimator that has been used to model observations of a signal contaminated with noise, which is Gaussian distributed and additive [7]. The Fourier Series estimator is used for patterned data that tend to repeat [2]. Among these estimators, the Fourier Series method was used to. This method is very specialized and well used in data cases in which the predictor and response variables exhibit a repeating pattern following a certain trend [8]. The Fourier Series estimator best optimizes the accuracy and computational cost of additive nonparametric regression models [9]. In addition to predictors using a single predictor variable (univariable), also known as multivariable predictors [10, 11].

First proposed by [2], the Fourier Series was further examined in nonparametric regression by [7]. Moreover, the Fourier Series was used in semiparametric regression by [12]. Using Fourier series, [13] created a birresponse semiparametric regression. [14, 15, 16] transformed it into an FSNR mixture estimator. and a Fourier Series Semiparametric Regression (FSSR) mixture estimator by [17]. However, previous studies that developed using this method only used quantitative data, such as [18, 19, 20, 21]. However, these approaches rely on specific basis functions and none have explored the use of Fourier Series Nonparametric Regression (FSNR) for categorical health data. This represents a significant gap in the current methodology, particularly given the complexity and periodic nature of many health-related phenomena.

Nonparametric regression estimators have been created by some researchers for handling categorical data, such as [22] using Local Likelihood Logit Estimation, [23] using the Decision Tree approach, and [24] using the B-Spline function and recently, [25, 26] have developed estimators for nonparametric regression using categorical data. These investigations only employed specific functions. No prior research has used a Fourier Series function to create an FSNR estimator for categorical response health data. Figure 1 shows the difference between nonparametric regression and simple logistic regression with categorical data. The Coronary Heart Disease (CHD) and age status of 100 patients serve as an illustration of the reference we employed.

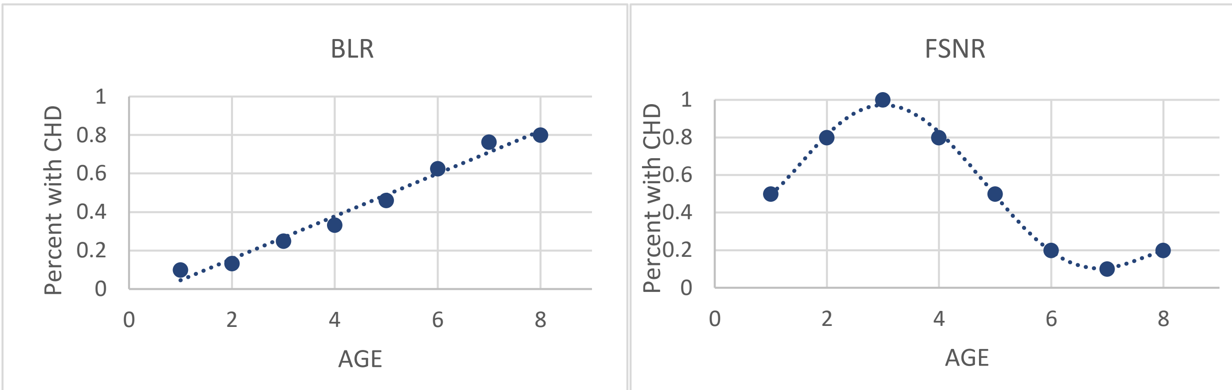

Figure 1. Percentage of subjects with CHD in each group

The plot may not always follow a linear trend, as shown in Figure 1, which occasionally leads to recurring and non-patterned patterns. For quantitative data, a traditional nonparametric regression estimator typically produces significant errors.

Unmet need refers to a condition in which a woman of reproductive age wishes to avoid or delay pregnancy but is not using any form of contraception. This situation represents a significant challenge for family planning programs, particularly in developing district such as East Java. In 2023, the unmet need rate in East Java, based on data from the Family Planning Information System (SIGA-YAN) , an information system used by National Population and Family Planning Agency (BKKBN) to collect and analyze data related to family planning as of June 16, 2023, was still high, reaching 12.97% (https://siga.bkkbn.go.id/). The unmet need target set is 11.74%. This indicates that there are still many couples of childbearing age in East Java who have unmet family planning needs. Despite ongoing efforts to promote reproductive health, the prevalence of unmet need remains high, indicating gaps in access, education, or service delivery. This issue not only impacts individual health and autonomy but also contributes to broader public health concerns, such as unplanned pregnancies, maternal mortality, and overpopulation. Some researchers who solve unmet need cases such [27, 28, 29].

In this study, the unmet need status among women in East Java Province in 2023 serves as the primary case for applying the newly developed Fourier Series Nonparametric Regression (FSNR) estimator. By modeling the logit function with a Fourier Series basis, our method is capable of capturing periodic and nonlinear trends in predictor variables. This approach holds significant potential for improving the accuracy of classification models in public health applications, particularly where patterns in data are cyclical or not well described by standard models.

2. PRELIMINARIES

2.1 Fourier Series Nonparametric Regression (FSNR)

FSNR is a model Fourier Series nonparametric functions. Given paired data (y_i, z_{li}) and the relationship between y_i, z_{li} is assumed to follow a FSNR model (1) [16].

$$y_i = g\left(z_{ji}\right) + \varepsilon_i \text{ }, i = 1,2, \dots, n \tag{1}$$

where y_i is response variable, $g\left(z_{ji}\right)$ are approximated using Fourier Series nonparametric function and ε_i is the error for the model.

The function $g(z_{1i}, \ \dots \ , z_{qi})$ is approximated by a nonparametric function using Fourier Series (2).

$$g\left(z_{1i}, \ \dots \ , z_{qi}\right) = \sum_{l=1}^{q} \left(b_l z_{li} + \frac{1}{2} a_{0l} + \sum_{s=1}^{S} a_{sl} \cos s z_{li} \right); \ i = 1,2, \dots n \tag{2}$$

Where, b_l, a_{0l} and a_{sl}, $l = 1,2, \dots, q$, $s = 1,2, \dots, S$ are the model parameters in the Fourier Series function.

2.2 Binary Logistic Regression (BLR)

The BLR model is a probability based-model where the response variable is expressed as the logit of the probability of a given outcome, contigent upon spesific predictor variables [30]. $\pi(\boldsymbol{x_i})$ represents the model (3).

$$\pi(\boldsymbol{x_i}) = \frac{e^{\delta_0 + \delta_1 x_{1i} + \delta_2 x_{2i} + \dots + \delta_w x_{wi}}}{1 + e^{\delta_0 + \delta_1 x_{1i} + \delta_2 x_{2i} + \dots + \delta_w x_{wi}}} \text{ }, i = 1,2, \dots, n \tag{3}$$

The response variable in BLR will have a probability of $\pi(\boldsymbol{x_i})$ if it is 1, and has a probability value of $1 - \pi(\boldsymbol{x_i})$ if it is 0, $\delta_0, \delta_1, \delta_2, \dots, \delta_w$ are the parameters model, $x_{1i}, x_{2i}, \dots, x_{wi}$ are the predictor variables, and w is the number of predictor variable.

2.3 Maximum Likelihood Estimation (MLE)

MLE is the joint probability function in a random sample through maximize the likelihood function [31]. Each part of data is considered independent so that the likelihood function is a combination of the distribution function for each pair. The likelihood function (4) is defined as.

$$l(\boldsymbol{\theta}) = \prod_{i=1}^{n} P(Y_i = y_i) \tag{4}$$

The estimator $\hat{\boldsymbol{\theta}}$ is obtained by deriving the likelihood function equation for $\boldsymbol{\theta}$ and then equaling by 0.

2.4 Unmet Need

When developing family planning policies, one of the key ideas used is the definition of unmet need. According to [32], the percentage of women who do not currently use a form of contraception and do not wish to delay pregnancy or have more children is known as the unmet need. According to BKKBN, unmet need for family planning is the percentage of women of childbearing age who do not want to have more children, or want to postpone their next birth, but do not use a contraceptive method or method. According to the Demographic and Health Survey, unmet need for family planning is the proportion of married or cohabiting (sexually active) women of childbearing age who do not want another child or who want to delay their next birth for at least 2 years but do not use contraceptives.

3. MAIN RESULTS

3.1 Parameter Estimation

Building an FSNR model, then generating a Log Likelihood function and deriving it for each model parameter are the necessary procedures in order to generate an FSNR estimator for categorical data. The Newton–Raphson iteration was used to conduct numerical iterations at the end.

Probability Distribution

Given $z_1, z_2, \ldots, z_q$, are as many as q predictor variables in FSNR. The variable Y is a random Bernoulli distribution variable with a specific probability. With a certain probability, the variable Y is a random Bernoulli distribution variable.

$$Y \sim B\big(1, \pi(\boldsymbol{z})\big), \boldsymbol{z} = z_1, z_2, \ldots, z_q$$

where the success probability is defined as

$$P(Y_i = 1) = \pi(\boldsymbol{z_i})$$

and the unsuccessful probability is defined as

$$P(Y_i = 0) = 1 - \pi(\boldsymbol{z_i})$$

The probability distribution function $P(Y_i = y_i)$ is defined by $\pi(\boldsymbol{z_i})$ for each i.

$$P(Y_i = y_i) = \pi(\boldsymbol{z_i})^{y_i}\left(1-\pi(\boldsymbol{z_i})\right)^{1-y_i} = \left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right)^{y_i}\left(1-\pi(\boldsymbol{z_i})\right) \tag{5}$$

Logit Function (Link Function)

Then, the equation (5) can be expressed as a natural logarithmic function (ln)

$$\ln P(Y_i = y_i) = y_i \ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right) + \ln(1-\pi(\boldsymbol{z_i})) \tag{6}$$

The ln function (6) creates the following exponential family distribution function when expressed in exponential form.

$$\exp(\ln P(Y_i = y_i)) = \exp\left(y_i \ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right) + \ln(1-\pi(\boldsymbol{z_i}))\right) \tag{7}$$

When the exponential family distribution function is defined as follows (8) after solving equation (7).

$$f(y_i,\theta) = \exp\left(\frac{y_i\,\theta - b(\theta)}{a(\emptyset)} + c(\theta,\emptyset)\right) \tag{8}$$

As a result, the exponential family of distribution functions includes its probability distribution function (9).

$$P(Y_i = y_i) = \exp\left(\frac{y_i \ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right)}{1} + \ln(1-\pi(\boldsymbol{z_i}))\right) \tag{9}$$

where,

$\theta \quad = \ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right)$ $\qquad a(\emptyset) \quad = 1$

$b(\theta) = \ln(1-\pi(\boldsymbol{z_i}))$ $\qquad c(\theta,\emptyset) = 0$

θ in function (9) is a logit function, then the logit function (10) for the regression obtained is

$$\theta = \ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right) \tag{10}$$

The logit function, often known as the link function, makes parameter estimation easier and streamlines lengthy regression models. Logit transformation is used to do this.

Logit Transformation Model

Logit transformation model (11) is defined as follows.

$$\ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right) = g\left(z_{1i}, \ \dots \ , z_{qi}\right) \tag{11}$$

where g is a regression equation or regression function (regression curve) that follows an additive model. The function is approximated non-parametrically. Using Fourier Series for $g(z_{1i}, \ \dots \ , z_{qi})$ in equation (2).

So, logit transformation of FSNR model (12) is defined as follows.

$$\ln\left(\frac{\pi(\boldsymbol{z_i})}{1-\pi(\boldsymbol{z_i})}\right) = \sum_{l=1}^{q}\left(b_l z_{li} + \frac{1}{2}a_{0l} + \sum_{s=1}^{S} a_{sl}\cos s z_{li}\right);\ i = 1,2,\dots n \quad (12)$$

By the function (12), FSNR model for categorical data (13) as follows.

$$\pi(\boldsymbol{x_i}) = \frac{e^{\sum_{l=1}^{q}\left(b_l z_{li}+\frac{1}{2}a_{0l}+\sum_{s=1}^{S} a_{sl}\cos s z_{li}\right)}}{1+e^{\sum_{l=1}^{q}\left(b_l z_{li}+\frac{1}{2}a_{0l}+\sum_{s=1}^{S} a_{sl}\cos s z_{li}\right)}} \quad (13)$$

Likelihood Function $l(\boldsymbol{\theta})$

The form of the likelihood function is obtained $l(\boldsymbol{\theta})$

where,

$$\boldsymbol{\theta} = (b_1 \quad a_{01} \quad a_{11} \quad \dots \quad a_{S1} \quad \vdots \quad \cdots \quad \vdots \quad b_q \quad a_{0q} \quad a_{1q} \quad \cdots \quad a_{Sq})$$

using the Maximum Likelihood Estimation (MLE) method.

$$l(\boldsymbol{\theta}) = \prod_{i=1}^{n} P(Y_i = y_i) = \pi(\boldsymbol{z_i})^{\sum_{i=1}^{n} y_i}(1-\pi(\boldsymbol{z_i}))^{n-\sum_{i=1}^{n} y_i} \quad (14)$$

By maximizing the log likelihood function's first derivative, the MLE approach can be used to estimate parameters in logistic regression. It is simple to maximize the likelihood function (14) as follows: $\ln l(\boldsymbol{\theta})$.

Log-Likelihood Function $L(\boldsymbol{\theta})$

$$\ln[l(\boldsymbol{\theta})] = L(\boldsymbol{\theta}) = \sum_{i=1}^{n} y_i \ln[\pi(\boldsymbol{z_i})] + \sum_{i=1}^{n}(1-y_i)\ln[1-\pi(\boldsymbol{z_i})]$$

$$= \sum_{i=1}^{n}\left\{y_i\left(g(z_{1i}, \dots, z_{qi})\right) - \ln\left[1+\exp(g(z_{1i}, \dots, z_{qi}))\right]\right\} \quad (15)$$

The estimator $\widehat{\boldsymbol{\theta}}$ is determined by computing the partial derivatives of equation (15) with respect to b_l, a_{0l}, and a_{sl}, and setting them to zero.

$$\frac{\partial L(\boldsymbol{\theta})}{\partial b_l} = 0\ ;\ l = 1,2,\dots,q$$

$$\frac{\partial L(\boldsymbol{\theta})}{\partial a_{0l}} = 0\ ;\ l = 1,2,\dots,q$$

$$\frac{\partial L(\boldsymbol{\theta})}{\partial a_{sl}} = 0\ ;\ s = 1,2,\dots,S\ ;\ l = 1,2,\dots,q$$

The estimator $\widehat{\boldsymbol{b}}$ will be obtained through equation (16).

$$\sum_{i=1}^{n}\left\{\left(y_i - \pi(\boldsymbol{z_i})\right)z_{li}\right\} = 0 \quad (16)$$

The estimator $\widehat{\boldsymbol{a_0}}$ will be obtained through equation (17).

$$\sum_{i=1}^{n}\left\{\frac{1}{2}\left(y_i - \pi(\boldsymbol{z_i})\right)\right\} = 0 \quad (17)$$

The estimator $\widehat{\boldsymbol{a_s}}$ will be obtained through equation (18).

$$\sum_{i=1}^{n}\left\{\sum_{s=1}^{S}\cos kz_{li}\left(y_i - \pi(\boldsymbol{z_i})\right)\right\} = 0 \quad (18)$$

Newton-Raphson Iteration

Since the derivative of $L(\boldsymbol{\theta})$ (15) with respect to b_l, a_{0l}, a_{sl} given by the implicit equation's derivative is not in closed form, numerical iteration with the Newton-Raphson method must be employed.

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \left(H(\boldsymbol{\theta})^{(t)}\right)^{-1} g(\boldsymbol{\theta})^{(t)} \quad (19)$$

where $\boldsymbol{\theta}^{(t)}$ is the $\boldsymbol{\theta}$ of the t-th iteration, *t=1,2,...,* converged.

$$\boldsymbol{\theta}^{(t)} = \begin{pmatrix} {b_q}^{(t)} & {a_{0q}}^{(t)} & {a_{1q}}^{(t)} & \ldots & {a_{Sq}}^{(t)} \end{pmatrix}$$

while $g(\boldsymbol{\theta})$ is the gradient vector of $\boldsymbol{\theta}$ and $H(\boldsymbol{\theta})$ is the Hessian matrix of $\boldsymbol{\theta}$ in function (19), with the following equation.

$$g(\boldsymbol{\theta}) = \left(\frac{\partial L(\boldsymbol{\theta})}{\partial b_1}, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{01}}, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{11}}, \cdots, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{S1}}, \cdots, \frac{\partial L(\boldsymbol{\theta})}{\partial b_q}, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{0q}}, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{1q}}, \cdots, \frac{\partial L(\boldsymbol{\theta})}{\partial a_{Sq}}\right)^T$$

$$H(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial^2 L(\boldsymbol{\theta})}{\partial {b_1}^2} & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial b_1 \partial a_{01}} & \cdots & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial b_1 \partial a_{Sq}} \\ \frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{01} \partial b_1} & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial {a_{01}}^2} & \cdots & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{01} \partial a_{Sq}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{Sq} \partial b_1} & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{Sq} \partial a_{01}} & \cdots & \frac{\partial^2 L(\boldsymbol{\theta})}{\partial {a_{Sq}}^2} \end{bmatrix}$$

The elements of vector $g(\boldsymbol{\theta})$ are obtained from the first derivative of function $L(\boldsymbol{\theta})$ with respect to b_l, a_{0l}, a_{sl}, while matrix $H(\boldsymbol{\theta})$ are obtained from the second derivative of function $L(\boldsymbol{\theta})$ with respect to b_w, a_{0w}, a_{sw}.

The first derivative of function $L(\boldsymbol{\theta})$ with respect to b_l, a_{0l}, a_{sl} yields the elements of vector g(θ), whereas the second derivative of function $L(\boldsymbol{\theta})$ with respect to b_w, a_{0w}, a_{sw} yields matrix H(θ).

Second Derivative of $L(\boldsymbol{\theta})$ Function with Respect to b_u

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial b_v \partial b_l} = -\sum_{i=1}^{n} x_{li} x_{vi}\, \pi(\boldsymbol{z_i})\left(1 - \pi(\boldsymbol{z_i})\right) \quad (20)$$

Following the same procedure (20), equation (21) yields the parameter's second derivative.

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{kv} \partial b_l} = -\sum_{i=1}^{n}\sum_{k=1}^{K} \pi(\boldsymbol{z_i})\left(1 - \pi(\boldsymbol{z_i})\right) x_{li} \cos kx_{vi} \quad (21)$$

Second Derivative of $L(\boldsymbol{\theta})$ Function with Respect to a_{0u}

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{0v} \partial a_{0l}} = -\frac{1}{4}\sum_{i=1}^{n} \pi(\boldsymbol{z_i})\left(1 - \pi(\boldsymbol{z_i})\right) \quad (22)$$

The second derivative of the parameter combination (21) is derived in the same way as in equation (22).

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{kv}\partial a_{0l}} = -\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{K}\pi(\boldsymbol{z_i})\big(1-\pi(\boldsymbol{z_i})\big)\cos kx_{vi} \tag{23}$$

Second Derivative of $L(\boldsymbol{\theta})$ Function with Respect to a_{ku}

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{kv}\partial a_{kl}} = -\sum_{i=1}^{n}\sum_{k=1}^{K}\cos kx_{li}\sum_{k=1}^{K}\cos kx_{vi}\,\pi(\boldsymbol{z_i})\big(1-\pi(\boldsymbol{z_i})\big) \tag{24}$$

Equation (25) yields the second derivative of the parameter combination in the same way as (24).

$$\frac{\partial^2 L(\boldsymbol{\theta})}{\partial a_{0v}\partial a_{kl}} = -\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{K}\pi(\boldsymbol{z_i})\big(1-\pi(\boldsymbol{z_i})\big)\cos kx_{li} \tag{25}$$

Estimator $\hat{\boldsymbol{\theta}}$

When employing function (26), $\hat{\boldsymbol{\theta}}$ will be derived from the Newton-Raphson iteration equation.

$$\left|\boldsymbol{\theta}^{(t+1)} - \boldsymbol{\theta}^{(t)}\right| < \varepsilon,\ \varepsilon = 0{,}000001 \tag{26}$$

Thus, the estimator $\hat{\boldsymbol{\theta}}$ is

$$\hat{\boldsymbol{\theta}} = \begin{pmatrix}\hat{b}_1 & \hat{a}_{01} & \hat{a}_{11} & \dots & \hat{a}_{S1} & \vdots & \cdots & \vdots & \hat{b}_q & \hat{a}_{0q} & \hat{a}_{1q} & \dots & \hat{a}_{Sq}\end{pmatrix}$$

A FSNR model for categorical data can be written using the estimator $\hat{\boldsymbol{\theta}}$.

$$\hat{\pi}(\boldsymbol{z_i}) = \frac{e^{\hat{b}_1 z_{1i}+\frac{1}{2}\hat{a}_{01}+\hat{a}_{11}\cos z_{1i}+\cdots+\hat{a}_{S1}\cos Sz_{1i}+\cdots+\hat{b}_q z_{qi}+\frac{1}{2}\hat{a}_{0q}+\hat{a}_{1q}\cos z_{qi}+\cdots+\hat{a}_{Sq}\cos Sz_{qi}}}{1+e^{\hat{b}_1 z_{1i}+\frac{1}{2}\hat{a}_{01}+\hat{a}_{11}\cos z_{1i}+\cdots+\hat{a}_{S1}\cos Sz_{1i}+\cdots+\hat{b}_q z_{qi}+\frac{1}{2}\hat{a}_{0q}+\hat{a}_{1q}\cos z_{qi}+\cdots+\hat{a}_{Sq}\cos Sz_{qi}}} \tag{27}$$

$\hat{b}_q, \hat{a}_{0q}$, and $\hat{a}_{sq}$ are the respective estimators for the Fourier Series functions in equation (27), given the predictor variables z_q, with oscillation parameter s and number of predictor variables of Fourier Series q.

3.2 Application of the FSNR Model

In applying the multivariable FSNR method for categorical data, we use application data the status of unmet need in East Java Province in 2023. The data used is secondary data sourced from dynamic tables on the BKKBN website as well as the Provincial publications in figures of East Java province. The data consists of 38 district with 1 response variable (y) and 3 predictor variables (x). The variables are detailed in Table 1.

Table 1. Variable Description

Variable	Notation	Description	Unit	Scale
Response	y	Status of Unmet Need	0 = Target Achieved 1 = Target not Achieved	Nominal
Predictor	x_1	Percentage of Family Heads with no Primary Education	Percent	Rasio
	x_2	Percentage of Couples of Childbearing Age with 2 Children	Percent	Rasio
	x_3	Percentage of Couples of Childbearing Age who Seek Services at FKTP (Health Centers or Equivalent, Doctor's Practices, Private Clinics or Equivalent and Class D private Hospitals or Equivalent)	Percent	Rasio

Descriptive Analytics

Descriptive analysis is used to determine the characteristics of the data for each predictor variable as follows Table 2.

Table 2. Descriptive Statistics

Variable	Mean	Variance	Min	Max
x_1	8.38	-0.41	5.07	11.82
x_2	7.06	-0.78	2.38	54.11
x_3	84.59	-2.65	50.30	98.18

The status of unmet demand in 38 East Java regencies and cities in 2023 exist of 13 regencies/cities with target not achieved and 25 regencies/cities with target achieved. Furthermore, it is determined that there is no multicollinearity amongst predictor variables and that none of the variables contain missing values. Figure 2 shows the conceptual predictor variable that was employed in this investigation.

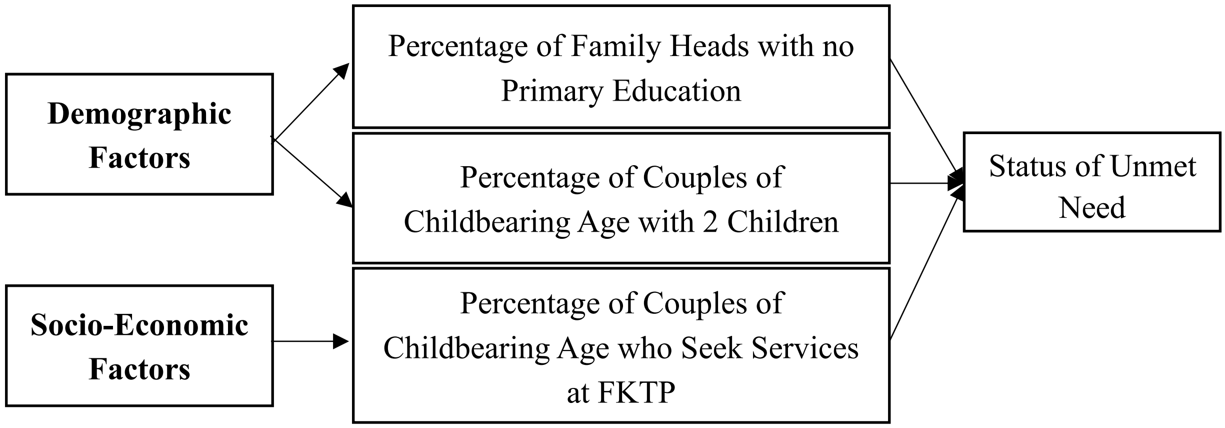

Figure 2. Conceptual Diagram of Variables [33]

Based on Figure 2, unmet need is a problem that covers many aspects including demographic factors and socio-economics factors. Most women of childbearing age who want to stop or adjust their spacing are unable to use family planning methods. Demographers and health experts refer to these women of childbearing age as having an unmet need for family planning services [34].

BLR Model

The parametric model using Binary Logistic Regression (BLR) model.

$$\pi(\boldsymbol{x}_i) = \frac{e^{\beta_0+\sum_{j=1}^{p}\beta_j x_{ji}}}{1+e^{\beta_0+\sum_{j=1}^{p}\beta_j x_{ji}}}, i = 1,2,\dots,n \tag{28}$$

Where p is the number of predictor variable.

Parameter Estimation

Based on BLR model (28), the results of parameter estimation in the BLR model for data on the status of unmet need in East Java in 2023 are as follows in equation (29).

$$\hat{\pi}(\boldsymbol{x}_i) = \frac{e^{15.5239-0.1773x_{1i}-0.4430x_{2i}-0.0184x_{3i}}}{1+e^{15.5239-0.1773x_{1i}-0.4430x_{2i}-0.0184x_{3i}}} \tag{29}$$

More details can be seen in Table 3.

Table 3. Parameter Estimation in BLR Model

Parameters	Estimations
β_0	15.5239
β_1	-0.1773
β_2	-0.4430
β_3	-0.0184

FSNR Model

To find the link that adhered to the FSNR model, we made a scatterplot in which the number of high unmet needs ($y = 1$) in each group was presented against each predictor variable that was included in multiple groups. To see the scatterplot, see Figure 3.

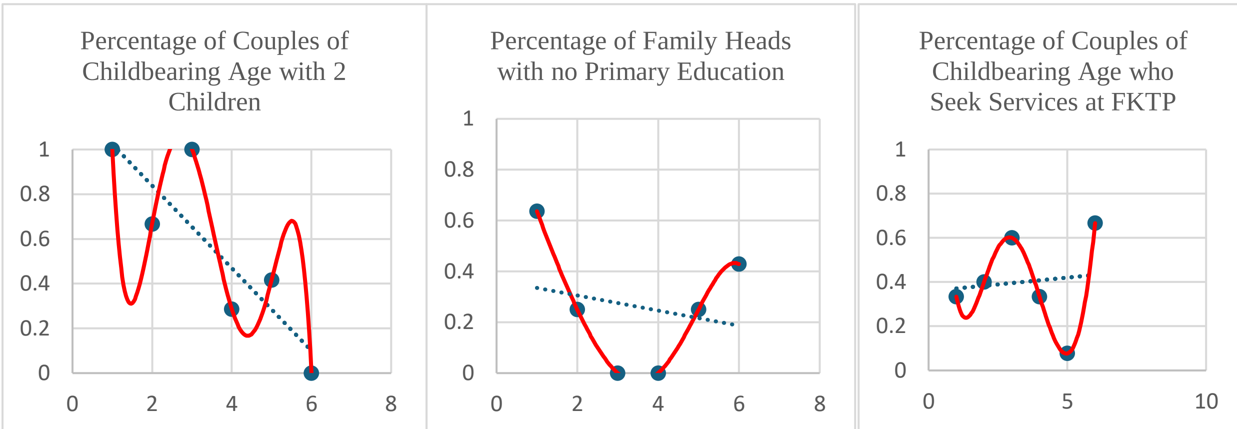

Figure 3. Scatterplots of several data groups versus the number of high unmet need in the group

Based on Figure 3, The probability of a high unmet need $(y = 1)$ for variable x_1, x_2, and x_3 it has a repeating pattern and follows a trend line.

The nonparametric model using FSNR model.

$$\pi(\boldsymbol{x_i}) = \frac{e^{\sum_{l=1}^{q}\left(b_l z_{li} + \frac{1}{2}a_{0l} + \sum_{s=1}^{S} a_{sl} \cos s z_{li}\right)}}{1 + e^{\sum_{l=1}^{q}\left(b_l z_{li} + \frac{1}{2}a_{0l} + \sum_{s=1}^{S} a_{sl} \cos s z_{li}\right)}} \; ; \; i = 1,2,\ldots,n \quad (30)$$

Where s is the number of oscillation parameter and q is the number of predictor variables in FSNR model.

Parameter Estimation

Based on FSNR model (30), parameter estimation results on the model with the optimal oscillation parameter combination of 1,1,2 for data on the status of unmet need in East Java in 2023 are as follows in equation (31).

$$\hat{\pi}(\boldsymbol{x_i}) = \frac{e^{59.64 - 0.89x_{1i} - 0.01\cos x_{1i} - 1.75x_{2i} - 4.12\cos x_{2i} - 0.07x_{3i} - 4.39\cos x_{3i} - 3.03\cos 2x_{3i}}}{1 + e^{59.64 - 0.89x_{1i} - 0.01\cos x_{1i} - 1.75x_{2i} - 4.12\cos x_{2i} - 0.07x_{3i} - 4.39\cos x_{3i} - 3.03\cos 2x_{3i}}} \quad (31)$$

More details in Table 4.

Table 4. Parameter Estimation in FSNR Model

Parameters	Estimations
a_0	59.6457
b_1	-0.8933
$a_{1,1}$	-0.0156
b_2	-1.7481
$a_{1,2}$	-4.1230
b_3	-0.0670
$a_{1,3}$	-4.3931
$a_{2,3}$	-3.0266

Selecting Optimal Oscillation Parameters

The least AIC value was used to determine the oscillation parameters in the FSNR model. In order to create a model that is not overly complex and yields results of suitable relevance, the number of oscillation parameters employed in this study was restricted. Table 5 displays the AIC values for each combination of oscillation parameters in the model using the R program.

Table 5. Minimum AIC Outcomes by Oscillation Parameter Number

Number of Oscillation Parameter	Oscillation Parameter Combination (K)			AIC (K)
	x_1	x_2	x_3	
K=1	1	1	1	36.2558
K=2	1	1	2	**34.9600**

Based on Table 5, the model with a combination of oscillation parameters $x_1 = 1, x_2 = 1, x_3 = 2$ is the Fourier Series model with optimal oscillation parameters because it has the smallest AIC value.

Comparison of BLR and FSNR Model

Finding the Best Model Using Deviance Value

The regression model with the lowest deviation value was chosen. The findings of the deviation statistical test are shown in Table 6 as follows.

Table 6. Comparison of Deviance Values

Methods	Deviance Values
BLR	30.9507
FSNR	**20.9598**

Based on Table 6, the deviance value for the FSNR (20.9598) was smaller than BLR (30.9507). Therefore, for such data, the FSNR model is the optimal choice on the status of unmet need in East Jawa in 2023 because has the smallest deviance value.

Fnding the Best Classification Using AUC & Press's Q Value

The FSNR model that was chosen showed the smallest Press's Q or the highest AUC. The results of the classification test are shown in Table 7 as follows.

Table 7. Comparison of AUC and Press's Q

Methods	Accuracy	Sensitivity	Specificity	AUC	Press's Q value	Chi Square
BLR	76.3158	84	61.5385	72.7692	10.5263	3.8415
FSNR	86.8421	92	76.9231	84.4615	20.6316	3.8415

According to Table 7, case 1, FSNR's AUC value (84.46%) is greater than BLR's (72.76%). Furthermore, the FSNR model can classify well and has a higher probability of rejecting H0 or Press's Q > Chi Square if its Press's Q value is higher (20.63).

4. CONCLUSION

The FSNR model for categorical data takes the form described in the discussion.

$$\hat{\pi}(\boldsymbol{x_i}) = \frac{e^{59.64-0.89x_{1i}-0.01\cos x_{1i}-1.75x_{2i}-4.12\cos x_{2i}-0.07x_{3i}-4.39\cos x_{3i}-3.03\cos 2x_{3i}}}{1+e^{59.64-0.89x_{1i}-0.01\cos x_{1i}-1.75x_{2i}-4.12\cos x_{2i}-0.07x_{3i}-4.39\cos x_{3i}-3.03\cos 2x_{3i}}}$$

The FSNR model (with a deviance value is 20.9598) outperforms BLR model (with larger deviance values) for estimating the data on the status of unmet need in East Java in 2023. The estimated value of the FSNR model in the plot is greater than BLR models. The FSNR model excels in forecasting unmet need status in East Java 2023. This superior performance is further supported by higher values of AUC (84.46%) and Press’s Q (20.63), indicating better classification accuracy and model reliability.

ACKNOWLEDGMENTS

We are thankful to the Indonesian Ministry of Education, Culture, Research, and Technology for financing our research under the PMDSU VII program," through Grant Number 038/E5/PG.02.00.PL/2024 and local Grant Number 1798/PKS/ITS/2024.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] R.L. Eubank, Spline Smoothing and Nonparametric Regression, Marcel Dekker, New York, (1998).

[2] M. Bilodeau, Fourier Smoother and Additive Models, Can. J. Stat. 20 (1992), 257-269. https://doi.org/10.2307/3315313.

[3] L. Li, Nonlinear Wavelet-Based Nonparametric Curve Estimation with Censored Data and Inference on Long Memory Processes, Thesis, Michigan State University, 2002.

[4] H. Okumura, K. Naito, Non-parametric Kernel Regression for Multinomial Data, J. Multivar. Anal. 97 (2006), 2009-2022. https://doi.org/10.1016/j.jmva.2005.12.008.

[5] L. Su, A. Ullah, Local Polynomial Estimation of Nonparametric Simultaneous Equations Models, J. Econ. 144 (2008), 193-218. https://doi.org/10.1016/j.jeconom.2008.01.002.

[6] A. Welsh, T. Yee, Local Regression for Vector Responses, J. Stat. Plan. Inference 136 (2006), 3007-3031. https://doi.org/10.1016/j.jspi.2004.01.024.

[7] A. Antoniadis, J. Bigot, T. Sapatinas, Wavelet Estimators in Nonparametric Regression: A Comparative Simulation Study, J. Stat. Softw. 6 (2001), 1-83. https://doi.org/10.18637/jss.v006.i06.

[8] A. Tripena, I.N. Budiantara, Fourier Estimator in Nonparametric Regression, in: International Conference on Natural Sciences and Applied Natural Sciences, Ahmad Dahlan University, Yogyakarta, (2007).

[9] U. Amato, A. Antoniadis, I. De Feis, Fourier Series Approximation of Separable Models, J. Comput. Appl. Math. 146 (2002), 459-479. https://doi.org/10.1016/s0377-0427(02)00398-9.

[10] J. Morton, L. Silverberg, Fourier Series of Half-Range Functions by Smooth Extension, Appl. Math. Model. 33 (2009), 812-821. https://doi.org/10.1016/j.apm.2007.12.009.

[11] D.D. Canditiis, I.D. Feis, Pointwise Convergence of Fourier Regularization for Smoothing Data, J. Comput. Appl. Math. 196 (2006), 540-552. https://doi.org/10.1016/j.cam.2005.10.009.

[12] L.J. Asrini, I.N. Budiantara, FSSR Models (Case study: The production of lowland rice irrigation in Central Java), ARPN J. Eng. Appl. Sci. 9 (2014), 1501-1506.

[13] I.N. Budiantara, V. Ratnasari, I. Zain, M. Ratna, M.F.F. Mardianto, Modeling of HDI and PQLI in East Java (Indonesia) Using Biresponse Semiparametric Regression with Fourier Series Approach, Asian Trans. Basic Appl. Sci. 5 (2015), 21-28.

[14] I.W. Sudiarsa, I.N. Budiantara, undefined. Suhartono, S.W. Purnami, Combined Estimator Fourier Series and Spline Truncated in Multivariable Nonparametric Regression, Appl. Math. Sci. 9 (2015), 4997-5010. https://doi.org/10.12988/ams.2015.55394.

[15] V. Ratnasari, I.N. Budiantara, I. Zain, M. Ratna, N.P.A.M. Mariati, Comparison of Truncated Spline and Fourier Series in Multivariable Nonparametric Regression Models (Application: Data Of Poverty In Papua, Indonesia), Int. J. Basic Appl. Sci. 15 (2015), 9–12.

[16] I.N. Budiantara, V. Ratnasari, M. Ratna, et al. Modeling Percentage of Poor People in Indonesia Using Kernel and Fourier Series Mixed Estimator in Nonparametric Regression, Investig. Oper. 40 (2019), 538–551.

[17] K. Nisa, I.N. Budiantara, A.T. Rumiati, Multivariable Semiparametric Regression Model with Combined Estimator of Fourier Series and Kernel, IOP Conf. Ser.: Earth Environ. Sci. 58 (2017), 012028. https://doi.org/10.1088/1755-1315/58/1/012028.

[18] A.T. Ampa, I.N. Budiantara, I. Zain, Modeling the Level of Drinking Water Clarity in Surabaya City Drinking Water Regional Company Using Combined Estimation of Multivariable Fourier Series and Kernel, Sustainability 14 (2022), 13663. https://doi.org/10.3390/su142013663.

[19] M. Ramli, I.N. Budiantara, V. Ratnasari, A Method for Parameter Hypothesis Testing in Nonparametric Regression with Fourier Series Approach, MethodsX 11 (2023), 102468. https://doi.org/10.1016/j.mex.2023.102468.

[20] P.H. Jou, S.H. Mirhashemi, Frequency Analysis of Extreme Daily Rainfall Over an Arid Zone of Iran Using Fourier Series Method, Appl. Water Sci. 13 (2022), 16. https://doi.org/10.1007/s13201-022-01823-z.

[21] L. Laome, I.N. Budiantara, V. Ratnasari, Poverty Modelling with Spline Truncated, Fourier Series, and Mixed Estimator Geographically Weighted Nonparametric Regression, AIP Conf. Proc. 3109 (2024), 070007. https://doi.org/10.1063/5.0206173.

[22] Suliyanto, M. Rifada, E. Tjahjono, Estimation of Nonparametric Binary Logistic Regression Model with Local Likelihood Logit Estimation Method (Case Study of Diabetes Mellitus Patients at Surabaya Hajj General Hospital), AIP Conf. Proc. 2264 (2020), 030007. https://doi.org/10.1063/5.0025807.

[23] H. Hamie, A. Hoayek, B. El-Ghoul, M. Khalifeh, Application of Non-Parametric Statistical Methods to Predict Pumpability of Geopolymers for Well Cementing, J. Pet. Sci. Eng. 212 (2022), 110333. https://doi.org/10.1016/j.petrol.2022.110333.

[24] T. Wang, W. Tang, Y. Lin, W. Su, Semi‐supervised Inference for Nonparametric Logistic Regression, Stat.

Med. 42 (2023), 2573-2589. https://doi.org/10.1002/sim.9737.

[25] M. Zulfadhli, I.N. Budiantara, V. Ratnasari, Nonparametric Regression Estimator of Multivariable Fourier Series for Categorical Data, MethodsX 13 (2024), 102983. https://doi.org/10.1016/j.mex.2024.102983.

[26] A.S. Suriaslan, I.N. Budiantara, V. Ratnasari, Nonparametric Regression Estimation Using Multivariable Truncated Splines for Binary Response Data, MethodsX 14 (2025), 103084. https://doi.org/10.1016/j.mex.2024.103084.

[27] R.S. Rachmaningrum, S.H. Wijaya, Determinan Unmet Need KB Wanita Usia Subur Berstatus Kawin di Provinsi DKI Jakarta Tahun 2017, Semin. Nas. Off. Stat. 2020 (2020), 957–966.

[28] S. Ajmal, A. Idris, B. Ajmal, Factors Affecting Contraceptive Use and Unmet Need Among Currently Married Women in Afghanistan: Further Analysis of the 2015 Afghanistan Demographic and Health Survey, J. Glob. Heal. Rep. 2 (2018), . https://doi.org/10.29392/joghr.2.e2018037.

[29] S. Ajmal, A. Idris, B. Ajmal, Factors Affecting Contraceptive Use and Unmet Need Among Currently Married Women in Afghanistan: Further Analysis of the 2015 Afghanistan Demographic and Health Survey, J. Glob. Heal. Rep. 2 (2018), e2023062286. https://doi.org/10.29392/joghr.2.e2018037.

[30] D.W. Hosmer, S. Lemeshow, Applied Logistic Regression, Wiley, (2000).

[31] R.V. Hogg, A.T. Craig, Introduction to Mathematical Statistics, Pearson Prentice Hall, (1995).

[32] S.E. Bradley, T.N. Croft, J.D. Fishel, C.F. Westoff, Revising Unmet Need for Family Planning, ICF International, Calverton, (2012).

[33] B.O. Ahinkorah, E.K. Ameyaw, A. Seidu, Socio-economic and Demographic Predictors of Unmet Need for Contraception Among Young Women in Sub-Saharan Africa: Evidence from Cross-Sectional Surveys, Reprod. Heal. 17 (2020), 163. https://doi.org/10.1186/s12978-020-01018-2.

[34] A. Kumar, A. Singh, Trends and Determinants of Unmet Need for Family Planning in Bihar (India): Evidence from National Family Health Surveys, Adv. Appl. Sociol. 03 (2013), 157-163. https://doi.org/10.4236/aasoci.2013.32021.