Available online at http://scik.org
Eng. Math. Lett. 2014, 2014:13
ISSN: 2049-9337

HYBRID BLOCK PREDICTOR-HYBRID BLOCK CORRECTOR FOR THE SOLUTION OF FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS

ADESANYA A. OLAIDE[1,*], JAMES A. ADEWALE[2] AND JOSHUA SUNDAY[3]

[1]Department of Mathematics, Modibbo Adama University of Technology, Yola, Nigeria

[2]Mathematics Division, American University of Nigeria, Yola, Nigeria

[3]Department of Mathematics, Adamawa State University, Mubi, Nigeria

Abstract. This paper considers the development of one step four, hybrid block method for the solution of first order initial value problems of ordinary differential equations. The method was developed by collocation and interpolation of power series approximate solution to generate a continuous implicit linear multistep method. Both the predictor and corrector are implemented in block method. The basic properties of the derived method are investigated and found to be convergent and the efficiency was tested on some numerical examples and found to give better approximation than the existing methods.

Keywords: hybrid points; collocation; interpolation; approximate solution; convergence.

2010 AMS Subject Classification: 65L05, 65L06, 65D30.

1. Introduction

In this paper, we consider the development of approximate solution to first-order initial value problems of the form,

*Corresponding author

Received August 14, 2013

$$y^{'} = f(x,y), \;\; y(a) = y_0, \; x \in [a,b], \tag{1}$$

where x_0 is the initial point, y_0 is the solution at the initial point and f is assumed to be continuous and satisfy Lipschitz's condition for the existence and uniqueness of solution. Most of the problems in Sciences, Medicine, Agriculture, etc. are modeled in the form (1), the few that are modeled in higher order are first reduced to systems of first order before appropriate method of solution is applied. Most often, this problem do not have a closed solution, hence an approximate solution is adopted to solve such problems.

Scholars have proposed different numerical methods for the solution of (1), these methods can be in form of single step method or multistep method. Multistep method can be in form of k-step method or hybrid method. Hybrid method has been reported to have circumvented Dahlquist barrier condition through the introduction of off step points, though this method is difficult to develop but it gives better approximation than the k-step method especially when the method is of low step length. Hydrid method is equally reported to give better stability condition especially when the problem is stiff or oscillatory (Adesanya *et al.* [1], Anake *et al.* [2], Sofoluwe *et al.* [3], Yakubu *et al.* [4]).

Scholars have also proposed different method of implementation ranging from predictor-corrector method to block method. Despite the success recorded by the predictor-corrector method, its major setback is that the predictors are in reducing order of accuracy especially when the value of the step lenght is high and moreover the results are given at an overlapping interval (Adesanya *et al.* [5], Ngawane and Jator [6], Olabode [7]). Block method which has advantage of being more efficient in terms of cost of implementation, time of execution and accuracy was developed to cater for some of the setbacks of predictor-corrector methods (Jator [8], Adesanya *et al.* [9], Majid *et al.* [10]).

James *et al.* [11], Adesanya *et al.* [12] and Adesanya *et al.* [13] revisited Milne's approach, where block method was first developed to serve as predictor to the predictor-corrector algorithms. These scholars concluded that though block method is cheaper to implement but the Milne's approach gives better approximation. They tagged Milne's approach as constant order predictor-corrector method. It should be noted that the Milne's approach gives results at an overlapping interval.

In this paper, we develop a method which combined the properties of hybrid methiod with the Milne's approach but gives the result at a non overlapping interval. This method was tagged block predictor-block corrector method. This paper is organized as follows; section two discusses the method involved in the development of both the block corrector and block predictor. Section three discusses the convergence of the corrector. Section four considers the numerical experiments and the discussion of result, finally section five gives conclusion and necessary recommendation.

2. Methods and materials

2.1. **Development of the block corrector.** We consider power series approximate solution in the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{2}$$

where r and s are the numbers of interpolation and collocation points respectively, x^j is the polynomial basis function of degree $r+s-1$, $a_j's \in \Re$ are the constants to be determined.

Substituting the firts derivative of (2) into (1), gives a differential system in the form

$$f(x,y) = \sum_{j=1}^{r+s-1} j a_j x^{j-1} \tag{3}$$

Interpolating (2) at $x_{n+r}, r = 0, \frac{1}{5}$, anad collocating (3) at $x_{n+s}, s = 0(\frac{1}{5})1$, gives a system of non-linear equation in the form

$$\mathbf{X}A = U \tag{4}$$

where

$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^T$

$$U = \begin{bmatrix} y_n & y_{n+\frac{1}{2}} & f_n & f_{n+\frac{1}{5}} & f_{n+\frac{2}{5}} & f_{n+\frac{3}{5}} & f_{n+\frac{4}{5}} & f_{n+1} \end{bmatrix}^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 1 & x_{n+\frac{1}{5}} & x_{n+\frac{1}{5}}^2 & x_{n+\frac{1}{5}}^3 & x_{n+\frac{1}{5}}^4 & x_{n+\frac{1}{5}}^5 & x_{n+\frac{1}{5}}^6 & x_{n+\frac{1}{5}}^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+\frac{1}{5}} & 3x_{n+\frac{1}{5}}^2 & 4x_{n+\frac{1}{5}}^3 & 5x_{n+\frac{1}{5}}^4 & 6x_{n+\frac{1}{5}}^5 & 7x_{n+\frac{1}{5}}^6 \\ 0 & 1 & 2x_{n+\frac{2}{5}} & 3x_{n+\frac{2}{5}}^2 & 4x_{n+\frac{2}{5}}^3 & 5x_{n+\frac{2}{5}}^4 & 6x_{n+\frac{2}{5}}^5 & 7x_{n+\frac{2}{5}}^6 \\ 0 & 1 & 2x_{n+\frac{3}{5}} & 3x_{n+\frac{3}{5}}^2 & 4x_{n+\frac{3}{5}}^3 & 5x_{n+\frac{3}{5}}^4 & 6x_{n+\frac{3}{5}}^5 & 7x_{n+\frac{3}{5}}^6 \\ 0 & 1 & 2x_{n+\frac{4}{5}} & 3x_{n+\frac{4}{5}}^2 & 4x_{n+\frac{4}{5}}^3 & 5x_{n+\frac{4}{5}}^4 & 6x_{n+\frac{4}{5}}^5 & 7x_{n+\frac{4}{5}}^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \end{bmatrix}.$$

Solving (4) for the unknown constants using Gaussian elimination method and substituting back into (2) gives a continuous hybrid linear multistep method of the form

$$y(t) = a_0(t)y_0 + a_{\frac{1}{5}}(t)y_{n+\frac{1}{5}} + h\left(\sum_{j=0}^{1} \beta_j(t)f_{n+j} + \beta_v(t)f_{n+v}\right),\ v = \frac{1}{5}(\frac{1}{5})\frac{4}{5}, \tag{5}$$

where

$$\alpha_0 = \frac{1}{863}(937500t^7 - 3281250t^6 + 4462500t^5 - 2953125t^4 + 959000t^3 - 126000t^2 + 863)$$

$$\alpha_{\frac{1}{5}} = -\frac{1}{863}(937500t^7 - 328125t^6 + 4462500t^5 - 2953125t^4 + 959000t^3 - 126000t^2)$$

$$\beta_0 = -\frac{1}{62136}(4453125t^7 - 15855625t^6 + 22167750t^5 - 15402750t^4 + 5526125t^3 - 951393t^2 + 62136t)$$

$$\beta_{\frac{1}{5}} = \frac{1}{62136}(13378125t^7 - 45475000^6 + 59149125t^5 - 36396750t^4 + 10362380t^3 - 1021320t^2)$$

$$\beta_{\frac{2}{5}} = \frac{1}{31068}(3740625t^7 - 11743750t^6 + 13598250t^5 - 7009500t^4 + 1517885t^3 - 114390t^2)$$

$$\beta_{\frac{3}{5}} = \frac{1}{31068}(2259375t^7 - 6559375t^6 + 6871125t^5 - 3152625t^4 + 628340t^3 - 44760t^2)$$

$$\beta_{\frac{4}{5}} = \frac{1}{62136}(1621875t^7 - 4328125t^6 + 4160250t^5 - 1791750t^4 + 342995t^3 - 23805t^2)$$

$$\beta_1 = \frac{1}{62136}\left(253125t^7 - 616250t^6 + 557625t^5 - 231000t^4 + 43180t^3 - 2952t^2\right)$$

$t = \frac{x-x_n}{h}$

Evaluating (5) at $t = \frac{2}{5}(\frac{3}{5})1$ its frst derivative at $t = 0$ and writing in block form gives the general block formula of the form

$$A^{(0)}Y_m = A^{(1)}Y_{m-1} + A^{(k)}Y_{m-2} + h[B^{(1)}F_{m-1} + B^{(k)}F_{m-2} + B^{(0)}F_m], \tag{6}$$

where $A^{(0)} = 4 \times 4$ identical matrix,

$$Y_{m-1} = [y_{n-\frac{2}{5}} \quad y_{n-\frac{3}{5}} \quad y_{n-\frac{4}{5}} \quad y_n\,]^T$$

$$Y_{m-2} = [y_{n-\frac{2}{5}} \quad y_{n-\frac{3}{5}} \quad y_{n-\frac{4}{5}} \quad y_{n+\frac{1}{5}}\,]^T$$

$$F_{m-1} = [f_{n-\frac{2}{5}} \quad f_{n-\frac{3}{5}} \quad f_{n-\frac{4}{5}} \quad f_n\,]^T$$

$$F_m = [f_{n+\frac{2}{5}} \quad f_{n+\frac{3}{5}} \quad f_{n+\frac{4}{5}} \quad f_{n+1}]^T$$

$$A^{(i)} = \begin{bmatrix} 0 & 0 & 0 & \frac{271}{863} \\ 0 & 0 & 0 & \frac{80}{863} \\ 0 & 0 & 0 & \frac{351}{863} \\ 0 & 0 & 0 & -\frac{512}{863} \end{bmatrix} \qquad A^{(k)} = \begin{bmatrix} 0 & 0 & 0 & \frac{592}{863} \\ 0 & 0 & 0 & \frac{783}{863} \\ 0 & 0 & 0 & \frac{512}{863} \\ 0 & 0 & 0 & \frac{1375}{863} \end{bmatrix}$$

$$B^{(i)} = \begin{bmatrix} 0 & 0 & 0 & \frac{6589}{388350} \\ 0 & 0 & 0 & \frac{84}{21575} \\ 0 & 0 & 0 & \frac{489}{21575} \\ 0 & 0 & 0 & \frac{-304}{7767} \end{bmatrix} \qquad B^{(k)} = \begin{bmatrix} 0 & 0 & 0 & \frac{29264}{194175} \\ 0 & 0 & 0 & \frac{4053}{43150} \\ 0 & 0 & 0 & \frac{144}{863} \\ 0 & 0 & 0 & \frac{-430}{7767} \end{bmatrix}$$

$$B^{(0)} = \begin{bmatrix} \frac{20804}{194175} & \frac{-2876}{194175} & \frac{1223}{388350} & \frac{-68}{194175} \\ \frac{5244}{21575} & \frac{1764}{21574} & \frac{-96}{21575} & \frac{3}{8630} \\ \frac{744}{4315} & \frac{1056}{4315} & \frac{66}{863} & \frac{-48}{21575} \\ \frac{2720}{7767} & \frac{520}{7767} & \frac{2320}{7767} & \frac{466}{7767} \end{bmatrix}$$

2.2. **Derivation of the block predictor.** Collocating (3) at $x_{n+s},\ s=0(\frac{1}{5})1$ and interpolating (2) at x_n gives a system of nonlinear equations in the form (4)

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^T$$

$$U = \begin{bmatrix} y_n & f_n & f_{n+\frac{1}{5}} & f_{n+\frac{2}{5}} & f_{n+\frac{3}{5}} & f_{n+\frac{4}{5}} & f_{n+1} \end{bmatrix}^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 \\ 1 & x_{n+\frac{1}{5}} & x_{n+\frac{1}{5}}^2 & x_{n+\frac{1}{5}}^3 & x_{n+\frac{1}{5}}^4 & x_{n+\frac{1}{5}}^5 & x_{n+\frac{1}{5}}^6 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 \\ 0 & 1 & 2x_{n+\frac{1}{5}} & 3x_{n+\frac{1}{5}}^2 & 4x_{n+\frac{1}{5}}^3 & 5x_{n+\frac{1}{5}}^4 & 6x_{n+\frac{1}{5}}^5 \\ 0 & 1 & 2x_{n+\frac{2}{5}} & 3x_{n+\frac{2}{5}}^2 & 4x_{n+\frac{2}{5}}^3 & 5x_{n+\frac{2}{5}}^4 & 6x_{n+\frac{2}{5}}^5 \\ 0 & 1 & 2x_{n+\frac{3}{5}} & 3x_{n+\frac{3}{5}}^2 & 4x_{n+\frac{3}{5}}^3 & 5x_{n+\frac{3}{5}}^4 & 6x_{n+\frac{3}{5}}^5 \\ 0 & 1 & 2x_{n+\frac{4}{5}} & 3x_{n+\frac{4}{5}}^2 & 4x_{n+\frac{4}{5}}^3 & 5x_{n+\frac{4}{5}}^4 & 6x_{n+\frac{4}{5}}^5 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 \end{bmatrix}.$$

Solving for the unknown constants and substituting back into (2) gives a continuous hybrid linear multistep method of the form

$$y(t) = a_0(t)y_n + h\left(\sum_{j=0}^{1}\beta_j(t)f_{n+j} + \beta_v(t)f_{n+v}\right),\ v = \frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5} \tag{7}$$

where $\alpha_0 = 1$

$$\beta_0 = \frac{1}{288}\left(1250t^6 - 4500t^5 + 6375t^4 - 4500t^3 + 1644t^2 - 288t\right)$$
$$\beta_{\frac{1}{5}} = \frac{1}{288}\left(6250t^6 - 21000t^5 + 26625t^4 - 15400t^3 + 3600t^2\right)$$
$$\beta_{\frac{2}{5}} = \frac{1}{144}\left(6250t^6 - 19500t^5 + 22125t^4 - 10700t^3 + 1800t^2\right)$$

$$\beta_{\frac{3}{5}} = \frac{1}{144}\left(6250t^6 - 18000t^5 + 18375t^4 - 7800t^3 + 1200t^2\right)$$

$$\beta_{\frac{4}{5}} = -\frac{1}{288}\left(6250t^6 - 16500t^5 + 15375t^4 - 6100t^3 + 900t^2\right)$$

$$\beta_1 = \frac{1}{288}(1250t^6 - 3000t^5 + 2625t^4 - 1000t^3 + 144t^2)$$

Evaluating (7) a $t = \frac{1}{5}(\frac{1}{5})1$ and writing in block gives the general block formula of the form,

$$A^{(0)}Y_m = A^{(i)}Y_{m-1} + h^2[B^{(i)}F_{m-1} + B^{(0)}F_m], \tag{8}$$

where $A^{(0)} = 5\times 5$ identical matrix

$$Y_m = [y_{n+\frac{1}{5}} \quad y_{n+\frac{2}{5}} \quad y_{n+\frac{3}{5}} \quad y_{n+\frac{4}{5}} \quad y_{n+1}\ ]^T$$

$$Y_{m-1} = [y_{n-\frac{1}{5}} \quad y_{n-\frac{2}{5}} \quad y_{n-\frac{3}{5}} \quad y_{n-\frac{4}{5}} \quad y_n]^T$$

$$F_{m-1} = [f_{n-\frac{1}{5}} \quad f_{n-\frac{2}{5}} \quad f_{n-\frac{3}{5}} \quad f_{n-\frac{4}{5}} \quad f_n\ ]^T$$

$$F_m = [f_{n+\frac{1}{5}} \quad f_{n+\frac{2}{5}} \quad f_{n+\frac{3}{5}} \quad f_{n+\frac{4}{5}} \quad f_{n+1}]^T$$

$$A^{(i)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad B^{(i)} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{19}{288} \\ 0 & 0 & 0 & 0 & \frac{14}{225} \\ 0 & 0 & 0 & 0 & \frac{51}{800} \\ 0 & 0 & 0 & 0 & \frac{14}{225} \\ 0 & 0 & 0 & 0 & \frac{19}{288} \end{bmatrix}$$

$$B^{(0)} = \begin{bmatrix} \frac{1427}{7200} & \frac{-133}{1200} & \frac{241}{3600} & \frac{-173}{7200} & \frac{3}{800} \\ \frac{43}{150} & \frac{7}{225} & \frac{7}{225} & \frac{-1}{75} & \frac{1}{450} \\ \frac{219}{800} & \frac{57}{400} & \frac{57}{400} & \frac{-21}{800} & \frac{3}{800} \\ \frac{64}{225} & \frac{8}{75} & \frac{64}{225} & \frac{14}{225} & 0 \\ \frac{25}{96} & \frac{25}{144} & \frac{25}{144} & \frac{25}{96} & \frac{19}{288} \end{bmatrix}$$

3. ANALYSIS OF THE BASIC PROPERTIES OF THE CORRECTOR

3.1. **Consistency.** The block method is said to be consistent if it has order $p \geq 1$

3.2. **Order.** Let the linear operator $L\{y(x);h\}$ associated with the block method (6) be defined as,

$$L\{y(x);h\} = A^{(0)}Y_m - A^{(1)}Y_{m-1} + A^{(k)}Y_{m-2} - h[B^{(1)}F_{m-1} + B^{(k)}F_{m-2} + B^{(0)}F] \tag{9}$$

Expanding (9) in Taylor expansion gives,

$$L\{y(x);h\} = C_0 y(x) + C_1 h y'(x) + C_2 h^2 y''(x) + ... + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + ...$$

Definition 1. *The linear operator L and associated block method are said to be of order p if* $C_0 = C_1 = ... = C_p = 0$, $C_{p+1} \neq 0$. *Then* C_{p+1} *is called the error constant and implies that the truncation error is given by*

$$t_{n+k} = C_{p+1} h^{p+1} y^{p+1}(x) + O\left(h^{p+2}\right)$$

The order of our method is seven with the error constant of

$$\left[\frac{4357}{2548546875000}, \frac{-13}{18878125000}, \frac{52}{117988281256}, \frac{-44}{2548546875}\right]^T$$

3.3. **Zero stability.** A block method is said to be zero stable if as $h \to 0$, the root $r_j, j = 1(1)k$ of the first characteristics polynomial $\rho(r) = 0$, that is $\rho(r) = \det\left[\sum A^0 R^{k-1}\right]$ satisfying $\mid R \mid = 1$, must be simple.

For our method,

$$\rho(r) = \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & \frac{271}{863} \\ 0 & 0 & 0 & \frac{80}{863} \\ 0 & 0 & 0 & \frac{351}{863} \\ 0 & 0 & 0 & \frac{-512}{863} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & \frac{592}{863} \\ 0 & 0 & 0 & \frac{783}{863} \\ 0 & 0 & 0 & \frac{512}{863} \\ 0 & 0 & 0 & \frac{1375}{863} \end{bmatrix}\right] = 0$$

Therefore, $R = 0,0,0,1$.Hence our method is zero stable.

3.4. **Convergence.** A block method is said to be convergent if it is consistent and zero-stable, hence it has been shown clearly that our method is convergent.

4. Numerical experiments

Problem I:

We consider a decaying problem given by,

$$y' = -y,\ y(0) = 1, 0 \leq x \leq 1, h = 0.1$$

Exact solution:

$$y(x) = e^{-x}$$

Source: James *et al.* [11]

Problem II:

We consider a nonlinear initial value problem

$$y' = x - y,\ y(0) = 0,\ 0 \leq x \leq 1,\ \ h = 0.1$$

Exact solution:

$$y(x) = x + e^{-x} - 1$$

Source: James *et al.* [11]

Problem III:

We consider the equation

$$y' = xy,\ y(0) = 1,\ \ h = 0.1$$

Exact solution:

$$y(x) = e^{\frac{1}{2}x^2}$$

Source: James *et al.* [11]

Note:

EBLM→Error in block method

EABM→Error in Areo *et al.* [14]

EJCPCM→Error in James *et al.* [11]

EBPBCM→Error in our new method

Table I: Showing results of problem I

x	EBLM	EABM	EJCPCM	EBPBCM
0.1	$1.9596(-11)$	$2.1(-10)$	$1.7444(-11)$	$2.520206(-14)$
0.2	$3.5462(-11)$	$2.2(-10)$	$1.5783(-11)$	$4.563017(-14)$
0.3	$4.8131(-11)$	$6.0(-10)$	$1.4281(-11)$	$6.183942(-14)$
0.4	$5.8068(-11)$	$1.0(-10)$	$1.2925(-11)$	$7.482903(-14)$
0.5	$6.5677(-11)$	$4.1(-10)$	$1.1676(-11)$	$8.455989(-14)$
0.6	$7.1313(-11)$	$7.0(-10)$	$1.0580(-11)$	$9.192647(-14)$
0.7	$7.5281(-11)$	$1.5(-10)$	$9.5701(-11)$	$9.697798(-14)$
0.8	$7.7848(-11)$	$7.0(-10)$	$8.6612(-11)$	$1.003087(-13)$
0.9	$7.9245(-11)$	$1.4(-10)$	$7.8371(-11)$	$1.021405(-13)$
1.0	$7.9671(-11)$	$8.0(-10)$	$7.0927(-11)$	$1.026956(-13)$

Table II: Showing results of problem II

x	EBLM	EABM	EJCPCM	EBPBCM
0.1	$1.9596(-11)$	$0.00+00$	$1.7443(-11)$	$2.528620(-14)$
0.2	$3.5462(-11)$	$0.00+00$	$1.5786(-11)$	$4.572731(-14)$
0.3	$4.8131(-11)$	$6.2(-10)$	$1.4283(-11)$	$6.196432(-14)$
0.4	$5.8068(-11)$	$2.0(-10)$	$1.2924(-11)$	$7.498169(-14)$
0.5	$6.5677(-11)$	$7.0(-10)$	$1.1694(-11)$	$8.461287(-14)$
0.6	$7.1313(-11)$	$1.0(-10)$	$1.0581(-11)$	$9.203749(-14)$
0.7	$7.5281(-11)$	$8.0(-10)$	$9.5739(-11)$	$9.706125(-14)$
0.8	$7.7848(-11)$	$2.0(-10)$	$8.6613(-11)$	$1.003087(-13)$
0.9	$7.9245(-11)$	$9.0(-10)$	$7.8396(-11)$	$1.021405(-13)$
1.0	$7.9671(-11)$	$4.0(-10)$	$7.0906(-11)$	$1.026956(-13)$

Table III: Showing results of problem III

x	EBLM	EABM	EJCPCM	EBPBCM
0.1	2.606759(−11)	2.6067(−11)	1.6554(−11)	1.532108(−13)
0.2	9.452580(−11)	8.4790(−11)	4.3981(−11)	6.263878(−13)
0.3	1.857241(−10)	1.8684(−10)	7.8451(−11)	1.470157(−12)
0.4	3.492937(−10)	3.5701(−10)	1.2662(−10)	2.778000(−12)
0.5	6.074909(−10)	6.1004(−10)	1.9709(−10)	4.697354(−12)
0.6	1.010818(−09)	1.6157(−09)	3.0180(−11)	7.480429(−12)
0.7	1.6369289(−09)	1.6445(−09)	4.5771(−10)	1.135025(−11)
0.8	2.604468(−09)	2.6158(−09)	6.8954(−10)	1.686473(−11)
0.9	4.094442(−09)	4.1110(−09)	1.0336(−09)	2.465228(−11)
1.0	6.383180(−09)	1.5435(−10)	1.5435(−09)	3.566325(−11)

4.1. **Discussion of Result.** We have considered three numerical examples in this paper. These problems were solved by James *et al.* [11] where they proposed order seven method implemented in constant order predictor-corrector method. This problems are equally solved by Areo *et al.* [14] where they proposed a method of order seven implemented in the block method and adopted classical Runge-Kutta method to provide the starting values. We also solved these problems using the block predictors. Tables I-III showed that the new method gives better results than the exixting methods.

5. Conclusion

In this paper, we have proposed a one-step block corrector method in this paper. We have established the claim of Olabode [7] that methods implemented at a non-overlapping interval gives better approximation than the overlapping interval. This new method enables us exhaust all the interpolation points without increasing the step lenght and enables us to understand the nature of the dynaical system at the selected points. It should be noted that this method is more costly to develop but gives better accuracy than the existing methods.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] A. O. Adesanya, M. R. Odekunle, A. A. James, Starting hybrid Stormer-Cowel more accurately by hybrid Adams method for the solution of first order ordinary differential equations, European J. Sci. Res. 77 (2012), 580-588.

[2] T. A. Anake, D. O. Awoyemi, A. O. Adesanya, One-step implicit hybrid block method for the direct solution of second order ordinary differential equations, IAENG, Int. J. Appl. Math. 42 2012, IJAM_42_4_04.

[3] A. B. Sofoluwe, S. A. Okunuga, J. O. Ehizie, Composite multi derivative multistep backward differentiation formula for solving stiff initial value problems, J. Math. Sci. 2 (2013), 227-239.

[4] D. G. Yakubu, A. G. Madaki, A. M. Kwami, Stable two step Runge-Kutta collocation methods for oscillatory system of initial value problems of ODEs, Amer. J. Comput. Appl. Math. 3 (2013), 119-130.

[5] A. O. Adesanya, M. R. Odekunle, M. O. Udoh, Four steps continuous method for the solution of $y''' = f(x,y,y',y'')$, Amer. J. Comput. Appl. Math. 3 (2013), 169-174.

[6] F. F. Ngwane, S. N. Jator, Block hybrid-second derivative method for stiff systems, Int. J. Pure Appl. Math. 80 (2012), 543-559.

[7] B. T. Olabode, Y. Yusuf, A new block method for special third order ordinary differential equations, J. Math. Stat. 534 (2009), 167-170.

[8] S. N. Jator, On a class of hybrid method for $y'' = f(x,y,y')$, Int. J. Pure Appl. Math. 59 (2010), 381-395.

[9] A. O. Adesanya, M. O. Udoh, A. M. Ajileye, A new hybrid method for the solution of general third order initial value problems of ordinary differential equations, Int. J. Pure Appl. Math. 86 (2010), 381-395

[10] Z. A. Majid, M. B. Suleiman, Z. Omar, 3 point implicit block method for solving ordinary differential equations, Bull. Malays. Math. Sci. 29 (2006), 23-31.

[11] A. A. James, A. O. Adesanya, M. R. Fasansi, Starting order seven method accurately for the solution of first order initial value problems of ordinary differential equations, Progress Appl. Math. 6 (2013), 30-39.

[12] A. O. Adesanya, M. R. Odekunle, O. Adeyeye, Continuous block hybrid predictor corrector method for the solution of $y'' = f(x,y,y')$, Int. J. Math. Soft Comput. 2 (2012), 35-42.

[13] A. O. Adesanya, M. R. Odekunle, M. A. Alkali, A. B. Abubakar, Starting five steps Stormer Cowell method by Adams-Bashforth method for the solution of first order ordinary differential equations, African J. Math. Comput. Res. 6 (2013), 89-93.

[14] E. A. Areo, R. A. Ademiluyi, P. O. Babatola, Three step hybrid linear multistep method for the solution of first order initial value problems in ordinary differential equations, J. N. A. M. P, 19, 261, 266.