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A HOMOTOPY PERTURBATION TECHNIQUE FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS IN FINITE DOMAINS

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Abstract. In this paper, a new homotopy perturbation technique is proposed to solve a class of intial-boundary value problems of partial differential equations over finite domains. The advantage of this technique is to admit both the initial and boundary conditions in the recursive relation so that we can obtain a good approximate solution for the problems. The effectiveness of the approach is verified by several examples.

Keywords: Partial differential equation; Initial-boundary value problems; Homotopy perturbation method.

2000 AMS Subject Classification: 35Q79; 35C10; 35G15

1. Introduction

In 1998, J. H. He proposed the homotopy perturbation method (HPM) for addressing linear and nonlinear problems [1] and [2]. This method has been the subject of extensive studies, and applied to different linear and nonlinear initial value problems [1]-[4]. The

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HPM has the advantage of dealing directly with the problem without transformations, linearization, discretization or any unrealistic assumption. The method yields a rapidly convergent series solution and usually a few iterations lead to accurate approximation of the exact solution [5].

Yet the classical HPM, among some other series solution methods, build the recurrence scheme of solution using only one type of the problem conditions: either the initial conditions or the boundary conditions. Recently, Lesnic [6]-[7] and El-Sayed et al [8] suggested a technique of Adomian decomposition method for solving partial differential equations (PDEs) over finite domains for the integer and fractional-order cases, respectively. Our aim here is to propose a new HPM technique that incorporates both types of conditions in the scheme to solve initial-boundary value problems (IBVP) over finite domains.

The article begins by presenting classical HPM in section two. In section three, we introduce the new HPM technique. In section four some examples are solved to illustrate the validity of this approach.

2. The Classical HPM

Consider the following equation

$$A\left(u\left(x,t\right)\right)-f(r)=0,\ \ r\in\Omega, \tag{2.1}$$

with boundary conditions

$$B(u,\partial u/\partial n)=0,\ \ r\in\Gamma, \tag{2.2}$$

where A is a general differential operator, $u(x,t)$ is the unknown function, B is a boundary operator, $g(r)$ is a known analytic function, and Γ is the boundary of the domain Ω, x and t denote the spatial and the temporal independent variables, respectively. The operator A can be generally divided into linear and nonlinear parts, say L and N. Therefore (2.1) can be written as

$$L\left(u\right)+N(u)-g(r)=0. \tag{2.3}$$

In [2], He constructed a homotopy $v(r,p):\Omega\times[0,1]\rightarrow\mathbb{R}$ which satisfies

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[L(v)+N(v)-g(r)\right]=0,\ \ r\in\Omega, \tag{2.4}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - g(r)] = 0, \ \ r \in \Omega, \tag{2.5}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial guess of $u(x,t)$ which satisfies the boundary conditions. Obviously, from (2.4) and (2.5) one has

$$H(v,0) = L(v) - L(u_0), \tag{2.6}$$

$$H(v,1) = L\left(u\right) + N(u) - g(r) = 0. \tag{2.7}$$

Changing p from zero to unity is just that change of $v(r,p)$ from $u_0(r)$ to $u\left(r\right)$. Expanding $v(r,p)$ in Taylor series with respect to p, one has

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \ .. \tag{2.8}$$

Setting $p = 1$ in equation (2.8) yields the approximate solution of (2.1) to be

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \ldots \ . \tag{2.9}$$

The basic assumption is that the solution of (2.4) and (2.5) can be written as a power series in p

$$u = u_0 + pu_1 + p^2 u_2 + \ldots. \tag{2.10}$$

Substituting (2.10) into (2.3) and equating the terms with identical powers of p, we obtain a series of linear equations in u_0, u_1, u_2, ..., which can be solved by symbolic computation softwares. The solution $u(x,t) = \sum_{i=0}^{\infty} u_i(x,t)$ is approximated by the truncated series

$$U_n(x,t) = \sum_{i=1}^{n-1} u_i(x,t). \tag{2.11}$$

3. The HPM technique for finite domains

In this section, we propose the HPM technique for solving IBVP over finite domains. Consider the PDE of the form

$$Qu + Mu + f = 0, \tag{3.1}$$

associated with initial and boundary conditions where Q denotes the highest-order partial derivative with respect to t, M denotes the highest-order partial derivative with respect to x and f is a function of x, t, u, and its temporal and spatial partial derivatives of order less than the order of Q and M, respectively. Then, to include both the initial and boundary conditions in the solution, we construct the two homotopies

$$u = Q^{-1}\left(-pMu - pf\right), \tag{3.2}$$

$$u = M^{-1}\left(-pQu - pf\right), \tag{3.3}$$

for each homotopy, the corresponding powers of p are compared to obtain two systems of partial differential equations with the prescribed conditions. We assume the solution of problem (3.1) in the form

$$u = \sum_{i=0}^{\infty} u_i, \tag{3.4}$$

where u_i is given by

$$u_i = \frac{\tilde{u}_i + \bar{u}_i}{2} \quad i = 0, 1, ..., \tag{3.5}$$

where $\tilde{u}_i$ and $\bar{u}_i$ are solutions of the ith equations in the PDE systems obtained from the homotopies (3.2) and (3.3), respectively.

4. Numerical implementation

In this section, some numerical examples are presented to validate the proposed solution scheme. The results are calculated using the symbolic software Mathematica.

Example 4.1 Consider the heat problem

$$\begin{cases} u_t = u_{xx}, & 0 < x < 1, t > 0, \\ u(x,0) = x^2, & \\ u(0,t) = 2t, \ u(1,t) = 1 + 2t. & \end{cases} \tag{4.1}$$

According to the homotopies (3.2) and (3.3), the two following systems of PDEs are obtained

$$\begin{aligned} p^0 &: \ u_{0t} = 0, \qquad u_0(x,0) = x^2, \\ p^1 &: \ u_{1t} = u_{0xx}, \ \ u_1(x,0) = 0, \\ p^2 &: \ u_{2t} = u_{1xx}, \ \ u_2(x,0) = 0, \\ &\ \vdots \end{aligned} \tag{4.2}$$

$$
\begin{aligned}
&p^0: \quad u_{0xx} = 0, \quad u_0(0,t) = 2t, \quad u_0(1,t) = 1 + 2t,\\
&p^1: \quad u_{1xx} = u_{0t}, \quad u_1(0,t) = 0, \quad u_1(1,t) = 0,\\
&p^2: \quad u_{2xx} = u_{1t}, \quad u_2(0,t) = 0, \quad u_2(1,t) = 0,\\
&\vdots
\end{aligned} \tag{4.3}
$$

Solving (4.2) and (4.3) for $\tilde{u}_0,\ \bar{u}_0,\ \tilde{u}_1,\ \bar{u}_1,\ \tilde{u}_2,\ \bar{u}_2, \ldots,$ the first few components of the homotopy perturbation solution for problem (4.1) are derived as follows

$$
\begin{aligned}
&\tilde{u}_0 = x^2, \quad \bar{u}_0 = 2t + x, \quad u_0 = \tfrac{1}{2}\left(x^2 + 2t + x\right),\\
&\tilde{u}_1 = t, \quad \bar{u}_1 = \tfrac{1}{2}\left(-x + x^2\right), \quad u_1 = \tfrac{1}{4}\left(x^2 + 2t - x\right),\\
&\tilde{u}_2 = \tfrac{t}{2}, \quad \bar{u}_2 = \tfrac{1}{4}\left(-x + x^2\right), \quad u_2 = \tfrac{1}{8}\left(x^2 + 2t - x\right),\\
&\vdots \qquad \vdots \qquad \vdots
\end{aligned}
$$

$$
u_{n+1} = \frac{1}{2^{n+1}}\left(\left(x^2 + 2t - x\right)\right), \ n \geq 1.
$$

Hence, we have

$$
u(x,t) = \left(x^2 + 2t\right)\sum_{n=0}^{\infty} 2^{-n-1} + \frac{x}{2}\left(1 - \sum_{n=1}^{\infty} 2^{-n}\right) = x^2 + 2t,
$$

which is the exact solution of problem (4.1) given by $u\left(x,t\right) = x^2 + 2t$ [9].

Example 4.2 Consider the heat conduction problem

$$
\begin{cases}
u_t = u_{xx}, \quad 0 < x < 1, t > 0,\\
u(x,0) = \sin\left(x\right),\\
u_x(0,t) = e^{-t}, \quad u_x(1,t) = \cos\left(1\right)e^{-t}.
\end{cases} \tag{4.4}
$$

According to the homotopies (3.2) and (3.3), the two following systems of PDEs are obtained

$$
\begin{aligned}
&p^0: \quad u_{0t} = 0, \quad u_0(x,0) = \sin\left(x\right),\\
&p^1: \quad u_{1t} = u_{0xx}, \quad u_1(x,0) = 0,\\
&p^2: \quad u_{2t} = u_{1xx}, \quad u_2(x,0) = 0,\\
&\vdots
\end{aligned} \tag{4.5}
$$

$$\begin{array}{llll} p^0: & u_{0xx}=0, & u_{0x}(0,t)=e^{-t}, & u_{0x}(1,t)=\cos(1)\,e^{-t}, \\ p^1: & u_{1xx}=u_{0t}, & u_{1x}(0,t)=0, & u_{1x}(1,t)=0, \\ p^2: & u_{2xx}=u_{1t}, & u_{2x}(0,t)=0, & u_{2x}(1,t)=0, \\ \vdots & & & \end{array} \tag{4.6}$$

Solving (4.5) and (4.6), the first few components of the homotopy perturbation solution for problem (4.4) are derived as follows

$$\begin{aligned}
\tilde{u}_0 &= \sin(x), \\
\bar{u}_0 &= e^{-t}x - 0.2298e^{-t}x^2, \\
u_0 &= \frac{1}{2}\left(e^{-t}x - 0.2298e^{-t}x^2 + \sin(x)\right), \\
\tilde{u}_1 &= -0.2298 + 0.2298\cosh(t) - 0.5t\sin(x) - 0.2298\sinh(t), \\
\bar{u}_1 &= 0.1058e^{-t}x^2 - 0.08333e^{-t}x^3 + 0.009577e^{-t}x^4, \\
u_1 &= -0.1149 + 0.05292e^{-t}x^2 - 0.04166e^{-t}x^3 + 0.0047e^{-t}x^4 + \\
&\quad 0.11492\cosh(t) - 0.25t\sin(x) - 0.11492\sinh(t), \\
\tilde{u}_2 &= (0.05742x^2 - 0.25x + 0.1058)(1 - \cosh(t) + \sinh(t)) + 0.125t^2\sin(x), \\
\bar{u}_2 &= -0.25x + \left(-0.0574e^{-t} + 0.05742 + 0.06155\,e^{-t}\right)x^2 + 0.25\sin(x) \\
&\quad -0.004e^{-t}x^4 + 0.002e^{-t}x^5 - 0.0001e^{-t}x^6, \\
u2 &= -e^{-t}(0.00007\,x^6 - 0.001030x^5 + 0.00218x^4 + 5.75\times 10^{-6}x^3 + \\
&\quad 0.02642x^2 - 0.1236x + 0.0522) + 0.057\,x^2 - 0.24795x + 0.05179 \\
&\quad + \left(0.125 + 0.0625t^2\right)\sin(x), \\
&\quad \vdots
\end{aligned}$$

and the solution is thus obtained as

$$u = u_0 + u_1 + u_2 + ...$$

Figure (1) gives the comparison at $t = 0.5$ between the HPM 3^{rd}-order approximate solution of problem (4.4) and the exact solution given in [6] by $u(x,t) = \sin(x)\,e^{-t}$.

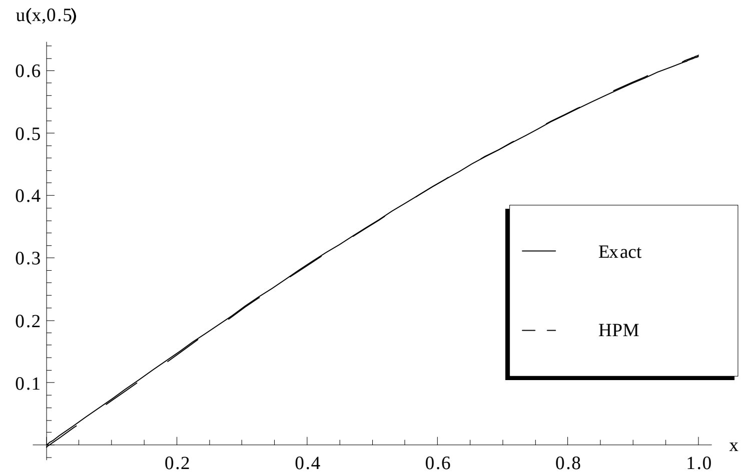

FIGURE 1. $u(x, 0.5)$ of Example (4.2) for 3^{rd}-order HPM approximation.

Example 4.3 Consider the Klien-Gordon problem

$$\begin{cases} u_{tt} - u_{xx} = u, \ 0 < x < \frac{\pi}{2}, \ t > 0, \\ u(x,0) = 1 + \sin(x), \ u_t(x,0) = 0, \\ u(0,t) = \cosh(t), \ u(\frac{\pi}{2}, t) = 1 + \cosh(t), \end{cases} \tag{4.7}$$

According to the homotopies (3.2) and (3.3), the two following systems of PDEs are obtained

$$\begin{aligned} &p^0: \quad u_{0tt} = 0, && u_0(x,0) = 1 + \sin(x), && u_{0t}(x,0) = 0, \\ &p^1: \quad u_{1tt} = u_{0xx} + u_0, && u_1(x,0) = 0, && u_{1t}(x,0) = 0, \\ &p^2: \quad u_{2tt} = u_{1xx} + u_1, && u_2(x,0) = 0, && u_{2t}(x,0) = 0, \\ &\vdots \end{aligned} \tag{4.8}$$

$$\begin{aligned} &p^0: \quad u_{0xx} = 0, && u_0(0,t) = \cosh(t), && u_0(\tfrac{\pi}{2}, t) = 1 + \cosh(t), \\ &p^1: \quad u_{1xx} = u_{0tt} - u_0, && u_1(0,t) = 0, && u_1(\tfrac{\pi}{2}, t) = 0, \\ &p^2: \quad u_{2xx} = u_{1tt} - u_1, && u_2(0,t) = 0, && u_2(\tfrac{\pi}{2}, t) = 0, \\ &\vdots \end{aligned} \tag{4.9}$$

Solving (4.8) and (4.9), the first few components of the homotopy perturbation solution for problem (4.7) are derived as follows

$$\begin{aligned}
\tilde{u}_0 &= 1+\sin(x),\\
\bar{u}_0 &= \cosh(t)+\frac{2x}{\pi},\\
u_0 &= 0.318\,31x+0.5\sin x+0.5\cosh t+0.5,\\
\tilde{u}_1 &= 0.5\cosh t+0.159\,15t^2x+0.25t^2-0.5,\\
\bar{u}_1 &= 0.5\sin x-0.02652\,x\left(2x^2+9.\,424\,8x-7.\,739\,2\right),\\
u_1 &= 0.25(\sin x+\cosh t-1)-0.156\,47x+0.07957\,t^2x+\\
&\quad 0.125\,t^2-0.01\,266\,x^2-0.02\,652\,6x^3-0.25,\\
\tilde{u}_2 &= -0.00663t^2\left(2x\left(x^2+12\right)-19.73x+9.\,4248x^2+37.\,699\right)+\\
&\quad 0.25(\cosh t-1)+0.00331\,t^4\,(2x+3.141\,6)\\
\bar{u}_2 &= 0.025\,1t^2x^2-0.01\,32\,t^2x^3+0.25x\sin x+0.25x^2\\
&\quad +0.00941x^3+0.01\,041x^4+0.001\,32x^5,\\
u2 &= 0.125\,(\sin x+\cosh t-\ t^2+\ x^2-1)-0.311\,7x+0.05\,13t^2x-0.02\,652tx^3+\\
&\quad 0.0033t^4x-0.062\,5t^2x^2+0.005\,2t^4+0.004\,7x^3+0.0052x^4+0.00066x^5,\\
&\quad \vdots
\end{aligned}$$

and the solution is thus obtained as

$$u=u_0+u_1+u_2+....$$

Figure (2) gives the comparison at $t=0.5$ between the HPM 3^{-rd}-order approximate solution of problem (4.7) and the exact solution given in [10] by $u\,(x,t)=\sin(x)+\cosh(t)$.

Example 4.4 Consider the telegraph problem

$$\begin{cases} u_{tt}+u_t=u_{xx}+\left(1+x^2+t^2\right),\ 0<x<1, t>0,\\ u(x,0)=x,\ u_t(x,0)=1+x^2,\\ u(0,t)=t+\frac{t^3}{3},\ u(1,t)=1+2t+\frac{t^3}{3}.\end{cases} \tag{4.10}$$

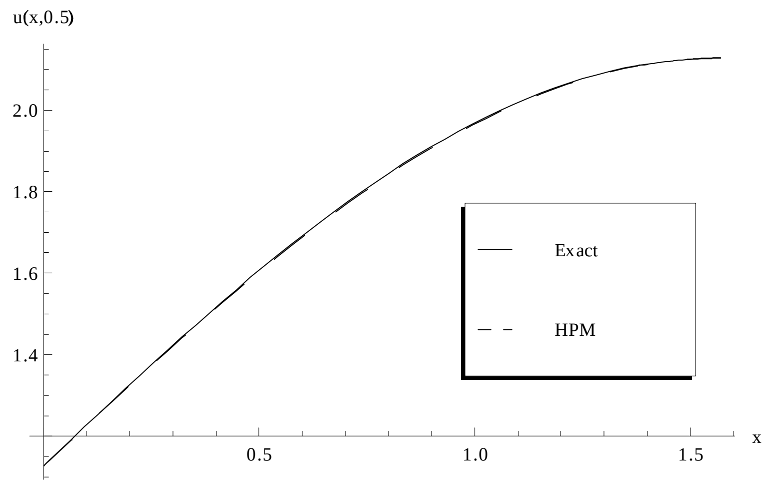

FIGURE 2. $u(x,0.5)$ of Example (4.3) for 3^{rd}-order HPM approximation.

According to the homotopies (3.2) and (3.3), the two following systems of PDEs are obtained

$$\begin{aligned} &p^0: \quad u_{0tt}=0, && u_0(x,0)=x, \quad u_{0t}(x,0)=1+x^2,\\ &p^1: \quad u_{1tt}=u_{0xx}-u_{0t}+(1+x^2+t^2)\,, && u_1(x,0)=0, \quad u_{1t}(x,0)=0,\\ &p^2: \quad u_{2tt}=u_{1xx}-u_{1t}, && u_2(x,0)=0, \quad u_{2t}(x,0)=0,\\ &\vdots \end{aligned} \tag{4.11}$$

$$\begin{aligned} &p^0: \quad u_{0xx}=0, && u_0(0,t)=t+\tfrac{t^3}{3}, \quad u_0(1,t)=1+2t+\tfrac{t^3}{3},\\ &p^1: \quad u_{1xx}=u_{0tt}+u_{0t}-(1+x^2+t^2)\,, && u_1(0,t)=0, \qquad u_1(1,t)=0,\\ &p^2: \quad u_{2xx}=u_{1tt}+u_{1t}, && u_2(0,t)=0, \qquad u_2(1,t)=0,\\ &\vdots \end{aligned} \tag{4.12}$$

Solving (4.11) and (4.12), the first few components of the homotopy perturbation solution for problem (4.10) are derived as follows

$$
\begin{aligned}
\tilde{u}_0 &= x + t\left(1 + x^2\right), \\
\bar{u}_0 &= t + \frac{t^3}{3} + x\left(1 + t\right), \\
u_0 &= \frac{1}{6}\left(t^3 + 6x + 3t\left(2 + x + x^2\right)\right), \\
\tilde{u}_1 &= \frac{t^3}{6} + \frac{t^4}{24} - \frac{t^2 x}{4} + \frac{t^2 x^2}{4}, \\
\bar{u}_1 &= -\left(\frac{1}{24} + \frac{t}{2} - \frac{t^2}{4}\right)x + \frac{tx^2}{2} - \frac{t^2x^2}{4} + \frac{x^3}{12} - \frac{x^4}{24}, \\
u_1 &= \frac{1}{48}\left(4t^3 + t^4 + 12t(-1+x)x - x\left(1 - 2x^2 + x^3\right)\right), \\
\tilde{u}_2 &= -\frac{1}{240}t^2\left(-20t + 5t^2 + t^3 + 60(-1+x)x\right), \\
\bar{u}_2 &= \frac{t^3}{2880}\left(120 + 12t^2 + t^3 - 15t\left(2 - x + x^2\right)\right), \\
u2 &= \frac{1}{480}\left(-5t^4 - t^5 + 60t(-1+x)x + 10t^3\left(2 - x + x^2\right) + 5\left(x - 2x^3 + x^4\right)\right), \\
&\vdots
\end{aligned}
$$

and the solution is thus obtained as

$$u = u_0 + u_1 + u_2 + \ldots.$$

Figure (3) gives the comparison at $t = 0.5$ between the HPM 3^{rd}-order approximate solution of problem (4.10) and the exact solution given in [7] by $u(x,t) = x + t\left(1 + x^2\right) + \frac{t^3}{3}$.

Example 4.5 Consider Schrodinger problem

$$
\begin{cases}
u_t + iu_{xx} = 0, \ 0 < x < 1, t > 0, \\
u(x,0) = 1 + \cosh(2x), \\
u(0,t) = 1 + e^{(-4it)}, \ u(1,t) = 1 + \cosh(2)\, e^{(-4it)}.
\end{cases}
\tag{4.13}
$$

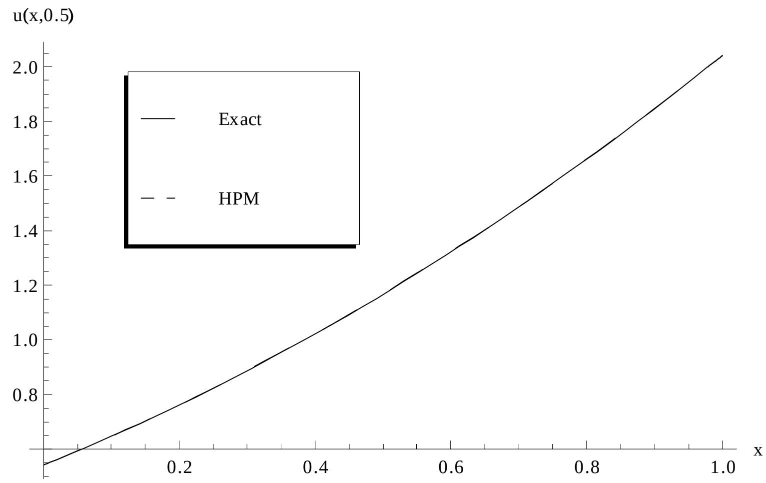

FIGURE 3. $u(x,0.5)$ of Example (4.4) for 3^{rd}-order HPM approximation.

According to the homotopies (3.2) and (3.3), the two following systems of PDEs are obtained

$$\begin{aligned} p^0: &\quad u_{0t} = 0, && u_0(x,0) = 1 + \cosh(2x), \\ p^1: &\quad u_{1t} = -iu_{0xx}, && u_1(x,0) = 0, \\ p^2: &\quad u_{2t} = -iu_{1xx}, && u_2(x,0) = 0, \\ &\vdots \end{aligned} \tag{4.14}$$

$$\begin{aligned} p^0: &\quad u_{0xx} = 0, && u_0(0,t) = 1 + e^{(-4it)}, && u_0(1,t) = 1 + \cosh(2)\, e^{(-4it)}, \\ p^1: &\quad u_{1xx} = iu_{0t}, && u_1(0,t) = 0, && u_1(1,t) = 0, \\ p^2: &\quad u_{2xx} = iu_{1t}, && u_2(0,t) = 0, && u_2(1,t) = 0, \\ &\vdots \end{aligned} \tag{4.15}$$

Solving (4.14) and (4.15), the first few components of the homotopy perturbation solution for problem (4.13) are derived as follows

$$
\begin{aligned}
\tilde{u}_0 &= 1+\cosh(2x), \\
\bar{u}_0 &= e^{-4it}\left(1+e^{4it}+x(-1+\cosh(2))\right), \\
u_0 &= \frac{1}{2}e^{-4it}\left(1+2e^{4it}-x+x\cosh(2)+e^{4it}\cosh(2x)\right), \\
\tilde{u}_1 &= \frac{1}{4}\left(\left(-1+e^{-4it}\right)(1+x(-1+\cosh(2)))-8t^2\cosh(2x)\right), \\
\bar{u}_1 &= \frac{1}{90}e^{-4it}\left(x\left(8+15x^3-45e^{4it}\left(-1+\cosh(1)^2\right)\right)+45e^{4it}\sinh(x)^2\right)+ \\
&\quad \frac{1}{90}e^{-4it}\left(3x^4(-1+\cosh(2))+7\cosh(2)-10x^2(2+\cosh(2))\right), \\
u_1 &= \frac{1}{6}e^{-4it}\left((-1+x)x(2+x(-1+\cosh(2))+\cosh(2))\right)- \\
&\quad t\cosh(2x), \\
\tilde{u}_2 &= \frac{1}{12}e^{-4it}\left(-\left(-1+e^{4it}\right)(-1+x)x(2+x(-1+\cosh(2))+\cosh(2))\right)+ \\
&\quad \frac{1}{12}e^{-4it}\left(2ie^{4it}t\left(-3+8t^2\right)\cosh(2x)\right), \\
\bar{u}_2 &= \frac{e^{-4it}}{3780}\left(x\left(-694+945x+42x^5+1890ie^{4it}t(-1+\cosh(2))+7x^2(-29+59\cosh(2))\right)-\right)+ \\
&\quad \frac{e^{-4it}}{3780}x\left(6x^6(-1+\cosh(2))-377\cosh(2)-42x^4(2+\cosh(2))+7x^2(-29+59\cosh(2))\right)- \\
&\quad it\sinh(x)^2, \\
u_2 &= \frac{1}{2}\left(-x\left(\frac{1}{2}\left(-1+(\cosh(1))^2\right)-\frac{1}{90}e^{-4it}(8+7\cosh(2))\right)\right)+ \\
&\quad \frac{1}{2}\left(+\frac{1}{4}\left(\left(-1+e^{-4it}\right)(1+x(-1+\cosh(2)))-8t^2\cosh(2x)\right)\right)+ \\
&\quad \frac{1}{2}\left(\frac{1}{90}e^{-4it}\left(x^3\left(15x+3x^2(-1+\cosh(2))-10(2+\cosh(2))\right)+45e^{4it}(\sinh(x))^2\right)\right), \\
&\quad \vdots
\end{aligned}
$$

and the solution is thus obtained as

$$u = u_0 + u_1 + u_2 + \ldots.$$

Figure (4) gives the comparison at $t = 0.5$ between the magintude of the 14^{th}-order HPM

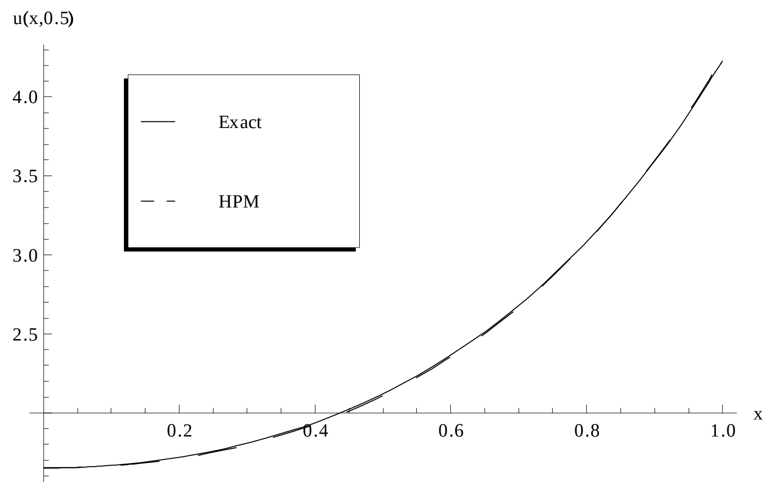

FIGURE 4. $|u(x,0.5)|$ of Example (4.5) for 14^{th}-order HPM approximation.

approximate solution of problem (4.13) and the magnitude of the exact solution given in [11] by $u\,(x,t) = 1 + \cosh\,(2x)\, e^{(-4it)}$.

5. Conclusion

We propose an analytical-numerical technique based on the HPM to solve IBVP over finite domains. The advantage of this technique is to include both the initial and boundary conditions in the recursive relation, so that we can obtain a good approximate solution for the problems. The results obtained in the numerical examples show good results.

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