Available online at http://scik.org
J. Math. Comput. Sci. 7 (2017), No. 3, 554-563
ISSN: 1927-5307

NUMERICAL APPLICATION OF ADOMIAN DECOMPOSITION METHOD TO FIFTH-ORDER AUTONOMOUS DIFFERENTIAL EQUATIONS

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Abstract. In this paper, Adomian Decomposition Method(ADM) is applied to Fifth-Order autonomous differential equations. The general concept of ADM to this class of equations was stated in relation to the general concept. Three test problems were used to validate the concept of the decomposition method, and the result in series form of only the first six terms were obtained. The absolute error were also obtained, similarly the plots of both the exact and ADM solutions. The series form solution by ADM gave almost the same result as those obtained by any known closed form method of the continuous function. Thus, justifying the excellent potentials of the decomposition method.

Keywords: fifth-order autonomous differential equations; Adomian decomposition method.

2010 AMS Subject Classification: 65L05.

1. Introduction

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Received November 8, 2016; Published May 1, 2017

Fifth-order Differential equations generally arise in modeling of visco-elastic flow. The existence and uniqueness of the solution to this class of linear autonomous differential equation is common everywhere [9]. Also, variation of parameter is applied to the linear case of this class of equations. For the nonlinear class of these equations, numerical methods are applied. Previous numerical methods applied to problems of visco-elastic flow are finite difference method, finite element method and finite volume method. In this paper, we explore the possibility of using the ADM in obtaining the solution to fifth-order autonomous differential equation.

Although the solution of the decomposition method is also an approximation, but it is one that does not change the problem. The method itself has some features in common with other methods. But, it is distinctly different on closer examination and it offers several significant advantages. For details on ADM see [2], [3], [4], [5], [7] and [10]. The general fifth-order differential equation is given as

$$\frac{d^n\xi}{dt^n} = f(t,\xi,\zeta,\alpha,\beta,\gamma), \qquad n=5 \tag{1}$$

where

$$\zeta = \frac{d\xi}{dt}, \alpha = \frac{d^2\xi}{dt^2}, \beta = \frac{d^3\xi}{dt^3}, \gamma = \frac{d^4\xi}{dt^4}$$

And $f(t,\xi,\zeta,\alpha,\beta,\gamma)$ is a continuous function of t,ξ,ζ,α,β and γ in some region.

2. Concept of ADM

ADM Consider equation (1.1) to be of the form

$$\xi - N(\xi) = f(t) \tag{2}$$

where N is a nonlinear operator and zero in this case with regards to equation (1.1). f(t) is a known function and seeks the solution ξ satisfying equation (1.1). We assume that for every f, equation (2.1) has one and only one solution. The ADM consist of approximating equation (2.1) as an infinite series.

$$\xi = \sum_{n=0}^{\infty} \xi_n(t) \tag{3}$$

and

$$N(\xi) = \sum_{n=0}^{\infty} A_n(t) \tag{4}$$

where $A_n(t)$ is known as the Adomian polynomial given as

$$A_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{i=0}^{\infty}\lambda^i \xi_i(t))]_{\lambda=0} \tag{5}$$

Where n = 0, 1, 2, ... and λ is a grouping parameter. For details of equation (2.4) see [8] and [11]. The proof of convergence of equations (2.2) and (2.3) are given in [1] and [6].

Substituting equations (2.2) and (2.3) in (2.1), yields

$$\sum_{n=0}^{\infty} \xi_n(t) - \sum_{n=0}^{\infty} A_n(t) = f(t) \tag{6}$$

From equation (2.5) the iterations are then determined in the following recursive way, where we identify

$$\xi_0(t) = f(t)$$

$$\xi_{n+1}(t) = A_n(\xi_0(t), \xi_1(t), \xi_2(t), ..., \xi_n(t))$$

Thus, all component of $\xi(t)$ are determined once the Adomian Polynomials are obtained. The polynomials are obtained based on the nonlinear term in the nonlinear functional. The accelerated form of this polynomial are given in literature and the usual Adomian form is given by [2]. The nth term approximation of equation (2.1) is given as

$$\phi_n(\xi) = \sum_{i=0}^{n-1} \xi_i(t) \tag{7}$$

with

$$lim_{n\to\infty}\phi_n(\xi) = \xi(t)$$

In the linear case of equation (1.1), the application of ADM is equivalent to a classical itereration method. But the posteriori calculation of the constant L^{-1} is by imposing each $\phi_n(\xi)$ to verify each initial/boundary condition. In this paper, L is a fifth-order differential and L^{-1} is a fifth-fold integral which determine a set suitable for good convergence.

3. Maple Computed Numerical Examples

In this section, we apply ADM to 5th-order linear autonomous differential equations.

3.1 Problem 1

In comparison to equation (1.1), consider

$$\xi^{(5)} + 3\gamma + 4\beta + 4\alpha + 3\zeta + \xi = 0 \qquad \xi = \xi(t), \qquad t\varepsilon[0,1]$$

and

$$\xi(0) = \xi''(0) = \xi^{(iv)}(0) = 0, \qquad \xi'(0) = \xi'''(0) = 1$$

The exact solution is

$$\xi(t) = e^{-t}(2 + 3t + t^2) - 2\cos t$$

Using the equations (2.1) through (2.6), we have

$$\xi_0 = t + \frac{t^3}{6}$$

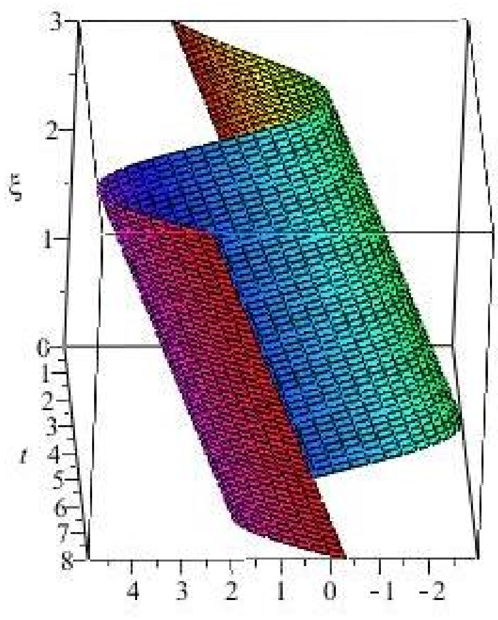
$$\xi_1 = -\frac{7t^5}{120} - \frac{t^6}{144} - \frac{t^7}{1680} - \frac{t^8}{40320}$$

$$\xi_2 = \frac{7t^6}{240} + \frac{43t^7}{5040} + \frac{19t^8}{13440} + \frac{19t^{10}}{181440} + \frac{t^{11}}{2217600} + \frac{t^{12}}{79833600} + \frac{t^{13}}{6227020800}$$

Proceeding in this order we obtain $\xi_5(t)$, the ADM result of $\xi(t) = \sum_{i=0}^{5} \xi_i(t)$ and exact result are shown in Table 1. With only 6 terms of the series solution considered the absolute error (E_A) is minimal as shown in Table 1. The similarities of the two results are also shown in Figures 1 and 2.

Table I: Exact versus ADM solution of Problem 1

t	ADM result	Exact result	E_A
0.0	0.0000000000	0.0000000000	0E-10
0.1	0.1006661051	0.1001661051	0E-10
0.2	0.2013160325	0.2013160323	2E-10
0.3	0.3043735016	0.3043734986	3E-09
0.4	0.4101533668	0.4101533090	5E-08
0.5	0.5193248501	0.5193242893	5E-07
0.6	0.6323851764	0.6323815582	3E-06
0.7	0.7496421699	0.7496245493	1E-05
0.8	0.8712045604	0.8711347180	6E-05
0.9	0.9969788887	0.9967423387	2E-04
1.0	1.1266720353	1.1259642953	7E-04



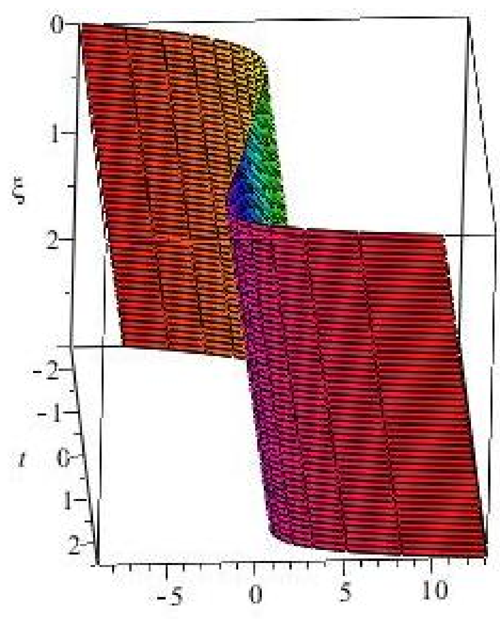


Figure 1: Exact solution of Problem 1

Figure 2: ADM solution of Problem 1

3.2 Problem 2

Also, in comparison to equation (1), consider

$$\xi^{(5)} + 5\gamma + 10\beta + 10\alpha + 5\zeta + \xi = 0 \qquad \xi = \xi(t), \qquad t\varepsilon(-1,1)$$

and

$$\xi(0) = -1, \xi'(0) = 1, \xi''(0) = \xi^{(iv)}(0) = \xi'''(0) = 0.$$

The exact solution is

$$\xi(t) = e^{-t}\left(-1 + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{8}\right)$$

Applying the equations (2.1) through (2.6), we have

$$\xi_0 = -1 + t$$

$$\xi_1 = -\frac{t^5}{30} - \frac{t^6}{720}$$

$$\xi_2 = \frac{t^6}{36} + \frac{t^7}{117} + \frac{5t^8}{4032} + \frac{t^9}{12096} + \frac{t^{10}}{403200} + \frac{t^{11}}{39916800}$$

$$\xi_3 = -\frac{5t^7}{252} - \frac{85t^8}{8064} - \frac{5t^9}{2016} - \frac{t^{10}}{3024} - \frac{109t^{11}}{3991680} - \frac{23t^{12}}{15966720} - \frac{17t^{14}}{17435658240} - \frac{t^{13}}{20756736} - \frac{t^{15}}{93405312000} - \frac{t^{16}}{20922789888000}$$

Similarly, we proceed in this order and obtain $\xi_5(t)$. The ADM result of $\xi(t) = \sum_{i=0}^{5} \xi_i(t)$ and exact result are shown in Table 2. With only 6 terms of the series solution considered the absolute error (E_A) is also very minimal as shown in Table 2. The similarities of the two results are shown in Figured 3 and 4.

Table 2: Exact versus ADM solution of Problem 2

t	ADM result	Exact result	E_A
-0.8	-1.7792457877	-1.7795001664	2E-04
-0.6	-1.5958116453	-1.5958273498	1E-05
-0.4	-1.3995304763	-1.3995307753	2E-07
-0.2	-1.1999874964	-1.1999874968	1E-09
0.0	-1.0000000000	-1.0000000000	0E-10
0.2	-0.8000091098	-0.8000011024	3E-10
0.4	-0.6002492572	-0.6002496949	4E-07
0.6	-0.4016203553	-0.4016481662	2E-02
0.8	-0.2058525761	-0.2063968927	5E-04

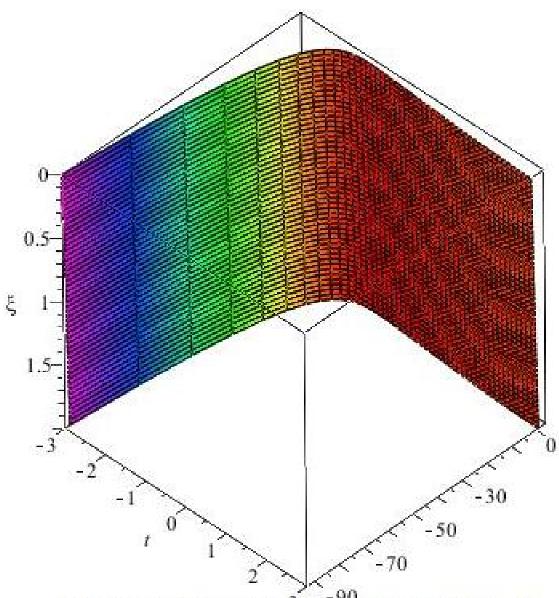


Figure 3: Exact solution of Problem 2

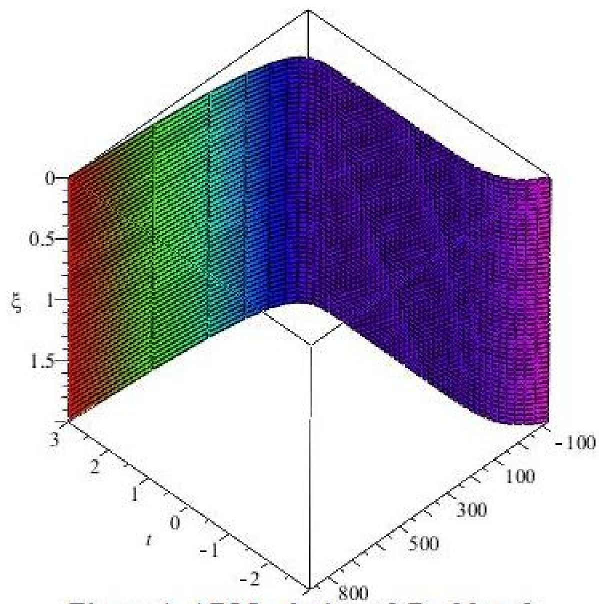


Figure 4: ADM solution of Problem 2

3.3 Problem 3

Similarly, in comparison to equation (1), consider

$$\xi^{(5)} - 2\beta + \zeta = 0 \qquad \xi = \xi(t), \qquad t\varepsilon[0,1]$$

and

$$\xi(0) = 1, \xi'(0) = 2, \xi''(0) = 1, \xi'''(0) = 3, \xi^{(iv)}(0) = 5.$$

The exact solution is

$$\xi(t) = 4 - \frac{e^{-t}}{4}(9+3t) - \frac{e^{t}}{4}(3-5t)$$

Also, applying the equations (2.1) through (2.6), we have

$$\xi_0 = 1 + 2t + \frac{t^2}{2} + \frac{t^3}{2} + \frac{5t^4}{24}$$
$$\xi_1 = \frac{t^5}{30} + \frac{t^6}{80} - \frac{t^7}{1680} - \frac{t^8}{8064}$$

Proceeding in this order we obtain $\xi_5(t)$. The ADM result of $\xi(t) = \sum_{i=0}^{5} \xi_i(t)$ and exact result are shown in Table 3. With only 6 terms of the series solution considered the absolute error (E_A) is also very very minimal as shown in Table 3. These is also obvious in Figures 5 and 6.

Table 3: Exact versus ADM solution of Problem 3

t	ADM result	Exact result	E_A
0.0	1.0000000000	1.0000000000	0E-10
0.1	1.2055211792	1.2055211792	0E-10
0.2	1.4243448136	1.4243448135	1E-10
0.3	1.6602778510	1.6602778510	0E-10
0.4	1.9177277082	1.9177277081	1E-10
0.5	2.2017668595	2.2017668595	0E-10
0.6	2.5182085825	2.5182085824	1E-10
0.7	2.8736948704	2.8736948703	1E-10
0.8	3.2757976843	3.2757976844	1E-10
0.9	3.7331349120	3.7331349120	0E-10
1.0	4.2555025907	4.2555025906	1E-10

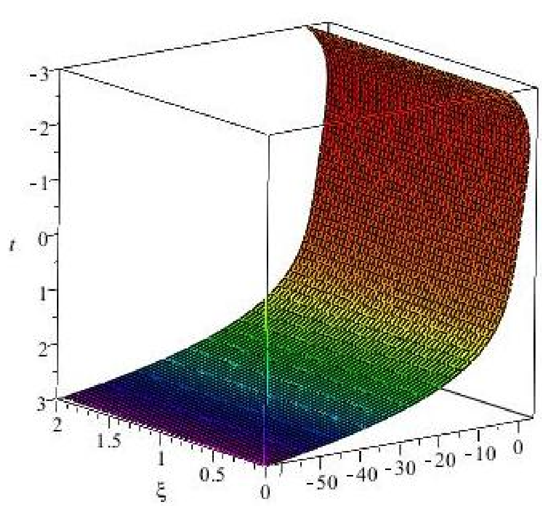


Figure 5: Exact solution of Problem 3

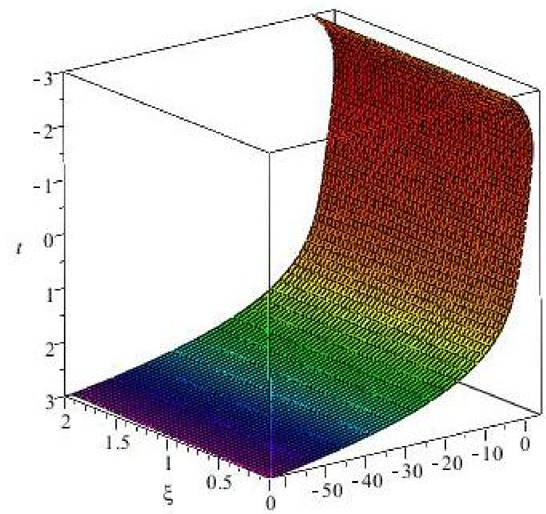


Figure 6: ADM solution of Problem 3

4. Conclusion

In this paper, we have successfully applied ADM to 5th order linear autonomous differential equations. The introduction consisted of real life areas were 5th differential equations are used as models. Followed by, the numerical methods that are often used for solving this class of equations and an overview of ADM. We also gave the general ADM for this class of equation and applied to concrete examples. The results were fantastic, nearly the same as those obtained by classical method. Although, we all know that in real life problems the laws of nature are nonlinear and stochastic in general. As such, one of the most relevant features of ADM is that the matching procedures are not necessary.

Conflict of Interests

The authors declare that there is no conflict of interests.

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