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CHANCE CONSTRAINT PROBLEM HAVING PARAMETERS AS PARETO RANDOM VARIABLES

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Abstract. Management and measurement of risk is an important issue in almost all areas that require decisions to be made under uncertain information. Chance constrained programming (CCP) has been used for modeling and analysis of risks in a number of application domains. This paper presents a deterministic reduction of linear and nonlinear chance constraint programming problem using geometric inequality, assuming the coefficients of the decision variables in the chance constraints as Pareto random variables. After implicative reduction of the proposed chance constraint programming problem into a deterministic problem, which is usually nonlinear, standard generic package is used to find the compromise solution. Then MATLAB programming code is used to verify the validity of solution as well as that of the reduced model. This method leads to an efficient reduced model as well as an optimal compromise solution.

Keywords: Stochastic programming; chance constraint programming; Pareto random variable; geometric inequality; deterministic implicative reduction; non-linear programming.

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1. Introduction

Addressing data uncertainty in mathematical programming models has been a central problem in optimization for a long time. There are two principal methods that have been proposed to address data uncertainty over the years: (a) Stochastic Programming (SP) [11, 8, 1], and (b) Robust Optimization (RO) [2,22]. SP models yield plans that are better able to hedge

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against losses and catastrophic failures. Such models have been developed for a variety of applications, including electric power generation [10], financial planning [23,7,24], telecommunication network planning [31,4], supply chain management [32], oil industry [25], vehicle manufacturers [12], electricity suppliers [20,33], environment [5], transportation [13,34], construction, energy, chemical processing [26], aerospace, and military systems [21].

Also several other models have been presented in the field of SP [16]. Contini [6] developed an algorithm for stochastic goal programming when the random variables are normally distributed with known means and variances. He transformed the stochastic problem into an equivalent deterministic quadratic programming problem, where the objective functions consisted of maximizing the probability of a vector of goals lying in the confidence region of a predefined size. Sullivan and Fitzsimmoms [30] suggested an algorithm using probabilistic goals based on the concept of chance constraints of Charnes and Cooper [1] where the goals can be stated in terms of probability of satisfying the aspiration levels. Teghem et al. [17] and Leclercq [19] have presented interactive methods in stochastic programming. Two major approaches to stochastic programming [3, 27] are recognized as:

1. Chance constrained programming,
2. Two-stage programming.

The CCP technique is one which can be used to solve problems involving constraints having finite probability of being violated. The CCP was originally developed by Charnes and Cooper [1] and has now in recent years been generalized in several directions and has various applications. This technique converts the chance constraint problem into a deterministic problem. Sarkar et al. [37] reduced the probabilistic constraints to deterministic constraints through implicative relationship using geometric inequality.

In this paper, we consider single-objective chance constraint programming problems with parameters as Pareto random variables in the chance constraints. The parameters considered as Pareto random variables because the Pareto distribution has a wide application [36] in various fields. The Pareto distribution was originally developed to describe the distribution of income and the allocation of wealth among individuals. Also applications of the Pareto distribution include insurance where it is used to model claims. In climatology it used to describe the occurrence of extreme weather. The Pareto distribution has been proposed a model for the oil and gas discoveries. The following examples [36] are sometimes seen as approximately Pareto-distributed:

- File size distribution of Internet traffic which uses the TCP protocol
- Hard disk drive error rates
- Clusters of Bose–Einstein condensate near absolute zero
- The values of oil reserves in oil fields
- The length distribution in jobs assigned supercomputers
- The standardized price returns on individual stocks
- Fitted cumulative Pareto distribution to maximum one-day rainfalls
- Sizes of sand particles
- Sizes of meteorites
- Numbers of species per genus
- Areas burnt in forest fires
- In hydrology the Pareto distribution is applied to extreme events such as annually maximum one-day rainfalls and river discharges.

Here we used some mathematical tools to convert the probabilistic model in to deterministic model. After converting into a deterministic model, which usually nonlinear, standard generic package is used to find the compromise solution. Then MATLAB programming code has been used to verify the validity of solution. This method leads to an efficient reduced model as well as an optimal compromise solution.

2: Mathematical model with linear constraints

2.1: Mathematical model with linear probabilistic constraints:

Let us consider an optimization problem having linear probabilistic constraints that can be modeled as follows:

To find $X = (x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n)$ so as to,

$$\text{Maximize / Minimize } f(X) \tag{2.1}$$

subject to chance constraints,

$$\Pr \text{ob}[\sum_{j=1}^{n} a_{ij} x_j \le b_i] \ge 1 - p_i \,, i = 1, 2, 3, ..., m \,, \tag{2.2}$$

$$x_j > 0 \,, \, j = 1, 2, 3, ..., n. \tag{2.3}$$

where $0 < p_i < 1$ and $b_i > 0$ for $i = 1, 2, 3, ..., m$, are given constants and a_{ij} 's are independently distributed Pareto random variables with parameters $\alpha_{ij} (> 0), \sigma_{ij} (> 0)$, which are respectively shape parameter and scale parameter for $i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n.$.

2.2: Deterministic reduction of the model:

Here we shall reduce the probabilistic liner constraints (2.2) to deterministic non-linear constraints as follows:

Let us consider the event $a_{ij} x_j \le b_i, i = 1, 2, 3, ..., m$.

Now applying geometric inequality (G.I.) to the L.H.S. of the above event, we get

$$n \prod_{j=1}^{n} (a_{ij} x_j)^{\frac{1}{n}} \le \sum_{j=1}^{n} (a_{ij} x_j) \,, \, i = 1, 2, 3, ..., m.$$

Therefore, $n \prod_{j=1}^{n} (a_{ij} x_j)^{\frac{1}{n}} \le b_i \,, \, i = 1, 2, 3, ..., m,$

i.e. $\prod_{j=1}^{n} (a_{ij} x_j) \le \left(\frac{b_i}{n} \right)^n \,, \, i = 1, 2, 3, ..., m,$

i.e. $\prod_{j=1}^{n} \{ (\frac{a_{ij}}{\sigma_{ij}})(\sigma_{ij} x_j) \} \le \left(\frac{b_i}{n} \right)^n \,, \, i = 1, 2, 3, ..., m.$

Now taking natural logarithm both sides of the above expression, we have

$$\sum_{j=1}^{n} \ln(\frac{a_{ij}}{\sigma_{ij}}) + \sum_{j=1}^{n} \ln(\sigma_{ij} x_j) \le n \, \ln\left(\frac{b_i}{n} \right), \, i = 1, 2, 3, ..., m.$$

Thus (2.2) becomes,

$$\text{Prob}[\sum_{j=1}^{n} \ln(\frac{a_{ij}}{\sigma_{ij}}) \le n \, \ln\left(\frac{b_i}{n} \right) - \sum_{j=1}^{n} \ln(\sigma_{ij} x_j)] \ge 1 - p_i, \, i = 1, 2, 3, ..., m \tag{2.4}$$

where, $\ln(\frac{a_{ij}}{\sigma_{ij}}) \sim \text{exponential}(\alpha_{ij}),\ \forall i, j$.

Let $a_{ij}{}' = \alpha_{ij} \ln(\frac{a_{ij}}{\sigma_{ij}})$, so it follows exponential distribution with parameter 1, $\forall i, j$.

Therefore (2.2) can be written as

$$\text{Prob}[\sum_{j=1}^{n} a_{ij}{}' \le (\max_{j=1}^{n} \alpha_{ij})\{n \ln\left(\frac{b_i}{n}\right) - \sum_{j=1}^{n} \ln(\sigma_{ij} x_j)\}] \ge 1 - p_i,\ i = 1, 2, 3, \ldots\ldots\ldots, m. \tag{2.5}$$

Now, $2\sum_{j=1}^{n} a_{ij}{}'$ follows χ^2_{2n} .

Also, if $\Pr\text{ob}[A_i \le t_i] = 1 - p_i$, where, A_i follows χ^2_{2n} , then $t_i = \chi^2_{2n}(1 - p_i), i = 1, 2, 3, \ldots\ldots., m$.

Therefore from (2.5), the deterministic form of (2.2) is

$$[n \ln\left(\frac{b_i}{n}\right) - \sum_{j=1}^{n} \ln(\sigma_{ij} x_j)] \ge \frac{1}{2 * \max_{j=1}^{n} \alpha_{ij}} \chi^2_{2n}(1 - p_i), i = 1, 2, 3, \ldots\ldots\ldots\ldots, m\ . \tag{2.6}$$

Thus the probabilistic model (2.1) - (2.3) reduced to the following deterministic form:

To find $X = (x_1, x_2, x_3, \ldots\ldots\ldots\ldots\ldots, x_n)$ so as to,

Maximize / Minimize $f(X)$ (2.7)

subject to chance constraints,

$$[n \ln\left(\frac{b_i}{n}\right) - \sum_{j=1}^{n} \ln(\sigma_{ij} x_j)] \ge \frac{1}{2 * \max_{j=1}^{n} \alpha_{ij}} \chi^2_{2n}(1 - p_i), i = 1, 2, 3, \ldots\ldots\ldots\ldots, m, \tag{2.8}$$

$$x_j > 0\,,\ j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, n. \tag{2.9}$$

3: Mathematical model with nonlinear constraints

3.1: Mathematical model with nonlinear probabilistic constraints:

Let us consider an optimization problem having nonlinear probabilistic constraints that can be modeled as follows:

To find $X = (x_1, x_2, x_3, \ldots\ldots\ldots\ldots\ldots, x_n)$, where $n = \max(n_1, n_2)$, so as to,

Maximize / Minimize $f(X)$ (3.1)

subject to chance constraints,

$$\Pr\text{ob}[\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a_{kij}x_i x_j \le b_k] \ge 1 - p_k \, , k = 1,2,3,.........., m \, , \tag{3.2}$$

$$x_l > 0 \, , l = 1,2,3,..............., n = \max(n_1, n_2) \tag{3.3}$$

where $0 < p_k < 1$ and $b_k > 0$ for $k = 1,2,3,.........., m$, are given constants and a_{kij} 's are independently distributed Pareto random variables with parameters $\alpha_{kij}(>0), \sigma_{kij}(>0)$ for $i = 1,2,3,.........., n_1$, $j = 1,2,3,.........., n_2$ and $k = 1,2,3,..............., m$.

3.2: Reduction of the probabilistic model to deterministic form:

Let us consider the event,

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a_{kij}x_i x_j \le b_k \, , \; k = 1,2,3,..............., m. \tag{3.4}$$

Now applying geometric inequality (G.I.) on the left hand side of the above expression, we have

$$\frac{1}{n_2}\sum_{j=1}^{n_2} a_{kij}x_j \ge \prod_{j=1}^{n_2}(a_{kij}x_j)^{\frac{1}{n_2}} \, , \forall k$$

i.e.,

$$\sum_{j=1}^{n_2} a_{kij}x_i x_j \ge n_2 \prod_{j=1}^{n_2}(a_{kij}x_i x_j)^{\frac{1}{n_2}} \, , \forall \, i, k.$$

Therefore,

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a_{kij}x_i x_j \ge n_2 \sum_{i=1}^{n_1}(\prod_{j=1}^{n_2}(a_{kij}x_i x_j)^{\frac{1}{n_2}}) \;\; \forall k.$$

Again applying G.I. on the right hand side of the above expression, we get

$$\begin{aligned}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a_{kij}x_i x_j &\ge n_2 \sum_{i=1}^{n_1}(\prod_{j=1}^{n_2}(a_{kij}x_i x_j)^{\frac{1}{n_2}}) \\ &\ge n_1 n_2 \prod_{i=1}^{n_1}(\prod_{j=1}^{n_2}(a_{kij}x_i x_j)^{\frac{1}{n_2}})^{\frac{1}{n_1}} \, , \forall k.\end{aligned} \tag{3.5}$$

Thus from (3.4) and (3.5), we have

$$b_k \ge n_1 n_2 \prod_{i=1}^{n_1}(\prod_{j=1}^{n_2}(a_{kij}x_i x_j)^{\frac{1}{n_2}})^{\frac{1}{n_1}} \, , \forall k.$$

i.e., $$\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(a_{kij}x_i x_j) \le \left(\frac{b_k}{n_1 n_2}\right)^{n_1 n_2}, \forall k$$

i.e., $$\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(a_{kij}) \prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(x_i x_j) \le \left(\frac{b_k}{n_1 n_2}\right)^{n_1 n_2}, \forall k$$

i.e., $$\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(a_{kij}) \prod_{i=1}^{n_1}(x_i^{n_2}) \prod_{j=1}^{n_2}(x_j^{n_1}) \le \left(\frac{b_k}{n_1 n_2}\right)^{n_1 n_2}, \forall k$$

i.e., $$\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(a_{kij}) \le \frac{\left(\frac{b_k}{n_1 n_2}\right)^{n_1 n_2}}{\prod_{i=1}^{n_1}(x_i^{n_2}) \prod_{j=1}^{n_2}(x_j^{n_1})}, \forall k.$$

Now dividing both sides of the above expression by $\sigma_{kij}\left(\neq 0,\ for\ all\ k, i, j\right)$, we have

$$\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}\left(\frac{a_{kij}}{\sigma_{kij}}\right) \le \frac{\left(\frac{b_k}{n_1 n_2}\right)^{n_1 n_2}}{\prod_{i=1}^{n_1}(x_i^{n_2}) \prod_{j=1}^{n_2}(x_j^{n_1}) \prod_{i=1}^{n_1}\prod_{j=1}^{n_2}(\sigma_{kij})}, \forall k.$$

Taking natural logarithm of both sides, we have from the above expression,

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln\left(\frac{a_{kij}}{\sigma_{kij}}\right) \le n_1 n_2(\ln(b_k) - \ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij}), \forall k \tag{3.6}$$

where, $\ln\left(\frac{a_{kij}}{\sigma_{kij}}\right)$ follows exponential $(\alpha_{kij}), \forall k, i, j$.

So, $a'_{kij} = (\alpha_{kij})\ln\left(\frac{a_{kij}}{\sigma_{kij}}\right)$ follows exponential distribution with parameter one, for all i, j, k.

Thus from (3.6), we get,

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left(\alpha_{kij}\ln\left(\frac{a_{kij}}{\sigma_{kij}}\right)\right)$$
$$\le \max_{i,j}\{\alpha_{kij}\}\left[n_1 n_2(\ln(b_k) - \ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij})\right], \forall k$$

i.e.,

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a'_{kij} \le \max_{i,j}\{\alpha_{kij}\}[n_1 n_2(\ln(b_k)-\ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij})]\ ,\forall k \ . \tag{3.7}$$

Using (3.7), we have from (3.2),

$$\Pr ob[\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a'_{kij} \le \max_{i,j}\{\alpha_{kij}\}\{n_1 n_2(\ln(b_k)-\ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij})\}] \ge 1 - p_k\ ,\forall k \tag{3.8}$$

Now, $\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} a'_{kij}$ follows $\chi^2_{2n_1 n_2}, \forall k$.

Also if $\Pr ob(A_k \le t_k) = 1 - p_k$, where A_k follows $\chi^2_{2n_1 n_2}, \forall k$, then $t_k = \chi^2_{2n_1 n_2}(1-p_k), \forall k$.

Therefore, from (3.8) the transformed deterministic form of (3.2) is

$$n_1 n_2(\ln(b_k)-\ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij}) \ge \frac{1}{2\max_{i,j}\{\alpha_{kij}\}}\chi^2_{2n_1 n_2}(1-p_k),$$

$$\forall k = 1,2,3,.....,m. \tag{3.9}$$

Thus the transformed deterministic form of the probabilistic model (3.1) - (3.3) is as follows:

To find $X = (x_1, x_2, x_3, ..., x_n)$, where $n = \max(n_1, n_2)$, so as to,

Maximize / Minimize $f(X)$ (3.10)

subject to chance constraints,

$$n_1 n_2(\ln(b_k)-\ln(n_1 n_2)) - n_2\sum_{i=1}^{n_1}\ln(x_i) - n_1\sum_{j=1}^{n_2}\ln(x_j) - \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\ln(\sigma_{kij}) \ge \frac{1}{2\max_{i,j}\{\alpha_{kij}\}}\chi^2_{2n_1 n_2}(1-p_k),$$

$$\forall k = 1,2,3,.....,m. \tag{3.11}$$

$$x_l > 0, l = 1, 2, 3, ..., n = \max(n_1, n_2). \quad (3.12)$$

4: Solution

The optimal solution for transformed deterministic form of both linear and nonlinear probabilistic model can be found using mathematical software LINGO 10 and then we can verify, using MATLAB 7.6, that whether this solution is also an optimal solution for the original probabilistic model.

5: Numerical Examples

5.1: Numerical example for the model having linear probabilistic constraints

Let us consider the following example for illustration.

To find x_1, x_2, x_3 so as to

Minimize $z = 2x_1 + 3x_2 + x_3$ (5.1)

subject to the chance constraints,
$$\begin{aligned} &Pr(a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \le b_1) \ge 1 - p_1 \\ &Pr(a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \le b_2) \ge 1 - p_2 \\ &Pr(a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \le b_3) \ge 1 - p_3 \end{aligned} \quad (5.2)$$

$$x_i > 0, i = 1, 2, 3, \quad (5.3)$$

where, a_{ij} follows $Pareto(\alpha_{ij}, \sigma_{ij})$; $i, j = 1, 2, 3$. Here α_{ij} 's and σ_{ij} 's are given in the following matrix:

α_{ij} 's

i \ j	1	2	3
1	0.7	0.1	0.2
2	0.3	0.5	0.6
3	0.2	0.8	0.03

σ_{ij} 's

i \ j	1	2	3
1	0.4	0.3	0.2
2	0.3	0.3	0.6
3	0.2	0.8	0.03

Also given that, $b_1 = 2, b_2 = 7, b_3 = 1$ and $p_1 = 0.01, p_2 = 0.05, p_3 = 0.1$.

5.1.1: Solution of the above problem

Using (2.7) – (2.9), the reduced implicative deterministic form of the given problem (5.1) – (5.3) can be written as follows:

To find x_1, x_2, x_3 so as to

Minimize $z = 2x_1 + 3x_2 + x_3$ (5.4)

subject to the constraints,

$$[3\ \ln\left(\frac{2}{3}\right) - (\ln(0.4 * x_1) + \ln(0.3 * x_2) + \ln(0.2 * x_3))] \geq \frac{1}{2*0.7}\chi_6^2(0.99)$$
$$[3\ \ln\left(\frac{7}{3}\right) - (\ln(0.3 * x_1) + \ln(0.3 * x_2) + \ln(0.6 * x_3))] \geq \frac{1}{2*0.6}\chi_6^2(0.95) \quad (5.5)$$
$$[3\ \ln\left(\frac{1}{3}\right) - (\ln(0.2 * x_1) + \ln(0.8 * x_2) + \ln(0.03 * x_3))] \geq \frac{1}{2*0.8}\chi_6^2(0.90)$$

$x_i > 0, i = 1, 2, 3.$ (5.6)

From the statistical table, we have $\chi_6^2(0.99) = 0.872$, $\chi_6^2(0.95) = 1.635$, $\chi_6^2(0.90) = 2.204$

Now using generic package LINGO 10.0 to solve (5.4) – (5.6), we get the following solution: $z = 0.2217644 \text{ x } 10^{-6}$, $x_1 = 0.3696073 \text{ x } 10^{-7}$, $x_2 = 0.2464049 \text{ x } 10^{-7}$, . $x_3 = 0.7392147 \text{ x } 10^{-7}$

After getting the above solution, using MATLAB 7.6, we can verify that, these solutions satisfy the given probabilistic constraints of the original problem (5.1) – (5.3) and thus may be considered as optimal solutions for the original probabilistic problem.

5.2: Numerical example for the model having nonlinear probabilistic constraints

Let us consider the following example for illustration.

To find x_1, x_2, x_3 so as to

Minimize $z = x_1^2 + x_2^2 + x_3^2$ (5.7)

subject to the chance constraints,

$$Pr(a_{111}x_1x_1 + a_{112}x_1x_2 + a_{121}x_2x_1 + a_{122}x_2x_2 + a_{131}x_3x_1 + a_{132}x_3x_2 \leq b_1) \geq 1 - p_1$$
$$Pr(a_{211}x_1x_1 + a_{212}x_1x_2 + a_{221}x_2x_1 + a_{222}x_2x_2 + a_{231}x_3x_1 + a_{231}x_3x_2 \leq b_2) \geq 1 - p_2 \quad (5.8)$$
$$Pr(a_{311}x_1x_1 + a_{312}x_1x_2 + a_{321}x_2x_1 + a_{322}x_2x_2 + a_{331}x_3x_1 + a_{332}x_3x_2 \leq b_3) \geq 1 - p_3$$

$x_i > 0, i = 1, 2, 3$ (5.9)

where a_{kij} follows $Pareto(\alpha_{kij}, \sigma_{kij})$; $i, k = 1, 2, 3$ *and* $j = 1, 2$. Here α_{kij} 's and σ_{kij} 's are given in the following matrix:

For $k = 1$

α_{kij} 's

i \ j	1	2
1	0.7	0.1
2	0.3	0.5
3	0.2	0.8

σ_{kij} 's

i \ j	1	2
1	0.4	0.3
2	0.3	0.3
3	0.2	0.8

For $k = 2$

α_{kij} 's

i \ j	1	2
1	0.6	0.4
2	0.5	0.5
3	0.9	0.8

σ_{kij} 's

i \ j	1	2
1	0.7	0.5
2	0.9	0.6
3	0.6	0.8

For $k = 3$

α_{kij} 's

i \ j	1	2
1	0.7	0.6
2	0.4	0.7
3	0.6	0.5

σ_{kij} 's

i \ j	1	2
1	0.9	0.5
2	0.9	0.7
3	0.3	0.8

Also given that, $b_1 = 2, b_2 = 7, b_3 = 1$ and $p_1 = 0.01, p_2 = 0.05, p_3 = 0.1$.

5.2.1: Solution of the above problem

Using (3.10) – (3.12), the reduced implicative deterministic form of the given problem (5.7) – (5.9) can be written as follows:

To find x_1, x_2, x_3 so as to

Minimize $z = x_1^2 + x_2^2 + x_3^2$ (5.10)

subject to the constraints,

$$3*2*(\ln b_1 - \ln(3*2)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(\sigma_{111}) + \ln(\sigma_{112}) + \ln(\sigma_{121}) + \ln(\sigma_{122}) + \ln(\sigma_{131}) + \ln(\sigma_{132}))$$
$$\geq \frac{1}{2\max_{i,j}\{\alpha_{1ij}\}} \chi^2_{2*3*2}(1-p_1)$$
$$3*2*(\ln b_2 - \ln(3*2)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(\sigma_{211}) + \ln(\sigma_{212}) + \ln(\sigma_{221}) + \ln(\sigma_{222}) + \ln(\sigma_{231}) + \ln(\sigma_{232}))$$
$$\geq \frac{1}{2\max_{i,j}\{\alpha_{2ij}\}} \chi^2_{2*3*2}(1-p_2)$$
$$3*2*(\ln b_3 - \ln(3*2)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(\sigma_{311}) + \ln(\sigma_{312}) + \ln(\sigma_{321}) + \ln(\sigma_{322}) + \ln(\sigma_{331}) + \ln(\sigma_{332}))$$
$$\geq \frac{1}{2\max_{i,j}\{\alpha_{3ij}\}} \chi^2_{2*3*2}(1-p_3)$$

(5.11)

$x_i > 0, i = 1, 2, 3.$ (5.12)

Thus entering all data, the problem becomes:

To find x_1, x_2, x_3 so as to

Minimize $z = x_1^2 + x_2^2 + x_3^2$ (5.13)

subject to the constraints,

$$6*(\ln 2 - \ln(6)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(0.4) + \ln(0.3) + \ln(0.3) + \ln(0.3) + \ln(0.2) + \ln(0.8)) \geq \frac{1}{2*0.8} \chi^2_{12}(0.99)$$
$$6*(\ln 7 - \ln(6)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(0.7) + \ln(0.5) + \ln(0.9) + \ln(0.6) + \ln(0.6) + \ln(0.8)) \geq \frac{1}{2*0.9} \chi^2_{12}(0.95)$$
$$6*(\ln 1 - \ln(6)) - 2*(\ln x_1 + \ln x_2 + \ln x_3) - 3*(\ln x_1 + \ln x_2) - (\ln(0.9) + \ln(0.5) + \ln(0.9) + \ln(0.7) + \ln(0.3) + \ln(0.8)) \geq \frac{1}{2*0.7} \chi^2_{12}(0.90)$$

(5.14)

$x_i > 0, i = 1, 2, 3.$ (5.15)

Also from the statistical table, we have

$$\chi^2_{12}(0.99) = 3.571, \chi^2_{12}(0.95) = 5.226, \chi^2_{12}(0.90) = 6.304.$$

Now using generic package LINGO 10.0 to solve (5.13) – (5.15), we get the following solution:

$z = 0.5988150 \text{ x } 10^{-15}, \; x_1 = 0.7406154 \text{ x } 10^{-8}, x_2 = 0.2324047 \text{ x } 10^{-7},$
$x_3 = 0.1960735 \text{ x } 10^{-8}$.

After getting the above solution, using MATLAB 7.6, we verified that, these solutions satisfy the given probabilistic constraints of the original problem (5.7) – (5.9) and thus may be considered as optimal solutions for the original probabilistic problem.

6: Conclusion

In this paper, we have transformed the probabilistic constraints to deterministic constraints for our convenience, which results in the change in the solution space. So one of the following cases may arise:

- modified solution space contains all the points of the original solution space
- modified solution space contains no points of the original solution space
- modified solution space contains some points of the original solution space and some other points

For this reason, we have verified that, whether the optimal solution obtained for the transformed problem, is an optimal solution for the given probabilistic model.

It can be noted that, there may be some other better optimal solutions, than what we have obtained because the solution region is changed at the time of deterministic reduction of the probabilistic constraints and some points, which are in original solution space, may not be in the reduced solution space. Moreover for CCP, exact optimal solution may not exist and the solution obtained is a compromised optimal solution.

Conflict of Interests

The authors declare that there is no conflict of interests.

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