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DEGREE EQUITABLE LINE DOMINATION IN GRAPHS

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Abstract: A line dominating set $D \subseteq V(L(G))$ is called a degree equitable line dominating set, if for every vertex $v \in V(L(G)) - D$ there exists a vertex $u \in D$ such that $uv \in E$ in $L(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of vertices in such a set is called a degree equitable line dominating set in $L(G)$ and is denoted by $\gamma_{el}(G)$. In this paper, we study the graph theoretic properties of $\gamma_{el}(G)$ and many bounds were obtained in terms of elements of G and its relationships with other domination parameters were found.

Keywords: graph; line graph; degree; dominating set; degree equitable line dominating set; degree equitable line domination number.

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1. Introduction

In this paper, we follow the notations of [2]. All the graphs considered here are simple and finite. As usual $p = |V|$ and $q = |E|$ denote the number of vertices and edges of a graph G respectively.

In general, we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X and $N(v)$ $(N[v])$ denote the open (closed) neighborhoods of a vertex v.

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The notation $\alpha_0(G)$ $(\alpha_1(G))$ is the minimum number of vertices (edges) in a vertex (edge) cover of G. The notation $\beta_0(G)$ $(\beta_1(G))$ is the minimum number of vertices (edges) in a maximal independent set of a vertex (edge) of G. Let $\deg(v)$ is the degree of vertex v and as usual $\delta(G)$ $(\Delta(G))$ is the minimum (maximum) degree. A vertex of degree one is called an end vertex. The degree of an edge $e=uv$ of G is defined by $\deg(e)=\deg(u)+\deg(v)-2$ and $\delta'(G)$ $(\Delta'(G))$ is the minimum (maximum) degree among the edges of G.

A line graph $L(G)$ is the graph whose vertices correspond to the edges of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent.

We begin by recalling some standard definitions from domination theory.

A set $S \subseteq V(G)$ is said to be a dominating set of G, if every vertex in $V-S$ is adjacent to some vertex in S. The minimum cardinality of vertices in such a set is called the domination number of G and is denoted by $\gamma(G)$. A dominating set S is called the total dominating set, if for every vertex $v \in V$, there exists a vertex $u \in S$, $u \neq v$ such that u is adjacent to v. The total domination number of G, denoted by $\gamma_t(G)$ is the minimum cardinality of total dominating set of G. A dominating set $S \subseteq V(G)$ is a connected dominating set, if the induced subgraph $\langle S \rangle$ has no isolated vertices. The connected domination number, $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set of G. A dominating set $S \subseteq V(G)$ is a restrained dominating set of G, if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G)-S$. The restrained domination number of a graph G, is denoted by $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set in G. The concept of restrained domination in graphs was introduced by Domke et. al., [1].

A set $D \subseteq V(L(G))$ is said to be line dominating set in $L(G)$, if every vertex not in D is adjacent to a vertex in D of $L(G)$. The line domination number in $L(G)$, is denoted by $\gamma_l(G)$ is the minimum cardinality of a line dominating set. The concept of domination with its many variations is now well studied in graph theory (see [3], [4]).

Let D be a line dominating set in $L(G)$, if $V(L(G))-D$ contains another dominating set D^{-1}, then D^{-1} is called an inverse line dominating set with respect to D. The minimum cardinality of vertices in such a set is called an inverse line domination number in $L(G)$ and is denoted by $\gamma_l^{-1}(G)$.

A Roman dominating function on a line graph $L(G)=\left(V^{'},E^{'}\right)$ is a function $f:V^{'}\rightarrow\{0,1,2\}$ satisfying the condition that every vertex $u^{'}$ for which $f\left(u^{'}\right)=0$ is adjacent to at least one vertex $v^{'}$ for which $f\left(v^{'}\right)=2$ in $L(G)$. The weight of a Roman dominating function is the value $f\left(V^{'}\right)=\sum_{u^{'}\in V^{'}} f\left(u^{'}\right)$. The minimum weight of a Roman dominating function on a line graph $L(G)$ is called the Roman domination number of $L(G)$ and is denoted by $\gamma_R\left(L(G)\right)$.

A dominating set D of $L(G)$ is a line cototal dominating set if the induced subgraph $\left\langle V\left(L(G)\right)-D\right\rangle$ has no isolated vertices. The line cototal domination number $\gamma_{lct}(G)$ in $L(G)$ is the minimum cardinality of a line cototal dominating set. The above line domination parameters were introduced by M.H.Muddebihal et. al., (see [5], [6] and [7]).

Analogously, a line dominating set $D\subseteq V\left(L(G)\right)$ is called a degree equitable line dominating set, if for every $v\in V\left(L(G)\right)-D$ there exists a vertex $u\in D$ such that $uv\in E$ in $L(G)$ and $\left|\deg(u)-\deg(v)\right|\leq 1$. The minimum cardinality of vertices in such a set is called a degree equitable line dominating set in $L(G)$ and is denoted by $\gamma_{el}(G)$. In this paper, we study the graph theoretic properties of $\gamma_{el}(G)$ and many bounds were obtained in terms of elements of G and its relationships with other domination parameters were found.

2. Results

Initially, we give the degree equitable line domination number for some standard graphs, which are straight forward in the following Theorem

Theorem 2.1.

a. For any cycle C_p with $p\geq 3$ vertices,

$$\gamma_{el}\left(C_p\right) = \frac{p}{3} \text{ for } p\equiv 0(\bmod 3).$$

$$= \left\lceil \frac{p}{3} \right\rceil \text{ otherwise.}$$

b. For any star $K_{1,n}$ with $1+n=p$ vertices,

$$\gamma_{el}(K_{1,n}) = 1.$$

c. For any wheel W_p with $p \geq 4$ vertices,

$$\gamma_{el}(W_p) = \frac{p}{2} \text{ if } p \text{ is even.}$$

$$= \left\lfloor \frac{p}{2} \right\rfloor \text{ if } p \text{ is odd.}$$

d. For any path P_p with p vertices,

$$\gamma_{el}(P_p) = n, \text{ if } p \equiv 0 (\bmod 3)+1, \text{ where } n = 1,2,3,\ldots$$

$$= \frac{p}{3} \text{ if } p \equiv 0 (\bmod 3).$$

$$= \left\lceil \frac{p}{3} \right\rceil \text{ otherwise.}$$

Theorem 2.2. *A degree equitable line dominating set D is minimal if and only if every vertex $u \in D$, one of the following condition holds:*

i. *Either $N(u) \cap D = \phi$ or $|\deg(v) - \deg(u)| \geq 2$ for all $N(u) \cap D$.*

ii. *There exists a vertex $v \in V(L(G)) - D$ such that $N(v) \cap D = \{u\}$ and $|\deg(v) - \deg(u)| \leq 1$.*

Proof. Assume that D be the minimal degree equitable line dominating set in $L(G)$. If (i) and (ii) does not holds, then for some vertex $u \in D$ there exists a vertex $v \in V(L(G)) - D$ such that $|\deg(v) - \deg(u)| \leq 1$. Further, for every vertex $v \in V(L(G)) - D$, either $N(v) \cap D \neq \{u\}$ or $|\deg(v) - \deg(u)| \geq 2$ or both. Therefore, $D - \{u\}$ forms a degree equitable line dominating set in $L(G)$, which is a contradiction to the minimality of the set D. Therefore, (i) and (ii) holds.

Conversely, suppose for every vertex $u \in D$, one of the statements (i) or (ii) holds. Further, if D is not minimal, then there exists a vertex $u \in D$ in $L(G)$ such that $D - \{u\}$ is a degree equitable line dominating set in $L(G)$ and there exists a vertex $v \in D - \{u\}$ such that v degree equitably dominates u. That is $v \in N(u)$ and $|\deg(v) - \deg(u)| \leq 1$. Therefore u does not satisfy (i), hence it must satisfy (ii). Then there exists a vertex $v \in V(L(G)) - D$ such that $N(v) \cap D - \{u\}$ and

$|\deg(v)-\deg(u)|\leq 1$. Since $D-\{u\}$ is a degree equitable line dominating set in $L(G)$, there exists a vertex $w\in D-\{u\}$ such that $w\in N(v)$ and w is a degree equitable with v. Therefore, $w\in N(v)\cap D$ and $|\deg(w)-\deg(v)|\leq 1$, where $w\neq u$, which is a contradiction to the fact that $N(v)\cap D=\{u\}$. Clearly, D is a minimal degree equitable line dominating set in $L(G)$.

Theorem 2.3. *Let F be the maximal degree equitable independent set. Then F is a minimal degree equitable line dominating set in $L(G)$.*

Proof. Let D be the unique minimal degree equitable line dominating set in $L(G)$. Suppose, $S=\{\, u\in V(L(G))/u$ is a degree equitable isolate}. Then $S\subseteq D$, we now prove that $S=D$. Assume $D-S\neq\phi$. Let $v\in D-S$. Since v is not a degree equitable isolate, $V(L(G))-v$ is a degree equitable line dominating set in $L(G)$. Hence, there exists a minimal degree equitable line dominating set $D_1\subseteq V(L(G))-\{v\}$ and $D_1\neq D$, which is a contradiction to the fact that $L(G)$ has a unique minimal degree equitable line dominating set. Therefore, $S=D$.

Converse is obvious.

The following Theorem characterizes the degree equitable line domination number and end vertices of trees.

Theorem 2.4. *If every non end vertex of a tree T is adjacent to at least one end vertex, then $\gamma_{el}(T)=m-1$, where m is the number of non end vertices in T.*

Proof. Let $C=\{v_1,v_2,...,v_n\}\subseteq V(T)$ be the set of all non end vertices with $|C|=m$ in T. Now, by definition of line graph, let $F=\{u_1,u_2,...,u_n\}\subseteq V(L(T))$ be the set of vertices corresponding to the edges which are incident with the vertices of C in T. Suppose, $D\subseteq F$ be the set of vertices with $N[D]=V(L(T))$ and $|\deg(x)-\deg(y)|\leq 1$, $\forall x\in D$, $y\in V(L(T))-D$. Then D forms a degree equitable line dominating set in $L(T)$. Further, if $|\deg(x)-\deg(y)|\not\leq 1$, then make $|\deg(x)-\deg(y)|$ ≤ 1 by considering the vertices $\{w_i\}\in V(L(T))-D$, $1\leq i\leq n$. Clearly, $D\cup\{w_i\}$ forms a minimal γ_{el}-set in $L(T)$. Therefore, it follows that $|D\cup\{w_i\}|=|C|-1$ and hence $\gamma_{el}(T)=m-1$.

The following Theorem relates the degree equitable line domination and domination number in terms of vertices of G.

Theorem 2.5. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)+\gamma(G)\le p$. *Equality holds if* $G\cong C_4$.

Proof. Let $S=\{v_1,v_2,...,v_n\}\subseteq V(G)$ be the set of vertices with $\deg(v_i)\ge 2$, $\forall v_i\in S$, $1\le i\le n$. Further, let there exists a set $S_1\subseteq S$ of vertices with $diam(u,v)\ge 3$, $\forall u,v\in S_1$, which covers all the vertices in G. Clearly, S_1 forms a dominating set of G. Otherwise, if $diam(u,v)<3$, then there exists at least one vertex $x\notin S_1$, such that $S^{'}=S_1\cup\{x\}$ forms a minimal γ- set of G. Now by definition of $L(G)$, let $F=\{u_1,u_2,...,u_k\}\subseteq V(L(G))$ be the set of vertices such that $\{u_j\}=\{e_j\}\in E(G)$, $1\le j\le k$, where $\{e_j\}$ are incident with the vertices of $S^{'}$. Further, let $D\subseteq F$ be the set of vertices with $\deg(w)\ge 2$ for every $w\in D$ such that $N[D]=V(L(G))$ and if $|\deg(x)-\deg(y)|\le 1$, $\forall x\in D$, $y\in V(L(G))-D$. Then D forms a minimal degree equitable line dominating set in $L(G)$. Clearly, it follows that $|D|\cup|S^{'}|\le|V(G)|$ and hence $\gamma_{el}(G)+\gamma(G)\le p$.

Suppose, $G\cong C_4$. Then in this case, $|D|=2=|S^{'}|$. Therefore, it follows that $\gamma_{el}(G)+\gamma(G)=p$.

The following Theorem relates the degree equitable line domination and total domination number in terms of vertices of G.

Theorem 2.6. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)\le p-\gamma_t(G)$.

Proof. Let $S=\{v_1,v_2,...,v_n\}$ be the minimum set of vertices which covers all the vertices in G. Suppose, $\deg(v_i)\ge 1$, $\forall v_i\in S$, $1\le i\le n$ in the subgraph $\langle S\rangle$, then S forms a γ_t- set of G. Otherwise, if $\deg(v_i)<1$, then attach the minimum number of vertices $\{u_i\}\in N(v_i)$ to the vertices of S having $\deg(v_i)<1$. Then $S\cup\{u_i\}$ forms a minimal total dominating set of G. Now in $L(G)$, let $F\subseteq V(L(G))$ be the set of vertices corresponding to the edges which are incident to the vertices of S in G. Let there exists a subset $D=\{u_1,u_2,...,u_k\}\subseteq F$ of vertices with $\deg(u_j)\ge 2$, $1\le j\le k$ and $N[u_j]=V(L(G))$. Further, $|\deg(u)-\deg(w)|\le 1$, $\forall u\in D$ and $w\in V(L(G))-D$. Clearly, D forms a minimal degree equitable line dominating set in $L(G)$. Therefore, it follows that, $|D|\le |V(G)|-|S\cup\{w_i\}|$ and hence $\gamma_{el}(G)\le p-\gamma_t(G)$.

The following Theorem relates the degree equitable line domination, connected domination and domination numbers of G.

Theorem 2.7. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)+\gamma_c(G)+2\le\alpha_0(G)+\beta_0(G)+\gamma(G)$.

Proof. Let $C=\{v_1,v_2,...,v_n\}\subseteq V(G)$ be the set of vertices with $\deg(v_i)\ge 2$, $\forall v_i\in C$, $1\le i\le n$, which are at distance at least two, covers all the edges in G. Clearly, $|C|=\alpha_0(G)$. Further, if for any vertex $x\in C$, $N(x)\in V(G)-C$. Then C itself is an independent vertex set. Otherwise, $C_1\cup C_2$, where $C_1\subseteq C$ and $C_2\subseteq V(G)-C$, forms a maximum independent set of G with $|C_1\cup C_2|=\beta_0(G)$. Now let $S=C^{'}\cup C^{"}$, where $C^{'}\subseteq C$ and $C^{"}\subseteq V(G)-C$, be the minimal set of vertices which covers all the vertices in G. Clearly, S forms a minimal γ- set of G. Suppose the subgraph $\langle S\rangle$ has only one component, then S itself is a connected dominating set of G. Otherwise, if the subgraph $\langle S\rangle$ has more than one component, then attach the minimum number of vertices $\{w_i\}\in V(G)-S$, where $\deg(w_i)\ge 2$, which are between the vertices of S such that $S_1=S\cup\{w_i\}$ forms exactly one component in the subgraph $\langle S_1\rangle$. Clearly, S_1 forms a minimal γ_c- set of G. Now by definition of $L(G)$, let $D=\{u_1,u_2,...,u_k\}\subseteq F$, where F is the set of vertices corresponding to the edges which are incident with the vertices of S in G, be the minimal set of vertices with $N[D]=V(L(G))$ and $|\deg(x)-\deg(y)|\le 1$, $\forall x\in D$, $y\in V(L(G))-D$. Clearly, D forms a minimal degree equitable line dominating set in $L(G)$. Therefore, it follows that $|D|\cup|S_1|+2\le |C|\cup |C_1\cup C_2|\cup|S|$ and hence $\gamma_{el}(G)+\gamma_c(G)+2\le\alpha_0(G)+\beta_0(G)+\gamma(G)$.

The following Theorem relates the degree equitable line domination and restrained domination number in terms of vertices of G.

Theorem 2.8. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)+\gamma_r(G)\le\alpha_1(G)+\beta_1(G)+\delta^{'}(G)$.

Proof. Let $B=\{e_1,e_2,...,e_n\}\subseteq E(G)$ be the maximal set of edges with $N(e_i)\cap N(e_j)=e$, for every $e_i,e_j\in B$, $1\le i\le n$, $1\le j\le n$ and $e\in E(G)-B$. Clearly, B forms a maximal independent edge set in G. Suppose $C=\{v_1,v_2,...,v_n\}$ be the set of vertices which are incident with the edges of B and if $|C|=p$, then B itself is an edge covering number. Otherwise, consider the minimum number of edges $\{e_m\}\subseteq E(G)-B$, such that $B_1=B\cup\{e_m\}$ forms a minimal edge covering set of G. Further, let $A=\{v_1,v_2,...,v_k\}\subseteq V(G)$ be the set of all end vertices, then $S=A\cup A'$, where $A'\subseteq V(G)-A$ be the set of vertices covering all the vertices with $diam(u,v)\geq 3$, $\forall u\in A$, $v\in A'$ or for every vertex $w\in V(G)-S$, there exists at least one vertex $z\in V(G)-S$ and $y\in S$. Clearly, S forms a minimal γ_r- set of G. Now by definition of line graph, let $D=\{u_1,u_2,...,u_k\}\subseteq V(L(G))$ be the minimum set of vertices with $N[u_j]=V(L(G))$, for every $u_j\in D$, $1\leq j\leq k$. Further, $|\deg(x)-\deg(y)|\leq 1$, $\forall x\in D$, $y\in V(L(G))-D$. Clearly, D forms a minimal degree equitable line dominating set in $L(G)$. Since for any graph G, there exists at least one edge e with $|\deg(e)|=\delta'(G)$, it follows that $|D|\cup|S'|\leq|B_1|$ $\cup|B|\cup|\deg(e)|$. Therefore, $\gamma_{el}(G)+\gamma_r(G)\leq\alpha_1(G)+\beta_1(G)+\delta'(G)$.

The following Theorem relates the degree equitable line domination and roman domination number in terms of G.

Theorem 2.9. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)+\gamma_R(L(G))\leq p+\Delta(G)$.

Proof. Let $f:V(L(G))\rightarrow\{0,1,2\}$ and partition the vertex set $V(L(G))$ into (V_0,V_1,V_2) induced by f with $|V_i|=n_i$ for $i=0,1,2$. Suppose the set V_2 dominates V_0, then $S=V_1\cup V_2$ forms a minimal roman dominating set of $L(G)$. Further, let $F=\{v_1,v_2,...,v_k\}\subseteq V(L(G))$ be the set of vertices with $\deg(v_j)\geq 2$. Suppose there exists a vertex set $D\subseteq F$ with $N[D]=V(L(G))$ and if $|\deg(x)-\deg(y)|$ ≤ 1, $\forall x\in D$, $y\in V(L(G))-D$. Then D forms a degree equitable line dominating set in $L(G)$. Otherwise, there exists at least one vertex $\{w\}\subseteq F$ where $\{w\}\notin D$, such that $D\cup\{w\}$ forms a minimal γ_{el}- set in $L(G)$. Since for any graph G, there exists at least one vertex $v\in V(G)$ of maximum degree $\Delta(G)$, it follows that $|D\cup\{w\}|\cup|S|\leq p\cup|\deg(v)|$. Clearly, $\gamma_{el}(G)+\gamma_R(L(G))\leq p+$ $\Delta(G)$.

The following Theorem relates the degree equitable line domination and line cototal domination number in terms of vertices of G.

Theorem 2.10. *For any connected* (p,q)*- graph* G, $\gamma_{el}(G)+\gamma_{lct}(G)\leq p$.

Proof. Let $D=\{v_1,v_2,...,v_n\}\subseteq V(L(G))$ be the minimal set of vertices which covers all the vertices in $L(G)$ and if $\deg(v)\geq 1$ for every $v\in V(L(G))-D$ in the sub graph $\langle V(L(G))-D\rangle$. Then D forms a γ_{lct} - set in $L(G)$. Otherwise, there exists a vertex $u \in V(L(G)) - D$ with $\deg(u) = 0$ in $\langle V(L(G)) - D \rangle$ then $D \cup \{u\}$ forms a minimal γ_{lct} - set in $L(G)$. Suppose, $|\deg(x) - \deg(y)| \leq 1$, $\forall x \in D$, $y \in V(L(G)) - D$. Then D forms a degree equitable line dominating set in $L(G)$. Otherwise, there exists at least one vertex $z \in V(L(G)) - D$ such that $D_1 = D \cup \{z\}$ forms a minimal γ_{el} - set in $L(G)$. Clearly, it follows that $|D_1| \cup |D| \leq p$ and hence $\gamma_{el}(G) + \gamma_{lct}(G) \leq p$.

The following Theorem relates the degree equitable line domination and inverse line domination number of G.

Theorem 2.11. *For any connected* (p,q) *- graph* G, $\gamma_{el}(G) + \gamma_l^{-1}(G) \leq \beta_1(G) + \Delta'(G) + 1$.

Proof. Let $B = \{e_1, e_2, ..., e_n\} \subseteq E(G)$ be the maximal set of edges which are not adjacent to each other with $|B| = \beta_1(G)$. Now in $L(G)$, let $F = \{v_1, v_2, ..., v_n\}$ forms the set of vertices in $L(G)$ corresponding to the edges of $N[B]$ in G. Suppose, there exists a vertex set $D^{-1} \subseteq V(L(G)) - D$, where $D \subseteq F$, is a γ - set covering all the vertices in $L(G)$. Then D^{-1} forms an inverse line dominating set in $L(G)$. Further, if $\deg(v_i) \geq 2$, $\forall v_i \in D$, $1 \leq i \leq k$ and $|\deg(x) - \deg(y)| \leq 1$, $\forall x \in D$, $y \in V(L(G)) - D$. Then D forms a degree equitable line dominating set in $L(G)$. Otherwise, if there exists a vertex w such that $|\deg(x) - \deg(w)| \not\leq 1$ or $\deg(w) \leq 1$, where $w \in V(L(G)) - D$, then $D \cup \{w\}$ forms a minimal degree equitable line dominating set in $L(G)$. Since for any graph G, there exists at least one edge $e \in E(G)$ with $|\deg(e)| = \Delta'(G)$, it follows that $|D \cup \{w\}| \cup |D^{-1}|$ $\leq |B| \cup |\deg(e)| + 1$. Therefore, $\gamma_{el}(G) + \gamma_l^{-1}(G) \leq \beta_1(G) + \Delta'(G) + 1$.

Conflict of Interests

The author declares that there is no conflict of interests.

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