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VECTOR BASIS S-CORDIAL LABELING OF GRAPHS WITH APPLICATIONS IN BIOLOGICAL MODELS

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Abstract. Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \rightarrow S$ be a function. For edge uv assign the label $< \phi(u), \phi(v) >$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. In this paper, we examine the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behaviour of Mongolian tent and parachute graph such a labeling finds applications in biological modelling.

Keywords: path; cycle; Mongolian tent; parachute graph; vector basis S-cordial.

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1. Introduction

Let G be a simple, finite and undirected graph with vertex set $V(G)$ and edge set $E(G)$ respectively. The order and size of a graph G are denoted by $p = |V(G)|$ and $q = |E(G)|$ respectively.

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In Graph Theory, graph labeling is one of the most studied subjects. Graph labeling was introduced by Rosa in 1967 [16]. It was further developed by Graham and Sloane in 1980. It is applied in the research area of computer science especially in networking, clustering, image segmentation, astronomy, circuit designing, data base management, data mining, image processing, cryptography, software testing, information security, coding theory, x-ray crystallography, radar and missile guidance [17]. S. Czerwinski et al. was first introduced the lucky labeling of graphs. Lucky labeling is coloring the vertices arbitrarily such that the sum of labels of all adjacent vertices of a vertex is not equal to the sum of labels of all adjacent vertices of any vertex which is adjacent to it. Lucky labeling of graphs were studied in recent times by S. Akbari et al [1]. Lucky labeling is applied in real life situations such as transportation network, where pair wise connections are given some numerical values. They are also applicable in computational biology to model protein structures. Lucky edge labeling of some graphs have studied in [9, 12, 10, 11]. The idea of cordial labeling was first invented by Cahit [2]. Lucky labeling and proper lucky labeling for bloom graph was discussed in [3].

Definition 1.1. [5] *The Mongolian tent $M_{m,n}$ is obtained by joining a vertex on top of $P_m \times P_n$ grid with the top row vertices of the grid.*

Definition 1.2. [5] *The parachute graph $P_{m,n}$ is obtained from the wheel W_{m+n}, $m \geq 3$ by deleting n consecutive spokes.*

In this paper, we consider the inner product space R^n and the standard inner product $<x,y>= x_1y_1 + x_2y_2 + \cdots + x_ny_n$ where $x = (x_1,x_2,\ldots,x_n), y = (y_1,y_2,\ldots,y_n)$, $x_i, y_i \in R$. Terms that are not specified in this paper are derived from Harary [7] and Herstein [8]. For standard terminology and notations, we follow the book of Gallian [5].

The vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0), (1,0,0,0)}-cordial labeling of various families of graphs has been illustrated in [13, 14, 15]. In this paper, we examine the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0), (1,0,0,0)}-cordial labeling behavior of Mongolian tent and parachute graph.

2. APPLICATION IN BIOLOGICAL MODEL

The techniques of graph theory find applications in biology by using networks to model systems of complex nature, enabling the researchers to carry out analysis of molecular structures, biological pathways and interactions between genes, proteins and metabolites. Gao et al. [18] and Pavlopoulos et al. [6] have considered various applications of graph theory in biological networks.

An interesting application of graph theory is its use in prey-predator modeling to represent the interactions of certain species as networks in food webs, wherein the vertices represent species and the edges correspond to predator-prey relationships. In a modelling of a system of food webs, the individual species within an ecosystem are represented by the vertices of a graph while the predator-prey interactions between species are represented by the edges of the graph. As an illustration, one can think of an edge from a predator node to a prey node as an indication of the predator feeding on the prey.

Food web analysis may be carried out by using graph theory is to examine real-world food webs from long-term monitoring programs to understand their resilience and functioning. By deriving graph theoretic criteria, it is possible to determine the stability and boundedness of population equilibria in various complex, multi-species predator-prey systems. The graph-theoretic method may be viewed in combination with other mathematical tools like coincidence degree theory to examine and understand the periodicity of coupled predator-prey systems in dispersed environments.

One can consider vertices in a graph as representing certain animals of the same type in a forest. The food available for the animals of the same type is represented by the set of vectors in the basis S. The energy of the two animals is the edge label of the corresponding vertices. The labeling is considered optimal if any pair of animals receive energy values of m and $m+1$ for some $m \in N$. The problem is to determine the labeling of the graph which would ensure optimal energy values to all the pairs of animals of the same type in the forest.

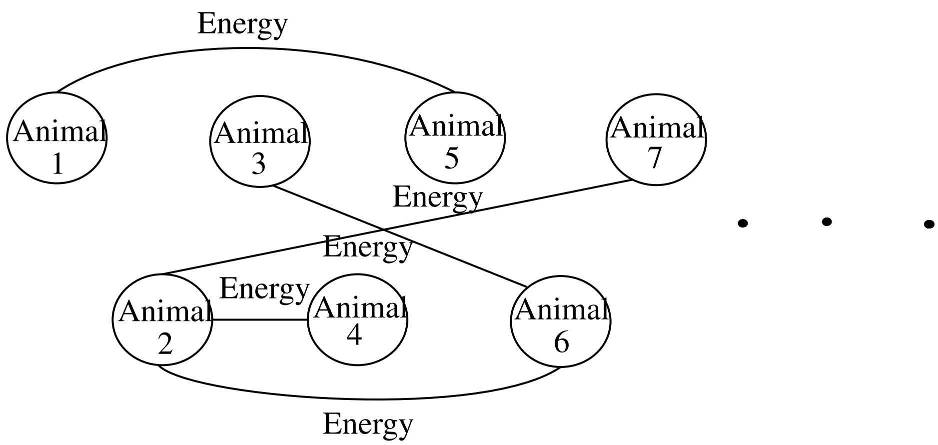

FIGURE 1. Food Energy Model

3. VECTOR BASIS S-CORDIAL LABELING

Definition 3.1. *Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \rightarrow S$ be a function.For edge uv assign the label $< \phi(u), \phi(v) >$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph.*

An example of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph is illustrated in Figure 2.

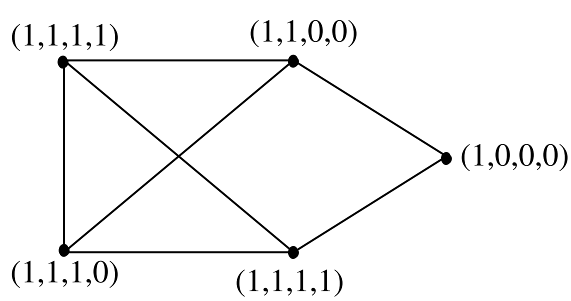

FIGURE 2. An example of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph

4. Main Results

In this section, we find the existence of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0), (1,0,0,0)}-cordial labeling of Mongolian tent and parachute graph.

Theorem 4.1. *The Mongolian tent $M_{m,n}$ is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph if and only if*

(1) $n \equiv 0 \pmod 4$ *and* $m \equiv 0,1,2,3 \pmod 4$

(2) $n \equiv 1 \pmod 4$ *and* $m \equiv 0 \pmod 4$

(3) $n \equiv 2 \pmod 4$ *and* $m \equiv 0,2 \pmod 4$

(4) $n \equiv 3 \pmod 4$ *and* $m \equiv 0 \pmod 4$

Proof. Let $M_{m,n}$, $m,n \geq 2$ be the Mongolian tent. Then the vertex set and edge set of $M_{m,n}$ are defined by $V(M_{m,n}) = \{u_0, u_{i,j} \mid 1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$ and $E(M_{m,n}) = \{u_0u_{1,j}, u_{i,j}u_{i+1,j} \mid 1 \leq i \leq m-1 \text{ and } 1 \leq j \leq n\} \cup \{u_{i,j}u_{i,j+1} \mid 1 \leq i \leq m \text{ and } 1 \leq j \leq n-1\}$ respectively. The number of vertices and edges of the Mongolian tent are given by $p = |V(M_{m,n})| = mn+1$ and $q = |E(M_{m,n})| = m(2n-1)$ respectively. Let us assign the vector to the vertices in the following order $u_0, u_{11}, u_{12}, \ldots, u_{1n}, u_{21}, u_{22}, \ldots, u_{2n}, u_{m1}, u_{m2}, \ldots, u_{mn}$.

Case (1): $n \equiv 0 \pmod 4$

Let $n = 4r_1$, $r_1 \in \mathbb{N}$. There are four subcases arises.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4r_2$, $r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2)+1$ and $q = 4(8r_1r_2 - r_2)$. Assign the vector (1,1,1,1) to the first $4r_1r_2+1$ vertices and (1,1,1,0) to the next $4r_1r_2$ vertices. Thereafter assign the vectors (1,1,0,0) to the next $4r_1r_2$ vertices and (1,0,0,0) to the next $4r_1r_2$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2+1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 8r_1r_2 - r_2$.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4r_2+1$, $r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2+r_1)+1$ and $q = 4(8r_1r_2 - r_2 + 2r_1) - 1$. Assign the vector (1,1,1,1) to the first $4r_1r_2 + r_1 + 1$ vertices and (1,1,1,0) to the next $4r_1r_2 + r_1$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2 + r_1$ vertices and (1,0,0,0) to the next $4r_1r_2 + r_1$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2 + r_1 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2 + r_1$. Hence $\gamma_4 = 8r_1r_2 - r_2 + 2r_1 - 1$ and $\gamma_3 = \gamma_2 = \gamma_1 = 8r_1r_2 - r_2 + 2r_1$.

Subcase (iii): $m \equiv 2 \pmod 4$

Take $m = 4r_2 + 2$, $r_2 \geq 0$. Then $p = 4(4r_1r_2 + 2r_1) + 1$ and $q = 4(8r_1r_2 + 4r_1 - r_2) - 2$. Assign the vector (1,1,1,1) to the first $4r_1r_2 + 2r_1 + 1$ vertices and (1,1,1,0) to the next $4r_1r_2 + 2r_1$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2 + 2r_1$ vertices and (1,0,0,0) to the next $4r_1r_2 + 2r_1$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2 + 2r_1 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2 + 2r_1$. Hence $\gamma_4 = \gamma_2 = 8r_1r_2 - r_2 + 4r_1 - 1$ and $\gamma_3 = \gamma_1 = 8r_1r_2 - r_2 + 4r_1$.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4r_2 + 3$, $r_2 \geq 0$. Then $p = 4(4r_1r_2 + 3r_1) + 1$ and $q = 4(8r_1r_2 + 6r_1 - r_2) - 3$. Assign the vector (1,1,1,1) to the first $4r_1r_2 + 3r_1 + 1$ vertices and (1,1,1,0) to the next $4r_1r_2 + 3r_1$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2 + 3r_1$ vertices and (1,0,0,0) to the next $4r_1r_2 + 3r_1$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2 + 3r_1 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2 + 3r_1$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = 8r_1r_2 - r_2 + 6r_1 - 1$ and $\gamma_1 = 8r_1r_2 - r_2 + 6r_1$.

Case (2): $n \equiv 1 \pmod 4$

Let $n = 4r_1 + 1$, $r_1 \in \mathbb{N}$. There are four subcases arises.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4r_2$, $r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2 + r_2) + 1$ and $q = 4(8r_1r_2 + r_2)$. Assign the vector (1,1,1,1) to the first $4r_1r_2 + r_2 + 1$ vertices and (1,1,1,0) to the next $4r_1r_2 + r_2$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2 + r_2$ vertices and (1,0,0,0) to the next $4r_1r_2 + r_2$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2 + r_2 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2 + r_2$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 8r_1r_2 + r_2$.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4r_2 + 1$, $r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2 + r_1 + r_2) + 2$ and $q = 4(8r_1r_2 + 2r_1 + r_2)$. From $4r_1r_2 + r_1 + r_2 + 1$ vertices with vertex label (1,1,1,1), we cannot get $8r_1r_2 + 2r_1 + r_2$ edges with edge label 4, a contradiction.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4r_2 + 2$, $r_2 \geq 0$. Then $p = 4(4r_1r_2 + 2r_1 + r_2) + 3$ and $q = 4(8r_1r_2 + 4r_1 + r_2) + 2$. From $4r_1r_2 + 2r_1 + r_2 + 1$ vertices with vertex label (1,1,1,1) we cannot get $8r_1r_2 + 4r_1 + r_2$ edges with edge label 4, a contradiction.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4r_2+3,\ r_2 \geq 0$. Then $p = 4(4r_1r_2+3r_1+r_2+1)$ and $q = 4(8r_1r_2+6r_1+r_2)+3$. From $4r_1r_2+3r_1+r_2+1$ vertices with vertex label (1,1,1,1) we cannot get $8r_1r_2+6r_1+r_2$ edges with edge label 4, a contradiction.

Case (3): $n \equiv 2 \pmod 4$

Let $n = 4r_1+2$, $r_1 \geq 0$. There are four subcases arises.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4r_2,\ r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2+2r_2)+1$ and $q = 4(8r_1r_2+3r_2)$. Assign the vector (1,1,1,1) to the first $4r_1r_2+2r_2+1$ vertices and (1,1,1,0) to the next $4r_1r_2+2r_2$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2+2r_2$ vertices and (1,0,0,0) to the next $4r_1r_2+2r_2$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2+2r_2+1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2+2r_2$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 8r_1r_2+3r_2$.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4r_2+1,\ r_2 \in \mathbb{N}$. Then $p = 4(4r_1r_2+r_1+2r_2)+3$ and $q = 4(8r_1r_2+2r_1+3r_2)+3$. From $4r_1r_2+r_1+2r_2+1$ vertices with vertex label (1,1,1,1), we cannot get $8r_1r_2+2r_1+3r_2$ edges with edge label 4, a contradiction.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4r_2+2$, $r_2 \geq 0$. Then $p = 4(4r_1r_2+2r_1+2r_2)+1$ and $q = 4(8r_1r_2+3r_2+4r_1+1)+2$. Assign the vector (1,1,1,1) to the first $4r_1r_2+2r_1+2r_2+2$ vertices and (1,1,1,0) to the next $4r_1r_2+2r_1+2r_2+1$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2+2r_1+2r_2+1$ vertices and (1,0,0,0) to the next $4r_1r_2+2r_1+2r_2+1$ vertices. We have $\phi_{(1,1,1,1)} = 4r_1r_2+2r_1+2r_2+2$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 4r_1r_2+2r_2+2r_1+1$. Hence $\gamma_4 = \gamma_2 = 8r_1r_2+3r_2+4r_1+1$ and $\gamma_3 = \gamma_1 = 8r_1r_2+3r_2+4r_1+2$.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4r_2+3$, $r_2 \geq 0$. Then $p = 4(4r_1r_2+3r_1+2r_2+1)+3$ and $q = 4(8r_1r_2+6r_1+3r_2+2)+1$. From $4r_1r_2+3r_1+2r_2+2$ vertices with vertex label (1,1,1,1), we cannot get $8r_1r_2+6r_1+3r_2+2$ edges with edge label 4, a contradiction.

Case (4): $n \equiv 3 \pmod 4$

Let $n = 4r_1+3$, $r_1 \geq 0$. There are four subcases arises.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m=4r_2$, $r_2\in\mathbb{N}$. Then $p=4(4r_1r_2+3r_2)+1$ and $q=4(8r_1r_2+5r_2)$. Assign the vector (1,1,1,1) to the first $4r_1r_2+3r_2+1$ vertices and (1,1,1,0) to the next $4r_1r_2+3r_2$ vertices. Then assign the vectors (1,1,0,0) to the next $4r_1r_2+3r_2$ vertices and (1,0,0,0) to the next $4r_1r_2+3r_2$ vertices. We have $\phi_{(1,1,1,1)}=4r_1r_2+3r_2+1$ and $\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=4r_1r_2+3r_2$. Hence $\gamma_4=\gamma_3=\gamma_2=\gamma_1=8r_1r_2+5r_2$.

Subcase (ii): $m\equiv 1 \pmod 4$

Let $m=4r_2+1$, $r_2\in\mathbb{N}$. Then $p=4(4r_1r_2+r_1+3r_2+1)$ and $q=4(8r_1r_2+2r_1+5r_2+1)+1$. From $4r_1r_2+r_1+3r_2+1$ vertices with vertex label (1,1,1,1), we cannot get $8r_1r_2+2r_1+5r_2+1$ edges with edge label 4, a contradiction.

Subcase (iii): $m\equiv 2 \pmod 4$

Let $m=4r_2+2$, $r_2\geq 0$. Then $p=4(4r_1r_2+2r_1+3r_2+1)+3$ and $q=4(8r_1r_2+5r_2+4r_1+2)+2$. From $4r_1r_2+2r_1+3r_2+2$ vertices with vertex label (1,1,1,1), we cannot get $8r_1r_2+4r_1+5r_2+2$ edges with edge label 4, a contradiction.

Subcase (iv): $m\equiv 3 \pmod 4$

Let $m=4r_2+3$, $r_2\geq 0$. Then $p=4(4r_1r_2+3r_1+3r_2+2)+2$ and $q=4(8r_1r_2+6r_1+5r_2+3)+3$. Assign the vector (1,1,1,1) to the first $4r_1r_2+3r_1+3r_2+3$, $\gamma_4=8r_1r_2+6r_1+5r_2+3$ vertices. If we assign the vector (1,1,1,0) to the next $4r_1r_2+3r_1+3r_2+2$ vertices then $\gamma_3=8r_1r_2+6r_1+5r_2+3$, a contradiction. If we assign the vector (1,1,1,0) to the next $4r_1r_2+3r_1+3r_2+3$ vertices then $\gamma_3=8r_1r_2+6r_1+5r_2+5$, a contradiction.

□

Example 4.2. *An illustrative example for the application in the food web of a forest is presented here. Consider a set of animals in a forest arranged as a Mongolian tent. A vector basis (1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)-cordial labeling of the Mongolian tent $M_{4,3}$ is illustrated in Figure 3.*

It is seen that the above biological model of a food web in a forest admits a vector basis-S cordial labeling.

Theorem 4.3. *The parachute graph $P_{m,n}$, $m,n\geq 2$ is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph if and only if*

(1) $m\equiv 0 \pmod 4$ *and* $n\neq 2,3$

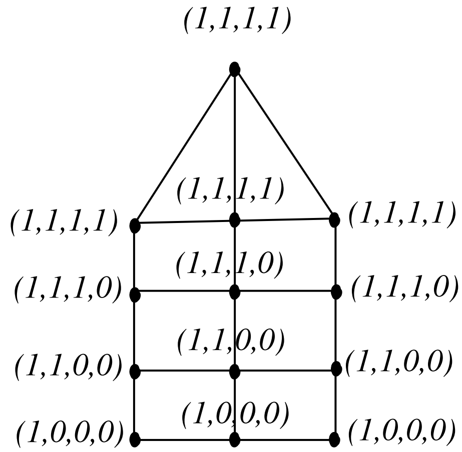

FIGURE 3. A vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of $M_{4,3}$

(2) $m \equiv 1 \pmod 4$ *and* $n \neq 2,6$

(3) $m \equiv 2 \pmod 4$ *and* $n \neq 4,5$

(4) $m \equiv 3 \pmod 4$ *and* $n \neq 2,3,4$

Proof. Consider the parachute graph $P_{m,n}$, $m,n \geq 2$. Denote by $V(P_{m,n}) = \{u_0, u_i, v_j \mid 1 \leq i \leq m$ and $1 \leq j \leq n\}$ and $E(P_{m,n}) = \{u_0u_i, u_iu_{i+1}, u_mv_n, u_1v_1, v_jv_{j+1} \mid 1 \leq i \leq m-1$ and $1 \leq j \leq n-1\}$ the vertex set and edge set of $P_{m,n}$ respectively. The number of vertices and edges of the parachute graph are given by $p = |V(P_{m,n})| = m+n+1$ and $q = |E(P_{m,n})| = 2m+n$ respectively. Assign the vector (1,1,1,1) to the vertices u_0, u_1 always.

Case (1): $m \equiv 0 \pmod 4$

Let $m = 4r_1$, $r_1 \geq 1$. Assign the vector (1,1,1,1) to the first t_1 vertices of P_m(from u_2) and (1,1,1,0) to the next t_1 vertices of P_m. Then assign the vector (1,1,0,0) to the next t_1 vertices of P_m and (1,0,0,0) to the next $t_1 - 1$ vertices of P_m.

Subcase (i): $n \equiv 0 \pmod 4$

Let $n = 4r_2$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2) + 1$ and $q = 4(2r_1 + r_2)$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next t_2 vertices of P_n. Then assign the vector (1,1,0,0) to the next t_2 vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = r_1 + r_2 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 2r_1 + r_2$.

Subcase (ii): $n \equiv 1 \pmod 4$

Let $n = 4r_2 + 1$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2) + 2$ and $q = 4(2r_1 + r_2) + 1$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next t_2 vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = r_1 + r_2 + 1$ and $\phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_2 = \gamma_1 = 2r_1 + r_2$ and $\gamma_3 = 2r_1 + r_2 + 1$.

Subcase (iii): $n \equiv 2 \pmod 4$

Let $n = 4r_2 + 2$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2) + 3$ and $q = 4(2r_1 + r_2) + 2$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 1$ and $\phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_1 = 2r_1 + r_2$ and $\gamma_3 = \gamma_2 = 2r_1 + r_2 + 1$.

Subcase (iv): $n \equiv 3 \pmod 4$

Let $n = 4r_2 + 3$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2 + 1)$ and $q = 4(2r_1 + r_2) + 3$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 2$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = 2r_1 + r_2$ and $\gamma_3 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 1$.

Case (2): $m \equiv 1 \pmod 4$

Let $m = 4r_1 + 1$, $r_1 \geq 1$. Assign the vector (1,1,1,1) to the first t_1 vertices of P_m(from u_2) and (1,1,1,0) to the next t_1 vertices of P_m. Then assign the vector (1,1,0,0) to the next t_1 vertices of P_m and (1,0,0,0) to the next t_1 vertices of P_m.

Subcase (i): $n \equiv 0 \pmod 4$

Let $n = 4r_2$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2) + 2$ and $q = 4(2r_1 + r_2) + 2$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next t_2 vertices of P_n and (1,0,0,0) to the next t_2 vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = r_1 + r_2 + 1$ and $\phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_2 = 2r_1 + r_2$ and $\gamma_3 = \gamma_1 = 2r_1 + r_2 + 1$.

Subcase (ii): $n \equiv 1 \pmod 4$

Let $n = 4r_2 + 1$, $r_2 \in \mathbb{N}$. Then $p = 4(r_1 + r_2) + 3$ and $q = 4(2r_1 + r_2) + 3$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next t_2 vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 1$ and $\phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_2 = \gamma_1 = 2r_1 + r_2$ and $\gamma_3 = 2r_1 + r_2 + 1$.

Subcase (iii): $n \equiv 2 \pmod 4$

Let $n = 4r_2 + 2$, $r_2 \geq 2$. Then $p = 4(r_1 + r_2 + 1)$ and $q = 4(2r_1 + r_2 + 1)$. Assign the vector (1,1,1,1) to the vertex u_2. Assign the vector (1,1,1,1) to the first t_1 vertices of P_m (from u_3)and (1,1,1,0) to the next t_1 vertices of P_m. Then assign the vector (1,1,0,0) to the next t_1 vertices of P_m and (1,0,0,0) to the next $t_1 - 1$ vertices of P_m.

Further, assign the vector (1,1,1,1) to the first $t_2 - 2$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 2$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 1$.

Subcase (iv): $n \equiv 3 \pmod 4$

Let $n = 4r_2 + 3$, $r_2 \geq 1$. Then $p = 4(r_1 + r_2 + 1) + 1$ and $q = 4(2r_1 + r_2 + 1) + 1$. Assign the vectors to the vertices of P_m as in subcase (ii) of case (2).

Further, assign the vector (1,1,1,1) to the first t_2 vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = r_1 + r_2 + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = 2r_1 + r_2 + 1$ and $\gamma_1 = 2r_1 + r_2 + 2$.

Case (3): $m \equiv 2 \pmod 4$

Let $m = 4r_1 + 2$, $r_1 \geq 1$. Assign the vector (1,1,1,1) to the first t_1 vertices of P_m(from u_2) and (1,1,1,0) to the next t_1 vertices of P_m. Then assign the vector (1,1,0,0) to the next t_1 vertices of P_m and (1,0,0,0) to the next t_1 vertices of P_m.

Subcase (i): $n \equiv 0 \pmod 4$

Let $n = 4r_2$, $r_2 \geq 2$. Then $p = 4(r_1 + r_2) + 3$ and $q = 4(2r_1 + r_2 + 1)$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next t_2 vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 1$ and $\phi_{(1,0,0,0)} = r_1 + r_2$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 1$.

Subcase (ii): $n \equiv 1 \pmod 4$

Let $n = 4r_2 + 1$, $r_2 \geq 2$. Then $p = 4(r_1 + r_2 + 1)$ and $q = 4(2r_1 + r_2 + 1) + 1$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next t_2 vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_1 = 2r_1 + r_2 + 1$ and $\gamma_3 = \gamma_2 = 2r_1 + r_2 + 1$.

Subcase (iii): $n \equiv 2 \pmod 4$

Let $n = 4r_2 + 2$, $r_2 \geq 0$. Then $p = 4(r_1 + r_2 + 1) + 1$ and $q = 4(2r_1 + r_2 + 1) + 2$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = r_1 + r_2 + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_1 = 2r_1 + r_2 + 2$ and $\gamma_3 = \gamma_2 = 2r_1 + r_2 + 1$.

Subcase (iv): $n \equiv 3 \pmod 4$

Let $n = 4r_2 + 3$, $r_2 \geq 1$. Then $p = 4(r_1 + r_2 + 1) + 2$ and $q = 4(2r_1 + r_2 + 1) + 3$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 2$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 2$ and $\gamma_3 = 2r_1 + r_2 + 1$.

Case (4): $m \equiv 3 \pmod 4$

Let $m = 4r_1 + 3$, $r_1 \geq 1$. Assign the vector (1,1,1,1) to the first t_1 vertices of P_m(from u_2) and (1,1,1,0) to the next $t_1 + 1$ vertices of P_m. Then assign the vector (1,1,0,0) to the next t_1 vertices of P_m and (1,0,0,0) to the next t_1 vertices of P_m.

Subcase (i): $n \equiv 0 \pmod 4$

Let $n = 4r_2$, $r_2 \geq 2$. Then $p = 4(r_1 + r_2 + 1)$ and $q = 4(2r_1 + r_2 + 1) + 2$. Assign the vector (1,1,1,1) to the first $t_2 - 2$ vertices of P_n and (1,1,1,0) to the next t_2 vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_2 = 2r_1 + r_2 + 1$

and $\gamma_3 = \gamma_1 = 2r_1 + r_2 + 2$.

Subcase (ii): $n \equiv 1 \pmod 4$

Let $n = 4r_2 + 1$, $r_2 \geq 1$. Then $p = 4(r_1 + r_2 + 1) + 1$ and $q = 4(2r_1 + r_2 + 1) + 3$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next t_2 vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 1$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = r_1 + r_2 + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_3 = \gamma_1 = 2r_1 + r_2 + 2$ and $\gamma_2 = 2r_1 + r_2 + 1$.

Subcase (iii): $n \equiv 2 \pmod 4$

Let $n = 4r_2 + 2$, $r_2 \geq 1$. Then $p = 4(r_1 + r_2 + 1) + 2$ and $q = 4(2r_1 + r_2 + 2)$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next t_2 vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 2$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_3 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 2$.

Subcase (iv): $n \equiv 3 \pmod 4$

Let $n = 4r_2 + 3$, $r_2 \geq 1$. Then $p = 4(r_1 + r_2 + 1) + 3$ and $q = 4(2r_1 + r_2 + 2) + 3$. Assign the vector (1,1,1,1) to the first $t_2 - 1$ vertices of P_n and (1,1,1,0) to the next $t_2 + 1$ vertices of P_n. Then assign the vector (1,1,0,0) to the next $t_2 + 2$ vertices of P_n and (1,0,0,0) to the next $t_2 + 1$ vertices of P_n. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = r_1 + r_2 + 2$ and $\phi_{(1,0,0,0)} = r_1 + r_2 + 1$. Hence $\gamma_4 = \gamma_2 = \gamma_1 = 2r_1 + r_2 + 2$ and $\gamma_1 = 2r_1 + r_2 + 3$.

□

Example 4.4. *A vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of parachute graph* $P_{5,5}$ *illustrated in Figure 4.*

5. Conclusion

In this paper, we have discussed the vector basis {(1,1,1,1), (1,1,1,0), (1,1,0, 0), (1,0,0,0)}-cordial labeling of Mongolian tent and parachute graph. The exploration of vector basis S-cordial labeling behaviour of more family of graphs and properties are the open problems for future research work. It would be an interesting problem to identify different biological models which admit vector basis-S cordial labeling.

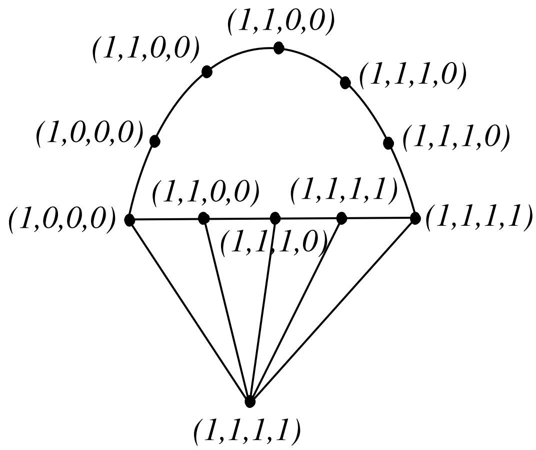

FIGURE 4. A vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of $P_{5,5}$

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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