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ON A NEW GENERAL INTEGRAL TRANSFORM: SOME PROPERTIES AND REMARKS

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Abstract. Integral transform method is widely used to solve the several differential equations with the initial values or boundary conditions which are represented by integral equations. With this purpose in this paper we suggest a new kind of general integral transform and establish related main theorems. The suggested transform which is called Ramadan Group (RG) transform and denoted by RGT is a hybrid of both Laplace and Sumudu transforms. The formula of the transform is defined and adopted as a standard general form. A table of transformations to most popular functions is presented where we are in agreement with the case of Laplace and Sumudu transforms.

Keywords: Integral transforms, Laplace transform, Sumudu transform, Ramadan group transform.

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1-Introduction

An integral transform is a particular kind of mathematical operator. In mathematics , an integral transform is any transform T of the following form

$$T(f(u)) = \int_{t_1}^{t_2} K(t,u) f(t)\ dt$$

The input of this transform is a function f, and the output is another function Tf. There are numerous useful integral transforms, each is specified by a choice of the function K of two variables , the kernel function or nucleus of the transform. Some kernels have an associated inverse kernel $K^{-1}(u, t)$ which yields an inverse transform:

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$$f(t)=\int_{u_1}^{u_2} K^{-1}(u,t)(T(f(u))\,du$$

A symmetric kernel is one that is unchanged when the two variables are permuted .

There are many classes of problems that are difficult to solve or at least quite unwieldy algebraically in their original representations. An integral transform maps an equation from its original domain into another domain. Manipulating and solving the equation in the target domain can be much easier than manipulation and solution in the original domain. The solution is then mapped back to the original domain with the inverse of the integral transform. Also there are many applications of probability that rely on integral transforms, such as pricing kernel or stochastic discount factor , or the smoothing of data recovered from robust statistics.

In the literature, there are several works on integral transforms such as Sumudu, Fourier Laplace, Mellin, Hankel and many others, but very little work on the power series transformation such as Sumudu transform. This is probably, because it is not widely used yet. The Sumudu transform was recently presented by Watugala; see [1], [2]. The properties of Sumudu transform were proposed and established in [3]. The precursor of the transforms was the Fourier series to express functions in finite intervals. Later the Fourier transform was developed to remove the requirement of finite intervals.

Although the properties of integral transforms vary widely, they have some properties in common. For example, every integral transform is a linear operator, since the integral is a linear operator, and in fact if the kernel is allowed to be a generalized function then all linear operators are integral transforms. Recently some applications for Sumudu transform is used to solving different types of differential equations, the interested reader is referred to [4-10]. The Laplace transform can be used to solve differential equations. In addition, being a different and efficient alternative to variation of parameters and undetermined coefficients, the Laplace method is particularly advantageous for input terms that are piecewise-defined, periodic or impulsive.

The direct Laplace transform or the Laplace integral of a function $f(t)$ defined for $0 < t < \infty$ is the ordinary calculus integration problem

$$\int_0^\infty f(t)e^{-st}dt\,,$$

succinctly denoted $L[(f(t))]$ in science and engineering literature. The L-notation recognizes that integration always proceeds over $t=0$ to $t=\infty$ and that the integral involves an integrator

$e^{-st}dt$ instead of the usual dt. These minor differences distinguish Laplace integrals from the ordinary integrals found on the inside covers of calculus texts. For more details about the properties and applications of Laplace transform, see [6, 11, 12]

2- The Proposed Ramadan Group Transform (RGT)

A new integral RG transform defined for functions of exponential order, is proclaimed. We consider functions in the set A, defined by:

$$A = \left\{ f(t) : \exists M, t_1, t_2 > 0 \ \ s.t. \ |f(t)| < M\, e^{\frac{|t|}{t_n}}, if \ \ t \in (-1)^n \times [0, \infty) \right\}, \tag{1}$$

The RG transform is defined by

$$K(s,u) = RG(f(t)) = \begin{cases} \int_0^\infty e^{-st} \ f(ut)\,dt, & 0 \le u < t_2, \\ \int_0^\infty e^{-st} \ f(ut)\,dt, & t_1 < u \le 0, \end{cases} \tag{2}$$

Consider $F(s) = L(f(t)) = \int_0^\infty e^{-st} \ f(t)\,dt$ and $G(u) = L(f(t)) = \int_0^\infty e^{-t} \ f(ut)\,dt$ are the Laplace and Sumudu integral transforms respectively, then we can write the following theorem

THEOREM 1

$$K(s,1) = F(s), \tag{3a}$$

$$K(1,u) = G(u), \tag{3b}$$

$$K(s,u) = \frac{1}{u} F\left(\frac{s}{u}\right) \tag{3c}$$

Proof:

The proof of the first two parts follows directly from the definitions, and the proof of $K(s,u) = \int_0^\infty e^{-st} \ f(ut)\,dt = \frac{1}{u} F\left(\frac{s}{u}\right)$, is as:

Let $f(t) \in A$, then for $-t_1 < u < t_2$, if we set $w = ut$, then we get:

$$\int_0^\infty e^{-\frac{s}{u}w} \ f(w) \frac{1}{u}\,dw = \frac{1}{u} F\left(\frac{s}{u}\right).$$

THEOTREM 2: Suppose K(s, u) is the Ramadan Group transform of the function $f(t)$ then we can prove the following

$$RG(\frac{df(t)}{dt}) = \frac{s\,RG(f(t)) - f(0)}{u},$$

$$RG(\frac{d^2 f(t)}{dt^2}) = \frac{s^2\,RG(f(t)) - s\,f(0) - u\,\dfrac{df(0)}{dt}}{u^2},$$

and in general

$$RG(\frac{d^n f(t)}{dt^n}) = \frac{s^n\,RG(f(t))}{u^n} - \sum_{k=0}^{n-1} \frac{s^{n-k-1}\,f^{(k)}(0)}{u^{n-k}}.$$

THEOREM 3: Let $K'(u,s)$ denote the RG transform of the definite integral of f , $W(t) = \int_0^t f(\tau)\,d\tau$.

Then

$$K'(u,s) = RG(W(t)) = \frac{u}{s} K(u,s)$$

THEOREM 4: For the Dirac delta function $\delta(t-a)$ and the Heaviside function $H(t-a)$, we have

$$\text{RG}\,(\delta(t-a)) = \frac{e^{\frac{-a\,s}{u}}}{u}, \text{and RG}\,(H(t-a)) = \frac{e^{\frac{-a\,s}{u}}}{s}.$$

THEOREM 5: Let $f(t) \in A$ with RG transform $K(u,s)$.

Then

$$RG(t\,\frac{d\,f(t)}{dt}) = u\,\frac{\partial K(u,s)}{\partial u},$$

$$RG(t\,\frac{d\,f(t)}{dt}) = -u\,\frac{\partial K(u,s)}{\partial s},$$

$$RG(t^2\,\frac{d^2\,f(t)}{dt^2}) = -u^2\,\frac{\partial^2 K(u,s)}{\partial s\,\partial u}.$$

THEOREM 6: Let $f(t) \in A$ with RG transform $K(u,s)$.

Then

$$RG(e^{a\,t}\,f(t)) = K(s-a\,u,u).$$

THEOREM 7: Let $f(t) = t^{x-1} \in A$ with RG transform $K(u,s)$.

Then

$$RG(t^{x-1}) = K(s,u) = \frac{u^{x-1}}{s^x}\,\Gamma(x).$$

THEOREM 8: Let $f(t) \in A$ with RG transform $K(u,s)$. Then

$$K(s,\frac{1}{s}) = s\,F(s),$$

$$K(\frac{1}{u},u) = u\,G(u^2).$$

THEOREM 9: Let $f(t) \in A$ with RG transform $K(u,s)$.

Then

$$RG(f(a\,t)) = K(s,u) = \frac{1}{s}G(\frac{a\,u}{s}),$$

$$RG(f(a\,t)) = K(s,u) = \frac{1}{a\,u}F(\frac{s}{a\,u}).$$

Table A.1 Ramadan Group transform of some functions

	$f(t)$	$RG(f(t)) = K(s,u)$
1	1	$K(s,u) = \frac{1}{s}$
2	t	$K(s,u) = \frac{u}{s^2}$
3	$\frac{t^{n-1}}{(n-1)!}$	$K(s,u) = \frac{u^{n-1}}{s^n}$
4	$\frac{1}{\sqrt{\pi t}}$	$K(s,u) = \frac{1}{\sqrt{s\,u}}$
5	$2\sqrt{\frac{t}{\pi}}$	$K(s,u) = \frac{\sqrt{u}}{s\sqrt{s}}$
6	$\frac{t^{a-1}}{\Gamma(a)}$	$K(s,u) = \frac{u^{a-1}}{s^a}$

Table A.1. Continued.

7	e^{at}	$K(s,u)=\dfrac{1}{s-au}$
8	$t\,e^{at}$	$K(s,u)=\dfrac{u}{(s-au)^2}$
9	$\dfrac{t^{n-1}\,e^{at}}{(n-1)!}$	$K(s,u)=\dfrac{u^{n-1}}{(s-au)^n}$
10	$\dfrac{e^{at}-e^{bt}}{a-b}$	$K(s,u)=\dfrac{1}{a-b}\left(\dfrac{1}{s-au}+\dfrac{1}{bu-s}\right)$
11	$\dfrac{a\,e^{at}-b\,e^{bt}}{a-b}$	$K(s,u)=\dfrac{1}{a-b}\left(\dfrac{a}{s-au}+\dfrac{b}{bu-s}\right)$
12	$\dfrac{\sin(wt)}{w}$	$K(s,u)=\dfrac{u}{s^2+u^2\,w^2}$
13	$\cos(wt)$	$K(s,u)=\dfrac{s}{s^2+u^2\,w^2}$
14	$\dfrac{\sinh(at)}{a}$	$K(s,u)=\dfrac{u}{s^2-u^2\,a^2}$
15	$\cosh(at)$	$K(s,u)=\dfrac{s}{s^2-u^2\,a^2}$
16	$\dfrac{e^{at}\sin(wt)}{w}$	$K(s,u)=\dfrac{u}{(s-au)^2+u^2\,w^2}$
17	$e^{at}\cos(wt)$	$K(s,u)=\dfrac{s-au}{(s-au)^2+u^2\,w^2}$
18	$\dfrac{1-\cos(wt)}{w^2}$	$K(s,u)=\dfrac{u^2}{s(s^2+u^2\,w^2)}$
19	$\dfrac{wt-\sin(wt)}{w^3}$	$K(s,u)=\dfrac{u^3}{s^2\,(s^2+u^2\,w^2)}$
20	$\dfrac{\sin(wt)-wt\cos(wt)}{2\,w^3}$	$K(s,u)=\dfrac{u^3}{(s^2+u^2\,w^2)^2}$
21	$\dfrac{t\sin(wt)}{2\,w}$	$K(s,u)=\dfrac{s\,u^2}{(s^2+u^2\,w^2)^2}$

From the table we can say that RGT is the general case of Laplace and Sumudu transformations and the next paper we show some application of this new transformation for solving differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests.

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