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# THE ROLE OF MEDIA COVERAGE IN THE DYNAMICS OF THE SPREAD OF DRUG ADDICTION

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**Abstract:** A mathematical epidemiological model is developed to investigate the influences of media and assess its efficacy in reducing and declining drug use among new users, whether as a direct or indirect beneficial influence. A nonlinear differential equations system serves as the foundation for this model. Numerical simulations and mathematical analytic techniques are used to study the dynamic behavior of the suggested model. Under certain restrictions, the local and global stability of the equilibrium points is analytically asymptotically stable. Additionally, the model exhibits a forward bifurcation, which was examined using central manifold theory, demonstrating notable advancements in the fight against drug use in society resulting from media influence. To validate these analytical results, numerical simulations were carried out using MATLAB version R2021a code. The findings show which parameters should be improved and which should be targeted.

**Keywords:** Addiction mathematical model; media; stability; bifurcation; basic reproduction number (BRN),  $\mathcal{R}_0$ .

**2020 AMS Subject Classification:** 92D30; 34D20; 34C23.

## 1. INTRODUCTION

One of the biggest issues facing civilizations is drug usage and addiction, particularly in light of the obvious rise in the number of addicts worldwide, both in established and developing nations, particularly among youth. Due to its psychological, material, social, and even political aspects, drug addiction necessitates the cooperation of numerous stakeholders to investigate its origins and potential remedies [1-3]. Substances that alter a person's awareness, focus, and perception are

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known as drugs. Because drug consumption spreads so quickly throughout society, it is regarded as an epidemic. One of the most common and significant topics covered by the media is the prevalence of drugs. The issue of how bulletins, reports, and news are delivered has long drawn attention due to its significance. Drug trafficking, its effects on individual health and community cohesion, its effects on moral, religious, and economic values, and the deaths caused by drug use are all issues that are taken into consideration among the topics covered in various media outlets when we concentrate on what is presented in the media regarding the spread of drugs [4-6]. It goes without saying that young people nowadays are the biggest users of digital and social media. Young people's body image standards and exposure to unhealthy items and habits can be significantly influenced by modern media [7]. The media significantly shape perceptions regarding the dangers of substance use. Increases in youth substance use were linked to high levels of advertising and promotion of alcohol and tobacco products, the entertainment industry's pro-use messaging, and a decline in the media, parents, and educational institutions' warning and anti-drug messaging [8-9]. Use is a significant issue that needs to be assessed in all of its aspects and risk factors. These dimensions include social, legal, medical, educational, and security aspects [10].

Adolescents and young adults today are growing up surrounded by social media sites that promote user-generated content, and user interactions include Facebook and Twitter. [11]. It is well recognized that one socializing mechanism that promotes addiction is the media. The use of the motifs of "fear" and "anxiety" is said to have evolved from an incentive to an exclusionary strategy, particularly in recent years. Therefore, it should be considered that the media is a very powerful tool in influencing and directing society about substance usage, just like in other social concerns. Instead of focusing on increasing circulation and rating, social goals should be pursued, and the significance of the messages and perceptions formed in society should be carefully questioned [12]. The application of mathematical modeling to physical problems has been extensively researched, see [13-15] and the references therein. Akanni et al. [16] investigate the role of drug use in a community using a mathematical modeling approach. After a thorough analysis of the model, the following conclusions were drawn: the model shows a forward bifurcation property, which suggests that the problem can be readily managed once attempts are made to lower  $\mathcal{R}_0$ . To predict drug abuse developments and qualitatively investigate the dynamics of substance abuse, Kalula and Nyabadza [17] developed a multi-state compartmental model that has a core and non-core group with fast and gradual progression to addiction. They proposed that raising the uptake rate of addicts in treatment and implementing intervention programs aimed at light drug users could help curb the epidemic of substance misuse. According to Siddiqui et al. [18], media coverage plays a

crucial role in treating drug addiction as a socially transmitted illness. According to mathematical models, media coverage dramatically changes the number of addicts by lowering the "transmission rate" from susceptible persons to users, even if it may not alter the basic stability of addiction in a community. Additionally, a three-dimensional drug model is built to study how media coverage affects the development and management of drug addiction [19-20]. The BRN ( $\mathcal{R}_0$ ) is used to analyze the model's dynamical behavior. If  $\mathcal{R}_0 < 1$ , the drug-free equilibrium is asymptotically stable globally; if  $\mathcal{R}_0 > 1$ , the drug addiction equilibrium is locally stable. The findings show that while the media's impact on the populace cannot alter the equilibrium's stability, it can influence the number of drug users. Sensitivity analyses are used to find efficient drug treatment control strategies.

A compartment model is typically used in epidemiology to explain how an infectious disease spreads. In these epidemiological models, the infection process is dependent on contact with infectious persons, and the population is separated into various classes based on people's state in relation to the disease (susceptible to getting the disease, infected, or removed); see [21-23]. Drug phenomena and other related subjects have been the subject of much research using the epidemiological models [24–26] and the references therein. Recently, the authors of [12, 27] attempted to find a practical way to lower the number of heavy drug users, light drug users, heavy drug sellers and providers, and temporary drug quitters. They employed four control strategies: media and educational awareness campaigns, security campaigns to prohibit contact, treatment, psychological support, and a follow-up.

Our work in this paper is a little different because we will try to lower the number of drug addicts by using the media to persuade and intimidate the greatest number of addicts to stop using drugs. We will also make sure that those who stop using drugs don't start using them again.

## 2. MODEL FORMULATION

The creation of a mathematical model that mimics the impact of the media on the dynamics of the epidemiological system, which characterizes the phenomenon of drug abuse in society, was covered in this part. The purpose of this study is to raise awareness to lessen the prevalence of drug usage. It assumed that the community is made up of a sample  $N(t)$ , which includes individuals who do not use drugs and are at risk of becoming drug users (represented by the symbol  $S(t)$ ), those who use drugs and can be convinced to stop through the media (represented by the symbol  $D(t)$ ), drug abusers (represented by the symbol  $A(t)$ ), addicts with intellectual disabilities (represented by the symbol  $Q(t)$ ), and recovering addicts (represented by  $R(t)$ ). In

the proposed model, it is suggested that susceptible individuals join the population through a recruitment rate of people, denoted as  $\Lambda$ . When susceptible individuals interact with drug abusers, there is a probability of adopting the drug ideology, denoted by  $\beta$ , with a fraction  $\theta$  that is affected by drug users too. However, this group can be influenced by the media and avoid the drug with the rate  $\frac{\beta(\theta D+A)S}{1+\pi(D+A)}$ , where  $\pi$  represents the media rate. Some of the drug users may transition into the recovering group without engaging in drug addiction, a rate denoted by  $\rho_1$ , or they may transition into the drug abusers group at the rate  $\alpha$ . A drug abuser can also relocate to the recovery stage at a rate of  $\rho_2$ , or they may relocate to the addiction stage at a rate of  $\sigma$ , where they are transferred to rehabilitation centers. Those in rehabilitation centers have the potential to recover and join the recovering group at a rate of  $\rho_3$ . Finally,  $\mu$  represents the natural mortality rate within each group, while  $\delta$  represents the death rate as a result of drug addiction. The differential equations (1) are the governing system:

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda - \frac{\beta(\theta D+A)S}{1+\pi(D+A)} - \mu S, \\ \frac{dD}{dt} &= \frac{\beta(\theta D+A)S}{1+\pi(D+A)} - (\alpha + \rho_1 + \mu)D, \\ \frac{dA}{dt} &= \alpha D - (\rho_2 + \mu + \sigma)A, \\ \frac{dQ}{dt} &= \sigma A - (\rho_3 + \mu + \delta)Q, \\ \frac{dR}{dt} &= \rho_1 D + \rho_2 A + \rho_3 Q - \mu R, \end{aligned} \right\} \quad (1)$$

with nonnegative initial circumstances  $S(0) > 0, D(0) \geq 0, A(0) \geq 0, Q(0) \geq 0, R(0) \geq 0$ .

According to this hypothesis, drug users are the group most impacted by the media. Let's hope that the media and its fundamental principles will cause them to retreat without making them abusers or junkies.

System (1) states that all of the interaction functions on the right side are Lipschitzian functions since they are continuous and have continuous partial derivatives. Therefore, the basic existence and uniqueness theorem ensures the existence of a unique solution for each initial value  $(S(0), D(0), A(0), Q(0), R(0)) \in \mathbb{R}_+^5$ . Moreover, since the first four equations are independent of the last variable ( $R$ ). Therefore, from now onward, we study only the reduced system:

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda - \frac{\beta(\theta D+A)S}{1+\pi(D+A)} - \mu S, \\ \frac{dD}{dt} &= \frac{\beta(\theta D+A)S}{1+\pi(D+A)} - (\alpha + \rho_1 + \mu)D, \\ \frac{dA}{dt} &= \alpha D - (\rho_2 + \mu + \sigma)A, \\ \frac{dQ}{dt} &= \sigma A - (\rho_3 + \mu + \delta)Q, \end{aligned} \right\} \quad (2)$$

with nonnegative initial circumstances  $S(0) > 0, D(0) \geq 0, A(0) \geq 0, Q(0) \geq 0$ .

However, the solution of the last equation in system (1) can be obtained after substituting the solution of system (2) directly.

### 3. MATHEMATICAL ANALYSIS

Making sure that every solution is still relevant and biologically viable is the first stage in analyzing system (2)'s internal consistency. First, analyzing the positivity of the solutions of system (2) will be carried out, and then, to make sure that the model functions within a reasonable and limited scope. It will also look at the system's limits. Negative values have no biological significance because each variable reflects a subpopulation (susceptible, drug users, drug abusers, and drug addicts). This property's verification ensures that the trajectories stay within the feasible region, reflecting the actual dynamics of drug addiction under favorable media influence.

**Theorem (1):** Consider the system (1) with its initial circumstances. Then every solution  $(S(t), D(t), A(t), Q(t), R(t))$  will be positive for future time.

**Proof:** From the third equation of system (1):

$$\frac{dA}{dt} = \alpha D - (p_2 + \delta + \mu)A \geq -(p_2 + \delta + \mu)A$$

Then, by applying the theories on the differential inequality, we reached:

$$A(t) \geq A(0) e^{-(p_2 + \mu + \delta)t}.$$

Hence, for every  $A(0) > 0$ ,  $A(t) > 0$  for all future times. Similarly, it is discovered that:

$$Q(t) \geq Q(0) e^{-(p_3 + \mu + \delta)t} > 0.$$

$$D(t) \geq D(0) e^{-((\alpha + \rho_1 + \mu)t)} > 0.$$

$$S(t) \geq S(0) e^{-\left(\frac{\beta(\theta D + A)}{1 + \pi(D + A)} + \mu\right)t} > 0.$$

$$R(t) \geq R(0) e^{-\mu t} > 0.$$

Thus, the proof is complete.

In order to reflect actual social and biological situations, the verification of positivity guarantees that the system trajectories stay inside the non-negative portion of the state space. This outcome provides a solid basis for further mathematical studies by confirming that all model variables retain significant values throughout the simulation.

**Theorem (2):** System (1) has uniformly bounded solutions.

**Proof:** Let  $N = S + D + Q + A + R$ , then it is discovered that:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dQ}{dt} + \frac{dA}{dt} + \frac{dR}{dt}.$$

Therefore

$$\frac{dN}{dt} = \Lambda - \mu S - \mu D - \mu A - \mu Q - \delta Q - \mu R \leq \Lambda - \mu(S + D + A + Q + R) = \Lambda - \mu N.$$

Then, by applying the theories on the differential inequality, it is discovered that:

$$N(t) \leq \frac{\Lambda}{\mu}(1 - e^{-\mu t}) + N(0)e^{-\mu t}.$$

So, as  $t \rightarrow \infty$ , results that  $N(t) \leq \frac{\Lambda}{\mu}$ .

By establishing boundedness, the suggested addiction model's state variables are guaranteed to remain finite throughout time. This condition sets the stage for stability and bifurcation analysis by preventing unchecked population expansion and preserving the model's viability within reasonable demographic and behavioral bounds.

#### 4. EQUILIBRIUM POINTS AND BRN

The drug addiction-free equilibrium (DAFE) point of the system (2) can be acquired via setting the functions on the right-hand side equal to zero and assuming the drug is absent, which means the infected variables  $D, A, Q$  equal zero, and solving for  $S$ , we have the DEFE point is  $e_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$ .

Therefore, in the following, the effective BRN, which represents a critical threshold parameter determining whether the addictive behavior persists or vanishes within the population, will be computed. In terms of mathematics,  $\mathcal{R}_0$  represents the anticipated number of new cases of addiction that one addict would create in a community that is otherwise devoid of addiction. The addiction-free equilibrium is both locally as well as globally stable when  $\mathcal{R}_0 < 1$ , which eventually results in the eradication of addiction; on the other hand,  $\mathcal{R}_0 > 1$  suggests that the addicted subpopulation increases, pushing the system toward an endemic equilibrium.

Because it offers a quantitative standard for successful control techniques, determining this threshold is essential for evaluating the effects of awareness and therapy treatments. To calculate  $\mathcal{R}_0$  and examine its sensitivity to model inputs, the next-generation matrix technique and its ensuing advancements are used [28].

The spectral radius of the next generation matrix  $FV^{-1}$ , where  $F$  and  $V^{-1}$  are the transmission and inverse of the transition matrices, respectively, is represented by the BRN, which is the

prevailing eigenvalue of  $\rho(FV^{-1})$ . The definitions of the  $F$  and  $V^{-1}$  matrices for this system (2) are as follows:

$$F = \begin{bmatrix} \frac{\beta\theta\Lambda}{\mu} & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

$$V = \begin{bmatrix} \alpha + \rho_1 + \mu & 0 & 0 \\ -\alpha & \rho_2 + \mu + \sigma & 0 \\ 0 & -\sigma & \rho_3 + \mu + \delta \end{bmatrix}. \quad (4)$$

$$V^{-1} = \begin{bmatrix} \frac{1}{\alpha + \mu + \rho_1} & 0 & 0 \\ \frac{\alpha}{(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)} & \frac{1}{\mu + \sigma + \rho_2} & 0 \\ \frac{\alpha\sigma}{(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)(\delta + \mu + \rho_3)} & \frac{\sigma}{(\mu + \sigma + \rho_2)(\delta + \mu + \rho_3)} & \frac{1}{\delta + \mu + \rho_3} \end{bmatrix}.$$

Accordingly, it is obtained that:

$$FV^{-1} = \begin{bmatrix} \frac{\beta\theta\Lambda}{\mu(\alpha + \mu + \rho_1)} + \frac{\alpha\beta\Lambda}{\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)} & \frac{\beta\Lambda}{\mu(\mu + \sigma + \rho_2)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, the BRN ( $\mathcal{R}_0$ ) of the system (2) computed as the spectral radius  $\rho(FV^{-1})$  of the matrix  $FV^{-1}$ , is obtained as:

$$\mathcal{R}_0 = \frac{\beta\theta\Lambda(\mu + \sigma + \rho_2) + \alpha\beta\Lambda}{\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)}. \quad (5)$$

The point at which drug addiction endures in the population is known as the drug addiction equilibrium (DAE) point, which is a non-disease-free state of the model. It is acquired by solving for the state variables after setting the derivatives in the model system (2) to zero.  $e_1$  represents the DAE point, and equation (6) defines the elements of  $e_1 = (\hat{S}, \hat{D}, \hat{A}, \hat{Q})$ .

$$\left. \begin{aligned} \hat{D} &= \frac{(\sigma + \rho_2 + \mu)}{\alpha} \hat{A}, \\ \hat{Q} &= \frac{\sigma}{(\rho_3 + \mu + \delta)} \hat{A}, \\ \hat{S} &= \frac{\alpha\Lambda + \hat{A}\pi\alpha\Lambda + \hat{A}\pi\Lambda(\mu + \sigma + \rho_2)}{\hat{A}\beta\alpha + \mu\alpha + \hat{A}\pi\mu\alpha + \hat{A}(\mu + \sigma + \rho_2)(\beta\theta + \pi\mu)}, \\ \hat{A} &= \frac{\alpha\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)}{B_1} \left[ \frac{\beta\Lambda(\alpha + \theta(\mu + \sigma + \rho_2))}{\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)} - 1 \right], \end{aligned} \right\} \quad (6)$$

where:

$$\begin{aligned}
B_1 = & \alpha^2\beta\mu + \pi\alpha^2\mu^2 + \alpha\beta\mu^2 + \alpha\beta\theta\mu^2 + 2\pi\alpha\mu^3 + \beta\theta\mu^3 + \pi\mu^4 + \alpha^2\beta\sigma + \pi\alpha^2\mu\sigma \\
& + \alpha\beta\mu\sigma + 2\alpha\beta\theta\mu\sigma + 3\pi\alpha\mu^2\sigma + 2\beta\theta\mu^2\sigma + 2\pi\mu^3\sigma + \alpha\beta\theta\sigma^2 + \pi\alpha\mu\sigma^2 \\
& + \beta\theta\mu\sigma^2 + \pi\mu^2\sigma^2 + \alpha\beta\mu\rho_1 + \pi\alpha\mu^2\rho_1 + \beta\theta\mu^2\rho_1 + \pi\mu^3\rho_1 + \alpha\beta\sigma\rho_1 \\
& + \pi\alpha\mu\sigma\rho_1 + 2\beta\theta\mu\sigma\rho_1 + 2\pi\mu^2\sigma\rho_1 + \beta\theta\sigma^2\rho_1 + \pi\mu\sigma^2\rho_1 + \alpha^2\beta\rho_2 \\
& + \pi\alpha^2\mu\rho_2 + \alpha\beta\mu\rho_2 + 2\alpha\beta\theta\mu\rho_2 + 3\pi\alpha\mu^2\rho_2 + 2\beta\theta\mu^2\rho_2 + 2\pi\mu^3\rho_2 \\
& + 2\alpha\beta\theta\sigma\rho_2 + 2\pi\alpha\mu\sigma\rho_2 + 2\beta\theta\mu\sigma\rho_2 + 2\pi\mu^2\sigma\rho_2 + \alpha\beta\rho_1\rho_2 + \pi\alpha\mu\rho_1\rho_2 \\
& + 2\beta\theta\mu\rho_1\rho_2 + 2\pi\mu^2\rho_1\rho_2 + 2\beta\theta\sigma\rho_1\rho_2 + 2\pi\mu\sigma\rho_1\rho_2 + \alpha\beta\theta\rho_2^2 + \pi\alpha\mu\rho_2^2 \\
& + \beta\theta\mu\rho_2^2 + \pi\mu^2\rho_2^2 + \beta\theta\rho_1\rho_2^2 + \pi\mu\rho_1\rho_2^2
\end{aligned}$$

Obviously, the drug equilibrium points exist if and only if  $\mathcal{R}_0 > 1$ .

## 5. STABILITY ANALYSIS

The goal of local stability analysis is to ascertain how the system responds to slight perturbations around its equilibrium points. One can forecast whether the nonlinear model will return to equilibrium or diverge following small disturbances by linearizing it and examining the Jacobian matrix at each steady state. This method is frequently used to confirm short-term stability conditions in social dynamics and epidemiology. Direct computation yields that the general Jacobian matrix of the model system (2) can be expressed as:

$$J = \begin{bmatrix} -\frac{\beta(A+D\theta)}{1+(A+D)\pi} - \mu & -\frac{S\beta(A\pi(-1+\theta)+\theta)}{(1+(A+D)\pi)^2} & \frac{S\beta(-1+D\pi(-1+\theta))}{(1+(A+D)\pi)^2} & 0 \\ \frac{\beta(A+D\theta)}{1+(A+D)\pi} & -\alpha + \frac{S\beta(A\pi(-1+\theta)+\theta)}{(1+(A+D)\pi)^2} - \mu - \rho_1 & \frac{S\beta(1+D(\pi-\pi\theta))}{(1+(A+D)\pi)^2} & 0 \\ 0 & \alpha & -\mu - \sigma - \rho_2 & 0 \\ 0 & 0 & \sigma & -\delta - \mu - \rho_3 \end{bmatrix} \quad (7)$$

**Theorem (4):** If  $\mathcal{R}_0 < 1$ , the DAFE point is locally asymptotically stable; if  $\mathcal{R}_0 > 1$ , it is unstable.

**Proof:** The Jacobean matrix given by equation (7) at the DAFE point becomes:

$$J(e_0) = \begin{bmatrix} -\mu & -\frac{\beta\theta\Lambda}{\mu} & -\frac{\beta\Lambda}{\mu} & 0 \\ 0 & -\alpha + \frac{\beta\theta\Lambda}{\mu} - \mu - \rho_1 & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & \alpha & -\mu - \sigma - \rho_2 & 0 \\ 0 & 0 & \sigma & -\delta - \mu - \rho_3 \end{bmatrix}. \quad (8)$$

Therefore, the characteristic equation of equation (8) can be expressed as:

$$(-\mu - \lambda)[\lambda^2 - Tr \lambda + De](-(\rho_3 + \mu + \delta) - \lambda) = 0, \quad (9)$$

where:

$$\begin{aligned}
Tr &= \frac{\beta\theta\Lambda}{\mu} - (\alpha + \rho_1 + \mu) - (\rho_2 + \mu + \sigma) \\
&= (\alpha + \rho_1 + \mu) \left[ \frac{\beta\theta\Lambda}{\mu(\alpha + \rho_1 + \mu)} - 1 \right] - (\rho_2 + \mu + \sigma)
\end{aligned} \quad (10)$$

$$De = \left( \frac{\beta\theta\Lambda}{\mu} - (\alpha + \rho_1 + \mu) \right) (-(\rho_2 + \mu + \sigma)) - \frac{\alpha\beta\Lambda}{\mu}. \quad (11)$$

$$= (\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)[1 - \mathcal{R}_0].$$

Obviously, the eigenvalues of  $J(e_0)$  are  $\lambda_{01} = -\mu < 0$ , and  $\lambda_{04} = -(\rho_3 + \mu + \delta) < 0$  with the other two roots of the second-order equation.

Therefore, if  $\mathcal{R}_0 < 1$ , then we obtain that  $Tr < 0$  and  $De > 0$ , which leads to having two negative real parts of eigenvalues  $\lambda_{02}, \lambda_{03} = \frac{Tr}{2} \pm \frac{1}{2}\sqrt{(Tr)^2 - 4De}$ . This makes DAFE locally asymptotically stable.

Otherwise, when  $\mathcal{R}_0 > 1$ , it is obtained that  $De < 0$ . Hence, the DAFE point will be unstable.

Now, to study the local stability of DAE, the matrix (7) at this point becomes:

$$J(e_1) = [\zeta_{ij}]_{4 \times 4}, \quad (12)$$

where:

$$\zeta_{11} = -\mu - \frac{\beta(\hat{A} + \theta\hat{D})}{1 + \pi(\hat{A} + \hat{D})}, \quad \zeta_{12} = -\frac{\beta(\theta - \pi(1 - \theta)\hat{A})\hat{S}}{(1 + \pi(\hat{A} + \hat{D}))^2}, \quad \zeta_{13} = -\frac{\beta(1 + \pi(1 - \theta)\hat{D})\hat{S}}{(1 + \pi(\hat{A} + \hat{D}))^2}, \quad \zeta_{21} = \frac{\beta(\hat{A} + \theta\hat{D})}{1 + \pi(\hat{A} + \hat{D})},$$

$$\zeta_{22} = -\alpha - \mu + \frac{\beta(\theta - \pi(1 - \theta)\hat{A})\hat{S}}{(1 + \pi(\hat{A} + \hat{D}))^2} - \rho_1, \quad \zeta_{23} = \frac{\beta(1 + \pi(1 - \theta)\hat{D})\hat{S}}{(1 + \pi(\hat{A} + \hat{D}))^2}, \quad \zeta_{32} = \alpha, \quad \zeta_{33} = -\mu - \sigma - \rho_2,$$

$$\zeta_{43} = \sigma, \quad \zeta_{44} = -\delta - \mu - \rho_3. \quad \zeta_{14} = \zeta_{24} = \zeta_{34} = \zeta_{31} = \zeta_{41} = \zeta_{42} = 0.$$

The characteristic equation of  $J(e_1)$  can be expressed as:

$$(\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3)(\zeta_{44} - \lambda) = 0, \quad (13)$$

where:

$$B_1 = -(\zeta_{11} + \zeta_{22} + \zeta_{33}).$$

$$B_2 = \zeta_{11}\zeta_{22} - \zeta_{12}\zeta_{21} + \zeta_{11}\zeta_{33} + \zeta_{22}\zeta_{33} - \zeta_{23}\zeta_{32}.$$

$$B_3 = -(\zeta_{11}\zeta_{22}\zeta_{33} + \zeta_{13}\zeta_{21}\zeta_{32} - \zeta_{11}\zeta_{23}\zeta_{32} - \zeta_{12}\zeta_{21}\zeta_{33}).$$

With:

$$\Delta = B_1B_2 - B_3 = -(\zeta_{11} + \zeta_{22})(\zeta_{11}\zeta_{22} - \zeta_{12}\zeta_{21})$$

$$- (\zeta_{22} + \zeta_{33})(\zeta_{22}\zeta_{33} - \zeta_{23}\zeta_{32})$$

$$- \zeta_{11}\zeta_{33}(\zeta_{11} + 2\zeta_{22} + \zeta_{33}) + \zeta_{13}\zeta_{21}\zeta_{32}.$$

Now, according to the Ruth-Hurwitz criterion, all eigenvalues of the polynomial of the third degree in equation (13) possess negative real parts under the conditions  $B_1 > 0$ ,  $B_3 > 0$ , and  $\Delta > 0$ . Direct calculations show that the Routh-Hurwitz requirement is fulfilled if the conditions of the following theorem are fulfilled, which makes the DAE point of the system (2) locally asymptotically stable.

**Theorem (5):** If the following requirements are met, the DAE point is locally asymptotically stable.

$$0 < \frac{\beta(\theta - \pi(1 - \theta)\hat{A})\hat{S}}{(1 + \pi(\hat{A} + \hat{D}))^2} < \alpha + \mu + \rho_1. \quad (14)$$

$$\zeta_{23}\zeta_{32} < \zeta_{22}\zeta_{33}. \quad (15)$$

$$-\zeta_{11}\zeta_{33}(\zeta_{11} + 2\zeta_{22} + \zeta_{33}) + \zeta_{13}\zeta_{21}\zeta_{32} > 0. \quad (16)$$

## 6. GLOBAL STABILITY ANALYSIS OF THE DAF

The Castillo-Chavez et al. [29] outlined two requirements for determining the global asymptotic stability of the DAFE point of a system (2), which is expressed in the form:

$$\begin{aligned} \frac{dX}{dt} &= F(X, Y), \\ \frac{dY}{dt} &= G(X, Y), \end{aligned} \quad (17)$$

where  $X \in \mathbb{R}^n$  represents the number of non-drug-addiction individuals,  $Y \in \mathbb{R}^m$  represents the number of drug-addiction individuals and  $e_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) = (\bar{S}, 0)$  represents the DAFE point.

The following presumptions need to be met for a DAFE point of a system (2) to be globally asymptotically stable:

$$H_1: \frac{dX}{dt} = F(X, 0), \quad X = \bar{S} \text{ is globally asymptotically stable.}$$

$$H_2: \frac{dY}{dt} = G(X, Y) = AY - G_*(X, Y), \text{ where } G_*(X, Y) \geq 0 \text{ for } (X, Y) \text{ belongs to the system domain, and } A = \frac{\partial G}{\partial Y}(\bar{S}, 0) \text{ is an M-matrix (the off-diagonal elements are nonnegative).}$$

**Theorem (6):** If  $\mathcal{R}_0 < 1$  and the conditions  $H_1$  and  $H_2$  are met, the DAFE point of the system (2) is globally asymptotically stable.

**Proof:** From the systems (2) and (17), it is obtained that  $F(X, Y)$ , and  $G(X, Y)$  as:

$$\begin{aligned} F(X, Y) &= \Lambda - \frac{\beta(\theta D + A)S}{1 + \pi(D + A)} - \mu S \\ G(X, Y) &= \begin{pmatrix} \frac{\beta(\theta D + A)S}{1 + \pi(D + A)} - (\alpha + \rho_1 + \mu)D \\ \alpha D - (\rho_2 + \mu + \sigma)A \\ \sigma A - (\rho_3 + \mu + \delta)Q \end{pmatrix} \end{aligned}$$

Therefore, when  $D = A = Q = 0$ , it is obtained:

$$\frac{dY}{dt} = F(X, 0) = \Lambda - \mu S.$$

Thus, this reduced system satisfies  $S(t) = \frac{\Lambda}{\mu} + \left(S(0) - \frac{\Lambda}{\mu}\right) e^{-\mu t}$ .

Hence, when  $t \rightarrow \infty$ , the solution  $S(t) \rightarrow \frac{\Lambda}{\mu}$ , which means,  $\bar{S} = \frac{\Lambda}{\mu}$  is a globally asymptotically stable point.

Now, for condition  $H_2$ , we have:

$$\frac{dY}{dt} = G(X, Y) = AY - G_*(X, Y),$$

where:

$$A = \frac{\partial G}{\partial Y} (\bar{S}, 0) = \begin{pmatrix} \frac{\beta\theta\Lambda}{\mu} - (\alpha + \rho_1 + \mu) & \frac{\beta\Lambda}{\mu} & 0 \\ \alpha & -(\rho_2 + \mu + \sigma) & 0 \\ 0 & \sigma & -(\rho_3 + \mu + \delta) \end{pmatrix}.$$

Therefore, it is obtained that:

$$G_*(X, Y) = \begin{pmatrix} \frac{\beta\theta\Lambda}{\mu} D - \frac{\beta(\theta D + A)S}{1 + \pi(D + A)} + \frac{\beta\Lambda}{\mu} A \\ 0 \\ 0 \end{pmatrix}.$$

Clearly,  $G_*(X, Y) \geq 0$  for all  $(X, Y)$  belonging to the system (2) domain, which means assumption  $H_2$  is satisfied. Therefore, the DAFE point is globally asymptotically stable.

## 7. BIFURCATION ANALYSIS

A bifurcation is a qualitative shift in a system's behavior that occurs when a parameter crosses a critical value. It connects mathematical theory to real-world transitions by explaining how and why a system's behavior changes when parameters change. The center manifold theory is applied in accordance with Castillo-Chavez and Song's [30] paradigm to investigate whether the proposed model exhibits forward or backward bifurcation. This method reduces the nonlinear system to a one-dimensional manifold near the critical threshold, where  $\mathcal{R}_0 = 1$  is the fundamental reproduction number.

Now, let  $\beta = \beta^*$ , where  $\beta^* = \frac{\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)}{\theta\Lambda(\mu + \sigma + \rho_2) + \alpha\Lambda}$ . Clearly, this is equivalent to

$$\mathcal{R}_0 = \frac{\beta\theta\Lambda(\mu + \sigma + \rho_2) + \alpha\beta\Lambda}{\mu(\alpha + \mu + \rho_1)(\mu + \sigma + \rho_2)} = 1.$$

Expressed the model system (2) as  $\frac{dZ}{dt} = F(Z)$ , where  $Z = (X_1, X_2, X_3, X_4, X_5)^T$ , the variables  $S = X_1, D = X_2, A = X_3, Q = X_4, R = X_5$ , and  $F = (f_1, f_2, f_3, f_4, f_5)^T$ .

Accordingly, the DAFE point becomes  $Z_0 = e_0$ . Therefore, system (2) can be expressed as:

$$\left. \begin{aligned} \frac{dX_1}{dt} &= \Lambda - \frac{\beta(\theta X_2 + X_3)X_1}{1 + \pi(X_2 + X_3)} - \mu X_1. \\ \frac{dX_2}{dt} &= \frac{\beta(\theta X_2 + X_3)X_1}{1 + \pi(X_2 + X_3)} - (\alpha + \rho_1 + \mu)X_2 \\ \frac{dX_3}{dt} &= \alpha X_2 - (\rho_2 + \mu + \sigma)X_3. \\ \frac{dX_4}{dt} &= \sigma X_3 - (\rho_3 + \mu + \delta)X_4. \end{aligned} \right\} \quad (18)$$

Moreover, the Jacobian at  $X_0 = e_0$  is given by Equation (8), while its characteristic equation was given by Equation (9).

Solving Equation (9) for  $\lambda$ , using the value  $\beta = \beta^*$ , gives that:

$$\begin{aligned} \lambda_{01} &= -\mu < 0, \quad \lambda_{02} = (\alpha + \rho_1 + \mu) \left[ \frac{\beta^* \theta \Lambda}{\mu(\alpha + \rho_1 + \mu)} - 1 \right] - (\rho_2 + \mu + \sigma) < 0, \\ \lambda_{03} &= 0, \quad \lambda_{04} = -(\rho_3 + \mu + \delta) < 0. \end{aligned}$$

Thus, the existence of a simple zero eigenvalue allows us to use the center manifold theorem.

Consider, the right eigenvector  $W = (w_1, w_2, w_3, w_4)^T$  corresponding to  $\lambda_{03} = 0$ , then it is obtained that:

$$\begin{bmatrix} -\mu & -\frac{\beta\theta\Lambda}{\mu} & -\frac{\beta\Lambda}{\mu} & 0 \\ 0 & -\alpha + \frac{\beta\theta\Lambda}{\mu} - \mu - \rho_1 & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & \alpha & -\mu - \sigma - \rho_2 & 0 \\ 0 & 0 & \sigma & -\delta - \mu - \rho_3 \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Direct calculation gives that:

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} -\frac{\beta\Lambda}{\mu^2} \left( \frac{\theta(\rho_2 + \mu + \sigma)}{\alpha} + 1 \right) w_3 \\ \frac{(\rho_2 + \mu + \sigma)w_3}{\alpha} \\ w_3 \\ \frac{\sigma w_3}{(\rho_3 + \mu + \delta)} \end{pmatrix},$$

where  $w_3 > 0$ . Now, the transportation of the Jacobean matrix gives:

$$[J(e_0, \beta^*)]^T = \begin{bmatrix} -\mu & 0 & 0 & 0 \\ -\frac{\beta^*\theta\Lambda}{\mu} & -\alpha + \frac{\beta^*\theta\Lambda}{\mu} - \mu - \rho_1 & \alpha & 0 \\ -\frac{\beta^*\Lambda}{\mu} & \frac{\beta^*\Lambda}{\mu} & -\mu - \sigma - \rho_2 & \sigma \\ 0 & 0 & 0 & -\delta - \mu - \rho_3 \end{bmatrix}.$$

Therefore, the left eigenvector associated  $V = (v_1, v_2, v_3, v_4)$  with the  $[J(e_0, \beta^*)]^T$  can be computed as:

$$\begin{bmatrix} -\mu & 0 & 0 & 0 \\ -\frac{\beta^*\theta\Lambda}{\mu} & -\alpha + \frac{\beta^*\theta\Lambda}{\mu} - \mu - \rho_1 & \alpha & 0 \\ -\frac{\beta^*\Lambda}{\mu} & \frac{\beta^*\Lambda}{\mu} & -\mu - \sigma - \rho_2 & \sigma \\ 0 & 0 & 0 & -\delta - \mu - \rho_3 \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Direct computation gives:

$$V^T = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\mu(\mu+\sigma+\rho_2)v_3}{\beta^*\Lambda} \\ v_3 \\ 0 \end{pmatrix},$$

where  $v_3 > 0$ .

Then we calculate the second partial derivatives of the  $f_i$  with respect to the  $X_i$  at  $\beta = \beta^*$ :

$$\frac{\partial^2 f_1}{\partial x_1 \partial x_2} = -\beta\theta, \quad \frac{\partial^2 f_2}{\partial x_2 \partial x_1} = \beta\theta, \quad \frac{\partial^2 f_1}{\partial x_1 \partial x_3} = -\beta, \quad \frac{\partial^2 f_2}{\partial x_2 \partial x_3} = \frac{-\pi\beta\Lambda}{\mu}, \quad \frac{\partial^2 f_2}{\partial x_2 \partial \beta} = \frac{\theta\Lambda}{\mu}.$$

While the other partial derivatives are all zero, after some calculation, it is obtained:

$$A = \sum_{k,i,j=1}^n w_k v_i v_j \frac{\partial^2 f_k}{\partial x_i \partial x_j} = \frac{-2\pi\beta\Lambda(\rho_2+\mu+\sigma)w_3 v_3}{\beta\Lambda\theta - \mu(\alpha+\rho_1+\mu)} < 0. \quad (19)$$

$$B = \sum_{k,i=1}^n w_k v_i \frac{\partial^2 f_k}{\partial x_i \partial \beta} = w_2 v_2 \frac{\theta\Lambda}{\mu} = \frac{\theta(\rho_2+\mu+\sigma)w_3 v_3}{\beta\Lambda\theta - \mu(\alpha+\rho_1+\mu)} > 0. \quad (20)$$

We use the center manifold theorem, and from our values of A and B, we have forward bifurcation. Based on the computed values of the bifurcation coefficients A and B, the system exhibits a forward bifurcation at the critical threshold  $\mathcal{R}_0 = 1$ .

This result indicates that as the BRN decreases below unity, the endemic equilibrium smoothly disappears and the system transitions toward the addiction-free equilibrium. In this case, enhancing awareness campaigns and maintaining treatment rates above their critical levels are sufficient to eliminate addiction.

The occurrence of a forward bifurcation reflects a desirable and stable behavioral transition, ensuring that positive media interventions can effectively suppress the spread of addiction in the population.

## 8. SENSITIVITY ANALYSIS

In mathematical modeling, sensitivity analysis is a precise methodological approach used to compute how responsive the model's outputs are to relative changes in its parameters. By identifying the variables that have the greatest impact on the dynamics of infection transmission, this research improves knowledge of the system's epidemiological structure. It guarantees the model's stability and consistency in the face of data fluctuations. Based on the analytical formula for  $\mathcal{R}_0$ , this study used sensitivity analysis to evaluate the effect of  $\mathcal{R}_0$  on the relative changes in the parameters of the suggested model.

To precisely expose the advantages and disadvantages of the epidemic's development and to

determine which parameters have the biggest influence on the dynamics of the addiction phenomenon's spread, a system sensitivity test for parameters and their values is carried out. In our effort to lower the threshold level and improve the parameters that contribute to lowering it, the significance of our findings resides in focusing efforts on lowering the numbers represented by certain parameters.

Sensitivity analysis of parameters, therefore, offers a scientific framework that enables more successful community interventions and gives decision-makers a better understanding of how to regulate drug usage to safeguard society's advantages and disadvantages in the disease's progression. The research pinpoints the parameters that have the biggest effects on the dynamics of infection.

The sensitivity index of  $\mathcal{R}_0$  for the model's parameter  $\omega$  is calculated using the following form [31].

$$\Lambda_{\omega}^{\mathcal{R}_0} = \left( \frac{\partial \mathcal{R}_0}{\partial \omega} \right) \left( \frac{\omega}{\mathcal{R}_0} \right). \quad (21)$$

It also  $\Lambda_{\omega}^{\mathcal{R}_0}$  stands for the degree of susceptibility to alter in  $\omega$ . The value of  $\mathcal{R}_0$  either rises or falls in response to an increase in the parameter value.

Now, can calculate  $\Lambda_{\omega}^{\mathcal{R}_0}$ , where we obtain:

$$\Lambda_{\beta}^{\mathcal{R}_0} = 1$$

$$\Lambda_{\lambda}^{\mathcal{R}_0} = 1$$

$$\Lambda_{\theta}^{\mathcal{R}_0} = \frac{\theta(\mu+\sigma+\rho_2)}{\alpha+\theta(\mu+\sigma+\rho_2)}.$$

$$\Lambda_{\sigma}^{\mathcal{R}_0} = -\frac{\alpha\sigma}{(\mu+\sigma+\rho_2)(\alpha+\theta(\mu+\sigma+\rho_2))}.$$

$$\Lambda_{\rho_1}^{\mathcal{R}_0} = -\frac{\rho_1}{(\alpha+\rho_1+\mu)}.$$

$$\Lambda_{\rho_2}^{\mathcal{R}_0} = -\frac{\alpha\beta}{\rho_2(\mu+\sigma+\rho_2)(\alpha+\theta(\mu+\sigma+\rho_2))}.$$

$$\Lambda_{\mu}^{\mathcal{R}_0} = \frac{-\theta(\alpha+2\mu+\rho_1)(\mu+\sigma+\rho_2)^2 - \alpha\mu(\alpha+\mu+\rho_1) - \alpha(\alpha+2\mu+\rho_1)(\mu+\sigma+\rho_2)}{(\alpha+\mu+\rho_1)(\mu+\sigma+\rho_2)(\alpha+\theta(\mu+\sigma+\rho_2))}.$$

$$\Lambda_{\alpha}^{\mathcal{R}_0} = \frac{\alpha(\mu-\theta(\mu+\sigma+\rho_2)+\rho_1)}{(\alpha+\mu+\rho_1)(\alpha+\theta(\mu+\sigma+\rho_2))}.$$

$$\Lambda_{\rho_3}^{\mathcal{R}_0} = 0.$$

$$\Lambda_{\pi}^{\mathcal{R}_0} = 0.$$

Thus, we looked at System (2)'s BRN. Therefore, based on the following set of fictitious parameter values:

$$\begin{aligned} \Lambda = 10, \beta = 0.01, \theta = 0.05, \pi = 1, \alpha = 0.2, \mu = 0.04, \\ \rho_1 = 0.5, \rho_2 = 0.4, \rho_3 = 0.1, \sigma = 0.2, \delta = 0.1. \end{aligned} \quad (22)$$

Therefore, the sensitivity analysis regarding the parameters set (22) is obtained and summarized in Table 1 and is drawn in Figure 1:

Table 1: Sensitivity of  $\mathcal{R}_0$  according to the set of hypothetical parameter values (22).

Parameter	Unit	sensitivity
$\Lambda$	people/day	1
$\beta$	1/(people×day)	1
$\theta$	-	0.137
$\sigma$	1/day	-0.269
$\rho_1$	1/day	-0.332
$\rho_2$	1/day	-0.538
$\mu$	1/day	-1.107
$\alpha$	1/day	0.591
$\rho_3$	1/day	0
$\pi$	1/people	0
$\delta$	1/day	0

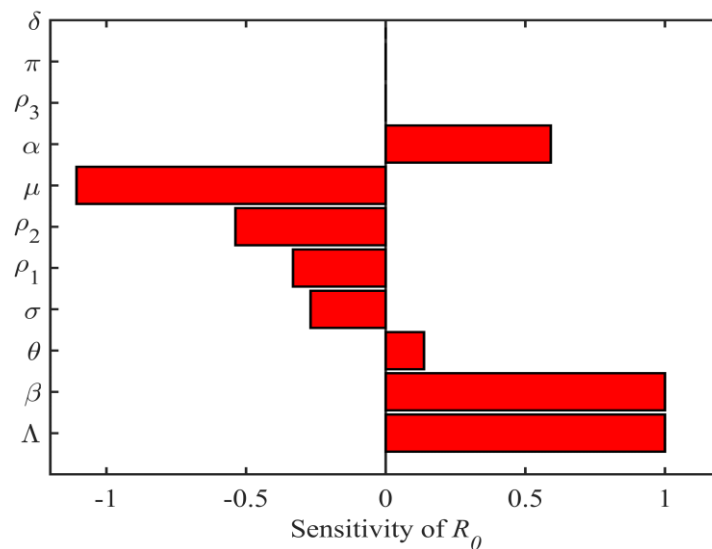


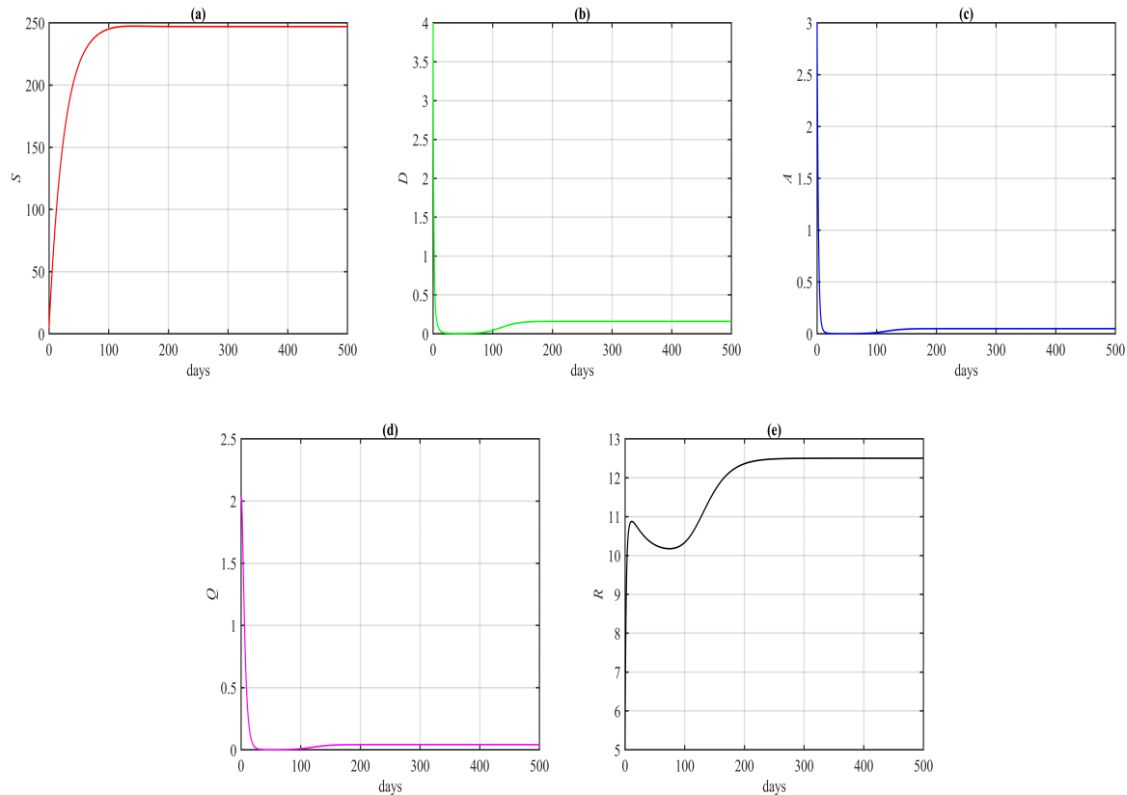
Figure 1: Sensitivity diagram of  $\mathcal{R}_0$ .

From both Table 1 and Fig.1, it is obtained that the parameters  $\Lambda$ ,  $\beta$ ,  $\theta$ , and  $\alpha$  have a positive effect on the outbreak of drug addiction, while the parameters  $\sigma$ ,  $\rho_1$ ,  $\rho_2$ , and  $\mu$  have a negative effect on the outbreak of drug addiction. Finally, the other parameters ( $\rho_3$ ,  $\pi$ , and  $\delta$ ) have no effect on the outbreak of drug addiction.

## 9. NUMERICAL SIMULATION

In this section, we will use numerical simulation to analyze the dynamic behavior of the suggested epidemiological model in order to confirm our earlier findings and show the beneficial effect of media in slowing the spread of medications that are categorized as global epidemics. As a result, using MATLAB version R2021a code and the Runge-Kutta of order four method, the simulation was carried out using suitable initial values and the assumed parameter values provided by Equation (22).

It is obtained that, for the data given by Equation (22), the value of  $\mathcal{R}_0 = 1.22$  and the trajectory of the system approaches the DAE point  $e_1 = (247.03, 0.16, 0.05, 0.04, 12.5)$  as shown in Fig. 2.

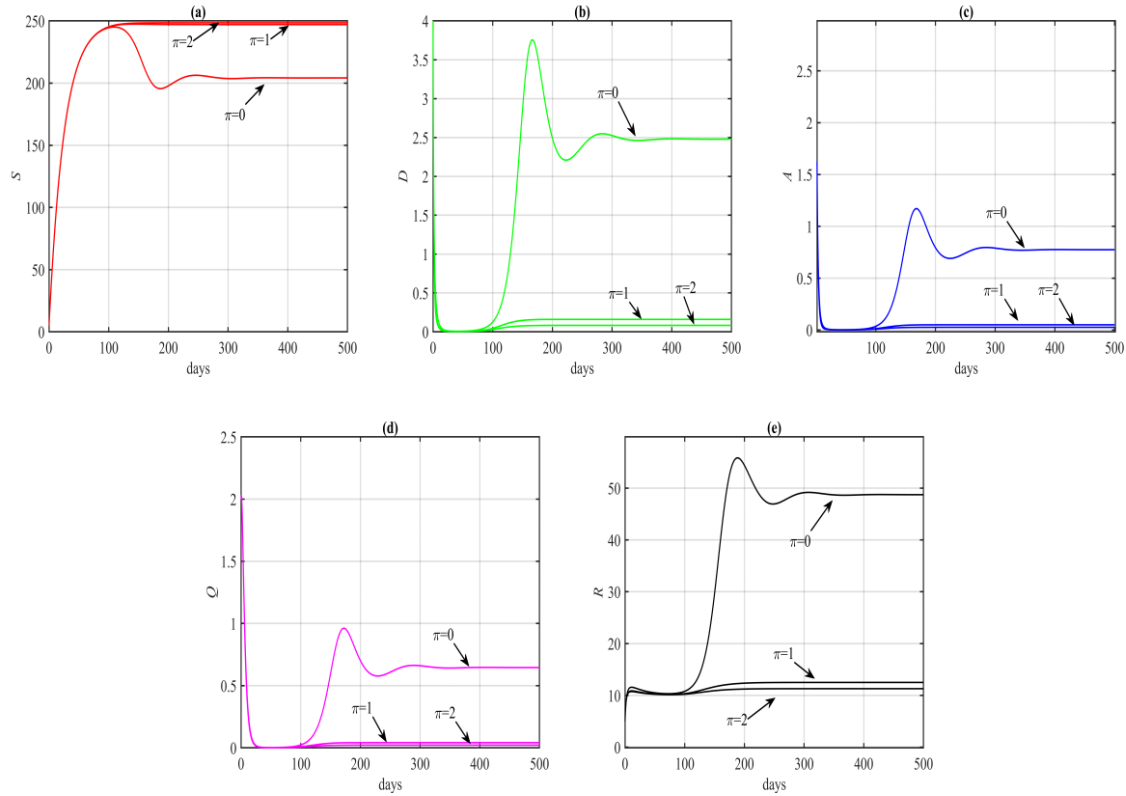


**Figure 2:** The trajectory of system (1) for the data set (22) approaches the DAE point when  $\mathcal{R}_0 = 1.22$ .

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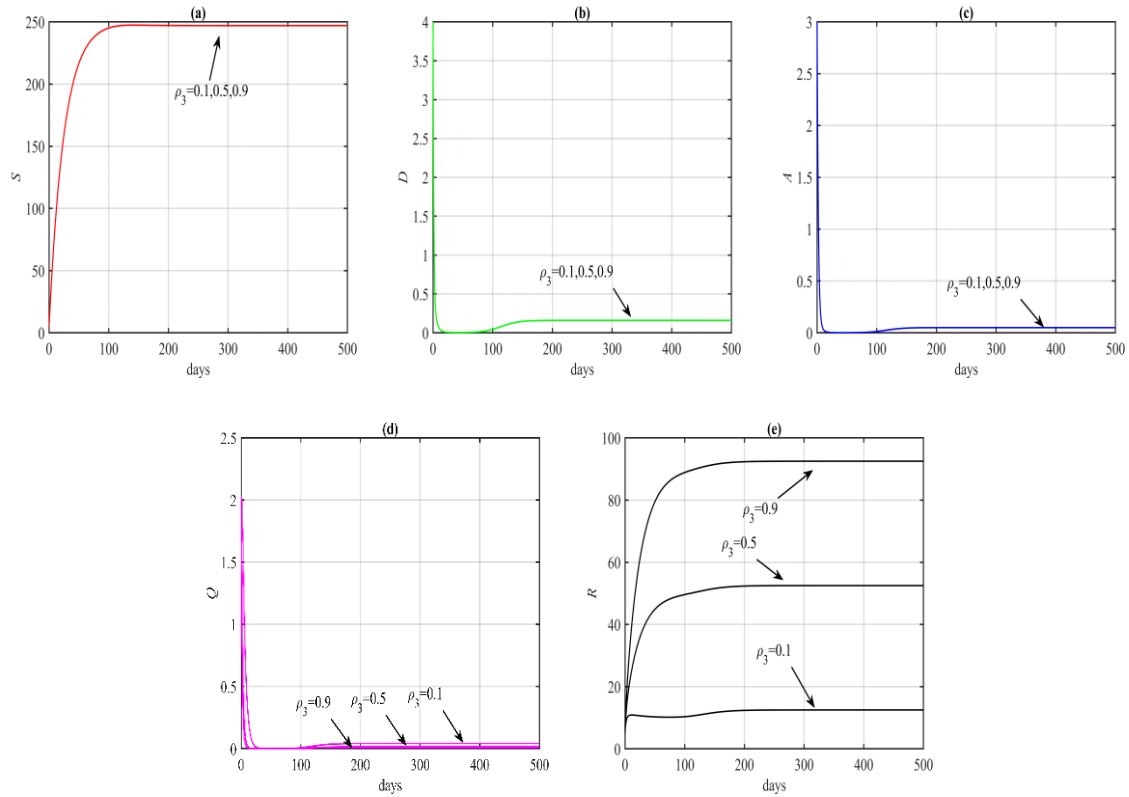
According to Fig. 2, the system diverges from the DAFE point and approaches the DAE point whenever  $\mathcal{R}_0 > 1$ , which confirms our finding.

The system (1) is now numerically solved for the data set (22) with various values of  $\pi = 0,1,2$ , and the resulting trajectories are shown in Fig. 3.



**Figure 3:** The trajectory of system (1) for the data set (22) with different values of  $\pi = 0,1,2$  approaches the DAE point when  $\mathcal{R}_0 = 1.22$ .

Although the parameter  $\pi$  (the media rate) does not affect the value of  $\mathcal{R}_0$ , increasing its value reduces the drug-addiction outbreak, as shown in Fig. 3. Moreover, increasing the value of the recovery rate in rehabilitation centers  $\rho_3$  increases the population in the recovery group, with decreases in those existing in the rehabilitation centers even when it does not affect  $\mathcal{R}_0$ , see Fig. 4.

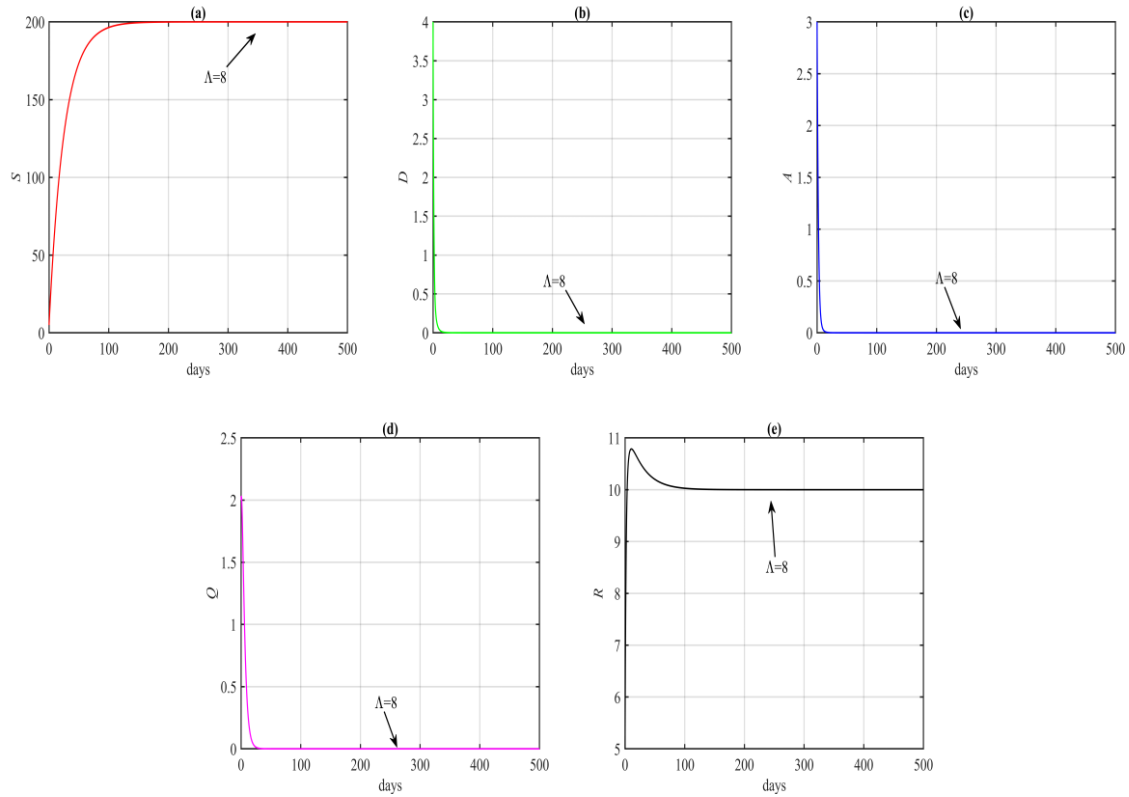


**Figure 4:** The trajectory of system (1) for the data set (22) with different values of  $\rho_3 = 0.1, 0.5, 0.9$  approaches the DAE point when  $\mathcal{R}_0 = 1.22$ .

Finally, it is observed that increasing the death rate as a result of drug addiction  $\delta$  leads to a decrease in the population of drug addiction without changing  $\mathcal{R}_0$  and other groups.

On the other hand, decreasing the value of  $\Lambda$  so that  $\mathcal{R}_0 < 1$  causes a path shift of the system (1) from the DAE point to the DAFE point, as shown in Fig. 5, confirming our sensitivity results, which show that  $\Lambda$  has a positive impact on the outbreak of drug addiction.

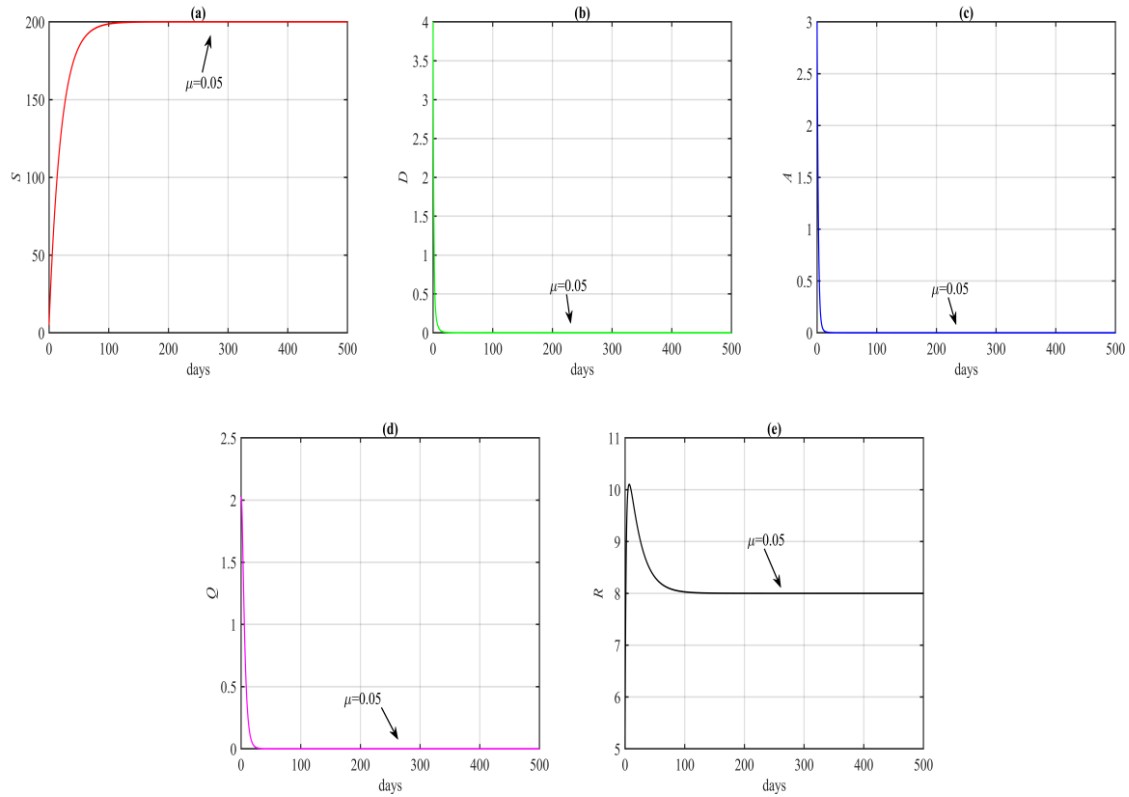
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**Figure 5:** The trajectory of system (1) for the data set (22) with  $\Lambda = 8$  approaches the DAFE point  $e_0 = (200, 0, 0, 0, 10)$  when  $\mathcal{R}_0 = 0.9797$ .

In the same way, we observed that a decrease in the division of parameters  $\beta$ ,  $\theta$ , and  $\alpha$  leads to a transformation of the path of system (1) from the DAE point to the DAFE point with a decrease in the value  $\mathcal{R}_0$  to become  $\mathcal{R}_0 < 1$ . Again, this confirms the sensitivity results.

Finally, increasing the value of  $\mu$  so that  $\mathcal{R}_0 < 1$  causes a path shift of the system (1) from the DAE point to the DAFE point, as shown in Fig. 6, confirming our sensitivity results, which show that  $\mu$  has a negative impact on the outbreak of drug addiction. Similar results were obtained to those of  $\mu$  when increasing each of  $\sigma$ ,  $\rho_1$ , and  $\rho_2$ .



**Figure 6:** The trajectory of system (1) for the data set (22) with  $\mu = 0.05$  approaches the DAFE point  $e_0 = (200,0,0,0,8)$ , when  $\mathcal{R}_0 = 0.95$ .

## 10. CONCLUSION

The dynamic behavior of a model of drug-using social groups is clarified by this study. It looks at how the media may help the public become more aware of the risks associated with this epidemic. Due to the system's sensitivity to these variables, a qualitative change in the behavior of the suggested system is seen when their values are altered. These variables also affect the stability of the suggested model, and changing the values of these variables alters the nature of its equilibrium points. As it was noted that lowering the values of the transition coefficients ( $\beta$ ,  $\theta$ , and  $\alpha$ ) produces results consistent with the sensitivity test for the parameters, resulting in a decrease in the BRN to less than one, the numerical results demonstrated a clear agreement with the theoretical analysis. Simultaneously, scenarios that are challenging to investigate analytically could be explored thanks to numerical simulations. These simulations showed that while the media coefficient ( $\pi$ ) does not influence the fundamental reproduction number, raising its value clearly results in a large reduction in the prevalence of drug addiction, as seen in Fig. 3. This suggests that, even in the absence of a

direct impact on the prevalence threshold, the media can be used as a non-pharmaceutical intervention strategy to slow the growth of this epidemic. The significance of including specific behavioral elements in epidemiological models is confirmed by the outcomes of both theoretical studies and numerical simulations. This model can be used as a mathematical basis for more thorough investigations in the future. Additionally, the model can be expanded by adding more significant social characteristics, and its forecast accuracy can be increased by using real-world data.

### CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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