Available online at http://scik.org
J. Math. Comput. Sci. 6 (2016), No. 3, 437-450
ISSN: 1927-5307

SOFT L-UNIFORMITIES AND SOFT L-NEIGHBORHOOD SYSTEMS

JUNG MI KO, YONG CHAN KIM*

Department of Mathematics, Gangneung-Wonju National University, Gangneung 210-702, Korea

Abstract. In this paper, we study the notions of soft L-quasi-uniformities in complete residuated lattices. We investigate the relations among soft L-topology, soft L-neighborhood systems and soft L-quasi-uniformities. We give their examples.

Keywords: Complete residuated lattices; Soft L- quasi-uniformities; L-neighborhood systems; Soft L-topologies

2000 AMS Subject Classification: 54A40, 03E72, 03G10, 06A15.

1. Introduction

Hájek [5] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structure of fuzzy contexts [6,7,12-15,28]. Many researcher introduced the notion of fuzzy uniformities in unit interval [0,1] [3,16], complete distributive lattices [8,31]. Recently, Molodtsov [22] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,4,12-14, 18-22, 29,30]. Pawlak's rough set [23,24] can be viewed as a

*Corresponding author.

Received October 27, 2015

special case of soft rough sets [5]. The topological structures of soft sets have been developed by many researchers [4,12-14,26-27].

Kim [13] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. Kim [12-14] introduced the soft topological structures, L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

In this paper, we study the notions of soft L-quasi-uniformities in complete residuated lattices. We investigate the relations among soft L-topology, soft L-neighborhood systems and soft L-quasi-uniformities. We give their examples.

2. Preliminaries

Definition 2.1. [2,6.7,28] An algebra $(L,\wedge,\vee,\odot,\to,0,1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L,\leq,\vee,\wedge,1,0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L,\odot,1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x,y,z \in L$.

In this paper, we assume that $(L,\leq,\odot,\to)$ is a complete residuated lattice and we denote $L_0 = L - \{0\}$.

Lemma 2.1. [2,6.7,28] For each $x,y,z,x_i,y_i,w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot (x \to y) \leq y$ and $x \to y \leq (y \to z) \to (x \to z)$,

(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$.

Definition 2.3. [13,14] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F,A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \to L^X$ is a mapping. We denote $S(X,A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 2.4.[13,14] Let (F,A) and (G,A) be two fuzzy soft sets over a common universe X.

(1) (F,A) is a fuzzy soft subset of (G,A), denoted by $(F,A) \leq (G,A)$ if $F(a) \leq G(a)$, for each $a \in A$.

(2) $(F,A) \wedge (G,A) = (F \wedge G,A)$ if $(F \wedge G)(a) = F(a) \wedge G(a)$ for each $a \in A$.

(3) $(F,A) \vee (G,A) = (F \vee G,A)$ if $(F \vee G)(a) = F(a) \vee G(a)$ for each $a \in A$.

(4) $(F,A) \odot (G,A) = (F \odot G,A)$ if $(F \odot G)(a) = F(a) \odot G(a)$ for each $a \in A$.

(6) $\alpha \odot (F,A) = (\alpha \odot F,A)$ for each $\alpha \in L$.

Definition 2.5. [13,14] Let $S(X,A)$ and $S(Y,B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X,A) \to S(Y,B)$ is a soft mapping where $f : X \to Y$ and $\phi : A \to B$ are mappings.

(1) The image of $(F,A) \in S(X,A)$ under the mapping f_ϕ is denoted by $f_\phi((F,A)) = (f_\phi(F),B)$ where

$$f_\phi(F)(b)(y) = \begin{cases} \bigvee_{a \in \phi^{-1}(\{b\})} f^{\to}(F(a))(y), & \text{if } \phi^{-1}(\{b\}) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

(2) The inverse image of $(G,B) \in S(Y,B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G,B)) = (f_\phi^{-1}(G),A)$ where

$$f_\phi^{-1}(G)(a)(x) = f^{\leftarrow}(G(\phi(a)))(x),\ \forall a \in A, x \in X.$$

(3) The soft mapping $f_\phi : S(X,A) \to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 2.6. [13,14] Let $f_\phi : S(X,A) \to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A),(F_i,A) \in S(X,A)$ and $(G,B),(G_i,B) \in S(Y,B)$,

(1) $(G,B) \geq f_\phi(f_\phi^{-1}((G,B)))$ with equality if f is surjective,

(2) $(F,A) \leq f_\phi^{-1}(f_\phi((F,A)))$ with equality if f is injective,

(3) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B)) = \bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,

(4) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B)) = \bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,

(5) $f_\phi(\bigvee_{i\in I}(F_i,A)) = \bigvee_{i\in I} f_\phi((F_i,A))$,

(6) $f_\phi(\bigwedge_{i\in I}(F_i,A)) \leq \bigwedge_{i\in I} f_\phi((F_i,A))$ with equality if f is injective,

(7) $f_\phi^{-1}((G_1,B)\odot(G_2,B)) = f_\phi^{-1}((G_1,B))\odot f_\phi^{-1}((G_2,B))$,

(8) $f_\phi((F_1,A)\odot(F_2,A)) \leq f_\phi((F_1,A))\odot f_\phi((F_2,A))$ with equality if f is injective.

Definition 2.7. [13,14] A map $\tau \subset S(X,A)$ is called a soft L topology on X if it satisfies the following conditions.

(ST1) $(0_X,A),(1_X,A) \in \tau$, where $0_X(a)(x) = 0, 1_X(a)(x) = 1$ for all $a \in A, x \in X$,

(ST2) If $(F,A),(G,A) \in \tau$, then $(F,A)\odot(G,A)) \in \tau$,

(ST3) If $(F_i,A) \in \tau$ for each $i \in I$, $\bigvee_{i\in I}(F_i,A) \in \tau$.

The triple (X,A,τ) is called a soft L-topological space.

A soft L-topology is called enriched if $\alpha\odot(F,A) \in \tau$ for each $(F,A) \in \tau$ and $\alpha \in L$.

Let (X,A,τ_1) and (X,A,τ_2) be soft L-fuzzy topological spaces. Then τ_1 is finer than τ_2 if $(F,A) \in \tau_1$, for all $(F,A) \in \tau_2$.

Let (X,A,τ_X) and (Y,B,τ_Y) be soft L-topological spaces and $f_\phi : S(X,A) \to S(Y,B)$ be a soft map. Then f_ϕ is called a continuous soft map if

$$f_\phi^{-1}((G,B)) \in \tau_X,\ \forall (G,B) \in \tau_Y.$$

Definition 2.8. [12] A map $N : X \to (L^A)^{S(X,A)}$ is called a soft L-neighborhood system on X if $N = \{N_x = N(x) \mid x \in X\}$ satisfies the following conditions

(SN1) $N_x((1_X,A)) = (1_X,A)(x) = 1_A$ and $N_x((1_X,A)) = (0_X,A)(x) = 0_A$,

(SN2) $N_x((F,A)\odot(G,A)) \geq N_x((F,A))\odot N_x((G,A))$ for each $(F,A),(G,A) \in S(X,A)$,

(SN3) If $(F,A) \leq (G,A)$, then $N_x((F,A)) \leq N_x((G,A))$,

(SN4) $N_x((F,A)) \leq (F,A)(x)$ for all $(F,A) \in S(X,A)$ where $(F,A)(x) = F(-)(x)$.

A soft L-neighborhood system is called stratified if

(S) $N_x(\alpha\odot(F,A)) \geq \alpha\odot N_x((F,A))$ for all $(F,A) \in S(X,A)$ and $\alpha \in L$.

The triple (X,A,N) is called a soft L-neighborhood space.

Let (X,A,N) and (Y,B,M) be soft L-neighborhood spaces. A mapping $f_\phi : (X,A,N) \to (Y,B,M)$ is said to be a continuous soft map iff $M_{f(x)}((G,B))(\phi(a)) \leq N_x(f_\phi^{-1}((G,B)))(a)$ for each $x \in X, a \in A, (G,B) \in S(Y,B)$.

3. Soft L-uniformities and soft L-neighborhood systems

Definition 3.1. A subset $\mathbf{U} \subset S(X \times X, A)$ is called a soft L-quasi-uniformity on X iff it satisfies the properties.

(SU1) $(1_{X\times X},A) \in \mathbf{U}$.

(SU2) If $(V,A) \leq (U,A)$ and $(V,A) \in \mathbf{U}$, then $(U,A) \in \mathbf{U}$.

(SU3) For every $(U,A),(V,A) \in \mathbf{U}$, $(U,A) \odot (V,A) \in \mathbf{U}$.

(SU4) If $(U,A) \in \mathbf{U}$ then $(1_\triangle,A) \leq (U,A)$ where

$$1_\triangle(a)(x,y) = \begin{cases} 1, & \text{if x = y,} \\ \perp, & \text{if } x \neq y, \end{cases}$$

(SU5) For every $(U,A) \in \mathbf{U}$, there exists $(V,A) \in \mathbf{U}$ such that $(V,A) \circ (V,A) \leq (U,A)$ where

$$((V,A) \circ (V,A))(a)(x,y) = (V(a) \circ V(a))(x,y)$$
$$= \bigvee_{z\in X}(V(a)(z,x) \odot V(a)(x,y)),\ \forall\, x,y \in X, a \in A.$$

The triple $(X,A,\mathbf{U})$ is called a soft L-quasi-uniform space.

A soft L-quasi-uniformity $\mathbf{U}$ is called stratified if $\alpha \odot (U,A) \in \mathbf{U}$ for all $\alpha \in L$ and $(U,A) \in \mathbf{U}$.

A soft L-quasi-uniformity $\mathbf{U}$ on X is said to be a soft L-uniformity if

(U) if $(U,A) \in \mathbf{U}$, then $(U^{-1},A) \in \mathbf{U}$ where $U^{-1}(a)(x,y) = U(a)(y,x)$.

Let $(X,A,\mathbf{U}_1)$ and $(Y,B,\mathbf{U}_2)$ be soft L-quasi-uniform spaces and $(f \times f)_\phi$ be a soft map. Then $(f \times f)_\phi$ is called an uniformly continuous soft map if, for all $(V,B) \in \mathbf{U}_2$, $(f \times f)_\phi^{-1}((V,B)) \in \mathbf{U}_2$.

Theorem 3.2. *Let $(X,A,\mathbf{U})$ be a soft L-quasi uniform space. Define two maps $rN^{\mathbf{U}}, lN^{\mathbf{U}} : X \to L^{L^X}$ by*

$$rN_x^{\mathbf{U}}((F,A))(a) = \bigvee_{(U,A)\in\mathbf{U}} (\bigwedge_{y\in X}(U(a)(y,x) \to F(a)(y))),\ \forall\, (F,A) \in S(X,A),\ x \in X,$$

$$lN_x^{\mathbf{U}}((F,A))(a) = \bigvee_{(U,A)\in\mathbf{U}} (\bigwedge_{y\in X} (U(a)(x,y) \to F(a)(y))), \ \forall\ (F,A) \in S(X,A),\ x \in X.$$

Then we have the following properties.

(1) $(X,A,rN^{\mathbf{U}})$ *is a stratified soft L-neighborhood space.*

(2) $(X,A,lN^{\mathbf{U}})$ *is a stratified soft L-neighborhood space.*

(3) $rN_x^{\mathbf{U}}((F,A))(a) = \bigvee \{\ G(a)(x) \mid (U,A)[(G,A)] \le (F,A) \mid (U,A) \in \mathbf{U}\ \} = \bigvee\{G(a)(x) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) \mid (U,A) \in \mathbf{U}\}$ *where*

$$(U,A)[(G,A)](a)(x) = \bigvee_{y\in X} U(a)(x,y) \odot (G,A)(y).$$

(4) $lN_x^{\mathbf{U}}((F,A)) = \bigvee \{\ (G,A)(x) \mid (U,A)[[(G,A)]] \le (F,A) \mid (U,A) \in \mathbf{U}\ \} = \bigvee\{G(a)(x) \odot \bigwedge_{y\in X}((U,A)[[(G,A)]](a)(y) \to F(a)(y)) \mid (U,A) \in \mathbf{U}\}$ *where*

$$(U,A)[[(G,A)]](x) = \bigvee_{y\in X} U(a)(y,x) \odot (G,A)(y).$$

Proof. (1) (SN1) For $(U,A)\mathbf{U}$, by (SU4), $(1_\triangle,A) \le (U,A)$. Then

$$\begin{aligned} rN_x^{\mathbf{U}}((0_X,A))(a) &= \bigvee_{(U,A)\in\mathbf{U}} \bigwedge_{y\in X}(U(a)(y,x) \to 0_X(y)) \\ &\le \bigvee_{(U,A)\in\mathbf{U}}(U(a)(x,x) \to 0) = \perp. \end{aligned}$$

Hence $rN_x^{\mathbf{U}}((0_X,A)) = (0_X,A)(x) = 0_A$. Also, $rN_x^{\mathbf{U}}((1_X,A)) = (1_X,A)(x) = 1_A$, because

$$rN_x^{\mathbf{U}}(1_X)(a) \ge \bigwedge_{y\in X} (1_\triangle(a)(x,y) \to 1_X(a)(y)) = 1.$$

(SN2) By Lemma 2.2 (11) , we have

$$\begin{aligned} & rN_x^{\mathbf{U}}((F,A))(a) \odot rN_x^{\mathbf{U}}((G,A))(a) = \big(\bigvee_{(U,A)\in\mathbf{U}} \bigwedge_{y\in X}(U(a)(y,x) \to F(a)(y))\big) \\ & \odot\big(\bigvee_{(V,A)\in\mathbf{U}} \bigwedge_{z\in X}(V(a)(z,x) \to G(a)(z))\big) \\ & \le \big(\bigvee_{U\odot(V,A)\in\mathbf{U}} \bigwedge_{y\in X}\big((U(a)(y,x) \to F(a)(y)) \odot (V(a)(y,x) \to G(a)(y))\big) \\ & \le \big(\bigvee_{U\odot(V,A)\in\mathbf{U}} \bigwedge_{y\in X}\big((U \odot V(a)(y,x) \to (F \odot G)(a)(y))\big) \\ & \le \bigvee_{W\in\mathbf{U}} \bigwedge_{y\in X}(W(a)(y,x) \to (F \odot G)(a)(y)) = rN_x^{\mathbf{U}}((F,A) \odot (G,A))(a). \end{aligned}$$

(SN3) It follows from the definition of $rN_x^{\mathbf{U}}$.

(SN4) For $(U,A)\in \mathbf{U}$, by (QU4), $(1_\triangle,A)\le (U,A)$. We have

$$\begin{aligned} rN_x^{\mathbf{U}}((F,A))(a) &= \bigvee_{(U,A)\in\mathbf{U}} \bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y)) \\ &\le \bigvee_{(U,A)\in\mathbf{U}}(U(a)(x,x)\to F(a)(x)) \le F(a)(x). \end{aligned}$$

(SN5)

$$\begin{aligned} & rN_x^{\mathbf{U}}((F,A))(a) = \bigvee_{(U,A)\in\mathbf{U}}(\bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y))) \\ & \le \bigvee_{(V,A)\in\mathbf{U}}(\bigwedge_{y\in X}((V\circ V)(a)(y,x)\to F(a)(y))) \\ & = \bigvee_{(V,A)\in\mathbf{U}}(\bigwedge_{y\in X}((\bigvee_{z\in X} V(a)(z,x)\odot V(a)(y,z))\to F(a)(y))) \\ & = \bigvee_{(V,A)\in\mathbf{U}}(\bigwedge_{y\in X}\bigwedge_{z\in X}((V(a)(z,x)\odot V(a)(y,z))\to F(a)(y))) \\ & \quad \text{(by Lemma 2.2 (9))} \\ & = \bigvee_{(V,A)\in\mathbf{U}}(\bigwedge_{y\in X}\bigwedge_{z\in X}(V(a)(z,x)\to (V(a)(y,z)\to F(a)(y)))) \\ & = \bigvee_{(V,A)\in\mathbf{U}}(\bigwedge_{z\in X}(V(a)(z,x)\to \bigwedge_{y\in X}(V(a)(y,z)\to F(a)(y)))). \end{aligned}$$

Put $G(a)(z)=\bigwedge_{y\in X}(V(a)(y,z)\to F(a)(y))$. Then $(G,A)(z)\le rN_z^{\mathbf{U}}((F,A))$ for all $z\in X$. Thus,

$$\begin{aligned} rN_x^{\mathbf{U}}((F,A))(a) &\le \bigvee_{(V,A)\in\mathbf{U}}\{\bigwedge_{z\in X}(V(a)(z,x)\to G(a)(z)) \mid (G,A)(z)\le N_z^{\mathbf{U}}((F,A))\} \\ &\le \bigvee_{(V,A)\in\mathbf{U}}\left\{rN_x^{\mathbf{U}}((G,A))(a) \mid (G,A)(z)\le N_z^{\mathbf{U}}((F,A))\right\}. \end{aligned}$$

Thus, $(X,A,rN^{\mathbf{U}})$ is a soft L-neighborhood space.

Since

$$\begin{aligned} & \alpha\odot U(a)(y,x)\odot \bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y)) \\ & \le \alpha\odot U(a)(y,x)\odot (U(a)(y,x)\to F(a)(y)) \le \alpha\odot F(a)(y), \end{aligned}$$

we have

$$\alpha\odot \bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y)) \le \bigwedge_{y\in X}(U(a)(y,x)\to \alpha\odot F(a)(y)).$$

Thus, $rN^{\mathbf{U}}$ is stratified from:

$$\begin{aligned} & \alpha\odot rN_x^{\mathbf{U}}((F,A))(a) = \alpha\odot \bigvee_{(U,A)\in\mathbf{U}}\bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y)) \\ & = \bigvee_{U\in\mathbf{U}}(\alpha\odot \bigwedge_{y\in X}(U(a)(y,x)\to F(a)(y))) \\ & \le \bigvee_{(U,A)\in\mathbf{U}}\bigwedge_{y\in X}(U(a)(y,x)\to (\alpha\odot F)(a)(y)) \\ & = rN_x^{\mathbf{U}}(\alpha\odot (F,A))(a). \end{aligned}$$

(2) It is similarly proved as (1).

(3) Put $\gamma=\bigvee\{G(a)(x)\mid (U,A)[(G,A)]\le (F,A)|(U,A)\in\mathbf{U}\}$. We show that $rN_x^{\mathbf{U}}((F,A))=\gamma$ from the following statements.

Let $G(a)(y) = \bigwedge_{x\in X}(U(a)(x,y) \to F(a)(x))$. Then

$$\begin{aligned}&(U,A)[(G,A)](a)(z) = \bigvee_{y\in X}(U(a)(z,y) \odot (G,A)(y))\\&= \bigvee_{y\in X}(U(a)(z,y) \odot (\bigwedge_{x\in X}(U(a)(x,y) \to F(a)(x))))\\&\le \bigvee_{y\in X}(U(a)(z,y) \odot (U(a)(z,y) \to F(a)(z))) \le F(a)(z).\end{aligned}$$

Hence $rN_x^{\mathbf{U}}((F,A)) \le \gamma$.

Let $(U,A)[(G,A)](a)(z) = \bigvee_{y\in X}(U(a)(z,y) \odot G(a)(y)) \le F(a)(z)$. Then

$$G(a)(y) \le \bigwedge_{z\in X}(U(a)(z,y) \to F(a)(z)).$$

Hence $rN_x^{\mathbf{U}}((F,A)) \ge \gamma$.

Put $\delta = \bigvee\{G(a)(x) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) \mid (U,A) \in \mathbf{U}\}$. We show that $\delta = \gamma$ from the following statements.

Let $(G,A) \in S(X,A)$ with $(U,A)[(G,A)] \le (F,A)$ and $(U,A) \in \mathbf{U}$. Then

$$\bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) = 1.$$

Hence $G(a)(x) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) = G(a)(x) \le \delta$. So, $\gamma \le \delta$.

Let $\delta = \bigvee\{G(a)(x) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) \mid (U,A) \in \mathbf{U}\}$. Since

$$\begin{aligned}&(U,A)[(G,A) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y))](a)(x)\\&= \bigvee_{y\in X}(U(a)(x,y) \odot (G,A)(y) \odot \bigwedge_{z\in X}((U,A)[(G,A)](a)(z) \to F(a)(z))\\&\le \bigvee_{y\in X}((U(a)(x,y) \odot (G,A)(y) \odot ((U,A)[(G,A)](a)(x) \to F(a)(x))\\&= (U,A)[(G,A)](a)(x) \odot ((U,A)[(G,A)](a)(x) \to F(a)(x)) \le F(a)(x).\end{aligned}$$

we have $(U,A)[(G,A) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y))] \le (F,A)$. Then

$$G(a)(x) \odot \bigwedge_{y\in X}((U,A)[(G,A)](a)(y) \to F(a)(y)) \le \gamma.$$

Thus, $\delta = \gamma$.

Theorem 3.3. *Let $(X,A,\mathbf{U})$ be a soft L-quasi-uniform space, $(X,A,rN^{\mathbf{U}})$ and $(X,A,lN^{\mathbf{U}})$ soft L-neighborhood spaces. Define $\tau^r_{\mathbf{U}}, \tau^l_{\mathbf{U}} \subset S(X,A)$ as follows*

$$\tau^r_{\mathbf{U}} = \{(F,A) \in S(X,A) \mid F(a)(x) = rN_x^{\mathbf{U}}((F,A))(a), \forall x \in X\},$$

$$\tau^l_{\mathbf{U}} = \{(F,A) \in S(X,A) \mid (F,A)(x) = lN^{\mathbf{U}}_x((F,A))(a), \forall x \in X\}.$$

Then,

(1) $\tau^r_{\mathbf{U}}$ is an enriched soft L-topology on X.

(2) $\tau^l_{\mathbf{U}}$ is an enriched soft L-topology on X.

(3) $rN^{\mathbf{U}} = N^{\tau^r_{\mathbf{U}}}$.

(4) $lN^{\mathbf{U}} = N^{\tau^l_{\mathbf{U}}}$.

Proof. (1) (ST1) Since $rN^{\mathbf{U}}_x((1_X,A)) = (1_X,A)(x) = 1_X(-)(x) = 1_A$ and $rN^{\mathbf{U}}_x((0_X,A)) = (0_X,A)(x) = 0_X(-)(x) = 0_A$, we have $(1_X,A),(0_X,A) \in \tau^r_{\mathbf{U}}$.

(ST2) Let $(F,A),(G,A) \in \tau^r_{\mathbf{U}}$. Since $rN^{\mathbf{U}}_x((F,A) \odot (G,A)) \geq rN^{\mathbf{U}}_x((F,A)) \odot rN^{\mathbf{U}}_x((G,A)) = ((F,A) \odot (G,A))(x)$ and (SN4), then $(F,A) \odot (G,A) \in \tau^r_{\mathbf{U}}$.

(ST3) Let $(F_i,A) \in \tau^r_{\mathbf{U}}$ for all $i \in \Gamma$. Since $rN^{\mathbf{U}}_x(\bigvee_{i\in\Gamma}(F_i,A)) \geq \bigvee_{i\in\Gamma} rN^{\mathbf{U}}_x((F_i,A)) = \bigvee_{i\in\Gamma}(F_i,A)$ and (SN4), then $\bigvee_{i\in\Gamma}(F_i,A) \in \tau^r_{\mathbf{U}}$.

Let $(F,A) \in \tau^r_{\mathbf{U}}$. Since $rN^{\mathbf{U}}_x(\alpha \odot (F,A)) \geq \alpha \odot rN^{\mathbf{U}}_x((F,A)) = \alpha \odot (F,A)(x)$ and (SN4), then $\alpha \odot (F,A) \in \tau^r_{\mathbf{U}}$. Hence $\tau^r_{\mathbf{U}}$ is an enriched soft L-topology on X.

(2) It is similarly proved as (1).

(3) Since $rN^{\mathbf{U}}_x((F,A)) \leq rN^{\mathbf{U}}_x(rN^{\mathbf{U}}_-((F,A))) \leq rN^{\mathbf{U}}_x((F,A))$ from (SN3) and (SN5), $rN^{\mathbf{U}}_x((F,A)) = rN^{\mathbf{U}}_x(rN^{\mathbf{U}}_-((F,A)))$ for all $x \in X$. Since $rN^{\mathbf{U}}_-((F,A)) \in \tau^r_{\mathbf{U}}$, by the definition of $N^{\tau^r_{\mathbf{U}}}$, $rN^{\mathbf{U}}_x((F,A)) \leq N^{\tau^r_{\mathbf{U}}}_x((F,A))$.

Since $N^{\tau^r_{\mathbf{U}}} = \bigvee\{(G_i,A)(x) \mid (G_i,A) \leq (F,A),\ (G_i,A) \in \tau^r_{\mathbf{U}}\}$ and $(G_i,A)(x) = rN^{\mathbf{U}}_x((G_i,A))$, then

$$\bigvee_i (G_i,A)(x) = \bigvee_i rN^{\mathbf{U}}_x((G_i,A)) \leq rN^{\mathbf{U}}_x(N^{\tau^r_{\mathbf{U}}}((F,A))) = rN^{\mathbf{U}}_x(\bigvee_i (G_i,A)) \leq \bigvee_i (G_i,A)(x).$$

Hence $rN^{\mathbf{U}}_x(N^{\tau^r_{\mathbf{U}}}((F,A))) = N^{\tau^r_{\mathbf{U}}}((F,A))$. Since $N^{\tau^r_{\mathbf{U}}}((F,A)) \leq (F,A)$, by (SN3), $N^{\tau^r_{\mathbf{U}}}((F,A)) = rN^{\mathbf{U}}_x(N^{\tau^r_{\mathbf{U}}}((F,A))) \leq rN^{\mathbf{U}}_x((F,A))$. So, $rN^{\mathbf{U}} = N^{\tau^r_{\mathbf{U}}}$.

(4) It is similarly proved as (3).

Theorem 3.4. *If $f_\phi : (X,A,\mathbf{U}) \to (Y,B,\mathbf{V})$ is an uniform continuous soft map, then*

(1) $f_\phi : (X,A,rN^{\mathbf{U}}) \to (Y,rN^{\mathbf{V}})$ is a continuous soft map.

(2) $f_\phi : (X,A,lN^{\mathbf{U}}) \to (Y,B,lN^{\mathbf{V}})$ is a continuous soft map.

(3) *a map $f_\phi : (X,A,\tau^r_{\mathbf{U}}) \to (Y,B,\tau^r_{\mathbf{V}})$ is a continuous soft map.*

(4) *a map* $f_\phi : (X,A,\tau^l_{\mathbf{U}}) \to (Y,B,\tau^l_{\mathbf{V}})$ *is a continuous soft map.*

Proof. (1) Since $(f\times f)^{-1}_\phi((V,B))(a)(x,z) = V(\phi(a))(f(x),f(z)))$, we have

$$\begin{aligned}
&\bigwedge_{y\in Y}(V(\phi(a))(y,f(x)) \to G(\phi(a))(y))\\
&\le \bigwedge_{z\in X}(V(\phi(a))(f(z),f(x)) \to G(\phi(a))(f(z)))\\
&= \bigwedge_{z\in X}((f\times f)^{-1}_\phi((V,B))(a)(z,x) \to f^{-1}_\phi((G,B))(a)(z))
\end{aligned}$$

$$\begin{aligned}
&rN^{\mathbf{V}}_{f(x)}((G,B))(\phi(a)) = \bigvee_{(V,A)\in\mathbf{V}}(\bigwedge_{z\in Y}(V(\phi(a))(z,f(x)) \to G(\phi(a))(z)))\\
&\le \bigvee_{(f\times f)^{-1}_\phi((V,A))\in\mathbf{U}}((f\times f)^{-1}_\phi((V,B))(a)(z,x) \to f^{-1}_\phi((G,B))(a)(z))\\
&\le rN^{\mathbf{U}}_x(f^{-1}_\phi((G,B)))(a).
\end{aligned}$$

(2) It is similarly proved as (1).

(3) Let $(G,B)\in\tau^r_{\mathbf{V}}$. Then $(G,B) = rN^{\mathbf{V}}_-((G,B))$. Then

$$f^{-1}_\phi((G,B)) = f^{-1}_\phi(rN^{\mathbf{V}}_-((G,B))).$$

Since

$$\begin{aligned}
&f^{-1}_\phi(rN^{\mathbf{V}}_-((G,B)))(a)(x) = rN^{\mathbf{V}}_-((G,B))(\phi(a))(f(x))\\
&= rN^{\mathbf{V}}_{f(x)}((G,B))(\phi(a)) \le rN^{\mathbf{U}}_x(f^{-1}_\phi((G,B)))(a) \ \text{(by (1))}\\
&= rN^{\mathbf{U}}_-(f^{-1}_\phi((G,B)))(a)(x),
\end{aligned}$$

then $f^{-1}_\phi(rN^{\mathbf{V}}_-((G,B))) \le rN^{\mathbf{U}}_-(f^{-1}_\phi((G,B)))$. Thus
$f^{-1}_\phi((G,B)) = f^{-1}_\phi(rN^{\mathbf{V}}_-((G,B))) \le rN^{\mathbf{U}}_-(f^{-1}_\phi((G,B)))$.
By (SN3), $f^{-1}_\phi((G,B)) = rN^{\mathbf{U}}_-(f^{-1}_\phi((G,B)))$. Hence $f^{-1}_\phi((G,B))\in\tau^r_{\mathbf{U}}$.

(4) It is similarly proved as (3).

Example 3.5. Let $X = \{h_i \mid i = \{1,...,4\}\}$ with h_i=house and $E_Y = \{e,b,w,c,i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Let $(L=[0,1],\odot,\to)$ be a complete residuated lattice defined by

$$x\odot y = x\wedge y,\ \ x\to y = \begin{cases} 1, & \text{if } x\le y,\\ y, & \text{otherwise.}\end{cases}$$

Let $X=\{x,y,z\}$ be a set and $W(e),W(b)\in L^{X\times X}$ such that

$$W(e)=\begin{pmatrix} 1 & 0.2 & 0.5 \\ 0.7 & 1 & 0.3 \\ 0.4 & 0.6 & 1 \end{pmatrix} W(b)=\begin{pmatrix} 1 & 0.6 & 0.8 \\ 0.4 & 1 & 0.4 \\ 0.5 & 0.5 & 1 \end{pmatrix}.$$

Define $\mathbf{U}=\{(U,A)\in S(X\times X,A)\mid (U,A)\geq (W,A)\}$.

(1) Since $W(e)\circ W(e)=W(e)$ and $W(b)\circ W(b)=W(b)$, $\mathbf{U}$ is a soft L-quasi-uniformity on X.

(2) Since $rN_x^{\mathbf{U}}((F,A))(e)=\bigvee_{(U,A)\in\mathbf{U}}\bigwedge_{y\in X}(U(e)(y,x)\to F(e)(y))$, we have

$$\begin{aligned}
rN_x^{\mathbf{U}}((F,A))(e)&=\bigvee_{(U,A)\in\mathbf{U}}\bigwedge_{y\in X}(U(e)(y,x)\to F(e)(y))\\
&=(F(e)(x)\wedge(0.7\to F(e)(y))\wedge(0.4\to F(e)(z)),\\
rN_y^{\mathbf{U}}((F,A))(e)&=(0.2\to F(e)(x))\wedge F(e)(y)\wedge(0.6\to F(e)(z)),\\
rN_z^{\mathbf{U}}((F,A))(e)&=(0.5\to F(e)(x))\wedge(0.3\to F(e)(y))\wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
rN_x^{\mathbf{U}}((F,A))(b)&=(F(b)(x)\wedge(0.4\to F(b)(y))\wedge(0.5\to F(b)(z)),\\
rN_y^{\mathbf{U}}((F,A))(b)&=(0.6\to F(b)(x))\wedge F(b)(y)\wedge(0.5\to F(b)(z)),\\
rN_z^{\mathbf{U}}((F,A))(b)&=(0.8\to F(b)(x))\wedge(0.4\to F(b)(y))\wedge F(b)(z),
\end{aligned}$$

(3) Since $lN_x^{\mathbf{U}}((F,A))(e)=\bigvee_{(U,A)\in\mathbf{U}}(\bigwedge_{y\in X}(U(e)(x,y)\to F(e)(y))$, we have

$$\begin{aligned}
lN_x^{\mathbf{U}}((F,A))(e)&=(F(e)(x)\wedge(0.2\to F(e)(y))\wedge(0.5\to F(e)(z)),\\
lN_y^{\mathbf{U}}((F,A))(e)&=(0.7\to F(e)(x))\wedge F(e)(y)\wedge(0.3\to F(e)(z)),\\
lN_z^{\mathbf{U}}((F,A))(e)&=(0.4\to F(e)(x))\wedge(0.6\to F(e)(y))\wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
lN_x^{\mathbf{U}}((F,A))(b)&=(F(b)(x)\wedge(0.6\to F(b)(y))\wedge(0.8\to F(b)(z)),\\
lN_y^{\mathbf{U}}((F,A))(b)&=(0.4\to F(b)(x))\wedge F(b)(y)\wedge(0.4\to F(b)(z)),\\
lN_z^{\mathbf{U}}((F,A))(b)&=(0.5\to F(b)(x))\wedge(0.5\to F(b)(y))\wedge F(b)(z),
\end{aligned}$$

(3) Since $\tau^r_{\mathbf{U}} = \{(F,A) \in L^X \mid (F,A)(x) = rN^{\mathbf{U}}_x((F,A)), \forall x \in X\}$, we have

$$(F,A) \in \tau^r_{\mathbf{U}} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \le 0.7 \to F(e)(y), F(e)(x) \le 0.4 \to F(e)(z), \\ F(b)(x) \le 0.4 \to F(b)(y), F(b)(x) \le 0.5 \to F(b)(z), \\ F(e)(y) \le 0.2 \to F(e)(x), (F(e)(y) \le 0.6 \to F(e)(z), \\ F(b)(y) \le 0.6 \to F(b)(x), F(b)(y) \le 0.5 \to F(b)(z), \\ F(e)(z) \le 0.5 \to F(e)(x), F(e)(z) \le 0.3 \to F(e)(y), \\ F(b)(z) \le 0.8 \to F(b)(x), F(b)(z) \le 0.4 \to F(b)(y). \end{cases}$$

$$(F,A) \in \tau^l_{\mathbf{U}} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \le 0.2 \to F(e)(y), F(e)(x) \le 0.5 \to F(e)(z), \\ F(b)(x) \le 0.6 \to F(b)(y), F(b)(x) \le 0.8 \to F(b)(z), \\ F(e)(y) \le 0.7 \to F(e)(x), (F(e)(y) \le 0.3 \to F(e)(z), \\ F(b)(y) \le 0.4 \to F(b)(x), F(b)(y) \le 0.4 \to F(b)(z), \\ F(e)(z) \le 0.4 \to F(e)(x), F(e)(z) \le 0.6 \to F(e)(y), \\ F(b)(z) \le 0.5 \to F(b)(x), F(b)(z) \le 0.5 \to F(b)(y). \end{cases}$$

For $F(e) = (0.6, 0.7, 0.6), F(b) = (0.6, 0.5, 0.6)$, we have $(F,A) \in \tau^r_{rN^{\mathbf{U}}}, (F,A) \notin \tau^l_{lN^{\mathbf{U}}}$ because

$$0.7 = F(e)(y) \not\le 0.7 \to F(e)(x) = 0.6.$$

For $G(e) = (0.6, 0.6, 0.7), G(b) = (0.5, 0.5, 0.6)$, we have $(G,A) \notin \tau^r_{rN^{\mathbf{U}}}, (G,A) \in \tau^l_{lN^{\mathbf{U}}}$ because

$$0.6 = G(b)(z) \not\le 0.8 \to G(b)(x) = 0.5.$$

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] K.V. Babitha, J.J. Sunil, Soft set relations and functions, Comput. Math. Appl. 60 (2010), 1840-1849.

[2] R. Bělohlávek, Fuzzy Relational Systems, Kluwer Academic Publishers, New York, 2002.

[3] M.H. Burton, The relationship between a fuzzy uniformity and its family of α-level uniformities, Fuzzy Sets and Systems, 54 (1993), 311-316.

[4] N. Cağman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. 62 (2011), 351-358.

[5] F. Feng, X. Liu, V.L. Fotea, Y.B. Jun, Soft sets and soft rough sets, Information Sci. 181 (2011), 1125-1137.

[6] P. Hájek, Metamathematices of Fuzzy Logic, Kluwer Academic Publishers, Dordrecht, 1998.

[7] U. Höhle, S.E. Rodabaugh, Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory, The Handbooks of Fuzzy Sets Series 3, Kluwer Academic Publishers, Boston, 1999.

[8] B. Hutton, uniformities in fuzzy topological spaces, J. Math. Anal. Appl. 58 (1977), 74-79.

[9] A. Kharal, B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. 7 (2011), 471-481.

[10] A.K. Katsaras, On fuzzy uniform spaces, J. Math. Anal. Appl. 101 (1984), 97-113.

[11] Y.C. Kim, A.A. Ramadan, M. A. Usama, L-fuzzy uniform Spaces, The Journal of Fuzzy Mathematics, 14 (2006), 821-850.

[12] Y.C. Kim and J.M. Ko, Soft L-topologies and soft L-neighborhood systems, (submit to) J. Math. Comput. Sci.

[13] Y.C. Kim and J.M. Ko, Soft L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders, (submit to) J. Intelligent and Fuzzy Systems.

[14] Y.C. Kim and J.M. Ko, Soft implicative L-fuzzy interior and closure operators, (submit to) J. Intelligent and Fuzzy Systems.

[15] W. Kotzé, uniform spaces, Chapter 8, 553-580 in [7].

[16] R. Lowen, Fuzzy uniform spaces, J. Math. Anal. Appl. 82 (1981), 370-385.

[17] R. Lowen, Fuzzy neighborhood spaces, Fuzzy Sets and Systems 7 (1982), 165-189.

[18] P.K. Maji, R. Biswas, R.Roy, Soft set theory, Comput. Math. Appl. 45 (2003), 555-562.

[19] P.K. Maji, R. Biswas, R.Roy, An application of soft sets in decision making problems, Comput. Math. Appl. 44 (2002), 1077-1083.

[20] P.K. Maji, R.Roy, A fuzzy set theoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2007), 412-418.

[21] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59 (2010), 1425-1432.

[22] D. Molodtsov, Soft set theory, Comput. Math. Appl. 37(1999), 19-31.

[23] Z. Pawlak, Rough sets, Int. J. Comput. Inf. Sci. 11 (1982), 341-356.

[24] Z. Pawlak, Rough probability, Bull. Pol. Acad. Sci. Math. 32 (1984), 607-615.

[25] S.E. Rodabaugh, E.P. Klement, Topological and Algebraic Structures In Fuzzy Sets, The Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Kluwer Academic Publishers, Boston, Dordrecht, London, 2003.

[26] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011), 1786-1799.

[27] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math. Appl. 61(10) (2011), 2952-2957.

[28] E. Turunen, Mathematics Behind Fuzzy Logic, A Springer-Verlag Co., Heidelberg, 1999.

[29] J. M. Zhan and Y. B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl. 59 (2010), 2037-2046.

[30] W.Z. Wu and W.X. Zhang, Neighborhood operator systems and approximations , Information Sci. 144 (2002), 201-217.

[31] D. Zhang, Stratified Hutton uniform spaces, Fuzzy Sets and Systems 131 (2002), 337-346.

[32] Hu Zhao and Sheng-Gang Li, L-fuzzifying soft topological spaces and L-fuzzifying soft interior operators, Ann. Fuzzy Math. Inform. 5(3) (2013), 493-503.

[33] Í. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2012), 171-185.