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L-FUZZY (K,E)-SOFT PREINTERIOR OPERATORS INDUCED BY L-FUZZY (K,E)-SOFT QUASI-UNIFORM SPACES

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Abstract. We investigate two L-fuzzy (K,E)-soft preinterior spaces induced by an L-fuzzy (K,E)-soft quasi-uniform space. Also, we study the relationship among L-fuzzy (K,E)-soft preinterior spaces, L-fuzzy (K,E)-soft preinterior spaces and L-fuzzy (K,E)-soft quasi-uniform space. Finally, we give their examples.

Keywords: complete residuated lattice, L-fuzzy (K,E)-soft preinterior space, L-fuzzy (K,E)-soft quasi-uniform space, L-fuzzy (K,E)-soft topological space

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1. Introduction

Molodtsov [15,16] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1-3,7].The topological structures of soft sets have been developed by many researchers [4,5,17-20,23].

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On the other hand, Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structures [9-10,21,22]. Ramadan et al.[19,20] investigated the relationships between L-fuzzy (K,E)-soft quasi-uniform structures and L-fuzzy (K,E)-soft topological structures in a complete residuated lattice.

We investigate two L-fuzzy (K,E)-soft preinterior spaces induced by an L-fuzzy (K,E)-soft quasi-uniform space. Also, we study the relationship among L-fuzzy (K,E)-soft preinterior spaces, L-fuzzy (K,E)-soft preinterior spaces and L-fuzzy (K,E)-soft quasi-uniform space. Finally, we give their examples.

2. Preliminaries

Definition 2.1. [8,9] An algebra $(L,\wedge,\vee,\odot,\to,0,1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $(L,\leq,\vee,\wedge,0,1)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L,\odot,1)$ is a commutative monoid;

(C3) $x\odot y\leq z$ iff $x\leq y\to z$ for $x,y,z\in L$.

In this paper, we assume that $L=(L,\vee,\wedge,\odot,\to,0,1)$ be a complete residuated lattice.

Lemma 2.2. [8,9] For each $x,y,z,w,x_i,y_i\in L$, the following properties hold.

(1) If $y\leq z$, then $x\odot y\leq x\odot z$.

(2) If $y\leq z$, then $x\to y\leq x\to z$ and $z\to x\leq y\to x$.

(3) $x\to y=1$ iff $x\leq y$.

(4) $x\to 1=1$ and $1\to x=x$.

(5) $x\odot y\leq x\wedge y$.

(6) $x\odot(\bigvee_{i\in\Gamma}y_i)=\bigvee_{i\in\Gamma}(x\odot y_i)$ and $(\bigvee_{i\in\Gamma}x_i)\odot y=\bigvee_{i\in\Gamma}(x_i\odot y)$.

(7) $x\to(\bigwedge_{i\in\Gamma}y_i)=\bigwedge_{i\in\Gamma}(x\to y_i)$ and $(\bigvee_{i\in\Gamma}x_i)\to y=\bigwedge_{i\in\Gamma}(x_i\to y)$.

(8) $\bigvee_{i\in\Gamma}x_i\to\bigvee_{i\in\Gamma}y_i\geq\bigwedge_{i\in\Gamma}(x_i\to y_i)$ and $\bigwedge_{i\in\Gamma}x_i\to\bigwedge_{i\in\Gamma}y_i\geq\bigwedge_{i\in\Gamma}(x_i\to y_i)$.

(9) $(x\to y)\odot x\leq y$ and $(x\to y)\odot(y\to z)\leq(x\to z)$.

(10) $x \to y \le (y \to z) \to (x \to z)$ and $x \to y \le (z \to x) \to (z \to y)$.

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$.

(12) $y \to z \le x \odot y \to x \odot z$ and $(x \to z) \odot (y \to w) \le x \odot y \to z \odot w$.

A lattice L is called s-compact if $\bigvee_{j \in \Gamma} c_j \ge a$ for $c_j, a \in L$, there exists $j_\circ \in \Gamma$ such that $c_{j_\circ} \ge a$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.3. [4-6] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \le g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(6) 0_X (resp. 1_X) is an L-fuzzy soft set if $(0_X)_e(x) = 0$ (resp. $(1_X)_e(x) = 1$), for each $e \in E$, $x \in X$.

Definition 2.4. [4] Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X,E) \to (Y,K)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X,E) into (Y,K).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by

$$\varphi(f)_b(y)=\bigvee_{\varphi(x)=y}\Big(\bigvee_{\psi(e)=b} f_e(x)\Big).$$

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x)=g_{\psi(e)}(\varphi(x)),\forall e\in E,\forall x\in X.$$

Definition 2.5. [4-6, 19-20] A mapping $\mathscr{T}:K\to L^{(L^X)^E}$ (where $\mathscr{T}_k:=\mathscr{T}(k):(L^X)^E\to L$ is a mapping for each $k\in K$) is called an L-fuzzy (K,E)-soft topology on X if it satisfies the following conditions for each $k\in K$.

(SO1) $\mathscr{T}_k(0_X)=\mathscr{T}_k(1_X)=1$,

(SO2) $\mathscr{T}_k(f\odot g)\geq \mathscr{T}_k(f)\odot\mathscr{T}_k(g)\ \forall\ f,g\in (L^X)^E$,

(SO3) $\mathscr{T}_k(\bigsqcup_i f_i)\geq \bigwedge_{i\in I}\mathscr{T}_k(f_i)\ \forall\ f_i\in (L^X)^E,\ i\in I$.

The pair $(X,\mathscr{T})$ is called an L-fuzzy (K,E)-soft topological space.

An L-fuzzy (K,E)-soft topology is called enriched if

(SR) $\mathscr{T}_k(\alpha\odot f)\geq\mathscr{T}_k(f)$ for all $f\in (L^X)^E$ and $\alpha\in L$.

Let $\varphi:X\to Y$, $\psi:E_1\to E_2$ and $\eta:K_1\to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X,\mathscr{T}^1)$ into $(Y,\mathscr{T}^2)$ is called L-fuzzy soft continuous if

$$\mathscr{T}^2_{\eta(k)}(f)\leq\mathscr{T}^1_k(\varphi^{-1}_{\psi,\eta}(f))\ \forall\ f\in (L^Y)^{E_2},k\in K_1.$$

Lemma 2.6. [19-20] Define a binary mapping $S:L^X\times L^X\to L$ by

$$S(f_e,g_e)=\bigwedge_{x\in X}(f_e(x)\to g_e(x)),\ \forall\ e\in E.$$

Then $\forall f,g,h,m,n\in (L^X)^E$ the following statements hold.

(1) $f\sqsubseteq g$ iff $S(f_e,g_e)=1,\ \forall\ e\in E$.

(2) If $f\sqsubseteq g$, then $S(h_e,f_e)\leq S(h_e,g_e)$ and $S(f_e,h_e)\geq S(g_e,h_e),\ \forall\ e\in E$.

(3) $S(f_e,h_e)\odot S(h_e,g_e)\leq S(f_e,g_e)$. Moreover, $\bigvee_{h_e\in L^X}(S(f_e,h_e)\odot S(h_e,g_e))=S(f_e,g_e),\ \forall\ e\in E$.

(4) $S(f_e,g_e)\odot S(m_e,n_e)\leq S(f_e\odot m_e,g_e\odot n_e),\forall\ e\in E$.

(5) If $\varphi_\psi : (X,E) \to (Y,F)$ is a fuzzy soft mapping, then $S(f_{\psi(e)}, g_{\psi(e)}) \leq S(\varphi_\psi^{-1}(f)_e, \varphi_\psi^{-1}(g)_e)$, for each $f,g \in (L^Y)^F$.

Definition 2.7. [19-20] An L- fuzzy (K,E)-soft quasi- uniformity is a mapping $\mathscr{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathscr{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathscr{U}_k(v) \leq \mathscr{U}_k(u)$.

(SU3) For every $u,v \in (L^{X\times X})^E, \mathscr{U}_k(u\odot v) \geq \mathscr{U}_k(u) \odot \mathscr{U}_k(v)$.

(SU4) If $\mathscr{U}_k(u) \neq 0$ then $1_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(1_\triangle)_e(x,y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathscr{U}_k(u) \leq \bigvee\{\mathscr{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x,z) = \bigvee_{y\in X} v_e(x,y) \odot w_e(y,z),$$

The pair $(X,\mathscr{U})$ is called an L-fuzzy (K,E)-soft quasi-uniform space.

An L-fuzzy (K,E)-soft quasi-uniform space $(X,\mathscr{U})$ is said to be an L-fuzzy (K,E)-soft uniform space if

(U) $\mathscr{U}_k(u) \leq \mathscr{U}_k(u^{-1})$, where $(u^{-1})_e(x,y) = u_e(y,x)$ for each $k \in K$ and $u \in (L^{X\times X})^E$.

An L-fuzzy (K,E)-soft quasi-uniformity $\mathscr{U}$ on X is said to be stratified if

(SR) $\mathscr{U}_k(\alpha \odot u) \geq \alpha \odot \mathscr{U}_k(u),\ \forall\, u \in (L^{X\times X})^E, \alpha \in L$.

Let $(X,\mathscr{U}^1)$ be an L-fuzzy (K_1,E_1)-soft quasi-uniform space and $(Y,\mathscr{U}^2)$ be an L-fuzzy (K_2,E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X,\mathscr{U}^1)$ into $(Y,\mathscr{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathscr{U}^2_{\eta(k)}(v) \leq \mathscr{U}^1_k((\varphi\times\varphi)_\psi^{-1}(v))\ \forall\, v \in (L^{Y\times Y})^{E_2}, k \in K_1.$$

3. L-fuzzy (K,E)-soft preinterior space and L-fuzzy (K,E)-soft quasi-uniform space

Definition 3.1. A map $\mathscr{I} : K \times (L^X)^E \to (L^X)^E$ is called an L-fuzzy (K,E)-soft pre-interior operator if it satisfies the following conditions:

(LI1) $\mathscr{I}(k,0_X) = 0_X$ and $\mathscr{I}(k,1_X) = 1_X$,

(LI2) $\mathscr{I}(k, f \odot g) \geq \mathscr{I}(k,f) \odot \mathscr{I}(k,g)$ for each $f,g \in (L^X)^E$,

(LI3) If $f \sqsubseteq g$, then $\mathscr{I}(k,f) \leq \mathscr{I}(k,g)$,

(LI4) $\mathscr{I}(k,f) \leq f_e(x)$ for all $f \in (L^X)^E$ and $e \in E$.

The pair $(X,\mathscr{I})$ is called an L-fuzzy (K,E)-soft pre interior space.

An L-fuzzy (K,E)-soft pre interior space is called stratified if

(R) $\mathscr{I}(k,\alpha \odot f) \geq \alpha \odot \mathscr{I}(k,f)$ for all $f \in (L^X)^E, k \in K$ and $\alpha \in L$.

Let $(X,\mathscr{I}_X)$ be (K_1,E_1)-soft interior space and $(X,\mathscr{I}_Y)$ be be (K_2,E_2)-soft interior space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta} : (X,\mathscr{I}_X) \to (X,\mathscr{I}_Y)$ is called an L- fuzzy soft interior map if

$$\varphi^{-1}_{\psi,\eta}(\mathscr{I}_Y(\eta(k),g)) \leq \mathscr{I}_X(k,\varphi^{-1}_{\psi,\eta}(g)),\ \forall\, g \in (L^Y)^{E_2}.$$

Theorem 3.2. Let $(X,\mathscr{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathscr{I}^{\mathscr{U}}_l : K \times (L^X)^E \to (L^X)^E$ by:

$$\mathscr{I}^{\mathscr{U}}_l(k,f)_e(x) = \bigvee_u \mathscr{U}_k(u) \odot S(u_e[x], f_e),\ \forall\, f \in (L^X)^E, e \in E,\ x \in X,$$

where $u_e[x](y) = u_e(y,x)$. Then, $(X,\mathscr{I}^{\mathscr{U}}_l)$ is an L-fuzzy (K,E)-soft preinterior space. If $\mathscr{U}$ is stratified, then $\mathscr{I}^{\mathscr{U}}_l$ is also stratified.

Proof. (LI1) For $\mathscr{U}_k(u) \neq 0$, $1_\triangle \sqsubseteq u$. For each $e \in E$,

$$\begin{aligned}\mathscr{I}^{\mathscr{U}}_l(k,0_X)_e(x) &= \bigvee_u \mathscr{U}_k(u) \odot S(u_e[x], (0_X)_e)\\ &\leq \bigvee_u \big(\mathscr{U}_k(u) \odot (u_e(x,x) \to 0)\big)\\ &= \bigvee_u \big(\mathscr{U}_k(u) \odot ((1_\triangle)_e(x,x) \to 0)\big) = 0.\end{aligned}$$

Hence, $\mathscr{I}_l^{\mathscr{U}}(k,0_X)=0_X$. Also, $\mathscr{I}_l^{\mathscr{U}}(k,1_X)=1_X$, because

$$\mathscr{I}_l^{\mathscr{U}}(k,1_X)_e(x)\geq \mathscr{U}_k(1_{X\times X})\odot\bigwedge_{y\in X}((1_{X\times X})_e(x,y)\to(1_X)_e(y))=1.$$

(LI2) By Lemma 2.6 (4) , we have

$$\begin{aligned}
&\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)\odot\mathscr{I}_l^{\mathscr{U}}(k,g)_e(x)\\
&=\big(\bigvee_u \mathscr{U}_k(u)\odot S(u_e[x],f_e)\big)\odot\big(\bigvee_v \mathscr{U}_k(v)\odot S(v_e[x],g_e)\big)\\
&=\bigvee_{u,v}\mathscr{U}_k(u)\odot\mathscr{U}_k(v)\odot S(u_e[x],f_e)\odot S(v_e[x],g_e)\\
&\leq\bigvee_{u,v}\mathscr{U}_k(u\odot v)\odot S((u\odot v)_e[x],f_e\odot g_e)\\
&\leq\bigvee_{w}\mathscr{U}_k(w)\odot S(w_e[x],(f\odot g)_e)=\mathscr{I}_l^{\mathscr{U}}(k,(f\odot g)_e)(x).
\end{aligned}$$

(LI3) By Lemma 2.6 (3), we have

$$\begin{aligned}
\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)&=\bigvee_u \mathscr{U}_k(u)\odot S(u_e[x],f_e)\\
&\leq\bigvee_u \mathscr{U}_k(u)\odot S(u_e[x],g_e)=\mathscr{I}_l^{\mathscr{U}}(k,g)_e(x).
\end{aligned}$$

(LI4) For $\mathscr{U}_k(u)\neq 0,\ 1_\triangle\sqsubseteq u$.

$$\begin{aligned}
\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)&=\bigvee_u \mathscr{U}_k(u)\odot\bigwedge_{y\in X}(u_e(y,x)\to f_e(y))\\
&\leq\bigvee_u\big\{\mathscr{U}_k(u)\odot(u_e(x,x)\to f_e(x))\big\}\leq f_e(x).
\end{aligned}$$

This implies that $(X,\mathscr{I}_l^{\mathscr{U}})$ is an L-fuzzy (K,E)-soft preinterior space.

(R)

$$\begin{aligned}
&\alpha \odot \mathscr{I}_l^{\mathscr{U}}(k,f)_e(x) = \alpha \odot \bigvee_u \mathscr{U}_k(u) \odot S(u_e[x], f_e) \\
&= \bigvee_u \alpha \odot \mathscr{U}_k(u) \odot S(\alpha,\alpha) \odot S(u_e[x], f_e) \\
&\leq \bigvee_u \mathscr{U}_k(\alpha \odot u) \odot S(\alpha \odot u_e[x], \alpha \odot f_e) \\
&\leq \mathscr{I}_l^{\mathscr{U}}(k, \alpha \odot f)_e(x).
\end{aligned}$$

Corollary 3.3. Let $(X,\mathscr{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathscr{I}_r^{\mathscr{U}} : K \times (L^X)^E \to (L^X)^E$ by:

$$\mathscr{I}_r^{\mathscr{U}}(k,f)_e(x) = \bigvee_u \mathscr{U}_k(u) \odot S(u_e[[x]], f_e),\ \forall\, f \in (L^X)^E, e \in E,\ x \in X,$$

where $u_e[[x]](y) = u_e(x,y)$. Then, $(X,\mathscr{I}_r^{\mathscr{U}})$ is an L-fuzzy (K,E)-soft preinterior space. If $\mathscr{U}$ is stratified, then $\mathscr{I}_r^{\mathscr{U}}$ is also stratified.

Theorem 3.4. (1) The L-fuzzy (K,E)-soft preinterior operator $\mathscr{I}_l^{\mathscr{U}}$ can be constructed from the cuts $\mathscr{U}_\alpha$, $\alpha > 0$, of the L-fuzzy quasi-uniformity by using the equality

$$\mathscr{I}_l^{\mathscr{U}}(k,f) = \bigvee_{\alpha \in L} \alpha \odot \mathscr{I}_l^{\mathscr{U}}(k,f,\alpha),$$

where $\mathscr{I}_l^{\mathscr{U}}(k,f,\alpha)$ is defined as

$$\mathscr{I}_l^{\mathscr{U}}(k,f,\alpha)_e(x) = \bigvee_{\mathscr{U}_k(u)\geq\alpha} (S(u_e[x], f_e).$$

(2)

$$\mathscr{I}_l^{\mathscr{U}}(k,f) \leq \mathscr{I}_l^{\mathscr{U}}(k, \mathscr{I}_l^{\mathscr{U}}(k,f,\mathscr{U}_k(v)).$$

Proof. (1) If for some $y \in X$ we have $A(y) \geq \alpha$, then we can write $A(y) \odot B(y) \geq \alpha \odot B(y)$ and

$$\bigvee\{A(x) \odot B(x) \mid A(x) \geq \alpha\} \geq \bigvee\{\alpha \odot B(y) \mid A(x) \geq \alpha\}.$$

Suppose

$$\bigvee\{A(x)\odot B(x)\mid x\in X\}\not\leq\bigvee_{\alpha\in L}\bigvee\{\alpha\odot B(x)\mid A(x)\geq\alpha\}.$$

There exists $x_0\in X$ such that

$$A(x_0)\odot B(x_0)\not\leq\bigvee_{\alpha\in L}\bigvee\{\alpha\odot B(x)\mid A(x)\geq\alpha\}.$$

It is a contradiction. Hence

$$\bigvee\{A(x)\odot B(x)\mid x\in X\}=\bigvee_{\alpha\in L}\bigvee\{\alpha\odot B(x)\mid A(x)\geq\alpha\}.$$

Applying this equality to the formula giving $\mathscr{I}_l^{\mathscr{U}}(k,f)$, we obtain

$$\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)=\bigvee_{\alpha\in L}\{\bigvee\alpha\odot S(u_e[x],f_e)\mid\mathscr{U}_k(u)\geq\alpha\}=\bigvee_{\alpha\in L}\alpha\odot\mathscr{I}_l^{\mathscr{U}}(k,f,\alpha)_e(x).$$

(2) For $u\in(L^{X\times X})^E$ and $f\in(L^X)^E$, we have

$$\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)=\bigvee_u\mathscr{U}_k(u)\odot S(u_e[x],f_e)$$

$$=\bigvee_u\big\{\mathscr{U}_k(u)\odot\bigwedge_{y\in X}(u_e(y,x)\to f_e(y))\big\}$$

$$\leq\bigvee_v\big\{\mathscr{U}_k(v)\odot\bigwedge_{y\in X}((v\circ v)_e(y,x)\to f_e(y))\big\}\text{ (by (SU5))}$$

$$=\bigvee_v\big\{\mathscr{U}_k(v)\odot\bigwedge_{y\in X}((\bigvee_{z\in X}v_e(z,x)\odot v_e(y,z))\to f_e(y))\big\}$$

$$=\bigvee_v\big\{\mathscr{U}_k(v)\odot\bigwedge_{y\in X}\bigwedge_{z\in X}((v_e(z,x)\odot v_e(y,z))\to f_e(y))\big\}$$

$$=\bigvee_v\big\{\mathscr{U}_k(v)\odot\bigwedge_{y\in X}\bigwedge_{z\in X}(v_e(z,x)\to(v_e(y,z)$$

$$\to f_e(y)))\big\}\text{ (by Lemma 2.2 (12))}$$

$$=\bigvee_v\big\{\mathscr{U}_k(v)\odot\bigwedge_{z\in X}(v_e(z,x)\to\bigwedge_{y\in X}(v_e(y,z)\to f_e(y))\big\}.$$

Put $\rho_e(z)=\bigwedge_{y\in X}(v_e(y,z)\to f_e(y))$. By the definition of $\mathscr{I}_l^{\mathscr{U}}(k,f,\mathscr{U}_k(v))_e$, $\rho_e(z)\le \mathscr{I}_l^{\mathscr{U}}(k,f,\mathscr{U}_k(v))_e(z)$ for all $z\in X, e\in E$ and $\mathscr{U}_k(v)\neq 0$. Thus,

$$\begin{aligned}
&\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)\\
&=\bigvee_v\left\{\mathscr{U}_k(v)\odot\bigwedge_{z\in X}(v_e(z,x)\to\rho_e(z))\mid \rho_e(z)\le\mathscr{I}_l^{\mathscr{U}}(f,\mathscr{U}_k(v))_e(z)\right\}\\
&\le\bigvee_v\left\{\mathscr{U}_k(v)\odot\bigwedge_{z\in X}(v_e(z,x)\to\mathscr{I}_l^{\mathscr{U}}(f,\mathscr{U}_k(v))_e(z))\right\}\\
&\le\mathscr{I}_l^{\mathscr{U}}(\mathscr{I}_l^{\mathscr{U}}(k,f,\mathscr{U}_k(v))_e(x).
\end{aligned}$$

Theorem 3.5. Let $(X,\mathscr{I})$ be an L-fuzzy (K,E)-soft preinterior space. Define a map $\mathscr{T}_k^{\mathscr{I}}:K\to L^{(L^X)^E}$ by:

$$\mathscr{T}_k^{\mathscr{I}}(f)=\bigwedge_{e\in E}S(f_e,\mathscr{I}(k,f)_e).$$

Then, $\mathscr{T}_k^{\mathscr{I}}$ is an L-fuzzy (K,E)-soft topology on X. If $\mathscr{I}$ is stratified, then $\mathscr{T}_k^{\mathscr{I}}$ is an enriched L-fuzzy (K,E)-soft topology.

Proof. (SO1)

$$\mathscr{T}_k^{\mathscr{I}}(1_X)=\bigwedge_{e\in E}\bigwedge_{x\in X}((1_X)_e(x)\to\mathscr{I}(k,1_X)_e(x))=1\to 1=1,$$

$$\mathscr{T}_k^{\mathscr{I}}(0_X)=\bigwedge_{e\in E}\bigwedge_{x\in X}((0_X)_e(x)\to\mathscr{I}(k,0_X)_e(x))=0\to 0=1.$$

(SO2) By Lemma 2.2(12), we havre

$$\begin{aligned}
&\mathscr{T}_k^{\mathscr{I}}(f\odot g)\\
&=\bigwedge_{e\in E}S((f\odot g)_e,\mathscr{I}(k,f_e\odot g_e))\\
&\ge\bigwedge_{e\in E}S((f\odot g)_e,\mathscr{I}(k,f)_e\odot\mathscr{I}(k,g)_e)\\
&\ge\bigwedge_{e\in E}S(f_e,\mathscr{I}(k,f)_e)\odot\bigwedge_{e\in E}S(g_e,\mathscr{I}(k,g)_e)\\
&=\mathscr{T}_k^{\mathscr{I}}(f)\odot\mathscr{T}_k^{\mathscr{I}}(g).
\end{aligned}$$

(SO3) By Lemma 2.2(8),we have

$$\mathscr{T}_k^{\mathscr{I}}(\sqcup_i f_i) = \bigwedge_{e\in E} S((\sqcup_i f_i)_e, \mathscr{I}(k, \sqcup_i f_i)_e)$$

$$\geq \bigwedge_{e\in E} S((\sqcup_i f_i)_e, \sqcup_i \mathscr{I}(k, f_i)_e)$$

$$\geq \bigwedge_{e\in E} \bigwedge_i S((f_i)_e, \mathscr{I}(k, f_i)_e) \geq \bigwedge_i \mathscr{T}_k^{\mathscr{I}}(f_i).$$

(R) By Lemma 2.2 (12)and Theorem 3.2(2), we have

$$\mathscr{T}_k^{\mathscr{I}}(\alpha \odot f) = \bigwedge_{e\in E} S((\alpha \odot f_e), \mathscr{I}(k, \alpha \odot f)_e)$$

$$\geq \bigwedge_{e\in E} S((\alpha \odot f_e), (\alpha \odot \mathscr{I}(k, f)_e))$$

$$\geq \bigwedge_{e\in E} S(f_e, \mathscr{I}(k, f)_e) = \mathscr{T}_k^{\mathscr{I}}(f).$$

From Theorems 3.2 and 3.5, we obtain the following corollary.

Corollary 3.6. Let $(X, \mathscr{U})$ be an L-fuzzy (K,E)-soft quasi-uniform space. Define a map $\mathscr{T}^{\mathscr{I}_l^{\mathscr{U}}}, \mathscr{T}^{\mathscr{I}_r^{\mathscr{U}}} : K \to L^{(L^X)^E}$ by:

$$\mathscr{T}^{\mathscr{I}_l^{\mathscr{U}}}(f) = \bigwedge_{e\in E} S(f_e, \mathscr{I}_l^{\mathscr{U}}(k, f)_e),$$

$$\mathscr{T}^{\mathscr{I}_r^{\mathscr{U}}}(f) = \bigwedge_{e\in E} S(f_e, \mathscr{I}_r^{\mathscr{U}}(k, f)_e).$$

Then, $\mathscr{T}_{\mathscr{U}}$ is an L-fuzzy (K,E)-soft topology on X. If $\mathscr{U}$ is stratified, then $\mathscr{T}_{\mathscr{U}}$ is an enriched L-fuzzy (K,E)-soft topology.

Theorem 3.7. Let $(X, \mathscr{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y, \mathscr{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X, \mathscr{U}) \to (Y, \mathscr{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta} : (X, \mathscr{I}_l^{\mathscr{U}}) \to (Y, \mathscr{I}_l^{\mathscr{V}})$ and $\varphi_{\psi,\eta} : (X, \mathscr{I}_r^{\mathscr{U}}) \to (Y, \mathscr{I}_r^{\mathscr{V}})$ are L-fuzzy soft continuous.

Proof. We have $\varphi^{-1}_{\psi,\eta}(v_{\psi(e)}[\varphi_{\psi,\eta}(x)]) = (\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v_{\psi(e)})[x]$ from

$$\varphi^{-1}_{\psi,\eta}(v_{\psi(e)}[\varphi_{\psi,\eta}(x)])(z) = v_{\psi(e)}[\varphi_{\psi,\eta}(x)](\varphi_{\psi,\eta}(z)) = v_{\psi(e)}(\varphi_{\psi,\eta}(z),\varphi_{\psi,\eta}(x))$$
$$= (\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v_{\psi(e)})(z,x) = (\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)_e[x](z).$$

Thus, by Lemma 2.6(4,5), we have

$$S(v_{\psi(e)}[\varphi_{\psi,\eta}(x)], f_{\psi(e)}) \leq S(\varphi^{-1}_{\psi,\eta}(v_{\psi(e)}[\varphi_{\psi,\eta}(x)]), \varphi^{-1}_{\psi,\eta}(f_{\psi(e)}))$$
$$= S((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)_e[x], \varphi^{-1}_{\psi,\eta}(f)_e).$$

$$\varphi^{-1}_{\psi,\eta}(\mathscr{I}_l^{\mathscr{U}}(\eta(k),g))_e(x) = \mathscr{I}_l^{\mathscr{U}}(\eta(k),g)_{\psi(e)}(\varphi_{\psi,\eta}(x))$$
$$= \bigvee_v \mathscr{V}_{\eta(k)}(v)\odot S(v_{\psi(e)}[\varphi_{\psi,\eta}(x)], f_{\psi(e)})$$
$$\leq \bigvee_v \mathscr{V}_{\eta(k)}(v)\odot S((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)_e[x], \varphi^{-1}_{\psi,\eta}(f)_e)$$
$$\leq \bigvee_u \mathscr{U}_k((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v))\odot S((\varphi_{\psi,\eta}\times\varphi_{\psi,\eta})^{-1}(v)_e[x], \varphi^{-1}_{\psi,\eta}(f)_e)$$
$$\leq \mathscr{I}_l^{\mathscr{U}}(k, \varphi^{-1}_{\psi,\eta}(f)).$$

Theorem 3.8. Let $(X,\mathscr{I}_X)$ be an L-fuzzy (K_1,E_1)-soft preinterior space and $(Y,\mathscr{I}_Y)$ be an L-fuzzy (K_2,E_2)-soft preinterior space. Let $\varphi: X\to Y$, $\psi: E_1\to E_2$ and $\eta: K_1\to K_2$ be mappings. If a map $\varphi_{\psi,\eta}: (X,\mathscr{I}_X)\to(Y,\mathscr{I}_Y)$ is L-fuzzy soft interior map, then a map $\varphi_{\psi,\eta}: (X,\mathscr{T}^{\mathscr{I}_X})\to(Y,\mathscr{T}^{\mathscr{I}_Y})$ is L-fuzzy soft continuous.

Proof. By Lemma 2.2, since $\varphi^{-1}_{\psi,\eta}(\mathscr{I}_Y(\eta(k),f)_{\psi(e)})(x) \le \mathscr{I}_X(\varphi^{-1}_{\psi,\eta}(f))_e(x)$ we have

$$\begin{aligned}
&\mathscr{T}^{\mathscr{I}_Y}(f) \to \mathscr{T}^{\mathscr{I}_X}(\varphi^{-1}_{\psi,\eta}(f))\\
&= \bigwedge_{e\in E}\bigwedge_{y\in Y}\Big(f_{\psi(e)}(y) \to \mathscr{I}_Y(\eta(k),f)_{\psi(e)}(y)\Big)\\
&\to \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\varphi^{-1}_{\psi,\eta}(f)_e(x) \to \mathscr{I}_X(k,\varphi^{-1}_{\psi,\eta}(f))_e(x)\Big)\\
&\ge \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\varphi^{-1}_{\psi,\eta}(f)_e(x) \to \varphi^{-1}_{\psi,\eta}(\mathscr{I}_Y(\eta(k),f)_{\psi(e)})(x)\Big)\\
&\to \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\varphi^{-1}_{\psi,\eta}(f)_e(x) \to \mathscr{I}_X(k,\varphi^{-1}_{\psi,\eta}(f))_e(x)\Big)\\
&\ge \bigwedge_{e\in E}\bigwedge_{x\in X}\Big(\varphi^{-1}_{\psi,\eta}(\mathscr{I}_Y(\eta(k),f)_{\psi(e)})(x) \to \mathscr{I}_X(k,\varphi^{-1}_{\psi,\eta}(f))_e(x)\Big).
\end{aligned}$$

Thus, if $\varphi^{-1}_{\psi,\eta}(\mathscr{I}_Y(\eta(k),f)) \le \mathscr{I}_X(k,\varphi^{-1}_{\psi,\eta}(f))$, then $\mathscr{T}^{\mathscr{I}_Y}(f) \le \mathscr{T}^{\mathscr{I}_X}(\varphi^{-1}_{\psi,\eta}(f))$.

From Theorems 3.5 and 3.8, we obtain the following corollary.

Corollary 3.9. Let $(X,\mathscr{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathscr{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathscr{U}) \to (Y,\mathscr{V})$ is L-fuzzy soft uniformly continuous, then two maps $\varphi_{\psi,\eta} : (X,\mathscr{T}^{\mathscr{I}_l^{\mathscr{V}}}) \to (Y,\mathscr{T}^{\mathscr{I}_l^{\mathscr{U}}})$ and $\varphi_{\psi,\eta} : (X,\mathscr{T}^{\mathscr{I}_r^{\mathscr{V}}}) \to (Y,\mathscr{T}^{\mathscr{I}_r^{\mathscr{U}}})$ are L-fuzzy soft continuous.

Example 3.10. Let $X = \{h_i \mid i = \{1,2,3\}\}$ with h_i=house and $E = \{e,b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0,1]$ by

$$x \odot y = \max\{0, x+y-1\},\ x \to y = \min\{1-x+y, 1\}$$

Then $([0,1],\wedge,\to,0,1)$ is a complete residuated lattice.

(1) Put $v, v\odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5\\ 0.3 & 1 & 0.5\\ 0.4 & 0.6 & 1\end{pmatrix} v_b = \begin{pmatrix} 1 & 0.5 & 0.3\\ 0.7 & 1 & 0.5\\ 0.6 & 0.6 & 1\end{pmatrix}$$

$$(v\odot v)_e=\begin{pmatrix}1&0.2&0\\0&1&0\\0&0.2&1\end{pmatrix}(v\odot v)_b=\begin{pmatrix}1&0&0\\0.4&1&0\\0.2&0.2&1\end{pmatrix}$$

$$w_e=\begin{pmatrix}1&0.4&0.5\\0.4&1&0.5\\0.4&0.6&1\end{pmatrix}w_b=\begin{pmatrix}1&0.5&0.3\\0.3&1&0.5\\0.2&0.3&1\end{pmatrix}$$

We define $\mathscr{U}:K=\{k_1,k_2\}\to[0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathscr{U}_{k_1}(u)=\begin{cases}1, & \text{if } u=1_{Y\times Y}\\ 0.6, & \text{if } v\sqsubseteq u\neq 1_{Y\times Y},\\ 0.3, & \text{if } v\odot v\sqsubseteq u\not\sqsupseteq v,\\ 0, & \text{otherwise.}\end{cases}$$

$$\mathscr{U}_{k_2}(u)=\begin{cases}1, & \text{if } u=1_{Y\times Y}\\ 0.5, & \text{if } w\sqsubseteq u\neq 1_{Y\times Y},\\ 0, & \text{otherwise.}\end{cases}$$

(2) From Theorem 3.2, since $\mathscr{I}_l^{\mathscr{U}}(k,f)_e(x)=\bigvee_u\mathscr{U}_k(u)\odot S(u_e[x],f_e),\ \forall\ f\in(L^X)^E, e\in E,\ x\in X$,

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_e(h_1)&=(\textstyle\bigwedge_{x\in X}f_e(x))\vee\Big(0.6\odot(f_e(h_1)\wedge(0.7+f_e(h_2))\\&\wedge(0.6+f_e(h_3))\Big)\vee(0.3\odot f_e(h_1))\end{aligned}$$

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_b(h_1)&=(\textstyle\bigwedge_{x\in X}f_b(x))\vee\Big(0.6\odot(f_b(h_1)\wedge(0.3+f_b(h_2))\\&\wedge(0.4+f_b(h_3))\Big)\vee\Big((0.3\odot(f_b(h_1)\wedge(0.6+f_b(h_2))\\&\wedge(0.8+f_b(h_3)))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_e(h_2)&=(\textstyle\bigwedge_{x\in X}f_e(x))\vee\Big(0.6\odot((0.4+f_e(h_1))\wedge f_e(h_2)\\&\wedge(0.4+f_e(h_3)))\Big)\vee\Big(0.3\odot(0.8+f_e(h_1))\\&\wedge f_e(h_2)\wedge(0.8+f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_b(h_2)&=(\textstyle\bigwedge_{x\in X}f_b(x))\vee\Big(0.6\odot((0.5+f_b(h_1))\wedge f_b(h_2)\\&\wedge(0.4+f_b(h_3))\Big)\vee\Big(0.3\odot(f_b(h_2)\wedge(0.8+f_b(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_e(h_3) &= (\bigwedge_{x\in X} f_e(x)) \vee \Big(0.6\odot((0.5+f_e(h_1))\wedge(0.5+f_e(h_2)) \\ &\wedge f_e(h_3))\Big)\vee\Big(0.3\odot f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_l^{\mathscr{U}}(k_1,f)_b(h_3) &= (\bigwedge_{x\in X} f_b(x)) \vee \Big(0.6\odot((0.7+f_b(h_1))\wedge(0.5+f_b(h_2)) \\ &\wedge f_b(h_3))\Big)\vee\Big(0.3\odot f_b(h_3)\Big)\end{aligned}$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_e(h_1) = S((1_{X\times X})_e[h_1],f_e)\vee(0.5\odot S(w_e[h_1],f_e))$$
$$= (\bigwedge_{x\in X} f_e(x))\vee\Big(0.5\odot(f_e(h_1)\wedge(0.6+f_e(h_2))\wedge(0.6+f_e(h_3))\Big)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_b(h_1) = S((1_{X\times X})_b[h_1],f_b)\vee(0.5\odot S(w_b[h_1],f_b))$$
$$= (\bigwedge_{x\in X} f_b(x))\vee\Big(0.5\odot(f_b(h_1)\wedge(0.7+f_b(h_2))\wedge(0.8+f_b(h_3))\Big)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_e(h_2) = (\bigwedge_{x\in X} f_e(x))$$
$$\vee\Big(0.5\odot((0.6+f_e(h_1))\wedge f_e(h_2)\wedge(0.4+f_e(h_3)))\Big)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_b(h_2) = (\bigwedge_{x\in X} f_b(x))$$
$$\vee\Big(0.5\odot((0.5+f_b(h_1))\wedge f_b(h_2)\wedge(0.7+f_b(h_3))\Big)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_e(h_3) = (\bigwedge_{x\in X} f_e(x))$$
$$\vee\Big(0.5\odot((0.5+f_e(h_1))\wedge(0.5+f_e(h_2))\wedge f_e(h_3))\Big)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_b(h_3) = (\bigwedge_{x\in X} f_b(x))$$
$$\vee\Big(0.5\odot((0.7+f_b(h_1))\wedge(0.5+f_b(h_2))\wedge f_b(h_3))\Big)$$

For $f_e = (0.5, 0.1, 0.7)$ and $f_b = (0.2, 0.5, 0.6)$,

$$\mathscr{I}_l^{\mathscr{U}}(k_1,f)_e = (0.5,0.1,0.2),\ \ \mathscr{I}_l^{\mathscr{U}}(k_1,f)_e = (0.2,0.2,0.2)$$

$$\mathscr{I}_l^{\mathscr{U}}(k_2,f)_e = (0.1,0.1,0.1),\ \ \mathscr{I}_l^{\mathscr{U}}(k_2,f)_e = (0.2,0.2,0.2)$$

$$\mathscr{T}_{k_1}^{\mathscr{I}_l^{\mathscr{U}}}(f) = 0.5,\ \mathscr{T}_{k_2}^{\mathscr{I}_l^{\mathscr{U}}}(f) = 0.4.$$

(3) From Corollary 3.3, since $\mathscr{I}_r^{\mathscr{U}}(k,f)_e(x) = \bigvee_u \mathscr{U}_k(u)\odot S(u_e[[x]],f_e)$,

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_e(h_1) &= (\bigwedge_{x\in X} f_e(x))\vee\Big(0.6\odot(f_e(h_1)\wedge(0.4+f_e(h_2)) \\ &\wedge(0.5+f_e(h_3))\Big)\vee(0.3\odot f_e(h_1))\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_b(h_1) &= (\bigwedge_{x\in X} f_b(x))\vee\Big(0.6\odot(f_b(h_1)\wedge(0.5+f_b(h_2))\\ &\wedge(0.7+f_b(h_3))\Big)\vee\Big(0.3\odot((0.8+f_b(h_2))\wedge f_b(h_1))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_e(h_2) &= (\bigwedge_{x\in X} f_e(x))\vee\Big(0.6\odot((0.7+f_e(h_1))\wedge f_e(h_2)\\ &\wedge(0.5+f_e(h_3)))\Big)\vee\Big(0.3\odot f_e(h_2)\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_b(h_2) &= (\bigwedge_{x\in X} f_b(x))\vee\Big(0.6\odot((0.3+f_b(h_1))\wedge f_b(h_2)\\ &\wedge(0.5+f_b(h_3))\Big)\vee\Big(0.3\odot((0.8+f_b(h_1))\wedge f_b(h_2))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_e(h_3) &= (\bigwedge_{x\in X} f_e(x))\vee\Big(0.6\odot((0.6+f_e(h_1))\wedge(0.4+f_e(h_2))\\ &\wedge f_e(h_3))\Big)\vee\Big(0.3\odot((0.8+f_e(h_2))\wedge f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_1,f)_b(h_3) &= (\bigwedge_{x\in X} f_b(x))\vee\Big(0.6\odot((0.4+f_b(h_1))\wedge(0.4+f_b(h_2))\\ &\wedge f_b(h_3))\Big)\vee\Big(0.3\odot((0.8+f_b(h_1))\\ &\wedge(0.8+f_b(h_2))\wedge f_b(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_e(h_1) &= (\bigwedge_{x\in X} f_e(x))\vee\\ &\Big(0.5\odot(f_e(h_1)\wedge(0.6+f_e(h_2))\wedge(0.5+f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_b(h_1) &= (\bigwedge_{x\in X} f_b(x))\\ &\vee\Big(0.5\odot(f_b(h_1)\wedge(0.5+f_b(h_2))\wedge(0.3+f_b(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_e(h_2) &= (\bigwedge_{x\in X} f_e(x))\\ &\vee\Big(0.5\odot((0.6+f_e(h_1))\wedge f_e(h_2)\wedge(0.5+f_e(h_3)))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_b(h_2) &= (\bigwedge_{x\in X} f_b(x))\\ &\vee\Big(0.5\odot((0.7+f_b(h_1))\wedge f_b(h_2)\wedge(0.5+f_b(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_e(h_3) &= (\bigwedge_{x\in X} f_e(x))\\ &\vee\Big(0.5\odot((0.6+f_e(h_1))\wedge(0.4+f_e(h_2))\wedge f_e(h_3))\Big)\end{aligned}$$

$$\begin{aligned}\mathscr{I}_r^{\mathscr{U}}(k_2,f)_b(h_3) &= (\bigwedge_{x\in X} f_b(x))\\ &\vee\Big(0.5\odot((0.8+f_b(h_1))\wedge(0.7+f_b(h_2))\wedge f_b(h_3))\Big)\end{aligned}$$

For $f_e=(0.5,0.1,0.7)$ and $f_b=(0.2,0.5,0.6)$,

$$\mathscr{I}_r^{\mathscr{U}}(k_1,f)_e=(0.1,0.1,0.2),\ \ \mathscr{I}_r^{\mathscr{U}}(k_1,f)_e=(0.2,0.2,0.2)$$

$$\mathscr{I}_r^{\mathscr{U}}(k_2,f)_e=(0.1,0.1,0.1),\ \mathscr{I}_r^{\mathscr{U}}(k_2,f)_e=(0.2,0.2,0.2)$$

$$\mathscr{T}_{k_1}^{\mathscr{I}_r^{\mathscr{U}}}(f)=0.5,\ \mathscr{T}_{k_2}^{\mathscr{I}_r^{\mathscr{U}}}(f)=0.4$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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