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DIFFERENCE CORDIAL LABELING OF CORONA GRAPHS

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Abstract. Let G be a (p,q) graph. Let f be a map from $V(G)$ to $\{1,2,\ldots,p\}$. For each edge xy, assign the label $|f(x)-f(y)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0)-e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordial labeling behavior of $G \odot P_n$, $G \odot mK_1$ $(m = 1,2,3)$ where G is either a unicycle or a tree and $G_1 \odot G_2$ where G_1 and G_2 are some more standard graphs.

Keywords: Corona, comb, tree, cycle, wheel.

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1. Introduction

Throughout this paper we have considered only simple and undirected graphs. Let $G = (V,E)$ be a (p,q) graph. The number $|V|$ is called the order of $G$and the number $|E|$ is called the size of G. The concept of difference cordial labeling has been introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [3]. In [3, 4], difference cordial labeling behaviour of several graphs such as path, cycle, complete graph, complete bipartite grpah, bistar, wheel, web and some more standard graphs have been investigated. In this paper we investigate the difference cordial labeling behaviour

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of $G \odot P_n$, $G \odot mK_1$ $(m = 1, 2, 3)$ where G is either a unicycle or a tree, crown $C_n \odot K_1$, comb $P_n \odot K_1$, $P_n \odot C_m$, $C_n \odot C_m$, $W_n \odot K_2$, $W_n \odot 2K_1$, $L_n \odot K_1$, $L_n \odot 2K_1$ and $L_n \odot K_2$. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms and definitions not defined here are follow from Harary [2].

2. Difference Cordial Graph

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, p\}$ be a bijection. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

3. Main results

Now we look into the corona of G with H. The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. $C_n \odot K_1$ is called the crown and $P_n \odot K_1$ is called the comb. Now we have the following.

Theorem 3.1. *Let G be a (p, q) graph. If G satisfies any one of the following then $G \odot P_n$ is difference cordial.*

(1) *G is a tree.*

(2) *G is a unicycle.*

(3) *q=p+1.*

Proof. Let $V(G) = \{u_i : 1 \leq i \leq p\}$ and $P_n^i : v_1^i v_2^i \dots v_n^i$ be the i^{th} copy of the path. $V(G \odot P_n) = V(G) \cup V(P_n^i)$ and $E(G \odot P_n) = E(G) \cup \bigcup_{i=1}^{p} \left\{u_i v_j^i : 1 \leq j \leq n\right\} \cup E(P_n^i)$. Clearly the order and size of $G \odot P_n$ are $(n+1)p$ and $2np - p + q$ respectively. We now define an injective map from the vertex set of $G \odot P_n$ to the set $\{1, 2 \dots (n+1)p\}$ as follows:

$$\begin{aligned} f(u_i) &= (n+1)i & & 1 \le i \le p \\ f(v_j^1) &= j & & 1 \le j \le n \\ f(v_j^i) &= f(u_{i-1}) + j & & 2 \le i \le p,\ 1 \le j \le n. \end{aligned}$$

Case 1: G is a tree.

In this case, $e_f(0) = np - 1$ and $e_f(1) = np$. Therefore, f is a difference cordial labeling.

Case 2: G is a unicycle.

Since $e_f(0) = np$ and $e_f(1) = np$, f is a difference cordial labeling.

Case 3: $q = p + 1$.

In this case, $e_f(0) = np + 1$ and $e_f(1) = np$. Hence, f is a difference cordial labeling.

Theorem 3.2. *Let G be a (p, q) graph. If G satisfies any one of the following then $G \odot mK_1$ $(m = 1, 2, 3)$ is difference cordial.*

(1) *G is a tree.*

(2) *G is a unicycle.*

(3) *q=p+1.*

Proof. Let $V(G) = \{u_i : 1 \le i \le p\}$.

Case 1: $m = 1$.

The proof follows from theorem 3.1.

Case 2: $m = 2$.

Let $V(G \odot 2K_1) = V(G) \cup \{v_i, w_i : 1 \le i \le p\}$ and $E(G \odot 2K_1) = E(G) \cup \{u_iv_i, u_iw_i : 1 \le i \le p\}$. Note that $G \odot 2K_1$ has $3p$ vertices and $2p + q$ edges.

Subcase 1: G is a tree.

Define a one to one map from the vertex set of $G \odot 2K_1$ to the set $\{1, 2, \ldots 3p\}$ as follows:

$$\begin{aligned} f(u_i) &= 3i - 2 & & 1 \le i \le \left\lceil \frac{p}{2} \right\rceil \\ f(v_i) &= 3i - 1 & & 1 \le i \le \left\lceil \frac{p}{2} \right\rceil \\ f(w_i) &= 3i & & 1 \le i \le \left\lceil \frac{p}{2} \right\rceil. \end{aligned}$$

$$\begin{aligned} f\left(u_{\lceil \frac{p}{2} \rceil + i}\right) &= 3\left\lceil \frac{p}{2} \right\rceil + 3i - 1 \quad 1 \le i \le \left\lfloor \frac{p}{2} \right\rfloor \\ f\left(v_{\lceil \frac{p}{2} \rceil + i}\right) &= 3\left\lceil \frac{p}{2} \right\rceil + 3i - 2 \quad 1 \le i \le \left\lfloor \frac{p}{2} \right\rfloor \\ f\left(w_{\lceil \frac{p}{2} \rceil + i}\right) &= 3\left\lceil \frac{p}{2} \right\rceil + 3i \qquad\quad 1 \le i \le \left\lfloor \frac{p}{2} \right\rfloor. \end{aligned}$$

Here $e_f(0) = p - 1 + \lceil \frac{p}{2} \rceil$ and $e_f(1) = p + \lfloor \frac{p}{2} \rfloor$. Hence, f is a difference cordial labeling.

Subcase 2: G is a unicycle.

Label the vertices of $G \odot 2K_1$ as in case 1. In this case, $e_f(0) = p + \lceil \frac{p}{2} \rceil$. $e_f(1) = p + \lfloor \frac{p}{2} \rfloor$. It follows that f is a difference cordial labeling.

Subcase 3: $q = p + 1$.

Label the vertices u_i, v_i and w_i $\left(1 \le i \le \lceil \frac{p}{2} \rceil - 1,\ \lceil \frac{p}{2} \rceil + 1 \le i \le p\right)$ as in case 1. Now assign the labels $3\lceil \frac{p}{2} \rceil - 1$, $3\lceil \frac{p}{2} \rceil - 2$ and $3\lceil \frac{p}{2} \rceil$ to the vertices $u_{\lceil \frac{p}{2} \rceil}$, $v_{\lceil \frac{p}{2} \rceil}$ and $w_{\lceil \frac{p}{2} \rceil}$ respectively. Here $e_f(0) = p + \lceil \frac{p}{2} \rceil$ and $e_f(1) = p + \lfloor \frac{p}{2} \rfloor + 1$.

Case 3: $m = 3$.

Let $V(G \odot 3K_1) = V(G) \cup \{v_i, w_i, z_i : 1 \le i \le p\}$ and $E(G \odot 3K_1) = E(G) \cup \{u_iv_i, u_iw_i, u_iz_i : 1 \le i \le p\}$. The order and size of $G \odot 3K_1$ are $4p$ and $4p + q$ respectively. Define a map $f : V(G \odot 3K_1) \to \{1, 2, \ldots 4p\}$ by

$$\begin{aligned} f(u_i) &= 4i - 2 \quad 1 \le i \le p \\ f(v_i) &= 4i - 3 \quad 1 \le i \le p \\ f(w_i) &= 4i - 1 \quad 1 \le i \le p \\ f(z_i) &= 4i \qquad\quad 1 \le i \le p. \end{aligned}$$

Subcase 1: G is a tree.

Now $e_f(0) = 2p - 1$ and $e_f(1) = 2p$. Therefore, f satisfies the edge condition of difference cordial labeling.

Subcase 2: G is a unicycle.

In this case, $e_f(0) = 2p$ and $e_f(1) = 2p$. Hence f is a difference cordial labeling.

Subcase 3: $q = p + 1$.

Here $e_f(0) = 2p + 1$ and $e_f(1) = 2p$. This implies, f is a difference cordial labeling.

Corollary 3.3. *The crown $C_n \odot K_1$ is difference cordial.*

Corollary 3.4. *The comb $P_n \odot K_1$ is difference cordial.*

Next we look into the graph $P_n \odot C_m$.

Theorem 3.5. *$P_n \odot C_m$ is difference cordial.*

Proof. Let P_n be the path $u_1u_2 \dots u_n$ and let $C_m^i : v_1^i v_2^i \dots v_n^i v_1^i$ be the i^{th} copy of the cycle C_m. Therefore $V(P_n \odot C_m) = V(P_n) \cup \bigcup_{i=1}^{n} V(C_m^i)$ and $E(P_n \odot C_m) = E(P_n) \cup \bigcup_{i=1}^{n} E(C_m^i) \cup \bigcup_{i=1}^{n} \left\{u_i v_j^i : 1 \le j \le m\right\}$. Clearly, $P_n \odot C_m$ has $n(m+1)$ vertices and $2mn+n-1$ edges. Define $f : V(P_n \odot C_m) \to \{1, 2, 3 \dots n(m+1)\}$ by $f(u_1) = m+1$, $f(u_2) = m+2$, $f\left(v_j^1\right) = j,\ 1 \le j \le m$, $f\left(v_j^2\right) = m+2+j,\ 1 \le j \le m$,

$$f(u_{2i+1}) = f(u_{2i-1}) + 2m + 2 \quad \begin{cases} 1 \le i \le \frac{n-1}{2} & \text{if } n \equiv 1 \pmod 2 \\ 1 \le i \le \frac{n-2}{2} & \text{if } n \equiv 0 \pmod 2. \end{cases}$$

$$f(u_{2i+2}) = f(u_{2i}) + 2m + 2 \quad \begin{cases} 1 \le i \le \frac{n-3}{2} & \text{if } n \equiv 1 \pmod 2 \\ 1 \le i \le \frac{n-2}{2} & \text{if } n \equiv 0 \pmod 2. \end{cases}$$

$$f\left(v_j^{2i+1}\right) = f\left(v_j^{2i-1}\right) + 2m + 2 \quad \begin{cases} 1 \le i \le \frac{n-1}{2} & \text{if } n \equiv 1 \pmod 2 \\ 1 \le i \le \frac{n-2}{2} & \text{if } n \equiv 0 \pmod 2. \end{cases}$$

$$f\left(v_j^{2i+2}\right) = f\left(v_j^{2i}\right) + 2m + 2 \quad \begin{cases} 1 \le i \le \frac{n-1}{2} & \text{if } n \equiv 1 \pmod 2 \\ 1 \le i \le \frac{n-2}{2} & \text{if } n \equiv 0 \pmod 2. \end{cases}$$

The table 1 shows that f is a difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{2mn+n-2}{2}$	$\frac{2mn+n}{2}$
$n \equiv 1 \pmod 2$	$\frac{2mn+n-1}{2}$	$\frac{2mn+n-1}{2}$

TABLE 1

Example 3.6. *The difference cordial labeling of $P_4 \odot C_3$ is given in figure 1.*

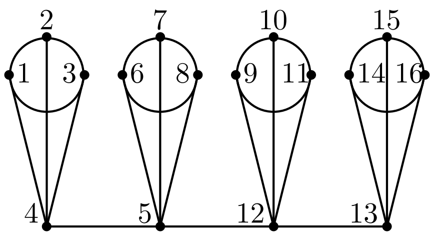

Figure 1

Theorem 3.7. *$C_n \odot C_m$ is difference cordial.*

Proof. The graph $C_n \odot C_m$ is obtained from $P_n \odot C_m$ by adding the edge u_1u_n. Assign the labels to the vertices of $C_n \odot C_m$ as in theorem 3.5. In this graph, when n is odd, $e_f(0) = \frac{2mn+n+1}{2}$ and $e_f(1) = \frac{2mn+n-1}{2}$; When n is even, $e_f(0) = e_f(1) = \frac{2mn+n}{2}$. Therefore f is a difference cordial labeling.

Now we investigate the difference cordiality of corona of W_n with K_2 and $2K_1$.

Theorem 3.8. *$W_n \odot K_2$ is difference cordial.*

Proof. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \ldots u_nu_1$ and $V(K_1) = \{u\}$. Let $V(W_n \odot K_2) = V(W_n) \cup \{v_i, w_i : 1 \le i \le n+1\}$ and $E(W_n \odot K_2) = E(W_n) \cup \{u_iv_i, u_iw_i, uv_{n+1}, uw_{n+1}, v_iw_i, v_{n+1}w_{n+1} : 1 \le i \le n\}$. Note that $W_n \odot K_2$ has $3n+3$ vertices and $5n+3$ edges. Define a one-one function f from $V(W_n \odot K_2)$ to the set $\{1, 2 \ldots 3n+3\}$ as follows:

Case 1: n is even.

$$\begin{aligned}
f(u_{2i-1}) &= 6i-3 \quad 1 \le i \le \frac{n}{2} \\
f(u_{2i}) &= 6i-2 \quad 1 \le i \le \frac{n}{2} \\
f(v_{2i-1}) &= 6i-5 \quad 1 \le i \le \frac{n}{2} \\
f(v_{2i}) &= 6i-1 \quad 1 \le i \le \frac{n}{2} \\
f(w_{2i-1}) &= 6i-4 \quad 1 \le i \le \frac{n}{2} \\
f(w_{2i}) &= 6i \qquad 1 \le i \le \frac{n}{2}
\end{aligned}$$

$f(u) = 3n+1$, $f(v_{n+1}) = 3n+2$ and $f(w_{n+1}) = 3n+3$.

Case 2: n is odd.

Assign the labels to the vertices u_i, v_i and w_i $(1 \le i \le n-1)$, u, v_{n+1} and w_{n+1} as in case 1. Then, label the vertices u_n, v_n and w_n by $3n-2$, $3n-1$ and $3n$ respectively. The following table 2 proves that f is a difference cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{5n+2}{2}$	$\frac{5n+4}{2}$
$n \equiv 1 \pmod 2$	$\frac{5n+3}{2}$	$\frac{5n+3}{2}$

TABLE 2

Theorem 3.9. *$W_n \odot 2K_1$ is difference cordial.*

Proof. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2\ldots u_nu_1$ and $V(K_1) = \{u\}$. Let $V(W_n \odot 2K_1) = V(W_n) \cup \{v_i, w_i : 1 \le i \le n+1\}$ and $E(W_n \odot 2K_1) = E(W_n) \cup \{u_iv_i, u_iw_i : 1 \le i \le n\} \cup \{uv_{n+1}, uw_{n+1}\}$. Define $f : V(W_n \odot 2K_1) \to \{1, 2\ldots 3n+3\}$ by

$$\begin{aligned} f(u_i) &= 3i-1 \quad && 1 \le i \le n \\ f(v_i) &= 3i-2 && 1 \le i \le n \\ f(w_i) &= 3i && 1 \le i \le n \end{aligned}$$

$f(u) = 3n+1$, $f(v_{n+1}) = 3n+2$ and $f(w_{n+1}) = 3n+3$. Since $e_f(0) = e_f(1) = 2n+1$, f is a difference cordial labeling of $W_n \odot 2K_1$.

The gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the cycle C_n. Let $V(G_n) = V(W_n) \cup \{v_i : 1 \le i \le n\}$ and $E(G_n) = E(W_n) \cup \{u_iv_i, v_ju_{j+1} : 1 \le i \le n,\ 1 \le j \le n\} - E(C_n)$.

Theorem 3.10. *$G_n \odot K_1$ is difference cordial.*

Proof. Let $V(G_n \odot K_1) = V(G_n) \cup \{w_i, x_i : 1 \le i \le n\} \cup \{w\}$ and $E(G_n \odot K_1) = E(G_n) \cup \{u_iw_i, v_ix_i : 1 \le i \le n\} \cup \{uw\}$. The order and size of $G_n \odot K_1$ are $4n+2$ and $5n+1$ respectively. Define a one-one map $f : V(G_n \odot K_1) \to \{1, 2, \ldots 4n+2\}$ as follows:

$$\begin{aligned} f(v_i) &= 4i-1 && 1 \le i \le n \\ f(x_i) &= 4i && 1 \le i \le n \\ f(u_i) &= 4i-2 && 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f(w_i) &= 4i-3 && 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(u_{\lfloor \frac{n}{2} \rfloor + i}\right) &= 4\left\lfloor \frac{n}{2} \right\rfloor + 4i - 3 && 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f\left(w_{\lfloor \frac{n}{2} \rfloor + i}\right) &= 4\left\lfloor \frac{n}{2} \right\rfloor + 4i - 2 && 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \end{aligned}$$

$f(u) = 4n+1$ and $f(w) = 4n+2$. The following table 3 proves that f is a difference cordial labeling of $G_n \odot K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{5n}{2}$	$\frac{5n+2}{2}$
$n \equiv 1 \pmod 2$	$\frac{5n+1}{2}$	$\frac{5n+1}{2}$

TABLE 3

Theorem 3.11. *$G_n \odot 2K_1$ is difference cordial.*

Proof. Let $V(G_n \odot 2K_1) = V(G_n) \cup \{w_i, w_i', x_i, x_i' : 1 \le i \le n\} \cup \{w, w'\}$ and $E(G_n \odot 2K_1) = E(G_n) \cup \{u_iw_i, u_iw_i', v_ix_i, v_ix_i' : 1 \le i \le n\} \cup \{uw, uw'\}$. The order and size of $G_n \odot 2K_1$ are $6n+3$ and $7n+2$ respectively. Define a map $f : V(G_n \odot 2K_1) \to \{1, 2, \dots 6n+3\}$ as follows:

$$
\begin{aligned}
f(u_i) &= 6i-4 && 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil \\
f(w_i) &= 6i-5 && 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil \\
f\left(w_i'\right) &= 6i-3 && 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil \\
f\left(u_{\lceil \frac{3n}{4} \rceil + i}\right) &= 6\left\lceil \frac{3n}{4} \right\rceil + 6i - 5 && 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \\
f\left(w_{\lceil \frac{3n}{4} \rceil + i}\right) &= 6\left\lceil \frac{3n}{4} \right\rceil + 6i - 4 && 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \\
f\left(w'_{\lceil \frac{3n}{4} \rceil + i}\right) &= 6\left\lceil \frac{3n}{4} \right\rceil + 6i - 3 && 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
f(v_i) = 6i-1 \qquad & 1 \le i \le \left\lfloor \tfrac{3n}{4} \right\rfloor && \text{if} \quad n \equiv 0, 2, 3 \pmod 4 \\
& 1 \le i \le \tfrac{3n+1}{4} && \text{if} \quad n \equiv 1 \pmod 4. \\
f(x_i) = 6i-2 \qquad & 1 \le i \le \left\lfloor \tfrac{3n}{4} \right\rfloor && \text{if} \quad n \equiv 0, 2, 3 \pmod 4 \\
& 1 \le i \le \tfrac{3n+1}{4} && \text{if} \quad n \equiv 1 \pmod 4. \\
f\left(x_i'\right) = 6i \qquad & 1 \le i \le \left\lfloor \tfrac{3n}{4} \right\rfloor && \text{if} \quad n \equiv 0, 2, 3 \pmod 4 \\
& 1 \le i \le \tfrac{3n+1}{4} && \text{if} \quad n \equiv 1 \pmod 4.
\end{aligned}
$$

Case 1: $n \equiv 0, 2, 3 \pmod 4$.

$$
\begin{aligned}
f\left(v_{\lfloor \frac{3n}{4} \rfloor + i}\right) &= 6\left\lfloor \frac{3n}{4} \right\rfloor + 6i - 2 && 1 \le i \le \left\lceil \frac{n}{4} \right\rceil \\
f\left(x_{\lfloor \frac{3n}{4} \rfloor + i}\right) &= 6\left\lfloor \frac{3n}{4} \right\rfloor + 6i - 1 && 1 \le i \le \left\lceil \frac{n}{4} \right\rceil
\end{aligned}
$$

$$f\left(x'_{\lfloor \frac{3n}{4}\rfloor+i}\right) = 6\left\lfloor\frac{3n}{4}\right\rfloor+6i \qquad 1\leq i\leq \left\lceil\frac{n}{4}\right\rceil.$$

Case 2: $n\equiv 1\,(\text{mod }4)$.

$$\begin{aligned} f\left(v_{\frac{3n+1}{4}+i}\right) &= \frac{9n-1}{2}+6i && 1\leq i\leq \frac{n-1}{4}\\ f\left(x_{\frac{3n+1}{4}+i}\right) &= \frac{9n+1}{2}+6i && 1\leq i\leq \frac{n-1}{4}\\ f\left(x'_{\frac{3n+1}{4}+i}\right) &= \frac{9n+3}{2}+6i && 1\leq i\leq \frac{n-1}{4}\end{aligned}$$

$f(u)=6n+1$, $f(w)=6n+2$ and $f\left(w'\right)=6n+3$. The following table 4 shows that f is a difference cordial labeling of $G_n\odot 2K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n\equiv 0\,(\text{mod }2)$	$\frac{7n+2}{2}$	$\frac{7n+2}{2}$
$n\equiv 1\,(\text{mod }2)$	$\frac{7n+1}{2}$	$\frac{7n+3}{2}$

TABLE 4

Theorem 3.12. *$G_n\odot K_2$ is difference cordial.*

Proof. Let $V(G_n\odot K_2)=V(G_n)\cup\left\{w_i,w_i',x_i,x_i':1\leq i\leq n\right\}\cup\left\{w,w'\right\}$ and $E(G_n\odot K_2)$ $=E(G_n)\cup\left\{u_iw_i,u_iw_i',w_iw_i',v_ix_i,v_ix_i',x_ix_i':1\leq i\leq n\right\}\cup\left\{uw,uw',ww'\right\}$. The order and size of $G_n\odot K_2$ are $6n+3$ and $9n+3$ respectively. Define a map $f:V(G_n\odot K_2)\rightarrow \{1,2,\ldots 6n+3\}$ as follows:

$$\begin{aligned} f(u_i) &= 6i-3 && 1\leq i\leq \left\lfloor\frac{n}{2}\right\rfloor\\ f(w_i) &= 6i-4 && 1\leq i\leq \left\lfloor\frac{n}{2}\right\rfloor\\ f\left(w_i'\right) &= 6i-5 && 1\leq i\leq \left\lfloor\frac{n}{2}\right\rfloor\\ f\left(u_{\lfloor\frac{n}{2}\rfloor+i}\right) &= 6\left\lfloor\frac{n}{2}\right\rfloor+6i-5 && 1\leq i\leq \left\lceil\frac{n}{2}\right\rceil\\ f\left(w_{\lfloor\frac{n}{2}\rfloor+i}\right) &= 6\left\lfloor\frac{n}{2}\right\rfloor+6i-4 && 1\leq i\leq \left\lceil\frac{n}{2}\right\rceil\\ f\left(w'_{\lfloor\frac{n}{2}\rfloor+i}\right) &= 6\left\lfloor\frac{n}{2}\right\rfloor+6i-3 && 1\leq i\leq \left\lceil\frac{n}{2}\right\rceil\\ f(v_i) &= 6i-2 && 1\leq i\leq n\end{aligned}$$

$$f(x_i) = 6i \qquad 1 \le i \le n$$
$$f\left(x_i'\right) = 6i-1 \qquad 1 \le i \le n$$

$f(u) = 6n+1$, $f(w) = 6n+2$ and $f\left(w'\right) = 6n+3$. The following table 5 shows that f is a difference cordial labeling of $G_n \odot K_2$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{9n+2}{2}$	$\frac{9n+4}{2}$
$n \equiv 1 \pmod 2$	$\frac{9n+3}{2}$	$\frac{9n+3}{2}$

TABLE 5

$C_n \times P_2$ is called a prism. Let $V(C_n \times P_2) = \{u_i, v_i : 1 \le i \le n\}$ and $E(C_n \times P_2) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\} \cup \{u_1 u_n, v_1 v_n\}$.

Theorem 3.13. *$(C_n \times P_2) \odot K_1$ is difference cordial.*

Proof. Let $V((C_n \times P_2) \odot K_1) = V(C_n \times P_2) \cup \{x_i, y_i : 1 \le i \le n\}$, $E((C_n \times P_2) \odot K_1)$ $= E(C_n \times P_2) \cup \{u_i x_i, v_i y_i : 1 \le i \le n\}$. Define $f : V((C_n \times P_2) \odot K_1) \to \{1, 2 \ldots 4n\}$ by

$$f(u_{2i-1}) = 4i-2 \qquad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(x_{2i-1}) = 4i-3 \qquad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(u_{2i}) = 4i-1 \qquad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(x_{2i}) = 4i \qquad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(v_i) = 2n+2i-1 \qquad 1 \le i \le n$$
$$f(y_i) = 2n+2i \qquad 1 \le i \le n.$$

The following table 6 shows that f is a difference cordial labeling of $(C_n \times P_2) \odot K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{5n}{2}$	$\frac{5n}{2}$
$n \equiv 1 \pmod 2$	$\frac{5n+1}{2}$	$\frac{5n-1}{2}$

TABLE 6

Theorem 3.14. *$(C_n \times P_2) \odot 2K_1$ is difference cordial.*

Proof. Let $V((C_n \times P_2) \odot 2K_1) = V(C_n \times P_2) \cup \left\{x_i, x_i^{'}, y_i, y_i^{'} : 1 \leq i \leq n\right\}$ and $E((C_n \times P_2) \odot 2K_1) = E(C_n \times P_2) \cup \left\{u_i x_i, u_i x_i^{'}, v_i y_i, v_i y_i^{'} : 1 \leq i \leq n\right\}$. Define a map $f : V((C_n \times P_2) \odot 2K_1) \to \{1, 2 \dots 6n\}$ by

$$
\begin{aligned}
f(u_i) &= 3i-1 && 1 \leq i \leq n \\
f(x_i) &= 3i-2 && 1 \leq i \leq n \\
f\left(x_i^{'}\right) &= 3i && 1 \leq i \leq n \\
f(v_i) &= 3n+3i-1 && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f(y_i) &= 3n+3i-2 && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f\left(y_i^{'}\right) &= 3n+3i && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f\left(v_{\lceil \frac{n}{2} \rceil + i}\right) &= 3\left\lceil \frac{n}{2} \right\rceil + 3n+3i-2 && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(y_{\lceil \frac{n}{2} \rceil + i}\right) &= 3\left\lceil \frac{n}{2} \right\rceil + 3n+3i-1 && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(y^{'}_{\lceil \frac{n}{2} \rceil + i}\right) &= 3\left\lceil \frac{n}{2} \right\rceil + 3n+3i && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor.
\end{aligned}
$$

The following table 7 shows that f is a difference cordial labeling of $(C_n \times P_2) \odot 2K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{7n}{2}$	$\frac{7n}{2}$
$n \equiv 1 \pmod 2$	$\frac{7n-1}{2}$	$\frac{7n+1}{2}$

TABLE 7

Theorem 3.15. *$(C_n \times P_2) \odot K_2$ is difference cordial.*

Proof. Let $V((C_n \times P_2) \odot K_2) = V(C_n \times P_2) \cup \left\{x_i, x_i^{'}, y_i, y_i^{'} : 1 \leq i \leq n\right\}$ and $E((C_n \times P_2) \odot K_2) = E(C_n \times P_2) \cup \left\{u_i x_i, u_i x_i^{'}, x_i x_i^{'}, v_i y_i, v_i y_i^{'}, y_i y_i^{'} : 1 \leq i \leq n\right\}$. The order and size of $(C_n \times P_2) \odot K_2$ are $6n$ and $9n$ respectively. Define a map $f : V((C_n \times P_2) \odot K_2) \to \{1, 2 \dots 6n\}$ by

$$
\begin{aligned}
f\left(u_{2i-1}\right) &= 6i-3 && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f\left(x_{2i-1}\right) &= 6i-4 && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f\left(x'_{2i-1}\right) &= 6i-5 && 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
f\left(u_{2i}\right) &= 6i-2 && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(x_{2i}\right) &= 6i && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(x'_{2i}\right) &= 6i-1 && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(v_{i}\right) &= 3n+3i-2 && 1 \leq i \leq n \\
f\left(y_{i}\right) &= 3n+3i-1 && 1 \leq i \leq n \\
f\left(y'_{i}\right) &= 3n+3i && 1 \leq i \leq n.
\end{aligned}
$$

The following table 8 shows that f is a difference cordial labeling of $(C_n \times P_2) \odot K_2$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{9n}{2}$	$\frac{9n}{2}$
$n \equiv 1 \pmod 2$	$\frac{9n+1}{2}$	$\frac{9n-1}{2}$

TABLE 8

$L_n = P_n \times P_2$ is called a ladder. We now investigate the difference cordial labeling behavior of the corona of L_n with K_n, $2K_1$ and K_2.

Theorem 3.16. *$L_n \odot K_1$ is difference cordial.*

Proof. Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_iv_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\}$. $V(L_n \odot K_1) = V(L_n) \cup \{w_i, x_i : 1 \leq i \leq n\}$ and $E(L_n \odot K_1) = E(L_n) \cup \{u_iw_i, v_ix_i : 1 \leq i \leq n\}$. Define a map $f : V(L_n \odot K_1) \rightarrow \{1, 2 \dots 4n\}$ as follows:

Case 1: n is odd.

$$\begin{aligned}
f(u_{2i-1}) &= 4i-2 & 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \\
f(u_{2i}) &= 4i-1 & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(w_{2i-1}) &= 4i-3 & 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \\
f(w_{2i}) &= 4i & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor \\
f(v_i) &= 2n+2+i & 1 \le i \le n-1 \\
f(x_i) &= 3n+1+i & 1 \le i \le n-1.
\end{aligned}$$

$f(u_n) = 2n,\ f(w_n) = 2n-1,\ f(v_n) = 2n+1$ and $f(x_n) = 2n+2$.

Case 2: n is even.

Label the vertices u_i and w_i $(1 \le i \le n-1)$ as in case 1. Define $f(u_n) = 2n-1$, $f(w_n) = 2n$, $f(v_n) = 3n$, $f(x_n) = 4n$, $f(v_i) = 2n+i$, $1 \le i \le n-1$, $f(x_i) = 3n+i$, $1 \le i \le n-1$. The following table 9 shows that f is a difference cordial labeling of $L_n \odot K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{5n-2}{2}$	$\frac{5n-2}{2}$
$n \equiv 1 \pmod 2$	$\frac{5n-3}{2}$	$\frac{5n-1}{2}$

TABLE 9

Theorem 3.17. *$L_n \odot 2K_1$ is difference cordial.*

Proof. $V(L_n \odot 2K_1) = V(L_n) \cup \{w_i, w_i', x_i, x_i' : 1 \le i \le n\}$ and $E(L_n \odot 2K_1) = E(L_n) \cup \{u_iw_i, u_iw_i', v_ix_i, v_ix_i' : 1 \le i \le n\}$. Define a map $f : V(L_n \odot 2K_1) \to \{1, 2 \ldots 6n\}$ as follows:

$$\begin{aligned}
f(u_i) &= 3i-1 & 1 \le i \le n \\
f(w_i) &= 3i-2 & 1 \le i \le n \\
f\left(w_i'\right) &= 3i & 1 \le i \le n
\end{aligned}$$

$$\begin{aligned}
f\left(v_i\right) &= 3n+3i-1 & 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil \\
f\left(v_{\left\lceil \frac{n-2}{2} \right\rceil + i}\right) &= 3n+3\left\lceil \frac{n-2}{2} \right\rceil + 3i-2 & 1 \leq i \leq \left\lfloor \frac{n+2}{2} \right\rfloor \\
f\left(x_i\right) &= 3n+3i-2 & 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil \\
f\left(x_{\left\lceil \frac{n-2}{2} \right\rceil + i}\right) &= 3n+3\left\lceil \frac{n-2}{2} \right\rceil + 3i-1 & 1 \leq i \leq \left\lfloor \frac{n+2}{2} \right\rfloor \\
f\left(x_i^{'}\right) &= 3n+3i & 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil \\
f\left(x^{'}_{\left\lceil \frac{n-2}{2} \right\rceil + i}\right) &= 3n+3\left\lceil \frac{n-2}{2} \right\rceil + 3i & 1 \leq i \leq \left\lfloor \frac{n+2}{2} \right\rfloor .
\end{aligned}$$

The following table 10 shows that f is a difference cordial labeling of $L_n \odot 2K_1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{7n-2}{2}$	$\frac{7n-2}{2}$
$n \equiv 1 \pmod 2$	$\frac{7n-3}{2}$	$\frac{7n-1}{2}$

TABLE 10

Theorem 3.18. *$L_n \odot K_2$ is difference cordial.*

Proof. $V(L_n \odot K_2) = V(L_n) \cup \left\{w_i, w_i^{'}, x_i, x_i^{'} : 1 \leq i \leq n\right\}$ and $E(L_n \odot K_2) = E(L_n) \cup \left\{u_iw_i, u_iw_i^{'}, w_iw_i^{'}, v_ix_i, v_ix_i^{'}, x_ix_i^{'} : 1 \leq i \leq n\right\}$. Define an injective map from the vertices of $L_n \odot K_2$ to the set $\{1, 2 \ldots 6n\}$ as follows:

Case 1: n is even.

$$\begin{aligned}
f\left(u_{2i-1}\right) &= 6i-3 & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(u_{2i}\right) &= 6i-2 & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(w_{2i-1}\right) &= 6i-4 & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(w_{2i}\right) &= 6i & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(w_{2i-1}^{'}\right) &= 6i-5 & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor
\end{aligned}$$

$$
\begin{aligned}
f\left(w_{2i}^{'}\right) &= 6i-1 && 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(v_i\right) &= 3n+i && 1 \leq i \leq n \\
f\left(x_i\right) &= 4n+2i-1 && 1 \leq i \leq n \\
f\left(x_i^{'}\right) &= 4n+2i && 1 \leq i \leq n.
\end{aligned}
$$

Case 2: n is odd.

Label the vertices u_i, w_i and $w_i^{'}$ $(1 \leq i \leq n-1)$ and v_i, x_i and $x_i^{'}$ $(1 \leq i \leq n)$ as in case 1. Define $f\left(u_n\right) = 3n-2$, $f\left(w_n\right) = 3n$ and $f\left(w_n^{'}\right) = 3n-1$. The following table 11 shows that f is a difference cordial labeling of $L_n \odot K_2$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{9n-2}{2}$	$\frac{9n-2}{2}$
$n \equiv 1 \pmod 2$	$\frac{9n-1}{2}$	$\frac{9n-3}{2}$

TABLE 11

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