Available online at http://scik.org
Eng. Math. Lett. 2022, 2022:1
https://doi.org/10.28919/eml/6643
ISSN: 2049-9337

DETERMINATION OF A BETTER NON-LINEAR MATHEMATICAL MODEL WITH TRIGONOMETRIC SINUSOIDAL BEHAVIOUR FOR THE PRICING OF LOCAL RICE IN NIGERIA MARKET

O.D. OGWUMU, O.S. ATARIBU*

Department of Mathematics and Statistics, Federal University Wukari, Wukari, Nigeria

Abstract: The intention of the study is to determine from a list of non-linear formulated model the best that can approximate to the exact/ real life data gathered from price of local Rice in Nigeria. This is obtained/ established in the model validation section. However, the outcome of the model validation upheld that, the purely sinusoidal model performed better with a minimal average difference error of 133 unit as compared to other models. Similarly, there were several highly non-linear models that was tested with the hope of high performance, but failed. Hence, this result showed that financial variables due to their stochastic and oscillating nature can be better modelled using the sine functions and functional.

Keywords: sinusoidal models; pricing, local rice; non-linear models; mathematical modelling; optimization theory; data analysis.

2010 AMS Subject Classification: 93A30.

*Corresponding author
E-mail address: samuelovuzonye@gmail.com
Received August 12, 2021

1. INTRODUCTION

The vulnerability of food as one of the primary source of survival has led to different channel of activity across the globe. Food channel from producers to consumers plays a non-disputable role that makes it either affordable or far expensive. The idea of food provision, readily affordable in time of need to satisfy both human and livestock purposes is called food security [1]. The distribution of various classes of food, pattern of consumption, nutritional value, cost of production and general acceptance promotes the critical examination of the food commodity; rice.

Rice is one of the most valuable cereal crops cultivated and consumed all over the world. It is a staple food in several African countries, mostly in Nigeria and constitutes a large portion of the diet on regular and ceremonial basis [2]. Nigeria is the continents leading consumers of rice, one of the highest producers of rice in Africa and simultaneously one of largest rice importers in the world. Rice is an important food security crop, it is an essential cash crop for it is mainly small-scale producers who commonly sell 80% of total production and consume only 20% [3]. The marketing strategy for rice and high demand causes scarcity for locally produced rice. On the other hand, series of factors influences the price of local rice in Nigeria market that led to the high in price and scarcity according to the population of the country [4]. These factors are classified as natural factors and human influences such as drought in the tropical and sub-tropical savannas where local rice production is on the increase [5], soil salinity which causes cause serious socioeconomic and environmental problems [6] and [7], pest and disease attribute to the local rice farming and storage [8], absence of mechanization that improve production [9] and [10], insufficient land space for farm practices [11], governmental policies, import and export band, insecurity in mist farmers, artificial scarcity in hand of marketers also causes the high in price of local rice in the market and increases the demand.

In spite of the challenges encountered in local rice production, market strategies, the pricing system varied with respect to time and space, due to other alternate food sources and managerial principle [12]. The shifting effect of local rice pricing and stock rating emerges different non-linear and sinusoidal model for analyzing and predicting the effect of market strategy for future recurrence.

Moreover in modelling, non-linear models tend to give the actual realistic picture of any situation according to [13], hence the choice of determining a model that is non-linear for this research is really cannot be overemphasized. Some studies had focused on consumer's acceptability of local rice brand and marketing [14], economic analysis of rice marketing [12], and determination of a better mathematical model for food security in Nigeria [1]. A review of studies related to local rice marketing shows a large number of market strategy with pricing scheme in order to promote food security via the population in the country.

2. METHODOLOGY

The research focused mainly on better skills to predict future pricing of local rice in Nigeria and measures to consider for improving the commodity. Due to the sinusoidal behavior of the price as location and time varies, higher order mathematical model and polynomial where analyzed, compared and combination models where viewed for approximate analysis.

Different prices of local rice from various market are considered on average scheme for different years. The deduction made is shown in the modelling tool below;

2.1 Quadratic model

We suppose that the quadratic price model of the local rice $p(t)$ certify the equation below,

$$p(t) = at^2 + bt + c \tag{1}$$

Hence, to minimize the large data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - at^2 - bt - c)^2$$

$$\left.\begin{aligned} \frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14} (p(t) - at^2 - bt - c) * t^2 = 0 \; (at\, turning\ po\text{int}) \\ \frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14} (p(t) - at^2 - bt - c) * t = 0 \; (at\, turning\ po\text{int}) \\ \frac{\partial I}{\partial c} &= -2\sum_{i=1}^{14} (p(t) - at^2 - bt - c) * 1 = 0 \; (at\, turning\ po\text{int}) \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned}\sum_{i=1}^{14} p(t)*t_i^2 &= a\sum_{i=1}^{14} t_i^4 + b\sum_{i=1}^{14} t_i^3 + c\sum_{i=1}^{14} t_i^2\\ \sum_{i=1}^{14} p(t)*t_i &= a\sum_{i=1}^{14} t_i^3 + b\sum_{i=1}^{14} t_i^2 + c\sum_{i=1}^{14} t_i\\ \sum_{i=1}^{14} p(t) &= a\sum_{i=1}^{14} t_i^2 + b\sum_{i=1}^{14} t_i + c\sum_{i=1}^{14} 1\end{aligned}\right\} \quad (3)$$

Data on average prices of local rice per mudu (bowl for measurement) mostly Northern Nigeria

Sign i	Years	T	$P(t)$	t^4	t^3	t^2	$P(t)*t^2$	$P(t)*t$
1	2008	0	278	0	0	0	0	0
2	2009	1	278	1	1	1	278	278
3	2010	2	278	16	8	4	1112	556
4	2011	3	300	81	27	9	2700	900
5	2012	4	300	256	64	16	4800	1200
6	2013	5	320	625	125	25	8000	1600
7	2014	6	320	1296	216	36	11520	1920
8	2015	7	330	2401	343	49	16170	2310
9	2016	8	380	4096	512	64	24320	3040
10	2017	9	380	6561	729	81	30780	3420
11	2018	10	390	10000	1000	100	39000	3900
12	2019	11	450	14641	1331	121	54450	4950
13	2020	12	500	20736	1728	144	72000	6000
14	2021	13	700	28561	2197	169	118300	9100
		$\sum_{i=1}^{14} t = 91$	$\sum_{i=1}^{14} p(t) = 5204$	$\sum_{i=1}^{14} t^4 = 89271$	$\sum_{i=1}^{14} t^3 = 8281$	$\sum_{i=1}^{14} t^2 = 819$	$\sum_{i=1}^{14} p(t)*t^2 = 383430$	$\sum_{i=1}^{14} p(t)*t = 39174$

SOURCE: Taraba State Statistical Year Book and Survey by the Researchers (2021)

Solving equation (3) simultaneously for the constant a, b and c with the substitution from table 2.1.1 the quadratic model becomes

$$P(t) = \frac{296}{91}t^2 - \frac{8544}{455}t + \frac{10622}{35} \quad (4)$$

The table below gives comparisons between the model results and gathered data

Table 2.1.2 Validation of the Model Result

Sign	Years	T	P(t)	P(t)(Model)	Absolute Error
1	2008	0	278	303.485714	25.485714
2	2009	1	278	287.96044	9.9604396
3	2010	2	278	278.940659	0.9406593
4	2011	3	300	276.426374	23.573626
5	2012	4	300	280.417582	19.582418
6	2013	5	320	290.914286	29.085714
7	2014	6	320	307.916484	12.083516
8	2015	7	330	331.424176	1.4241758
9	2016	8	380	361.437363	18.562637
10	2017	9	380	397.956044	17.956044
11	2018	10	390	440.98022	50.98022
12	2019	11	450	490.50989	40.50989
13	2020	12	500	546.545055	46.545055
14	2021	13	700	609.085714	90.914286
					$\cong 387.6043956$

2.2 cubic polynomial model

We suppose that the cubic polynomial price model of the local rice $p(t)$ certify the equation below,

$$p(t) = at^3 + bt^2 + ct + d \tag{5}$$

Hence, to minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - at^3 - bt^2 - ct - d)^2$$

$$\left.\begin{aligned}
\frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14}(p(t) - at^3 - bt^2 - ct - d) * t^3 = 0 \text{ (at turning point)} \\
\frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14}(p(t) - at^3 - bt^2 - ct - d) * t^2 = 0 \text{ (at turning point)} \\
\frac{\partial I}{\partial c} &= -2\sum_{i=1}^{14}(p(t) - at^3 - bt^2 - ct - d) * t = 0 \text{ (at turning point)} \\
\frac{\partial I}{\partial d} &= -\sum_{i=1}^{14}(p(t) - at^3 - bt^2 - ct - d) * 1 = 0 \text{ (at turning point)}
\end{aligned}\right\} \tag{6}$$

$$\left.\begin{array}{l}\sum_{i=1}^{14} p(t)*t_i^3 = a\sum_{i=1}^{14} t_i^6 + b\sum_{i=1}^{14} t_i^5 + c\sum_{i=1}^{14} t_i^4 + d\sum_{i=1}^{14} t_i^3 \\ \sum_{i=1}^{14} p(t)*t_i^2 = a\sum_{i=1}^{14} t_i^5 + b\sum_{i=1}^{14} t_i^4 + c\sum_{i=1}^{14} t_i^3 + d\sum t_i^2 \\ \sum_{i=1}^{14} p(t)*t_i = a\sum_{i=1}^{14} t_i^4 + b\sum_{i=1}^{14} t_i^3 + c\sum_{i=1}^{14} t_i^2 + d\sum_{i=1}^{14} t_i \\ \sum_{i=1}^{14} p(t) = a\sum_{i=1}^{14} t_i^3 + b\sum_{i=1}^{14} t_i^2 + \sum_{i=1}^{14} t_i + d\sum_{i=1}^{14} 1 \end{array}\right\} \quad (7)$$

Table 2.2.1 Computational details for Cubic Polynomial Model

Sign	Years	T	$P(t)$	t^6	t^5	$P(t)*t^3$	$P(t)*t^2$	$P(t)*t$
1	2008	0	278	0	0	0	0	0
2	2009	1	278	1	1	278	278	278
3	2010	2	278	64	32	2224	1112	556
4	2011	3	300	729	243	8100	2700	900
5	2012	4	300	4096	1024	19200	4800	1200
6	2013	5	320	15625	3125	40000	8000	1600
7	2014	6	320	46656	7776	69120	11520	1920
8	2015	7	330	117649	16807	113190	16170	2310
9	2016	8	380	262144	32768	194560	24320	3040
10	2017	9	380	531441	59049	277020	30780	3420
11	2018	10	390	1000000	100000	390000	39000	3900
12	2019	11	450	1771561	161051	598950	54450	4950
13	2020	12	500	2985984	248832	864000	72000	6000
14	2021	13	700	4826809	371293	1537900	118300	9100
		$\sum_{i=1}^{14} t = 91$	$\sum_{i=1}^{14} P(t) = 5204$	$\sum_{i=1}^{14} t^6 = 11562759$	$\sum_{i=1}^{14} t^5 = 1002001$	$\sum_{i=1}^{14} P(t)*t^3 = 4114542$	$\sum_{i=1}^{14} P(t)*t^2 = 383430$	$\sum_{i=1}^{14} P(t)*t = 39174$

Solving equation (7) simultaneously for the constant a, b, c and d with the substitution from table 2.2.1 and table 2.1.1, the cubic model becomes

$$P(t)=\frac{233}{442}t^{3}-\frac{43481}{6188}t^{2}+\frac{202499}{6188}t+\frac{61465}{238} \quad (8)$$

The table below gives comparisons between the model results and gathered data

Table 2.2.2 Validation of the Model Result

Sign	Year	T	P(t)	P(t) Model	Absolute Error
1	2008	0	278	258.2563	19.7437
2	2009	1	278	284.4813	6.481254
3	2010	2	278	299.8158	21.81577
4	2011	3	300	307.4228	7.422754
5	2012	4	300	310.4651	10.46509
6	2013	5	320	312.1057	7.894312
7	2014	6	320	315.5074	4.492566
8	2015	7	330	323.8332	6.166774
9	2016	8	380	340.246	39.75404
10	2017	9	380	367.9085	12.09147
11	2018	10	390	409.9838	19.98384
12	2019	11	450	469.6348	19.63478
13	2020	12	500	550.0242	50.02424
14	2021	13	700	654.3151	45.68487
					$\cong 271.6554622$

2.3 Fifth order polynomial model

We suppose that the order five polynomial price model of the local rice $p(t)$ certify the equation below,

$$p(t)=at^{5}+bt^{4}+ct^{3}+dt^{2}+et+f \quad (9)$$

Hence, to minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f)^2$$

$$\left.\begin{aligned}
\frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * t^5 = 0\ (at\,turning\ po\text{int}) \\
\frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * t^4 = 0\ (at\,turning\ po\text{int}) \\
\frac{\partial I}{\partial c} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * t^3 = 0\ (at\,turning\ po\text{int}) \\
\frac{\partial I}{\partial d} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * t^2 = 0\ (at\,turning\ po\text{int}) \\
\frac{\partial I}{\partial e} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * t = 0\ (at\,turning\ po\text{int}) \\
\frac{\partial I}{\partial f} &= -2\sum_{i=1}^{14}(p(t) - at^5 - bt^4 - ct^3 - dt^2 - et - f) * 1 = 0\ (at\,turning\ po\text{int})
\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}
\sum_{i=1}^{14} p(t) * t_i^5 &= a\sum_{i=1}^{14} t_i^{10} + b\sum_{i=1}^{14} t_i^9 + c\sum_{i=1}^{8} t_i^8 + d\sum_{i=1}^{14} t_i^7 + e\sum_{i=1}^{14} t_i^6 + f\sum_{i=1}^{14} t_i^5 \\
\sum_{i=1}^{14} p(t) * t_i^4 &= a\sum_{i=1}^{14} t_i^9 + b\sum_{i=1}^{14} t_i^8 + c\sum_{i=1}^{14} t_i^7 + d\sum_{i=1}^{14} t_i^6 + e\sum_{i=1}^{14} t_i^5 + f\sum_{i=1}^{14} t_i^4 \\
\sum_{i=1}^{14} p(t) * t_i^3 &= a\sum_{i=1}^{14} t_i^8 + b\sum_{i=1}^{14} t_i^7 + c\sum_{i=1}^{14} t_i^6 + d\sum_{i=1}^{14} t_i^5 + e\sum_{i=1}^{14} t^4 + f\sum_{i=1}^{14} t^3 \\
\sum_{i=1}^{14} p(t) * t^2 &= a\sum_{i=1}^{14} t_i^7 + b\sum_{i=1}^{14} t_i^6 + c\sum_{i=1}^{14} t_i^5 + d\sum_{i=1}^{14} t_i^4 + e\sum_{i=1}^{14} t_i^3 + f\sum_{i=1}^{14} t_i^2 \\
\sum_{i=1}^{14} p(t) * t &= a\sum_{i=1}^{14} t_i^6 + b\sum_{i=1}^{14} t_i^5 + c\sum_{i=1}^{14} t_i^4 + d\sum_{i=1}^{14} t_i^3 + e\sum_{i=1}^{14} t_i^2 + f\sum_{i=1}^{14} t_i \\
\sum_{i=1}^{14} p(t) &= a\sum_{i=1}^{14} t_i^5 + b\sum_{i=1}^{14} t_i^4 + c\sum_{i=1}^{14} t_i^3 + d\sum_{i=1}^{14} t_i^2 + e\sum_{i=1}^{14} t + f\sum_{i=1}^{14} 1
\end{aligned}\right\} \tag{11}$$

Table 2.3.1 Computational Details of fifth order Polynomial Model

Year	T	P(t)	t^7	t^8	t^9	t^{10}	$P(t)*t^4$	$P(t)*t^5$
2008	0	278	0	0	0	0	0	0
2009	1	278	1	1	1	1	278	278
2010	2	278	128	256	512	1024	4448	8896
2011	3	300	2187	6561	19683	59049	24300	72900
2012	4	300	16384	65536	262144	1048576	76800	307200
2013	5	320	78125	390625	1953125	9765625	200000	1000000
2014	6	320	279936	1679616	10077696	60466176	414720	2488320
2015	7	330	823543	5764801	40353607	282475249	792330	5546310
2016	8	380	2097152	16777216	134217728	1073741824	1556480	12451840
2017	9	380	4782969	43046721	387420489	3486784401	2493180	22438620
2018	10	390	10000000	100000000	1000000000	10000000000	3900000	39000000
2019	11	450	19487171	214358881	2357947691	25937424601	6588450	72472950
2020	12	500	35831808	429981696	5159780352	61917364224	10368000	124416000
2021	13	700	62748517	815730721	10604499373	1.37858E+11	19992700	259905100
	$\sum_{i=1}^{14} t = 91$	$\sum_{i=1}^{14} P(t) = 5204$	$\sum_{i=1}^{14} t^7 = 136147921$	$\sum_{i=1}^{14} t^8 = 1627802631$	$\sum_{i=1}^{14} t^9 = 19696532401$	$\sum_{i=1}^{14} t^{10} = 2.40628*10^{11}$	$\sum_{i=1}^{14} P(t)*t^4 = 46411686$	$\sum_{i=1}^{14} P(t)*t^5 = 54010844$

Solving equation (11) simultaneously for the constant a, b, c, d, e and f with the substitution from table 2.3.1, table 2.2.1 and table 2.1.1, then the model becomes

$$P(t) = 0.01740066429*t^5 - 0.4548332581*t^4 + 4.087498495*t^3$$
$$-13.48594466*t^2 + 18.92502769*t + 274.7266349 \quad (12)$$

The table below gives comparisons between the model results and gathered data

Table 2.3.2 Validation of the model Result

Sign	Year	T	P(t)	P(t) Model	Absolute Error
1	2008	0	278	274.7266	3.273365
2	2009	1	278	283.8158	5.815784
3	2010	2	278	284.6124	6.612389
4	2011	3	300	287.8775	12.12246
5	2012	4	300	297.6325	2.367499
6	2013	5	320	313.2468	6.753242
7	2014	6	320	331.5261	11.52613
8	2015	7	330	348.8008	18.80084
9	2016	8	380	363.0136	16.98643
10	2017	9	380	375.8076	4.192411
11	2018	10	390	394.6148	4.614789
12	2019	11	450	434.7438	15.25621
13	2020	12	500	521.468	21.46799
14	2021	13	700	692.1137	7.886298
					$\cong 137.6758$

2.4 simple Trigonometric involving sine and cosine model

We suppose that the simple trigonometrically price model of the local rice $p(t)$ certify the equation below,

$$p(t) = a\sin t + b\cos t \tag{13}$$

To minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - a\sin t - b\cos t)^2$$

$$\left.\begin{aligned} \frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14}(p(t) - a\sin t - b\cos t) * \sin t = 0\ (at\,turning\,po\text{int}) \\ \frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14}(p(t) - a\sin t - b\cos t) * \cos t = 0\ (at\,turning\,po\text{int}) \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} \sum_{i=1}^{14} p(t) * \sin t_i &= a\sum_{i=1}^{14}\sin^2 t_i + b\sum_{i=1}^{14}\cos t_i * \sin t_i \\ \sum_{i=1}^{14} p(t) * \cos t_i &= a\sum_{i=1}^{14}\sin t_i * \cos t_i + b\sum_{i=1}^{14}\cos^2 t_i \end{aligned}\right\} \tag{15}$$

Table 2.4.1 Computational Details of simple Trigonometric involving sine and cosine Model

Year	T	P(t)	Sint	P(t)*sint	$\sin^2 t$	Cost	$\cos^2 t$	P(t)*cost	sint*cost
2008	0	278	0	0	0	1	1	278	0
2009	1	278	0.017452	4.851769	0.000304586	0.999847695	0.999695	277.957659	0.017449748
2010	2	278	0.034899	9.70206	0.001217975	0.999390827	0.998782	277.83065	0.034878237
2011	3	300	0.052336	15.70079	0.002739052	0.998629535	0.997261	299.58886	0.052264232
2012	4	300	0.069756	20.92694	0.004865966	0.99756405	0.995134	299.269215	0.06958655
2013	5	320	0.087156	27.88984	0.007596123	0.996194698	0.992404	318.782303	0.086824089
2014	6	320	0.104528	33.44911	0.0109262	0.994521895	0.989074	318.247007	0.103955845
2015	7	330	0.121869	40.21688	0.014852137	0.992546152	0.985148	327.54023	0.120960948
2016	8	380	0.139173	52.88578	0.019369152	0.990268069	0.980631	376.301866	0.137818678
2017	9	380	0.156434	59.4451	0.024471742	0.987688341	0.975528	375.321569	0.154508497
2018	10	390	0.173648	67.72279	0.03015369	0.984807753	0.969846	384.075024	0.171010072
2019	11	450	0.190809	85.86405	0.036408073	0.981627183	0.963592	441.732233	0.187303297
2020	12	500	0.207912	103.9558	0.043227271	0.978147601	0.956773	489.0738	0.203368322
2021	13	700	0.224951	157.4657	0.050602977	0.974370065	0.949397	682.059045	0.219185573
	$\sum_{i=1}^{14} t = 91$	$\sum_{i=1}^{14} P(t) = 5204$	$\sum_{i=1}^{14} \sin t = 1.5809$	$\sum_{i=1}^{14} P(t)*\sin t = 680.08$	$\sum_{i=1}^{14} \sin^2 t = 0.24673$	$\sum_{i=1}^{14} \cos t = 13.8756$	$\sum_{i=1}^{14} \cos^2 t = 13.753$	$\sum_{i=1}^{14} P(t)*\cos t = 5145.8$	$\sum_{i=1}^{14} snt*\cos t = 1.5591$

Simultaneously solving equation (15) for the unknown constant a and b with the substitution from table 2.4.1, then the trigonometric of sine and cosine model becomes,

$$P(t) = 1382.148055*\sin t + 217.4648281*\cos t \qquad (16)$$

The table below gives comparisons between the model results and gathered data

Table 2.4.2 Validation of the model Result

Sign	Year	T	P(t)	P(t) Model	Absolute Error
1	2008	0	278	217.4648	60.53517
2	2009	1	278	241.5535	36.44648
3	2010	2	278	265.5686	12.43137
4	2011	3	300	289.5028	10.49716
5	2012	4	300	313.3489	13.34887
6	2013	5	320	337.0994	17.09945
7	2014	6	320	360.7473	40.74735
8	2015	7	330	384.2854	54.28535
9	2016	8	380	407.7063	27.70631
10	2017	9	380	431.0031	51.00307
11	2018	10	390	454.1685	64.16854
12	2019	11	450	477.1957	27.19567
13	2020	12	500	500.0774	0.077439
14	2021	13	700	522.8069	177.1931
					$\cong 592.7353$

2.5 second order Purely sinusoidal model

We suppose that the second order sinusoidal price model of the local rice $p(t)$ certify the equation below,

$$p(t) = a\sin^2 t + b\sin t + c \tag{17}$$

To minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - a\sin^2 t - b\sin t - c)^2$$

$$\left.\begin{aligned}
\frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14} (p(t) - a\sin^2 t - b\sin t - c) * \sin^2 t = 0 \ (at\ turning\ po\text{int}) \\
\frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14} (p(t) - a\sin^2 t - b\sin t - c) * \sin t = 0 \ (at\ turning\ po\text{int}) \\
\frac{\partial I}{\partial c} &= -2\sum_{i=1}^{14} (p(t) - a\sin^2 t - b\sin t - c) * 1 = 0 \ (at\ turning\ po\text{int})
\end{aligned}\right\} \tag{18}$$

$$\left.\begin{aligned}\sum_{i=1}^{14} p(t)*\sin^2 t_i &= a\sum_{i=1}^{14}\sin^4 t_i + b\sum_{i=1}^{14}\sin^3 t_i + c\sum_{i=1}^{14}\sin^2 t_i \\ \sum_{i=1}^{14} p(t)*\sin t_i &= a\sum_{i=1}^{14}\sin^3 t_i + b\sum_{i=1}^{14}\sin^2 t_i + c\sum_{i=1}^{14}\sin t_i \\ \sum_{i=1}^{14} p(t) &= a\sum_{i=1}^{14}\sin^2 t_i + b\sum_{i=1}^{14}\sin t_i + c\sum_{i=1}^{14}1\end{aligned}\right\} \quad (19)$$

Table 2.5.1 Computational Details for second order purely sinusoidal model

Years	T	P(t)	$\sin t$	$\sin^2 t$	$\sin^3 t$	$\sin^4 t$	$P(t)*\sin t$	$P(t)*\sin^2 t$
2008	0	278	0	0	0	0	0	0
2009	1	278	0.017452	0.000305	5.31577E-06	9.27729E-08	4.851769	0.08467504
2010	2	278	0.034899	0.001218	4.25067E-05	1.48346E-06	9.70206	0.33859701
2011	3	300	0.052336	0.002739	0.000143351	7.50241E-06	15.70079	0.82171569
2012	4	300	0.069756	0.004866	0.000339433	2.36776E-05	20.92694	1.45978969
2013	5	320	0.087156	0.007596	0.000662046	5.77011E-05	27.88984	2.43075952
2014	6	320	0.104528	0.010926	0.001142099	0.000119382	33.44911	3.49638388
2015	7	330	0.121869	0.014852	0.00181002	0.000220586	40.21688	4.90120516
2016	8	380	0.139173	0.019369	0.002695665	0.000375164	52.88578	7.36027777
2017	9	380	0.156434	0.024472	0.003828224	0.000598866	59.4451	9.2992619
2018	10	390	0.173648	0.030154	0.005236133	0.000909245	67.72279	11.7599389
2019	11	450	0.190809	0.036408	0.006946988	0.001325548	85.86405	16.3836327
2020	12	500	0.207912	0.043227	0.008987455	0.001868597	103.9558	21.6136356
2021	13	700	0.224951	0.050603	0.011383193	0.002560661	157.4657	35.4220838
	$\sum_{i=1}^{14} t = 91$	$\sum_{i=1}^{14} P(t) = 5204$	$\sum_{i=1}^{14}\sin t = 1.580925$	$\sum_{i=1}^{14}\sin^2 t = 0.246735$	$\sum_{i=1}^{14}\sin^3 t = 0.043222$	$\sum_{i=1}^{14}\sin^4 t = 0.008069$	$\sum_{i=1}^{14} P(t)*\sin t = 680.0767$	$\sum_{i=1}^{14} P(t)*\sin^2 t = 115.371957$

Simultaneously solving equation (19) for the unknown constant a, b and c with the substitution from table 2.5.1, then the second order purely sinusoidal model becomes,

$$P(t) = 1089.227894*\sin^2 t - 1097.823546*\sin t + 303.7193032 \quad (20)$$

The table below gives comparisons between the model results and gathered data

Table 2.5.2 Validation of the model Result

Sign	Years	t	P(t)	P(t)Model	Absolute Error
1	2008	0	278	303.7193032	25.7193
2	2009	1	278	287.8772815	9.877281
3	2010	2	278	278.672336	0.672336
4	2011	3	300	276.09818	23.90182
5	2012	4	300	280.1404588	19.85954
6	2013	5	320	290.7767726	29.22323
7	2014	6	320	307.9767092	12.02329
8	2015	7	330	331.701886	1.701886
9	2016	8	380	361.9060027	18.094
10	2017	9	380	398.5349025	18.5349
11	2018	10	390	441.5266433	51.52664
12	2019	11	450	490.811579	40.81158
13	2020	12	500	546.312449	46.31245
14	2021	13	700	607.9444781	92.05552
					$\cong 390.3138$

2.6 Concentric trigonometric (sine and cosine) model

We suppose that the concentric trigonometric model price model of the local rice $p(t)$ certify the equation below,

$$p(t) = a\sin^3 t + a\sin^2 t\cos t + c\cos t \tag{21}$$

To minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - a\sin^3 t - b\sin^2 t\cos t - c\cos t)^2$$

$$\left.\begin{aligned}
\frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14} (p(t) - a\sin^3 t - b\sin^2 t\cos t - c\cos t)^2 * \sin^3 t = 0 \ (at\, turning\ po\mathrm{int}) \\
\frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14} (p(t) - a\sin^3 t - b\sin^2 t\cos t - c\cos t) * \sin^2 t\cos t = 0 \ (at\, turning\ po\mathrm{int}) \\
\frac{\partial I}{\partial c} &+ -2\sum_{i=1}^{14} (p(t) - a\sin^3 t - b\sin^2 t\cos t - c\cos t) * \cos t = 0 \ (at\, turning\ po\mathrm{int})
\end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned}
\sum_{i=1}^{14} p(t)*\sin^3 t_i &= a\sum_{i=1}^{14}\sin^6 t_i + b\sum_{i=1}^{14}(\sin^2 t_i \cos t_i)\sin^3 t_i + c\sum_{i=1}^{14}\sin^3 t_i \cos t \\
\sum_{i=1}^{14} p(t)*\sin^2 t_i \cos t_i &= a\sum_{i=1}^{14}(\sin^2 t_i \cos t_i)\sin^3 t_i + b\sum_{i=1}^{14}(\sin^2 t_i \cos t_i)^2 + c\sum_{i=1}^{14}\cos t_i(\sin^2 t_i \cos t_i) \\
\sum_{i=1}^{14} p(t)*\cos t_i &= a\sum_{i=1}^{14}\sin^3 t_i \cos t_i + b\sum_{i=1}^{14}(\sin^2 t_i \cos t_i)\cos t_i + c\sum_{i=1}^{14}\cos^2 t_i
\end{aligned}\right\} \quad (23)$$

Table 2.6.1 Computational Details for Concentric trigonometric (sine and cosine) Model

$(\sin^3 t)$	$P(t)*\sin^3 t$	$\sin^6 t$	$\sin^2 t\cos t$	$\sin^3 t\cos^4 t$	$\sin^3 t\cos t$	$P(t)*$ $\sin^2 t\cos t$	$(\sin^2 t\cos t)^2$	$\sin^2 t\cos^2 t$
0	0	0	0	0	0	0	0	0
5.32E-06	0.00147778	2.8257E-11	0.0003045	1.62E-09	5.31496E-06	0.084662148	9.27447E-08	0.000304494
4.25E-05	0.01181687	1.8068E-09	0.0012172	5.17E-08	4.24808E-05	0.33839075	1.48166E-06	0.001216491
0.000143	0.04300528	2.0549E-08	0.0027353	3.92E-07	0.000143154	0.820589562	7.48186E-06	0.00273155
0.000339	0.10182978	1.1521E-07	0.0048541	1.65E-06	0.000338606	1.456233714	2.35624E-05	0.004842288
0.000662	0.21185465	4.383E-07	0.0075672	5.01E-06	0.000659527	2.421509744	5.72628E-05	0.007538422
0.001142	0.36547163	1.3044E-06	0.0108663	1.24E-05	0.001135842	3.477230326	0.000118077	0.010806818
0.00181	0.59730666	3.2762E-06	0.0147414	2.67E-05	0.001796529	4.864672324	0.00021731	0.014631551
0.002696	1.02435268	7.2666E-06	0.0191807	5.17E-05	0.002669431	7.288648054	0.000367897	0.018993988
0.003828	1.45472506	1.4655E-05	0.0241705	9.25E-05	0.003781092	9.184772559	0.000584211	0.023872876
0.005236	2.04209197	2.7417E-05	0.0296956	0.000155	0.005156585	11.58127905	0.000881828	0.029244445
0.006947	3.1261445	4.8261E-05	0.0357392	0.000248	0.006819352	16.08261924	0.001277287	0.035082525
0.008987	4.49372752	8.0774E-05	0.0422827	0.00038	0.008791058	21.14132579	0.001787823	0.041358674
0.011383	7.9682351	0.00012958	0.049306	0.000561	0.011091442	34.51421808	0.002431084	0.048042316
$SUM =$ 3.9512	**SUM= 21.4420395**	**SUM= 0.00031311**	**SUM= 0.2426607**	**SUM= 0.001535**	**SUM= 0.042430413**	**SUM= 113.2561514**	**SUM= 0.007755399**	**SUM= 0.238666437**

Simultaneously solving equation (23) for the unknown constant a, b and c with the substitution from table 2.5.1 and table 2.6.1 then the concentric trigonometric (sine and cosine) model becomes,

$$P(t) = 48999.33373*\sin^3 t - 4205.946653*\sin^2 t\cos t + 295.9686867*\cos t \quad (24)$$

The table below gives comparisons between the model results and gathered data

Table 2.6.2 Validation of the model Result

Sign	Years	t	P(t)	P(t) Model	Absolute Error
1	2008	0	278	295.9686867	17.96869
2	2009	1	278	294.9031989	16.9032
3	2010	2	278	292.7515743	14.75157
4	2011	3	300	291.0826518	8.917348
5	2012	4	300	291.4635556	8.536444
6	2013	5	320	295.4549238	24.54508
7	2014	6	320	304.6061559	15.39384
8	2015	7	330	320.4506897	9.54931
9	2016	8	380	344.501324	35.49868
10	2017	9	380	378.2455983	1.754402
11	2018	10	390	423.1412418	33.14124
12	2019	11	450	480.6117063	30.61171
13	2020	12	500	552.0417928	52.04179
14	2021	13	700	638.7733866	61.22661
					$\cong 330.8399$

2.7 Fifth order Purely sinusoidal model

We suppose that the Fifth order sinusoidal model price model of the local rice $p(t)$ certify the equation below,

$$p(t) = a\sin^5 t + b\sin^4 t + c\sin^3 t + d\sin^2 t + e\sin t + f \tag{25}$$

To minimize the cumbersome data for $p(t)$ using least square method, we have that

$$I_{\min} = \min \sum_{i=1}^{14} (p(t) - a\sin^5 t - b\sin^4 t - c\sin^3 t - d\sin^2 t - e\sin t - f)^2$$

$$\left.\begin{aligned}
\frac{\partial I}{\partial a} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*\sin^5 t=0\ (at\,turning\ po\mathrm{int})\\
\frac{\partial I}{\partial b} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*\sin^4 t=0\ (at\,turning\ po\mathrm{int})\\
\frac{\partial I}{\partial c} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*\sin^3 t=0\ (at\,turning\ po\mathrm{int})\\
\frac{\partial I}{\partial d} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*\sin^2 t=0\ (at\,turning\ po\mathrm{int})\\
\frac{\partial I}{\partial e} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*\sin t=0\ (at\,turning\ po\mathrm{int})\\
\frac{\partial I}{\partial f} &= -2\sum_{i=1}^{14}(p(t)-a\sin^5 t-b\sin^4 t-c\sin^3 t-d\sin^2 t-e\sin t-f)*1=0\ (at\,turning\ po\mathrm{int})
\end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned}
\sum_{i=1}^{14}p(t)*\sin^5 t_i &= a\sum_{i=1}^{14}\sin^{10} t_i+b\sum_{i=1}^{14}\sin^9 t_i+c\sum_{i=1}^{14}\sin^8 t_i+d\sum_{i=1}^{14}\sin^7 t_i+e\sum_{i=1}^{14}\sin^6 t_i+f\sum_{i=1}^{14}\sin^5 t_i\\
\sum_{i=1}^{14}p(t)*\sin^4 t_i &= a\sum_{i=1}^{14}\sin^9 t_i+b\sum_{i=1}^{14}\sin^8 t_i+c\sum_{i=1}^{14}\sin^7 t_i+d\sum_{i=1}^{14}\sin^6 t_i+e\sum_{i=1}^{14}\sin^5 t_i+f\sum_{i=1}^{14}\sin^4 t_i\\
\sum_{i=1}^{14}p(t)*\sin^3 t_i &= a\sum_{i=1}^{14}\sin^8 t_i+b\sum_{i=1}^{14}\sin^7 t_i+c\sum_{i=1}^{14}\sin^6 t_i+d\sum_{i=1}^{14}\sin^5 t_i+e\sum_{i=1}^{14}\sin^4 t_i+f\sum_{i=1}^{14}\sin^3 t_i\\
\sum_{i=1}^{14}p(t)*\sin^2 t_i &= a\sum_{i=1}^{14}\sin^7 t_i+b\sum_{i=1}^{14}\sin^6 t_i+c\sum_{i=1}^{14}\sin^5 t_i+d\sum_{i=1}^{14}\sin^4 t_i+e\sum_{i=1}^{14}\sin^3 t_i+f\sum_{i=1}^{14}\sin^2 t_i\\
\sum_{i=1}^{14}p(t)*\sin t_i &= a\sum_{i=1}^{14}\sin^6 t_i+b\sum_{i=1}^{14}\sin^5 t_i+c\sum_{i=1}^{14}\sin^4 t_i+d\sum_{i=1}^{14}\sin^3 t_i+e\sum_{i=1}^{14}\sin^2 t_i+f\sum_{i=1}^{14}\sin t_i\\
\sum_{i=1}^{14}p(t) &= a\sum_{i=1}^{14}\sin^5 t_i+b\sum_{i=1}^{14}\sin^4 t_i+c\sum_{i=1}^{14}\sin^3 t_i+d\sum_{i=1}^{14}\sin^2 t_i+e\sum_{i=1}^{14}\sin t_i+f\sum_{i=1}^{14}1
\end{aligned}\right\} \quad (27)$$

Table 2.7.1 Computational Details for Fifth order purely sinusoidal model

$\sin^5 t$	$\sin^6 t$	$\sin^7 t$	$\sin^8 t$	$\sin^9 t$	$\sin^{10} t$	$P(t)*\sin^3 t$	$P(t)*\sin^4 t$	$P(t)*\sin^5 t$
0	0	0	0	0	0	0	0	0
1.62E-09	2.83E-11	4.9316E-13	8.6068E-15	1.502E-16	2.62E-18	0.001477783	2.57909E-05	4.50113E-07
5.18E-08	1.81E-09	6.3057E-11	2.2007E-12	7.68E-14	2.68E-15	0.011816865	0.000412403	1.43926E-05
3.93E-07	2.05E-08	1.0755E-09	5.6286E-11	2.946E-12	1.54E-13	0.043005277	0.002250722	0.000117794
1.65E-06	1.15E-07	8.037E-09	5.6063E-10	3.911E-11	2.73E-12	0.101829781	0.007103286	0.0004955
5.03E-06	4.38E-07	3.8201E-08	3.3294E-09	2.902E-10	2.53E-11	0.211854651	0.018464349	0.001609274
1.25E-05	1.3E-06	1.3635E-07	1.4252E-08	1.49E-09	1.56E-10	0.365471634	0.038202188	0.003993216
2.69E-05	3.28E-06	3.9927E-07	4.8658E-08	5.93E-09	7.23E-10	0.597306655	0.07279337	0.00887128
5.22E-05	7.27E-06	1.0113E-06	1.4075E-07	1.959E-08	2.73E-09	1.024352681	0.142562339	0.019840843
9.37E-05	1.47E-05	2.2926E-06	3.5864E-07	5.61E-08	8.78E-09	1.454725061	0.227569137	0.035599656
0.000158	2.74E-05	4.7609E-06	8.2673E-07	1.436E-07	2.49E-08	2.042091968	0.354605549	0.061576607
0.000253	4.83E-05	9.2086E-06	1.7571E-06	3.353E-07	6.4E-08	3.1261445	0.596496492	0.113816896
0.000389	8.08E-05	1.6794E-05	3.4917E-06	7.26E-07	1.51E-07	4.49372752	0.934298487	0.194251578
0.000576	0.00013	2.9149E-05	6.557E-06	1.475E-06	3.32E-07	7.968235097	1.792462886	0.403216416
$SUM =$ 0.001568	**SUM= 0.000313**	**SUM= 6.3799E-05**	**SUM= 1.3199E-05**	**SUM= 2.763E-06**	**SUM= 5.84E-07**	**SUM= 21.44203947**	**SUM= 4.187246999**	**SUM= 0.843403904**

Simultaneously solving equation (23) for the unknown constant a, b, c, d, e, and f with the substitution from table 2.5.1 and table 2.7.1, then the Fifth order Purely sinusoidal model becomes,

$$P(t) = 7.698927579*10^6 * \sin^5 t - 3.005842648*10^6 * \sin^4 t + 3.586232146*10^5 * \sin^3 t - 7788.550772*\sin^2 t - 74.85079349*\sin t + 280.4518019 \quad (28)$$

The table below gives comparisons between the model results and gathered data

Table 2.7.2 Validation of the model Result

Sign	Years	t	P(t)	P(t) Model	Absolute Error
1	2008	0	278	280.4518019	2.4518019
2	2009	1	278	278.4131502	0.413150203
3	2010	2	278	279.5367147	1.536714714
4	2011	3	300	287.0820285	12.91797154
5	2012	4	300	300.6049286	0.604928637
6	2013	5	320	317.4676807	2.532319318
7	2014	6	320	334.3420234	14.3420234
8	2015	7	330	348.6894078	18.68940782
9	2016	8	380	360.2030252	19.79697479
10	2017	9	380	372.1966697	7.803330305
11	2018	10	390	392.9260545	2.926054502
12	2019	11	450	436.8288927	13.17110731
13	2020	12	500	525.6708571	25.67085713
14	2021	13	700	689.5854441	10.41455587
					$\cong 133.2711974$

The vary effect of each model is then properly estimated on average scale of different performance the light of is then analyzed as follows.

2.8 All model scale

This is the model consisting of mixture of polynomial, quadratic, trigonometric of sine and cosine and purely sinusoidal model equation

$$P_m = \frac{equation(4)+(8)+(12)+(16)+(20)+(24)+(28)}{7}$$

$$P_m(t) = \frac{1}{7}\begin{pmatrix} -17.25986192 * t^2 + 32.87147242 * t + 1420.639757 + 4.614647816 * t^3 + 0.01740066429 * t^5 \\ -0.4548332581 * t^4 + 3103.728168 \sin^2 t + 209.4737155 \sin t + 7.698927579 * 10^6 \sin^5 t \\ -3.005842648 * 10^6 \sin^4 t + 4.076225483 * 10^5 \sin^3 t + 513.4335148 * \cos t - 4205.946653 \sin^2 t \cos t \end{pmatrix} \quad (29)$$

Table 2.8.1 Validation of all model scaling

Sign	Years	t	P(t) Model	P(t)	Absolute Error
1	2008	0	278	276.2961817	1.703818314
2	2009	1	278	279.8578036	1.857803565
3	2010	2	278	282.8425817	4.842581667
4	2011	3	300	287.9274816	12.07251844
5	2012	4	300	296.2961413	3.703858653
6	2013	5	320	308.1522227	11.84777731
7	2014	6	320	323.2317546	3.231754602
8	2015	7	330	341.312225	11.31222505
9	2016	8	380	362.7162215	17.2837785
10	2017	9	380	388.8074861	8.807486062
11	2018	10	390	422.4773325	32.47733248
12	2019	11	450	468.6194712	18.61947119
13	2020	12	500	534.5914035	34.59140353
14	2021	13	700	630.6606759	69.33932405
					$\cong 231.6911334$

2.9 Quadratic, Polynomial and purely sinusoidal scale

This is the model consisting of mixture of polynomial, quadratic, and purely sinusoidal model equation

$$P_n(t) = \frac{equation(4)+(8)+(12)+(20)+(28)}{5}$$

$$P_n(t) = \frac{1}{5}\begin{pmatrix} \frac{296}{91}*t^2 - \frac{8544}{455}*t + \frac{10622}{35} + \frac{233}{442}*t^3 - \frac{43481}{6188}*t^2 + \frac{202499}{6188}*t + \frac{61465}{238} + 0.01740066429*t^5 \\ -0.4548332581*t^4 + 4.087498495*t^3 - 13.48594466*t^2 + 18.92502769*t + 274.7266349 \\ +10892.27894\sin^2 t - 1097.823546\sin t + 303.7193032 + 7.698927579*10^6\sin^5 t - 3.005842648* \\ 10^6\sin^4 t + 3.586232146*10^5\sin^3 t - 7788.550772\sin^2 t - 74.85079349\sin t + 280.4518019 \end{pmatrix} \quad (30)$$

2.9.1 Validation of Quadratic, Polynomial and purely sinusoidal scale

Sign	Years	t	P(t)	P(t) Model	Absolute Error
1	2008	0	278	284.1279514	6.127951361
2	2009	1	278	284.5095818	6.509581826
3	2010	2	278	284.3155742	6.31557425
4	2011	3	300	286.9813757	13.01862426
5	2012	4	300	293.8521129	6.147887099
6	2013	5	320	304.9022371	15.09776285
7	2014	6	320	319.4537562	0.546243794
8	2015	7	330	336.8899063	6.889906293
9	2016	8	380	357.3611841	22.63881592
10	2017	9	380	382.4807475	2.480747509
11	2018	10	390	416.0063092	26.00630922
12	2019	11	450	464.5057848	14.50578479
13	2020	12	500	538.0041187	38.00411874
14	2021	13	700	650.608893	49.39110699
					$\cong 213.680419$

3. DISCUSSION

From our general analysis, the models based on performance rating follows a sequence of descending order from non-linear to higher non-linear model. It is observe that the model of equation (16), (20), (4), (24), (8), (12) and (28) which produce an average difference error estimation of 592.7 unit, 390.3138 unit, 387.6 unit, 330.8 unit, 271.66 unit, 137.67 unit and 133.27 unit respectively and are all represented in their validation table of data computation. Models with smaller unit of average error difference like model of equation (12) and (28) are non-linear model of higher degree which makes them more vulnerable and perform better than others. Also, comparing the trigonometric models that include sine and cosine function perform weaker than that of purely sinusoidal model as seen from equation (29) and (30) with average error difference of 231.691 unit and 213.680 unit. The model of equation (30) appraise sinusoidal behavior when all models containing cosine function were removed and the preceding model summed up on the average point. This results of our model resonate with the remark of Ogwumu (2020) that non-linear sinusoidal models perform better in terms of finance and pricing computations.

4. CONCLUSION

In all the various aspect of our model validation analysis, the general result agree to the fact that as time goes on, the price of rice in Nigeria keep being on the increase. Thus, if all the various contending factors such as (drought, pest and diseases attribute, poor mechanization, governmental policies, import and export bond, insecurity in mist of farmers, artificial scarcity, flood, etc.) that leads high price of local rice in Nigeria is not well checked and managed, by the year 2025 according to our best model represented by equation (28), the Nigeria local Rice could be sold at the rate of #3,039 (three thousand, thirty-nine naira) per mudu measurement.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] O.D. Ebelechukwu, O.E. Onah, A.A. Oche, et al. Determination of a better mathematical model for food security in Nigeria (A case study of Taraba State). Int. J. Math. Anal. Appl. 4 (2017), 35-41.

[2] H.P. Lu, T. Luo, H.W. Fu, et al. Resistance of rice to insect pests mediated by suppression of serotonin biosynthesis, Nat. Plants 4 (2018), 338 -344.

[3] T.O. Ojo, A.A. Ogundeji, S.C. Babu, T. Alimi, Estimating financing gaps in rice production in Southwestern Nigeria, Economic Structures. 9 (2020), 12.

[4] U.A. Mohammed, S. Ibrahim, M. Hayatu, F.A. Mohammed, Rice (Oryza Sativa L.) Production in Nigeria: Challenges and Prospects, Dutse J. Pure Appl. Sci. 5 (2019), 67-75.

[5] U.A. Mohammed, Z.A. Abubakar, A. U. Jibo, et al. Response of some rice cultivars to abiotic stress, Int. J. Agric. Sci. 5 (2015), 375-380.

[6] R. Munns, Comparative Physiology of Salt and Water Stress. J. Plant Cell Environ. 25 (2002), 239–250.

[7] W. Chesworth, ed., Salt affected soils, in: Encyclopedia of Soil Science, Springer Netherlands, Dordrecht, 2008: pp. 611–611.

[8] F.E. Nwilene, S.O. Oikeh, T.A. Agunbiade, et al. Growing Lowland Rice: a Production handbook. Africa Rice Center, (WARDA). (2008). http://www.fao.org/fileadmin.

[9] IFPRI. International Food Policy Research Institute 2010 Annual Report. (2010). http://ciat.cgiar.org/wp-content/uploads/2012/12/ar10.

[10] W.A. Lamidi, L.O. Akande, A study of status, challenges and prospects of agricultural mechanization in Osun State, Nigeria. J. Educ. Arts Human. 1 (2013), 1-8.

[11] L. Gougong Ezra, H. Yahaya, Analysis of land tenure system among rice farmers in Awe local government area of Nasarawa State, Nigeria. Int. J. Agric. Manage. Develop. 3 (2013), 9-15.

[12] S.O.W. Toluwase, F.O. Osundare, A.O. Adekunmi, Economic Analysis of Rice Marketing in some selected Local Government Areas of Ekiti State, Nigeria. Eur. J. Food Sci. Technol. 7 (2019), 9-23.

[13] Ogwumu, Introduction to Mathematical Modelling (MTH328) Lecture Note (unpublished) of the Department of Mathematics and statistics, Federal, University Wukari, Nigeria. (2020).

[14] U. Obih, L. Baiyegunhi, Consumers' acceptability of local rice brands in Nigeria. which marketing functions really matter? J. Agribus Rural Develop. 2 (2018), 183–196.