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A DELAY-DEPENDENT STABILITY CRITERION FOR LINEAR NEUTRAL SYSTEMS WITH TIME-VARYING DELAY

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Abstract. This paper addresses exponential stabilization problem for a class of linear systems with time-varying delay. The time delay is any continuous function belonging to a given interval, but not necessary to be differentiable. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, new delay-dependent sufficient conditions for the exponential stabilization of the systems are first established in terms of LMIs. At the end, numerical example is given to indicate that the result presented in this research is effective and better some criteria of previous work.

Keywords: Neutral system; exponential stabilization; interval delay; lyapunov Theory; linear matrix inequalities.

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1. Introduction

Neutral system is one the important system used to describe practical applications in sciences and engineering such as lossless transmission lines, population ecology, chemical

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processes, etc [1–3]. Unlike other systems, the neutral has time-delay in both the state and derivative. However, it is well-known that time-delay in the system may be a source of instability or bad system performance. Thus many researchers try to study them to find stability criteria for such system with time-delay to be stable. (See for examples [4–20]).

In practice, systems may be disturbed by a small amount of perturbation in time. Thus systems with some uncertainties are common modeled for practical uses. Thus the problem of robust stability analysis has been widely investigated [1, 2, 6–23].

In the past decade, researchers have introduced new sufficient conditions for the system with time-delay. Conditions developed in the past can be classified into two types; delay-dependent and delay-independent. The later condition can be considered as an inappropriate since it provides stability of the system unrelated to the delay's size. Therefore, recently, increasing attention has been focused on delay-dependent of the delay system which, in general, can be considered as less conservative than delay-dependent ones.

Due to some applications needed speed of convergence, therefore many researches, recently, paid more attention to stability analysis is exponential stability which can guarantee stability with faster speed than asymptotic stability [2, 6–9].

Stability analysis of linear systems with time-varying delays $\dot{x}(t) = Ax(t)+Dx(t-h(t))$ is fundamental to many practical problems and has received considarable attention [24, 25]. Most of the known results on this problem are derived assuming only that the time-varing delay $h(t)$ is a continuously differentialbe function, satisfying some boundedness condition on its derivative: $\dot{h}(t) \leq \delta < 1$. In delay–dependent stability criteria, the main concerns is to enlarge the feasible region of stability criteria in given time-delay interval. Interval time-varying delay means that a time delay varies in an interval in which the lower bound is not restricted to be zero. By constracting a suitable argumented Lyapunov functionals and utilizing free weigtht matrices, some less conservative conditions for asymptotic stability are derived in [26–29] for systems with time delay varying in an interval. However, the shortcoming of the method used in these works is that the delay function is assumed to be differential and its derivative is still bounded: $\dot{h}(t) \leq \delta$.

This paper gives the improved results for the exponential stabilization of systems with interval time-varying delay. The time delay is assumed to be a time-varying continuous function belonging to a given interval, but not necessary to be differentiable. By constructing argumented Lyapunov functionals combined with LMI technique, we propose new criteria for the exponential stabilization of the system. The delay-dependent stabilization conditions are formulated in terms of LMIs, being thus solvable by utilizing Matlab's LMI Control Toolbox available in the literature to date.

The paper is organized as follows: Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stabilization conditions of the system with numerical examples showing the effectiveness of proposed method are presented in Section 3.

2. Preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the $n-$dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\| \, . \, \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions; A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\min/\max}(A) = \min/\max\{\mathrm{Re}\lambda; \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}, \|x_t\| = \sup_{s\in[-h,0]} \| \, x(t+s) \, \|$; $C([0,t], R^n)$ denotes the set of all R^n-valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0; A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix. Consider a linear control system with interval time-varying delay of the form

$$\dot{x}(t) = Ax(t) + Du(t), \quad t \in R^+, x(t) = \phi(t), t \in [-h_2, 0], \tag{2.1}$$

where $x(t) \in R^n$ is the state; $u(t) \in R^m, m \leq n$, is the control input, $A \in M^{n\times n}, D \in M^{n\times m}$ and $\phi(t) \in C([-h_2, 0], R^n)$ is the initial function with the norm $\| \, \phi \, \| = \sup_{s\in[-h_2,0]} \| \, \phi(s) \, \|$;

We consider a delayed feedback control law

$$u(t) = Fx(t - h(t)), \quad t \in R^+, \tag{2.2}$$

and F is the controller gain to be determined.

Remark 2.1. Note that to implement the delayed state feedback controller (2.1) the function $h(t)$ must be khown in advance. Applying the feedback controller (2.2) to the system (2.1), the closed-loop discrete-time delay system is

$$\dot{x}(t) = Ax(t) + DFx(t - h(t)), \quad t \in R^+. \tag{2.3}$$

The time-varying delay function $h(t)$ satisfies

$$0 \leq h_1 \leq h(t) \leq h_2, \quad t \in R^+.$$

Definition 2.1. Given $\alpha > 0$. The zero solution of system (2.1) is $\alpha-$exponentially stable if there exist a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\| x(t, \phi) \| \leq Ne^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1. (Cauchy inequality) *For any symmetric positive definite marix $N \in M^{n\times n}$ and $a, b \in R^n$ we have*

$$\pm a^T b \leq a^T Na + b^T N^{-1} b.$$

Proposition 2.2. [30] *For any symmetric positive definite matrix $M \in M^{n\times n}$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\Big(\int_0^\gamma \omega(s)\, ds \Big)^T M \Big(\int_0^\gamma \omega(s)\, ds \Big) \leq \gamma \Big(\int_0^\gamma \omega^T(s) M \omega(s)\, ds \Big).$$

Proposition 2.3. [31] *Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have*

$$EFH + H^T F^T E^T \leq \epsilon EE^T + \epsilon^{-1} H^T H.$$

Proposition 2.4. (Schur complement lemma [32]). *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

3. Main results

Let us set

$$\lambda_1 = \lambda_{\min}(P), \quad \lambda_2 = \lambda_{\max}(P) + 2h_2^2 \lambda_{\max}(U).$$

Theorem 3.1. *Given $\alpha > 0$. The zero solution of the system (2.1) is $\alpha-$exponentially stabilizable by the delayed feedback control (2.2), where $F = D^T[DD^T]^{-1}$, if there exist symmetric positive definite matrices P, U, and matrices R, S, such that the following LMI holds*

$$\mathcal{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & 0 & 0 \\ * & * & M_{33} & 0 & 0 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix}, \tag{3.1}$$

where

$$
\begin{aligned}
M_{11} &= PA + A^T P - e^{-2\alpha h_1} U - e^{-2\alpha h_2} U + 2\alpha P,\\
M_{12} &= e^{-2\alpha h_1} U,\\
M_{13} &= e^{-2\alpha h_2} U,\\
M_{14} &= P + A^T S,\\
M_{15} &= A^T R,\\
M_{22} &= -e^{-2\alpha h_1} U,\\
M_{33} &= -e^{-2\alpha h_2} U,\\
M_{44} &= S + S^T,\\
M_{45} &= R - S^T,\\
M_{55} &= (h_1^2 + h_2^2) U - R - R^T.
\end{aligned}
$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\| x(t, \phi) \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system (2.1)

$$V(t, x_t) = \sum_{i=1}^{3} V_i,$$

where

$$
\begin{aligned}
V_1 &= x^T(t) P x(t),\\
V_2 &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\alpha(\tau - t)} \dot{x}^T(\tau) U \dot{x}(\tau)\, d\tau\, ds,\\
V_3 &= h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{2\alpha(\tau - t)} \dot{x}^T(\tau) U \dot{x}(\tau)\, d\tau\, ds.
\end{aligned}
$$

It easy to check that

$$\lambda_1 \| x(t) \|^2 \leq V(t, x_t) \leq \lambda_2 \| x_t \|^2, \quad \forall t \geq 0. \tag{3.2}$$

Let us set $\xi(t) = [x(t), \dot{x}(t), x(t-h(t))]$, and

$$F = \begin{pmatrix} P & 0 & 0 \\ P & R & 0 \\ 0 & 0 & S \end{pmatrix}.$$

Taking the derivative of V_1 along the solution of system (2.1) we have

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2\xi^T(t)F^T \begin{pmatrix} \dot{x}(t) \\ 0 \\ 0 \end{pmatrix}, \tag{3.3}$$

because of

$$2\xi^T(t)F^T \begin{pmatrix} \dot{x}(t) \\ 0 \\ 0 \end{pmatrix} = 2x^T(t)P\dot{x}(t).$$

Using the expression of system (2.1)

$$0 = -\dot{x}(t) + Ax(t) + Ix(t-h(t)),$$

we have

$$2\xi^T(t)F^T \begin{pmatrix} \dot{x}(t) \\ -\dot{x}(t) + Ax(t) + Ix(t-h(t)) \\ -\dot{x}(t) + Ax(t) + Ix(t-h(t)) \end{pmatrix}$$

$$= \xi^T(t)F^T \begin{pmatrix} 0 & I & 0 \\ A & -I & I \\ A & -I & I \end{pmatrix} \xi(t) + \xi^T(t) \begin{pmatrix} 0 & A^T & A^T \\ I & -I & -I \\ 0 & I & I \end{pmatrix} F\xi(t).$$

Therefore, from (3.3) it follows that

$$\dot{V}_1 = \xi^T(t)W\xi^T(t),$$

where

$$W = F^T \begin{pmatrix} 0 & I & 0 \\ A & -I & I \\ A & -I & I \end{pmatrix} + \begin{pmatrix} 0 & A^T & A^T \\ I & -I & -I \\ 0 & I & I \end{pmatrix} F.$$

The derivative of V_2 and V_3 are given by

$$\dot{V}_2 = h_1^2 \dot{x}^T(t) U \dot{x}(t) - h_1 e^{-2\alpha h_1} \int_{t-h_1}^{t} \dot{x}^T(s) U \dot{x}(s)\, ds - 2\alpha V_2,$$
$$\dot{V}_3 = h_2^2 \dot{x}^T(t) U \dot{x}(t) - h_2 e^{-2\alpha h_2} \int_{t-h_2}^{t} \dot{x}^T(s) U \dot{x}(s)\, ds - 2\alpha V_3.$$

Applying Proposition 2.2 and the Leibniz-Newton formula, we have

$$\begin{aligned} -h_i \int_{t-h_i}^{t} \dot{x}^T(s) U \dot{x}(s)\, ds &\leq -\left[\int_{t-h_i}^{t} \dot{x}(s)\, ds\right]^T U \left[\int_{t-h_i}^{t} \dot{x}(s)\, ds\right] \\ &\leq -[x(t) - x(t-h_i)]^T U [x(t) - x(t-h_i)] \\ &= -x^T(t) U x(t) + 2x^T(t) U x(t-h_i) - x^T(t-h_i) U x(t-h_i). \end{aligned}$$

Therefore, we have

$$\dot{V}(.) + 2\alpha V(.) \leq x^T(t)[PA + A^T P - e^{-2\alpha h_1} U - e^{-2\alpha h_2} U + 2\alpha P] x(t)$$
$$+2x^T(t)[e^{-2\alpha h_1} U] x(t-h_1) + 2x^T(t)[e^{-2\alpha h_2} U] x(t-h_2)$$
$$+2x^T(t)[P + A^T S] x(t-h(t)) + 2x^T(t)[A^T R] \dot{x}(t)$$
$$-x^T(t-h_1)[e^{-2\alpha h_1} U] x(t-h_1) - x^T(t-h_2)[e^{-2\alpha h_2} U] x(t-h_2)$$
$$+x^T(t-h(t))[S + S^T] x(t-h(t)) + 2x^T(t-h(t))[R - S^T] \dot{x}(t)$$
$$+\dot{x}^T(t)[(h_1^2 + h_2^2) U - R - R^T] \dot{x}(t)$$
$$= \zeta^T(t) \mathcal{M} \zeta(t), \tag{3.4}$$

where

$$\zeta(t) = [x(t), x(t-h_1), x(t-h_2), x(t-h(t)), \dot{x}(t)],$$

$$\mathcal{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & 0 & 0 \\ * & * & M_{33} & 0 & 0 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

where

$$M_{11} = PA + A^T P - e^{-2\alpha h_1}U - e^{-2\alpha h_2}U + 2\alpha P,$$

$$M_{12} = e^{-2\alpha h_1}U,$$

$$M_{13} = e^{-2\alpha h_2}U,$$

$$M_{14} = P + A^T S,$$

$$M_{15} = A^T R,$$

$$M_{22} = -e^{-2\alpha h_1}U,$$

$$M_{33} = -e^{-2\alpha h_2}U,$$

$$M_{44} = S + S^T,$$

$$M_{45} = R - S^T,$$

$$M_{55} = (h_1^2 + h_2^2)U - R - R^T.$$

By condition (3.1), we obtain

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \in R^+. \tag{3.5}$$

Integrating both sides of (3.5) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+.$$

Furthermore, taking condition (3.2) into account, we have

$$\lambda_1 \parallel x(t,\phi) \parallel^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \parallel \phi \parallel^2,$$

then

$$\parallel x(t,\phi) \parallel \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \parallel \phi \parallel, \quad t \in R^+,$$

which concludes the proof by the Lyapunov stability theorem [33].

To illustrate the obtained result, let us give the following numerical example.

Example 3.1. Consider the following the linear control system with interval time-varying delay (2.1) , where the delay function $h(t)$ is given by

$$h(t) = 0.1 + 2.3237 sin^2 3t,$$

and

$$A = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}, D = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

It is worth noting that, the delay function $h(t)$ is non-differentiable and the exponent $\alpha \geq 1$. Therefore, the methods used is in [1, 2, 4–31, 34–37] are not applicable to this system. By using LMI Toolbox in MATLAB, we use the condition in the Theorem 3.1 for this eaxample. The solutions of LMI verify as follow of the form

$$P = \begin{pmatrix} 1.3399 & 0.3542 \\ 0.3542 & 0.9403 \end{pmatrix}, U = \begin{pmatrix} 1.6468 & 0.2037 \\ 0.2037 & 1.3638 \end{pmatrix},$$

$$R = \begin{pmatrix} -2.5817 & -1.1037 \\ -0.9758 & -1.1502 \end{pmatrix}, S = \begin{pmatrix} -1.8497 & -0.7148 \\ -0.9218 & -0.7859 \end{pmatrix}.$$

By Theorem 3.1, the system is stabilizable, the delayed feedback control is:

$$u(t) = \begin{bmatrix} -0.5x_1(t - h(t)) \\ 0.5x_2(t - h(t)) \end{bmatrix}.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\| x(t, \phi) \| \leq 30.1917e^{-1.4t} \| \phi \|, \quad \forall t \in R^+.$$

Therefore, the system (2.1) is $1.4-$exponentially stable.

4. Conclusions

In this paper, we have proposed new delay-dependent conditions for the exponential stabilization of linear systems with non-differentiable interval time-varying delay. Based on the improved Lyapunov-Krasovskii functionals and linear matrix inequality technique, the conditions for the exponential stabilization of the systems have been established in terms of LMIs.

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